Analytic bootstrap at large spin

Aninda Sinha
Indian Institute of Science, Bangalore

- Work done with Apratim Kaviraj, Kallol Sen arXiv:1502.0143 and to appear soon.

Summary of main results

- Given a (4d for most part) CFT with a scalar operator of dimension Δ_ϕ and a spin-2 (minimal) twist-2 operator there is an infinite sequence of large spin operators of dimension

$$\Delta = 2\Delta_\phi + 2n + \ell + \gamma(n,\ell)$$ — Anomalous dim.

$\ell \gg n \gg 1$ $$\gamma(n,\ell) = -\frac{160}{c_T}\frac{n^4}{\ell^2}$$

$n \gg \ell \gg 1$ $$\gamma(n,\ell) = -\frac{80}{c_T}\frac{n^3}{\ell}$$

$$\langle T_{ab}(x) T_{cd}(x')\rangle = \frac{c_T}{|x-x'|^{2d}} \mathcal{I}_{ab,cd}(x-x')$$

- We can think of these operators as double trace operators of the form

$$O_1 \partial_{\mu_1} \cdots \partial_{\mu_\ell} (\partial^2)^n O_1$$

- However the CFT bootstrap analysis of course only yields conformal dimension, spin and the OPE coefficients and not the precise form of these operators.

Why is this interesting?

- Result is universal. Does not depend on lagrangian or the dimension of the seed operator. Just assumes twist gap of these operators from other operators in the spectrum.

- Anomalous dimension of double trace operators is related to bulk Shapiro time delay. Sign of anomalous dimension is related to causality. Interplay between unitarity of CFT and causality of bulk.

  Camanho, Edelstein, Maldacena, Zhiboedov

- Can be extended to arbitrary (eg. 3d) dimensions. May be relevant for 3d Ising model at criticality.

  Rychkov et al

- Can compare with AdS/CFT. Two different ways to calculate the anomalous dimensions a) Eikonal approximation of 2-2 scattering b) Energy shift in a black hole background.
- Turns out that the result matches exactly with the AdS/CFT prediction.
- Another example where dynamics match without needing supersymmetry.

Quick review of bootstrap

“s-channel” “t-channel”

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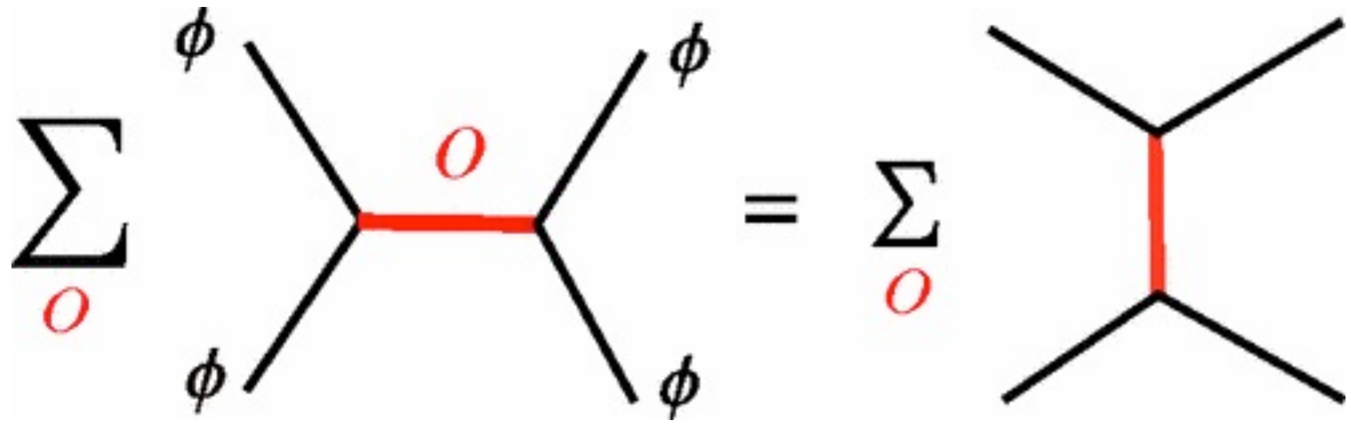

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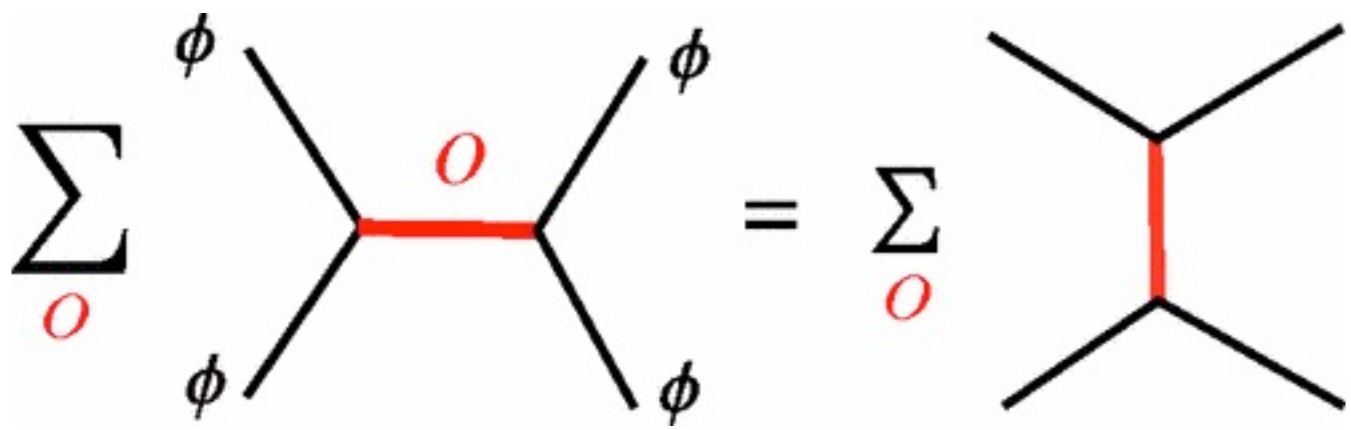

"s-channel" "t-channel" even spin

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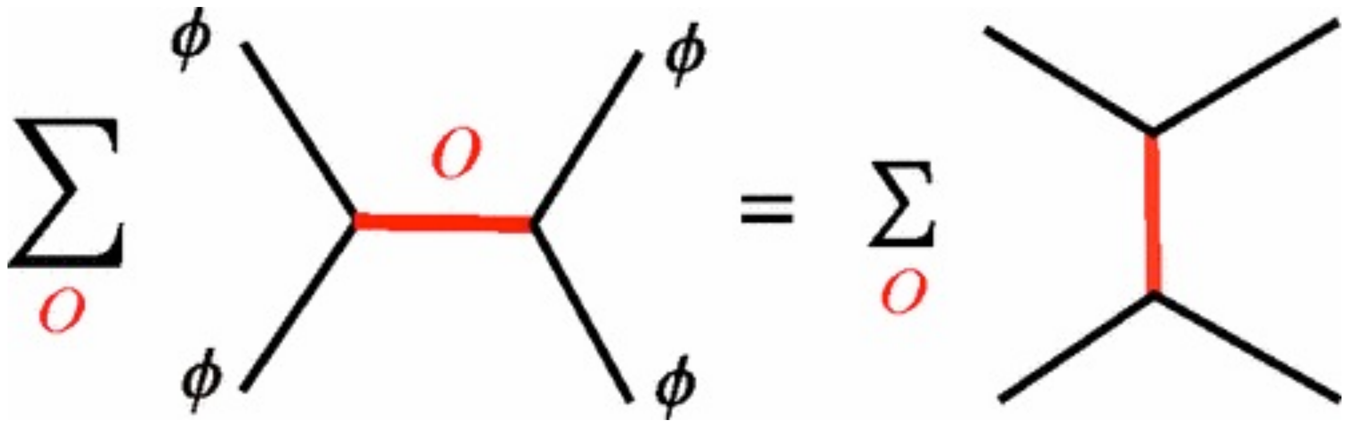

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$$1+\sum_{\tau,\ell} P_{\tau,\ell}\, g_{\tau,\ell}(u,v) = \left(\frac{u}{v}\right)^{\Delta_\phi}\left(1+\sum_{\tau,\ell} P_{\tau,\ell}\, g_{\tau,\ell}(v,u)\right)$$

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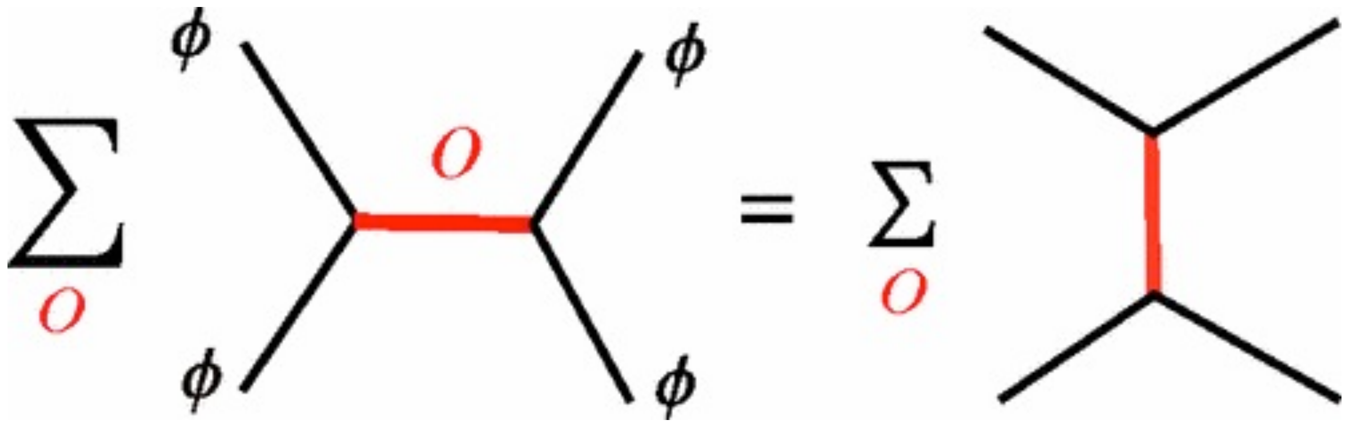

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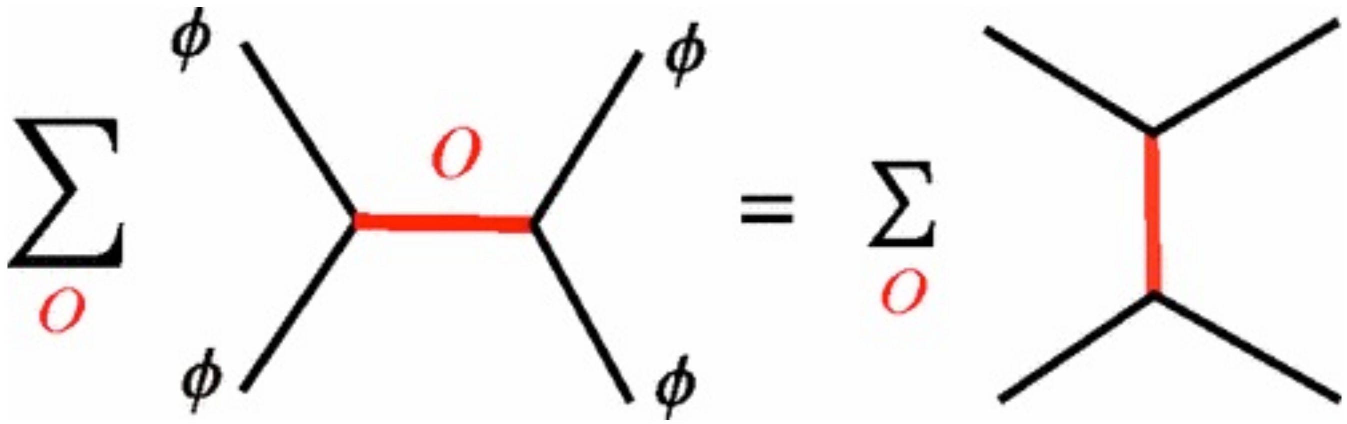

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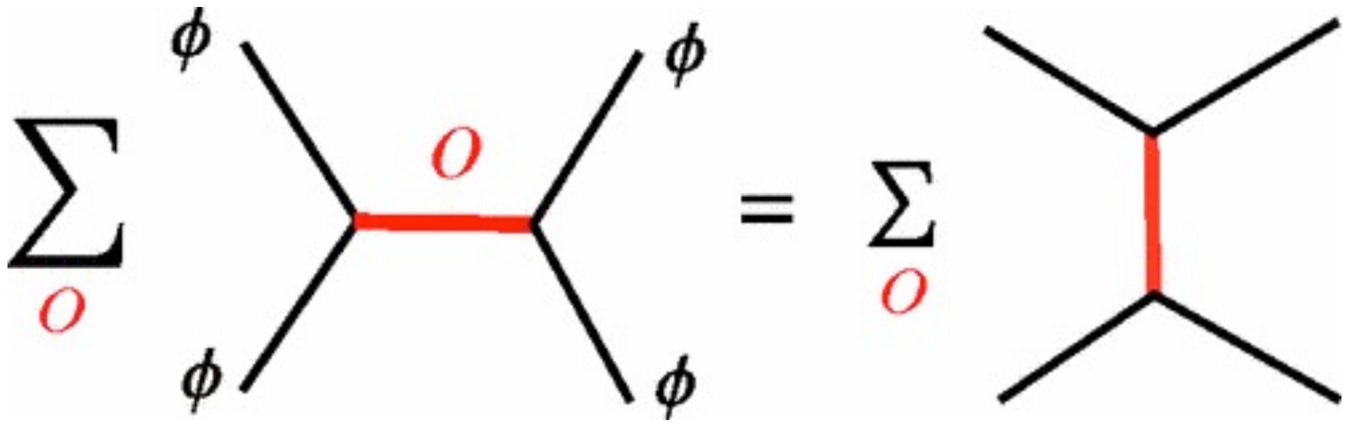

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Conformal cross ratios

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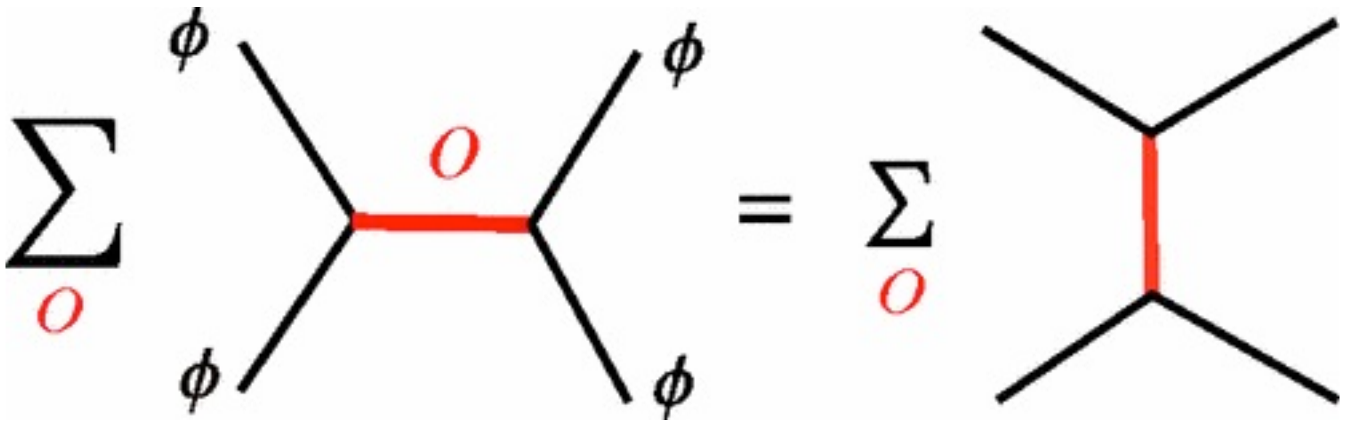

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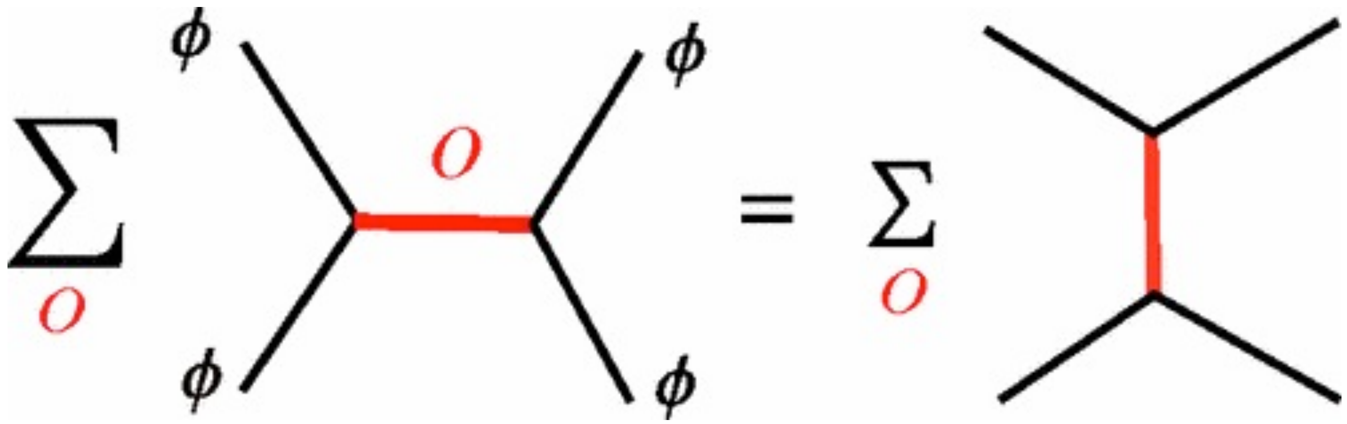

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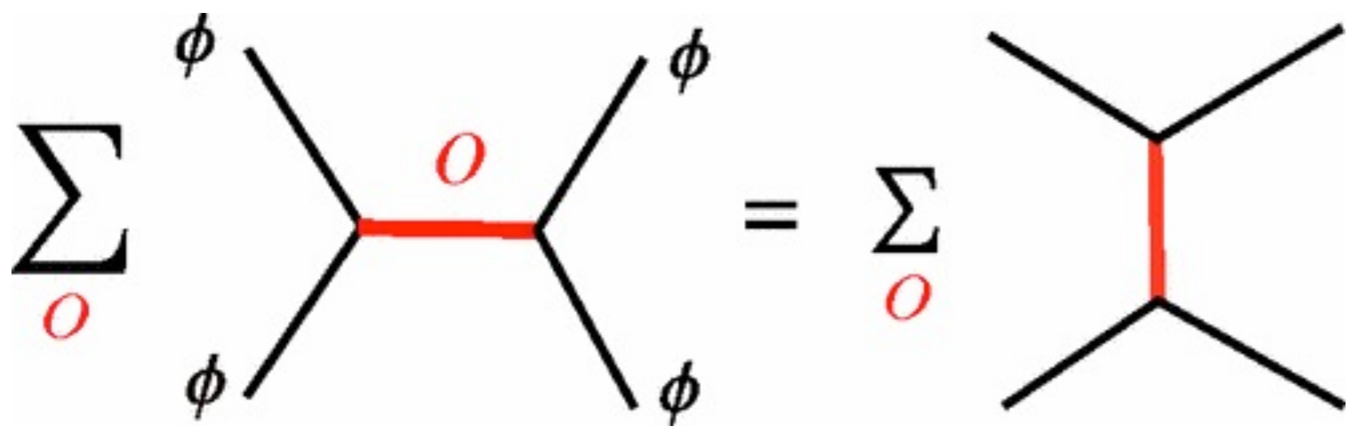

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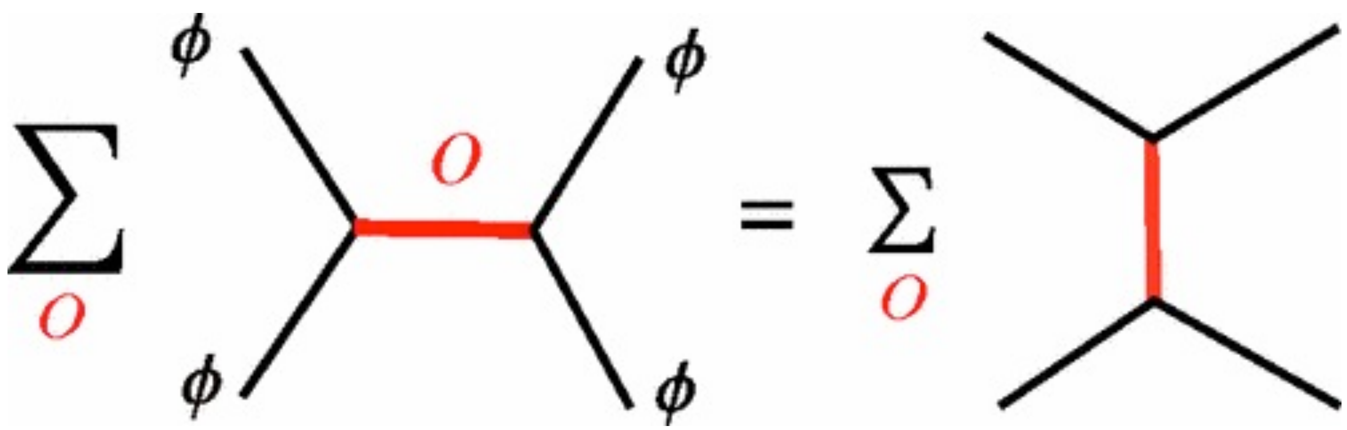

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$g_{\tau,\ell}(u,v)$

Dolan, Osborn;

Blocks

Closed form expressions for conformal blocks are known only in even dimensions.

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"factorizes"

(twist,spin,v) x (twist, u, d)

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Recursion relations for blocks in any dimension

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$$
\begin{aligned}
\left(\frac{\bar{z}-z}{(1-z)(1-\bar{z})}\right)^2 g^{(d)}_{\Delta,\ell}(v,u) =& g^{(d-2)}_{\Delta-2,\ell}(v,u) - \frac{4(\ell-2)(d+\ell-3)}{(d+2\ell-4)(d+2\ell-2)} g^{(d-2)}_{\Delta-2,\ell}(v,u) \\
&- \frac{4(d-\Delta-3)(d-\Delta-2)}{d-2\Delta-2)(d-2\Delta)} \left[\frac{(\Delta+\ell)^2}{16(\Delta+\ell-1)(\Delta+\ell+1)} g^{(d-2)}_{\Delta,\ell+2}(v,u) \right. \\
&\left. - \frac{(d+\ell-4)(d+\ell-3)(d+\ell-\Delta-2)^2}{4(d+2\ell-4)(d+2\ell-2)(d+\ell-\Delta-3)(d+\ell-\Delta-1)} g^{(d-2)}_{\Delta,\ell}(v,u) \right]
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$$(1-v)^2 F^{(d)}(\tau,v) = 16F^{(d-2)}(\tau-4,v) - 2vF^{(d-2)}(\tau-2,v) + \frac{(d-\tau-2)^2}{16(d-\tau-3)(d-\tau-1)} v^2 F^{(d-2)}(\tau,v)\,.$$

New results from bootstrap

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$$F^{(d)}(\tau, v) = \frac{2^\tau}{(1-v)^{\frac{d-2}{2}}} {}_2F_1\left(\frac{1}{2}(\tau - d + 2), \frac{1}{2}(\tau - d + 2), (\tau - d + 2), v\right)$$

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Same as what appears in MFT. OPE's known.

$$\Delta = 2\Delta_\phi + 2n + \ell + \gamma(n, \ell)$$

We can go to subleading order

$$P_m = P_{MFT} + c(n, \ell)$$

It can be shown that the anomalous dimension at large spin goes like an inverse power of the spin for dimension>2.

This means that we can treat the inverse spin as an expansion parameter and this result is true even for theories which does not have a "large N".

Our objective is to determine the n-dependence for the anomalous dimension.

After some clever detective work we find

$$\gamma(n,\ell)\ell^{\tau_m} = \sum_{m=0}^{n} C^{(d)}_{n,m} B^{(d)}_m$$

$$C^{(d)}_{n,m} = \frac{(-1)^{m+n}}{8}\left(\frac{\Gamma[\Delta_\phi]}{(\Delta_\phi - d/2+1)_m}\right)^2 \frac{n!}{m!(n-m)!}(2\Delta_\phi + n + 1 - d)_m$$

$$B^{(d)}_k = -\frac{16 P_m \Gamma[\tau_m + 2\ell_m]\Gamma[\tau_m/2 + \ell_m + k]^2}{\Gamma[1+k]^2\Gamma[\tau_m/2+\ell_m]^4}$$

$$\times {}_3F_2\left(-k,-k,-\frac{\tau_m}{2} - \ell_m - \frac{d-2}{2} + \Delta_\phi; 1-\ell_m - \frac{\tau_m}{2} - k, 1 - \ell_m - \frac{\tau_m}{2} - k; 1\right)$$

${}_3F_2$: Since k is a positive integer, this is a polynomial.

At this stage, no amount of pleading with mathematica helped!

Kernel running for hours!

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After a lot of hard work, mathematica produces A=A

Progress is possible in 4d (and similar techniques apply in even d)

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$$ {}_3F_2(-m, -m-4+\Delta_\phi; -2-m, -2-m; 1) = \sum_{k=0}^{m} \frac{(m+1-k)^2(m+2-k)^2}{(m+1)^2(m+2)^2} \frac{\Gamma[\Delta_\phi - 4 + k]}{\Gamma[k+1]\Gamma[\Delta_\phi - 4]} $$

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$$

Use basic defn

$$
=\frac{4[6m^2+6m(\Delta_\phi-1)+\Delta_\phi(\Delta_\phi-1)]}{(m+1)(m+2)\Gamma[m+3]}\frac{\Gamma[m+\Delta_\phi-1]}{\Gamma[\Delta_\phi+1]}
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$$ \gamma(n,\ell)\ell^2 = -(-1)^n \frac{80}{3c_T} \sum_{m=0}^{n} (-1)^m [6m^2+6m(\Delta_\phi-1)+\Delta_\phi(\Delta_\phi-1)] \times \frac{\Gamma[\Delta_\phi]\Gamma[n+1]\Gamma[2\Delta_\phi+m+n-3]}{\Gamma[m+1]\Gamma[n-m+1]\Gamma[\Delta_\phi+m-1]\Gamma[2\Delta_\phi+n-3]} $$

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$$
\times\frac{\Gamma[\Delta_\phi]\Gamma[n+1]\Gamma[2\Delta_\phi+m+n-3]}{\Gamma[m+1]\Gamma[n-m+1]\Gamma[\Delta_\phi+m-1]\Gamma[2\Delta_\phi+n-3]}
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Using the following:

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$$ \gamma(n,\ell)\ell^2=-(-1)^n\frac{80}{3c_T}\sum_{m=0}^{n}(-1)^m[6m^2+6m(\Delta_\phi-1)+\Delta_\phi(\Delta_\phi-1)] \times\frac{\Gamma[\Delta_\phi]\Gamma[n+1]\Gamma[2\Delta_\phi+m+n-3]}{\Gamma[m+1]\Gamma[n-m+1]\Gamma[\Delta_\phi+m-1]\Gamma[2\Delta_\phi+n-3]} $$

Using the following:

$$ \Gamma[x]\Gamma[1-x]=\frac{\pi}{\sin\pi x} $$

Progress is possible in 4d (and similar techniques apply in even d)

$$ {}_3F_2(-m,-m-4+\Delta_\phi;-2-m,-2-m;1)=\sum_{k=0}^{m}\frac{(m+1-k)^2(m+2-k)^2}{(m+1)^2(m+2)^2}\frac{\Gamma[\Delta_\phi-4+k]}{\Gamma[k+1]\Gamma[\Delta_\phi-4]} $$

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$$ \Gamma[x]=\int_0^\infty t^{x-1}e^{-t}dt $$

So effectively we just need to do the integral

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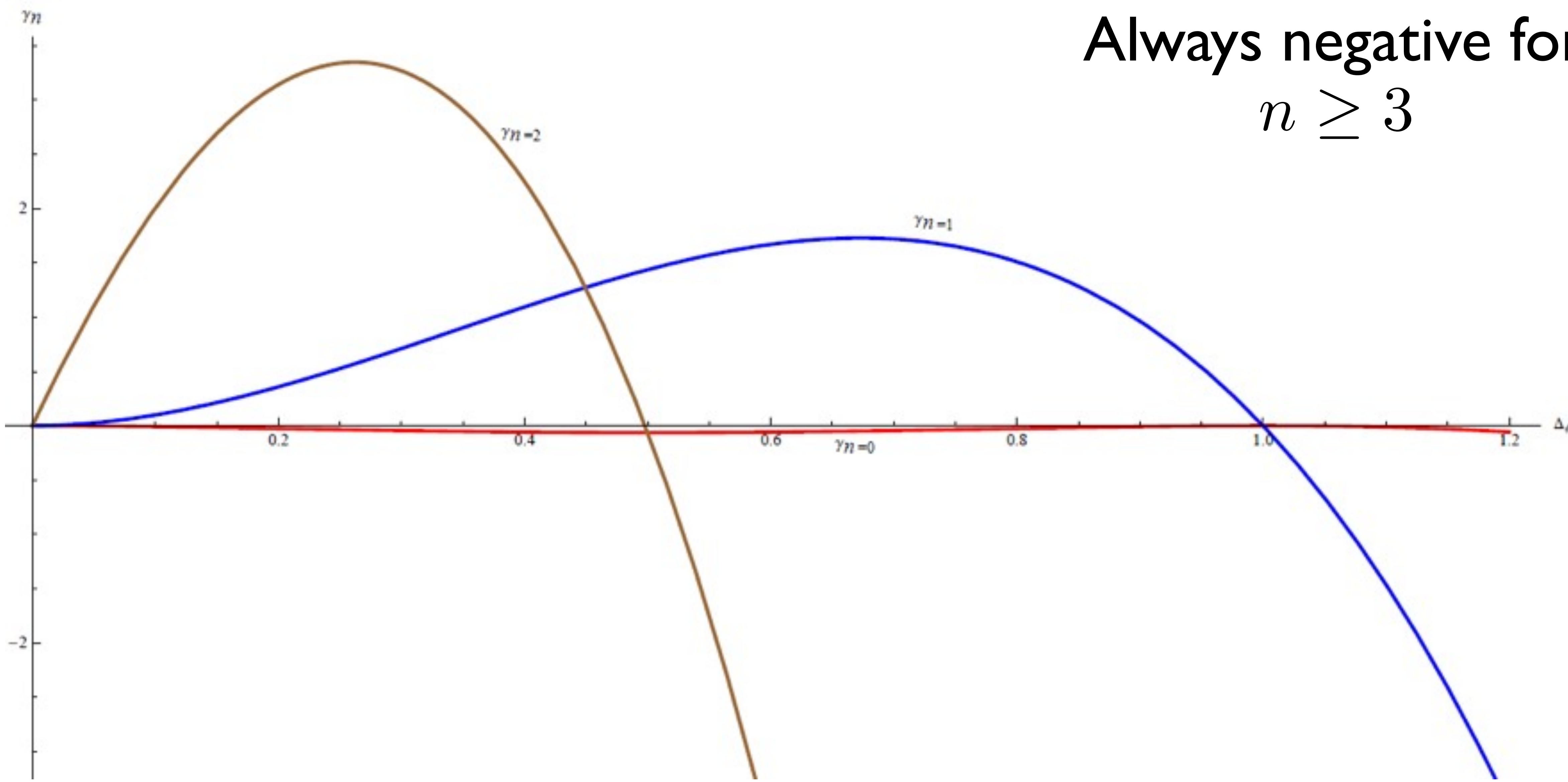
This is negative and monotonically decreasing with n for any conformal dimension satisfying the unitarity bound

Always negative for
$n \geq 3$



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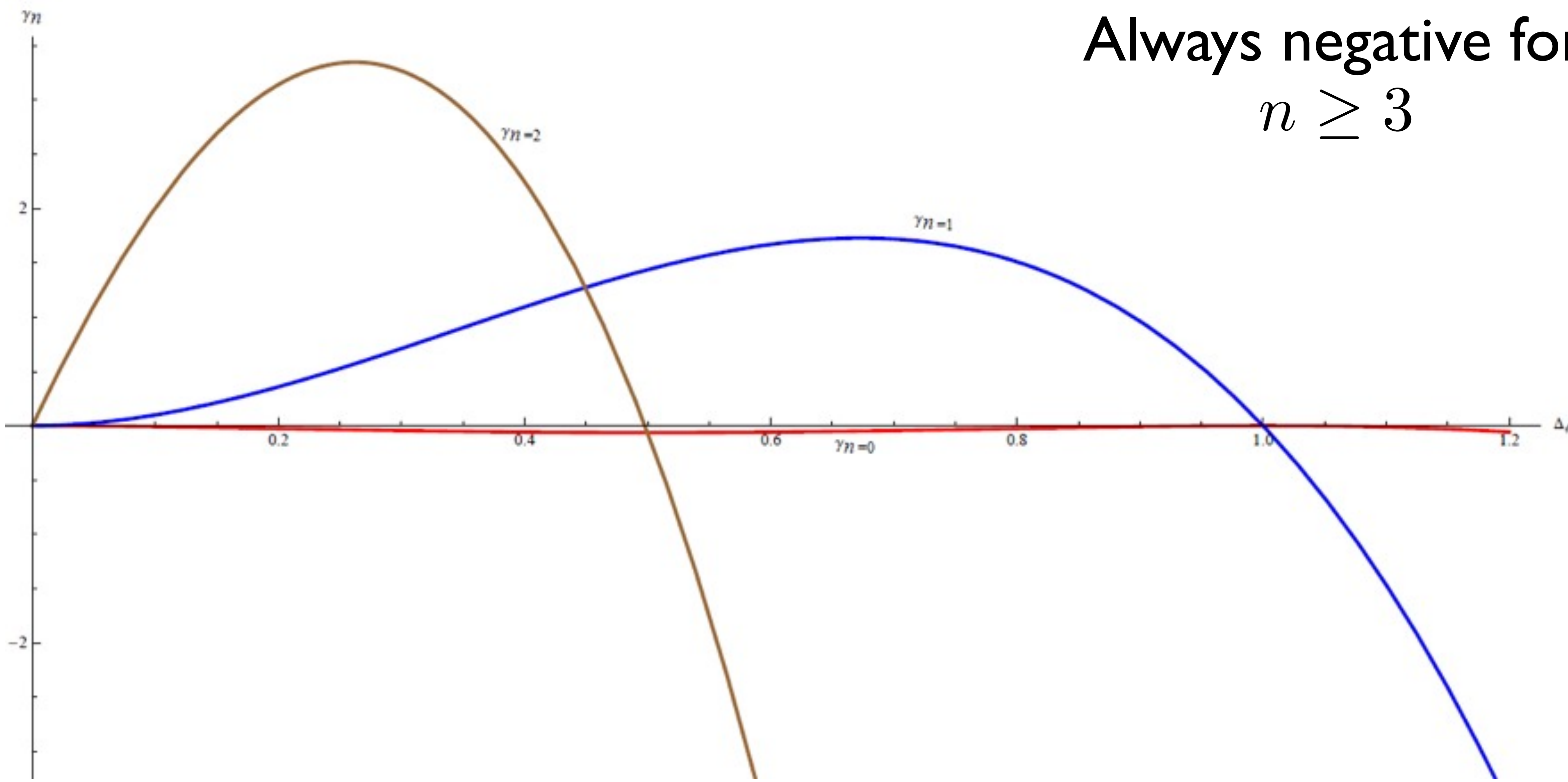

<u>If unitarity bound is violated</u> anomalous dimensions can be positive.

Comments on Nachtmann theorem

- Nachtmann in 1973 proved the following under certain plausible assumptions (unitarity, Regge behaviour of amplitudes)

$$\frac{\partial}{\partial \ell}\gamma(n=0,\ell) > 0$$

- This means that the leading twist operator for $\ell^{-\#}$should have negative anomalous dimension.

- Our results extends this to non-zero twists.

Large spin gymnastics from holography--
take I

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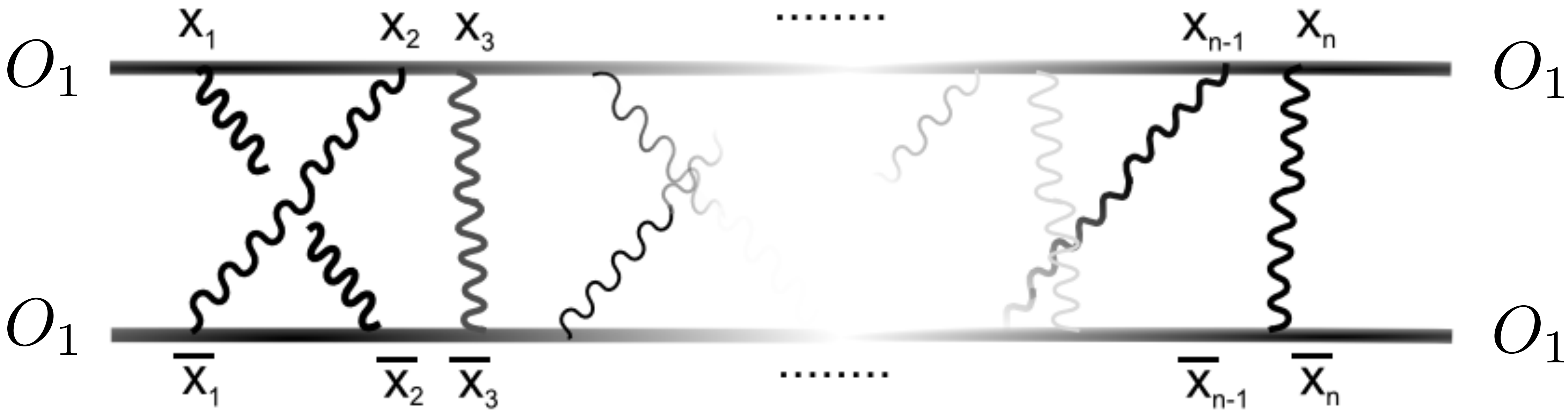
- Cornalba et al showed through a series of papers that the anomalous dimension of double trace operators can be calculated in the high energy Eikonal approximation of 2-2 scattering in AdS spacetime.
- The calculation is difficult but the bottom line is that in the t-channel the amplitude for double trace exchange is related to the exponential of the propagator of the exchanged particle (eg. graviton) in the s-channel.



Cornalba, Costa, Penedones

Ladder and crossed ladder diagrams can be resummed using Eikonal approximation.

Needs both large spin and twist

- In the other channel the amplitude is dominated by the composite state of two incoming particles dual to the double trace operators.

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- The Eikonal approximation determines a phase shift due to the exchange of a particle.
- This phase shift is related to the anomalous dimension.

4d CFT

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$$\ell \gg n \gg 1$$

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$$\gamma(n,\ell) = -\frac{1}{4N^2}\frac{(E-J)^4}{EJ} \approx -\frac{4}{N^2}\frac{n^4}{\ell^2} = -\frac{160}{c_T}\frac{n^4}{\ell^2}$$

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There is a prediction from AdS/CFT in this limit.

$$\gamma(n,\ell)_{AdS/CFT} = -\frac{2}{N^2}\frac{n^3}{\ell}$$

$\mathcal{N}$=4 normalizations

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$$\frac{\ell}{n} > \frac{\ell_0}{n_{max}}$$

- In the AdS/CFT language the impact parameter is related by

$$\rho \sim \frac{\ell}{n}$$

Cornalba, Costa, Penedones

- This means that there is a "mass gap" for this result to be valid.
- This is similar in spirit to the double expansion in α', g_s

Camanho, Edelstein, Maldacena, Zhiboedov

- There is a closely related recent discussion by Alday, Bissi and Lukowski.
- They discussed N=4 SYM bootstrap. By assuming that the leading spectrum is the same as in SUGRA (AdS/CFT input) they found closed form expressions for the anomalous dimensions for $\Delta_\phi = 4$
- Our results are in exact agreement with their findings in the two limits.
- Our derivation suggests that the result should hold universally in any CFT (with the caveats).

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- At least to me the AdS/CFT Eikonal method obscures the reason why the result should be universal in holography.
- In other words how do we see that higher derivative corrections will not spoil the universality?
- Namely why can't the overall factor depend on the 't Hooft coupling?

Large spin gymnastics from holography--take II

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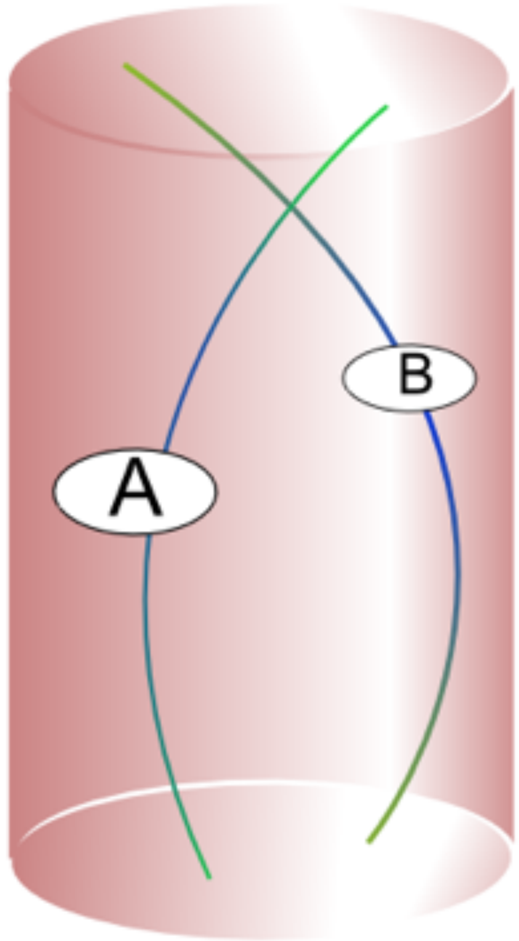
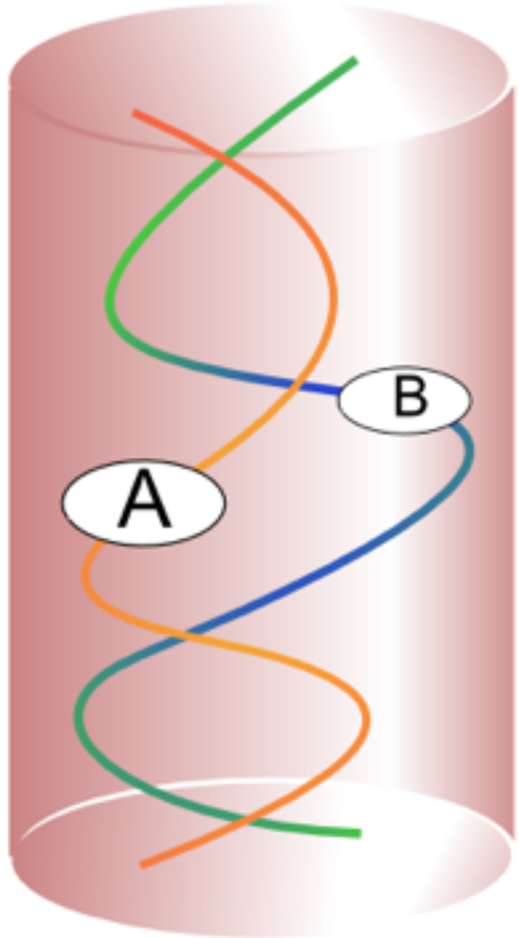
Large spin gymnastics from holography--take II

- Fitzpatrick, Kaplan and Walters suggested the following simple calculation.
- The double trace operators can be thought of as two massive particles in AdS rotating around each other. The anomalous dimension arises due to the interacting energy of these particles.
- Essential idea is to do perturbation theory in inverse distance corresponding to a Newtonian approximation in AdS.


B
A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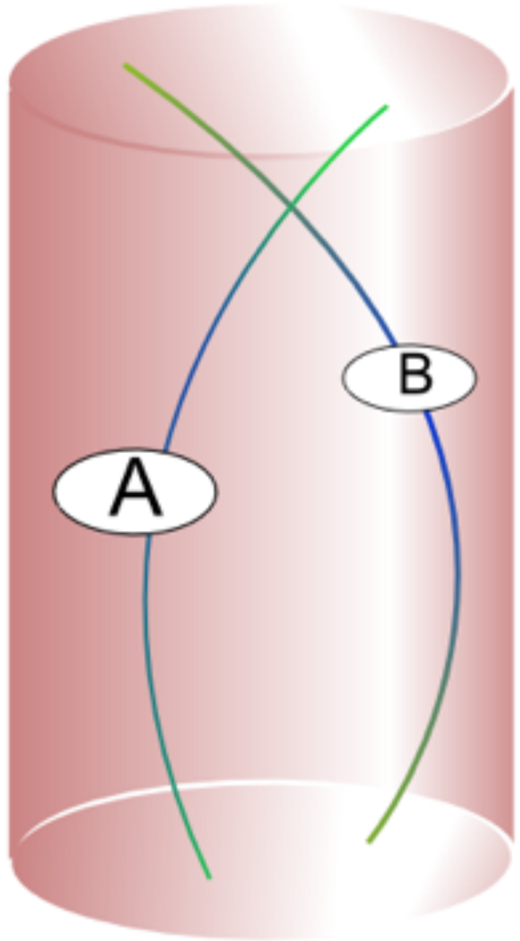
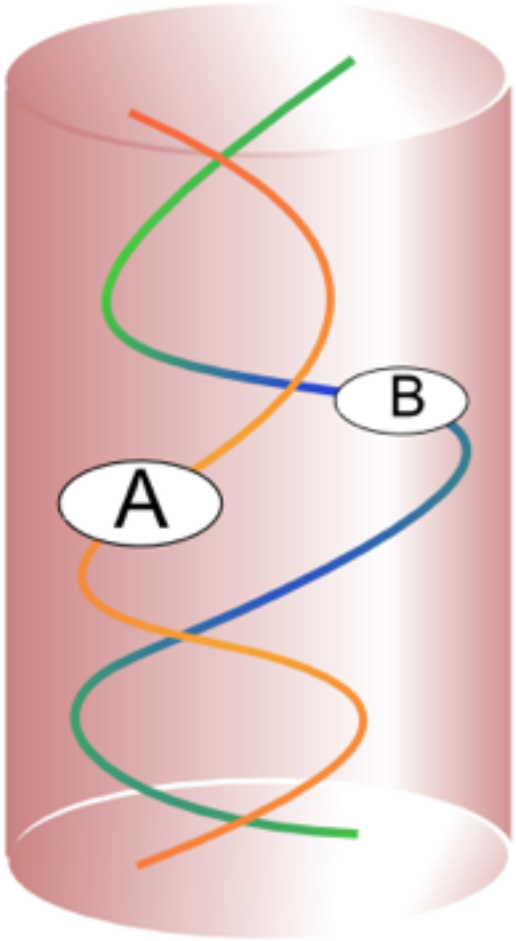
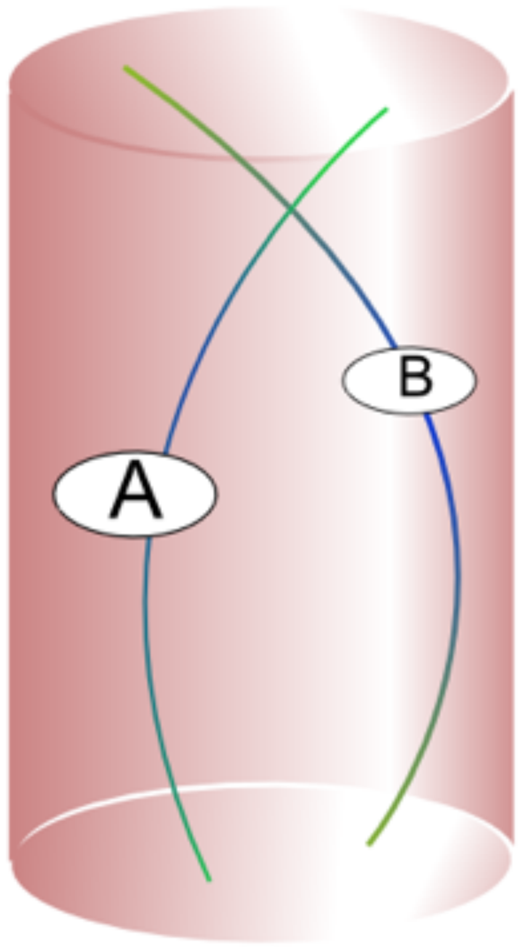
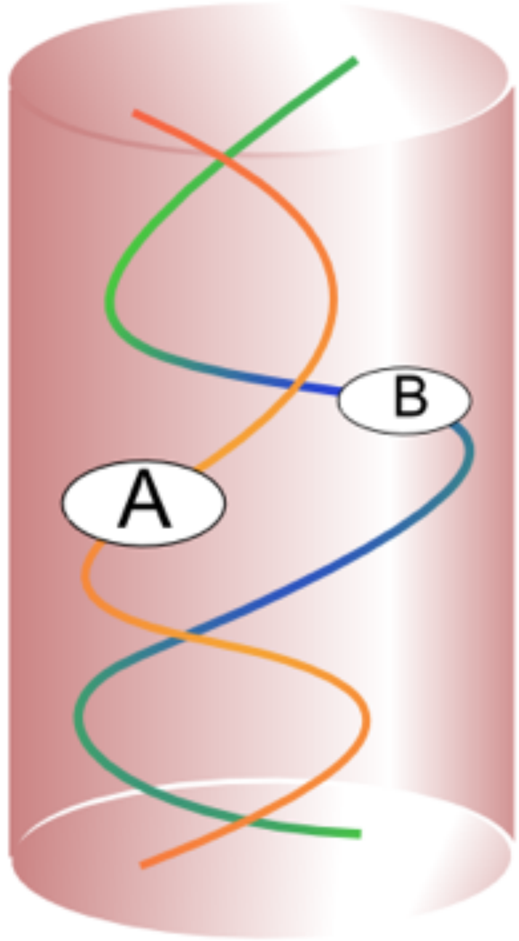




- So from the gravity side we ignore backreaction due to the “distant” scalar field.





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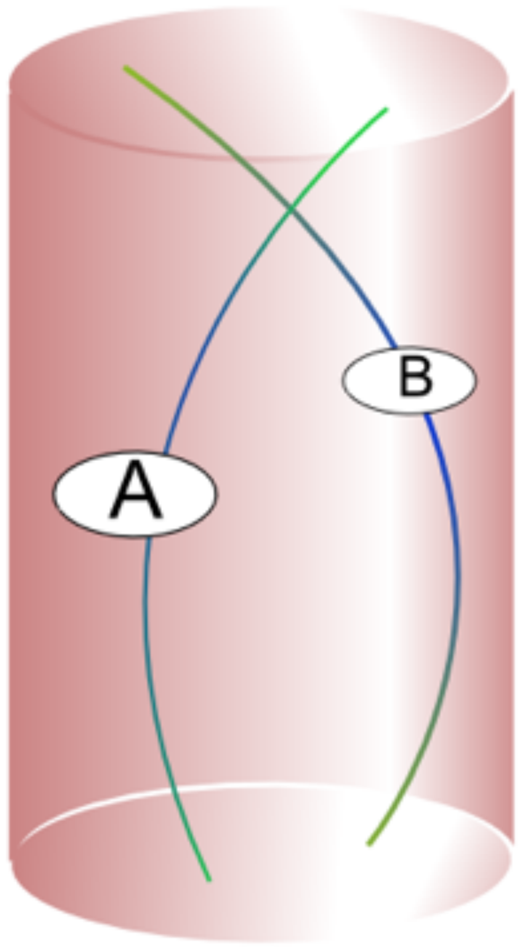
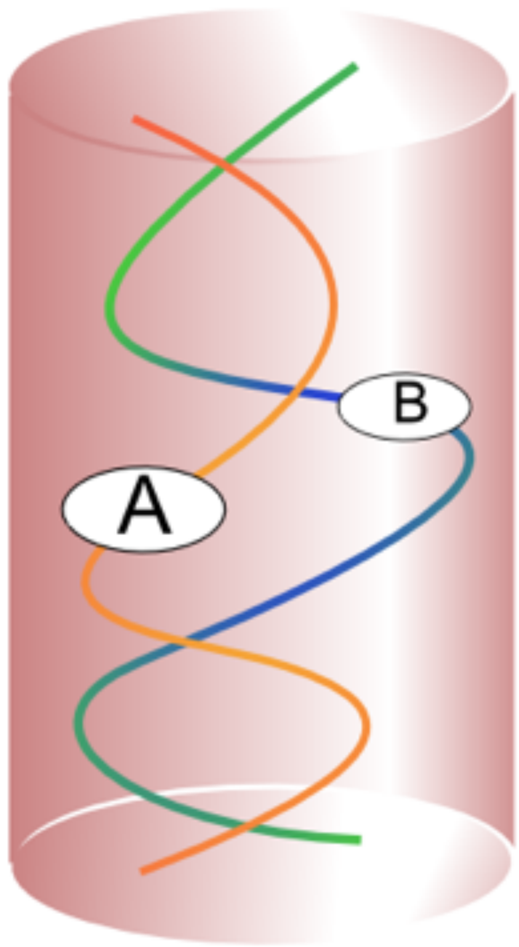
- We assume that the mass of this orbiting scalar field is big (in units of AdS radius).





- So from the gravity side we ignore backreaction due to the "distant" scalar field.
- We assume that the mass of this orbiting scalar field is big (in units of AdS radius).
- We assume that the mass of the black hole which corresponds to the dimension of the 2nd scalar field is big.

- It has been shown that for n=0, the result of the calculation agrees with the bootstrap prediction. (Unlike Eikonal where both spin and n needed to be large)
- Non-zero n is quite hard. However, we have been able to make progress (barring overall constants) at large n, i.e., $\ell \gg n \gg 1$
- It turns out to give exactly the same universal behaviour predicted by bootstrap!

Non-renormalization from holography

Higher derivative correction

$$\delta E^d_{n,\ell_{orb}} = -\frac{\mu}{2}\int r(1+\alpha'^h r^{-2h})dr \left[\sum_{k,\alpha=0}^{n}\left(\frac{E^2_{n,\ell}}{(1+r^2)^2}\psi_k(r)\psi_\alpha(r)+\partial_r\psi_k(r)\partial_r\psi_\alpha(r)\right)\right] = \mathcal{I}_1+\mathcal{I}_2\,,$$

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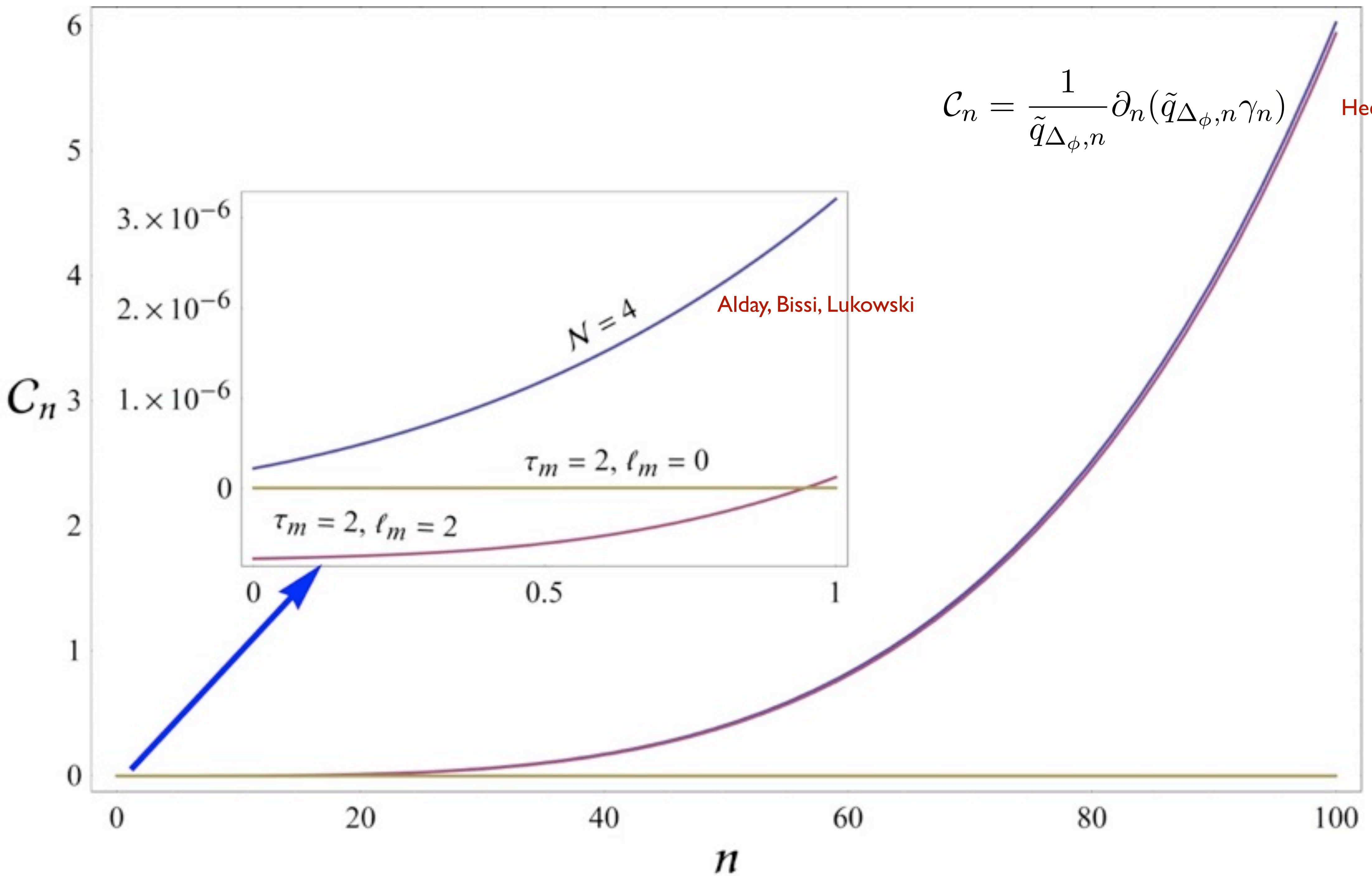
$$\delta E^{4}_{n,\ell_{orb}} = -\frac{\mu(\ell+2n)^2\Gamma(\ell+2+n)}{4\Gamma(\ell+2)\Gamma(n+\Delta-1)}\sum_{k=0}^{n}(-1)^k\frac{\Gamma(k+\ell+n+\Delta)}{\Gamma(\ell+2+k)\Gamma(n+1-k)\Gamma(2+k+\ell+\Delta)\Gamma(k+1)}$$
$$\times\Big[\Gamma(1+\ell+k)\Gamma(1+\Delta)\,{}_3F_2\Big(-n,k+\ell+1,\ell+n+\Delta;\ell+2,2+k+\ell+\Delta;1\Big)$$
$$+\,\alpha'^{h}\Gamma(1+\ell+k-h)\Gamma(1+\Delta+h)\,{}_3F_2\Big(-n,k+\ell+1-h,\ell+n+\Delta;\ell+2,2+k+\ell+\Delta;1\Big)\Big]$$

The spin dependence for the Einstein term can be shown to be $\frac{1}{\ell^2}$ while the higher derivative term gives $\frac{1}{\ell^{2h+2}}$. Thus no ’t Hooft coupling dependence!

Prediction for susy bootstrap: N=4 ’t Hooft coupling shows up at ℓ^{-8}

- It will be interesting to work out what happens in the other limit. $n \gg \ell \gg 1$
- Here we expect higher order corrections to play a role.
- It will be interesting to derive constraints on the higher derivative couplings by demanding that the anomalous dimension is negative.
- Do we need an infinite tower of higher spin massive particles for a consistent theory of quantum gravity?

Comments on OPE coefficients



$$\mathcal{C}_n = \frac{1}{\tilde{q}_{\Delta_\phi,n}}\partial_n(\tilde{q}_{\Delta_\phi,n}\gamma_n)$$

Heemskerk, Penedones, Polchinski, Sully;

Comments on general dimensions

Assume minimal twist for stress tensor exchange d-2

$$\ell \gg n \gg 1$$

$$\gamma(n,\ell)\ell^{d-2} = -P_m \frac{\Gamma[d+1]\Gamma[d+2]}{2\Gamma[1+\frac{d}{2}]\Delta_\phi^2} n^d$$

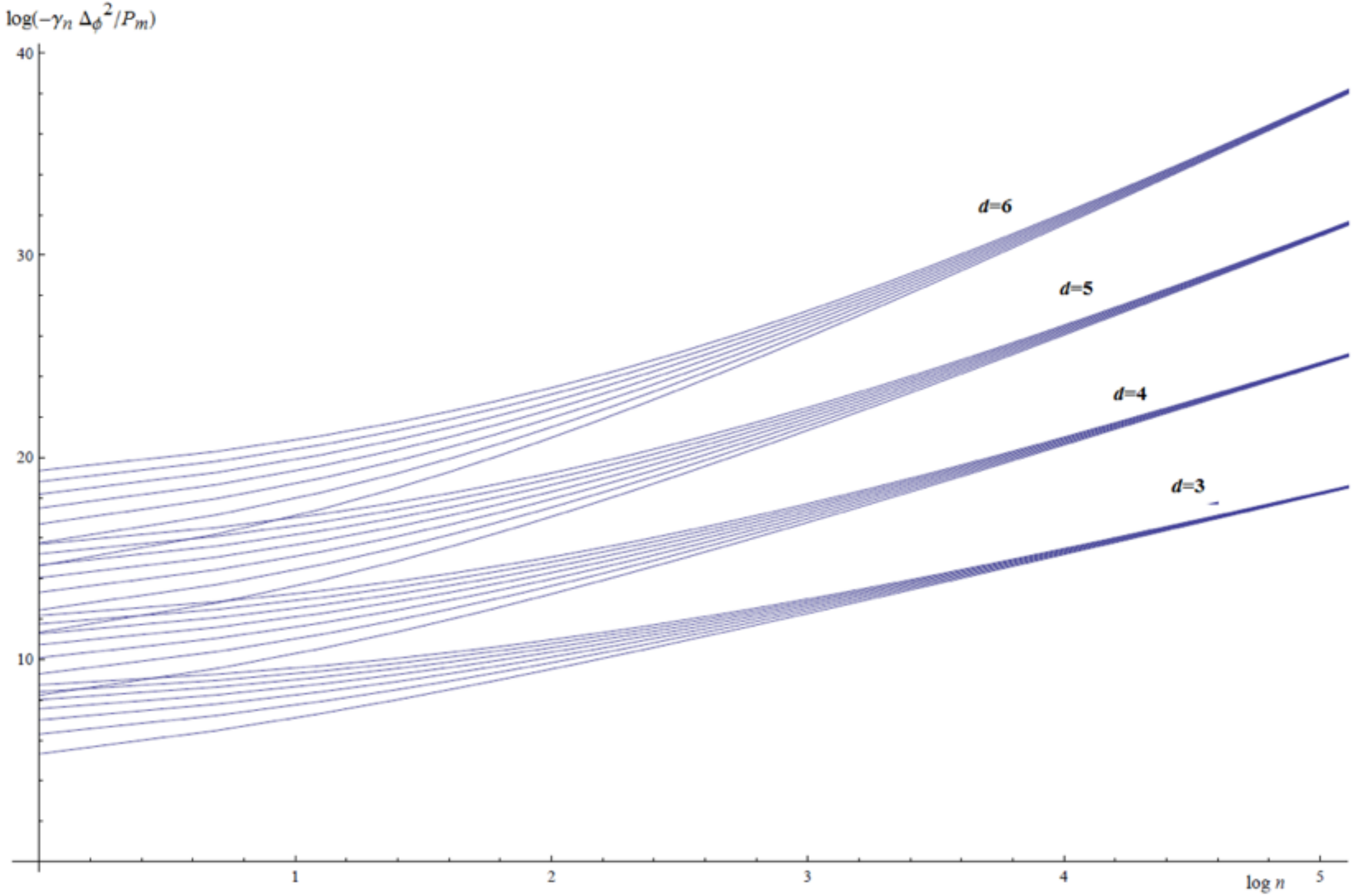
With some effort this can be derived analytically in all d. In terms of c_T:

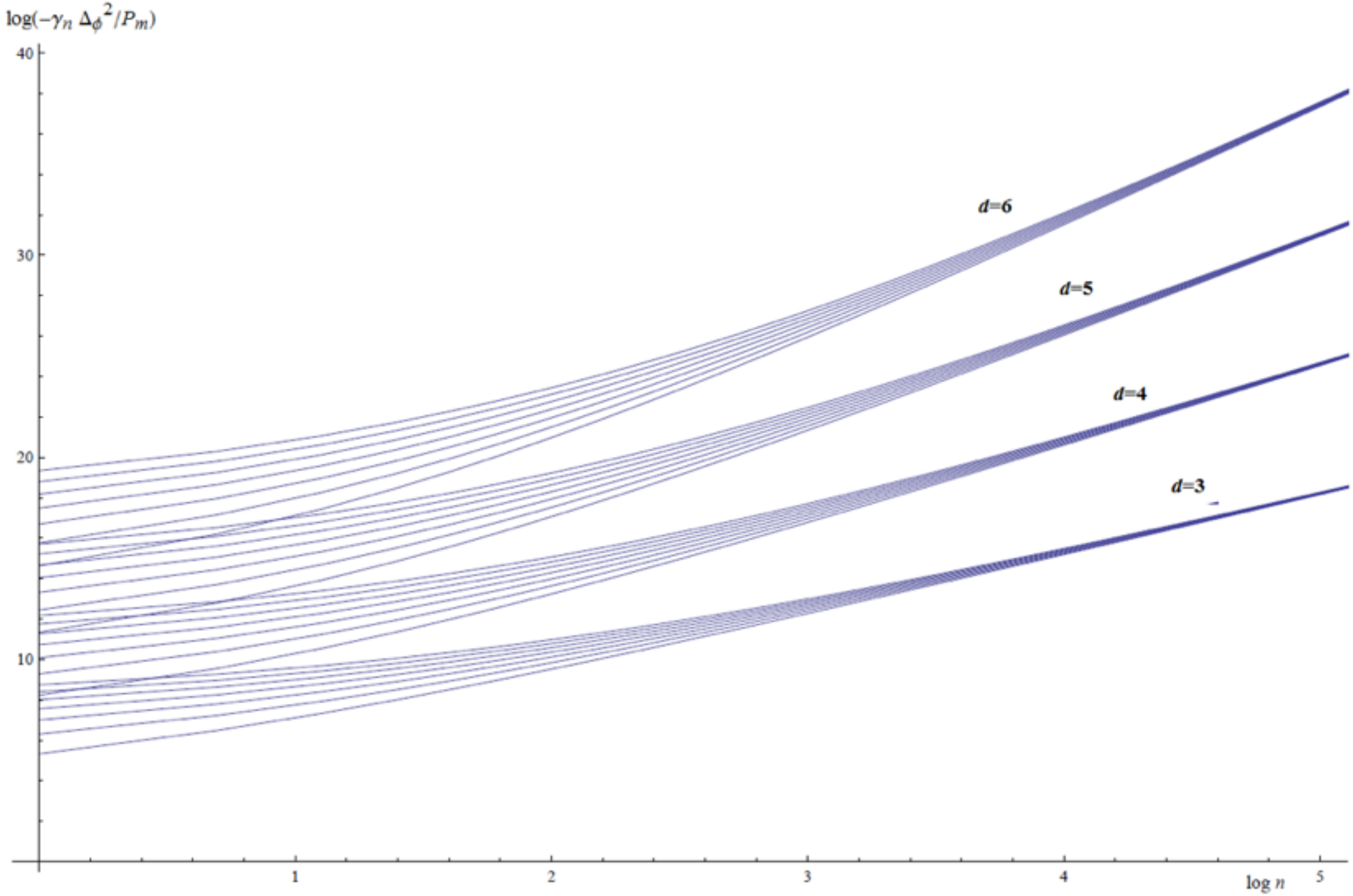
$$\gamma(n,\ell) = -\frac{d^2}{2(d-1)^2 c_T} \frac{\Gamma[d+1]\Gamma[d+2]}{\Gamma[1+\frac{d}{2}]^4} \frac{n^d}{\ell^{d-2}}$$

For this to match with the AdS Eikonal calculation, we need

$$P_m = \frac{16 G_N}{\pi} \frac{\Gamma(d-1)\Gamma(1+\frac{d}{2})^3}{\Gamma(d+1)\Gamma(d+2)} \Delta_\phi^2$$

Exactly expected from AdS/CFT


$\log(-\gamma_n \Delta_\phi^2/P_m)$
40
30
20
10
d=6
d=5
d=4
d=3
1
2
3
4
5
log n



Plots in diverse spacetime dimensions for various conformal dimensions. Asymptotes indicate same intercept independent of conformal dimension.

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- Will be very interesting to explore this further.

Thank you for listening