Moduli, Inflation & the CMB

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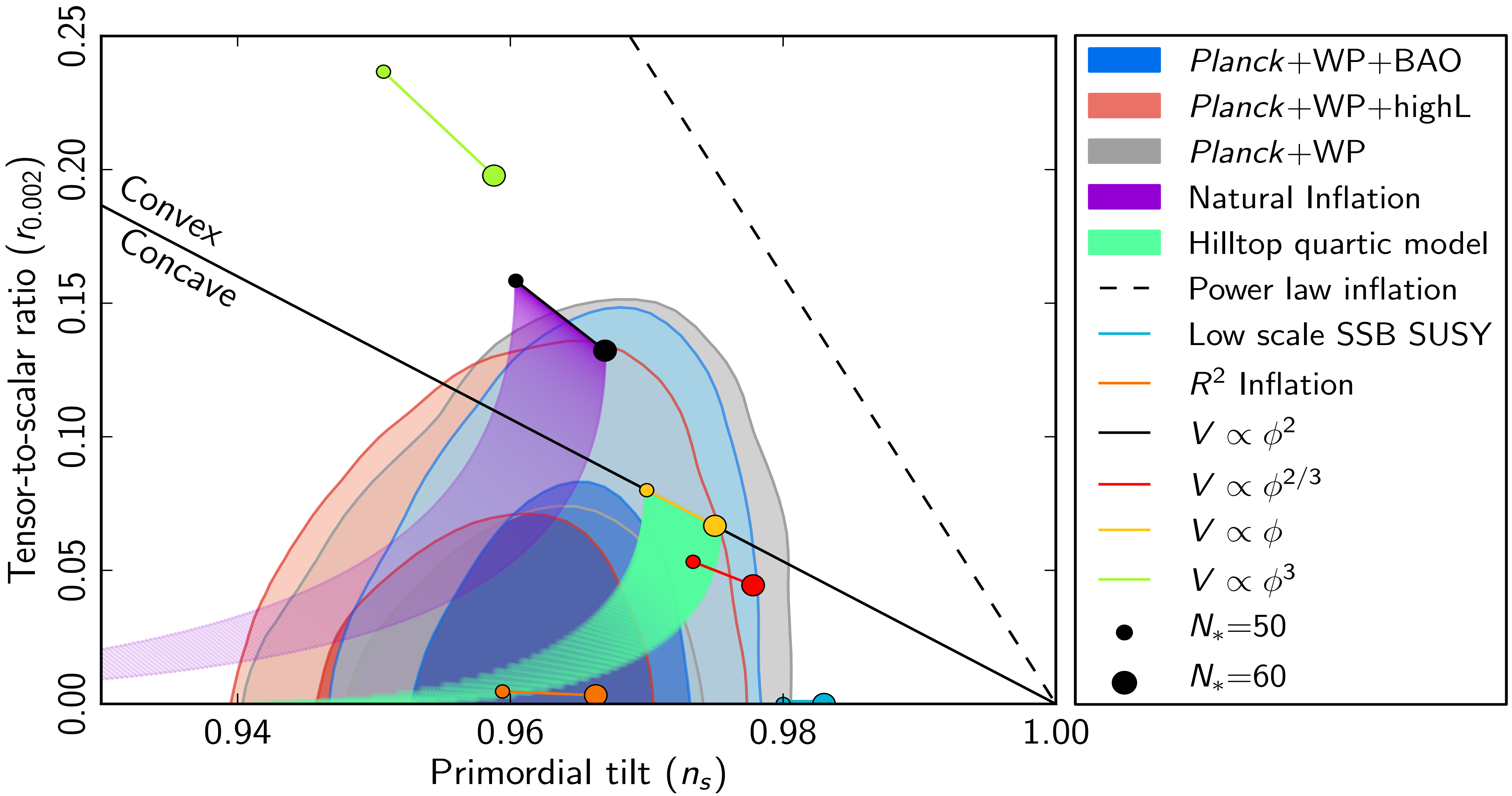
Introduction

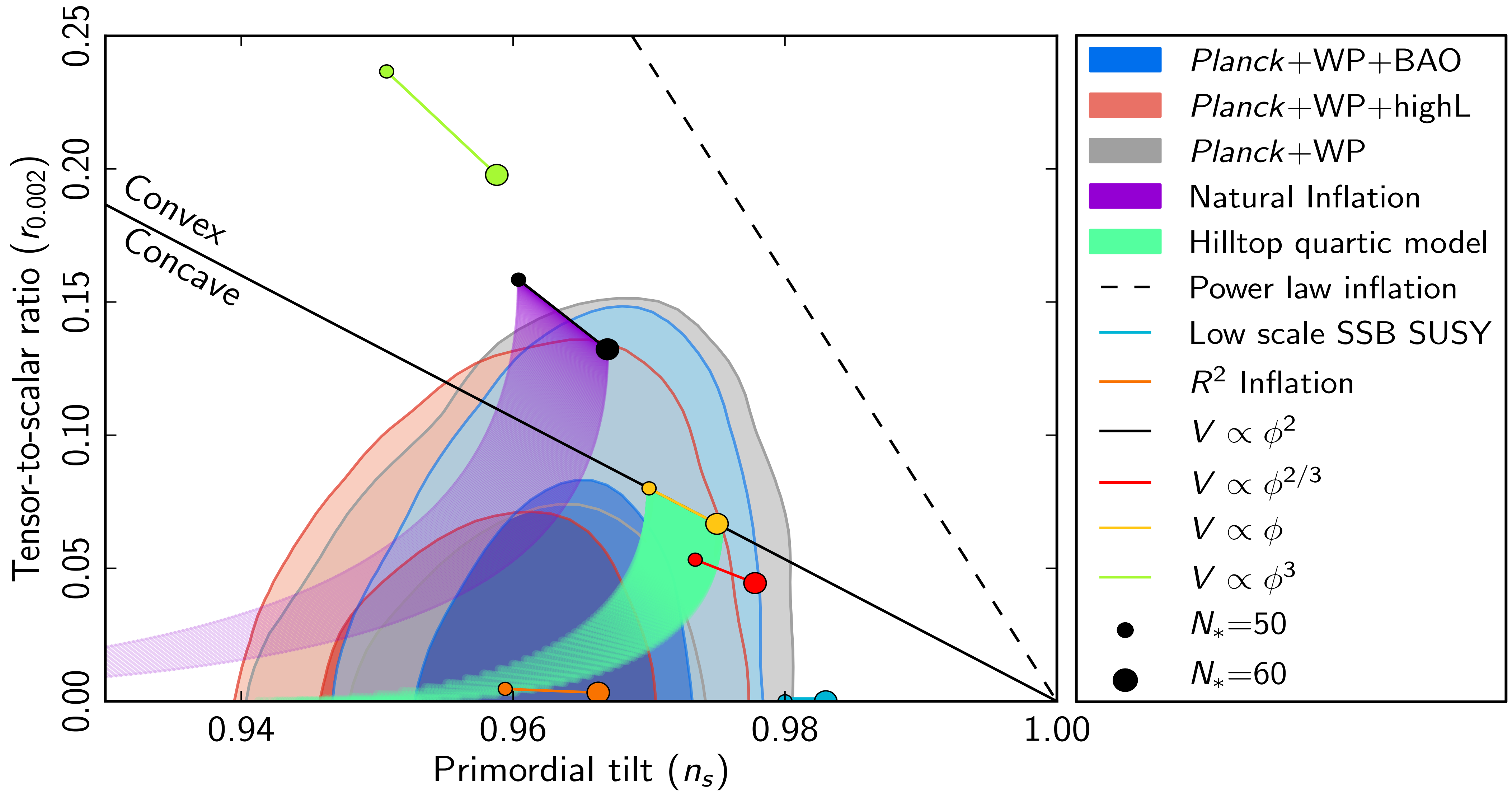
- Inflation relates late time cosmological observables (CMB) to the physics of very early times (i.e. very high energies).

- Natural to ask: What can we learn about strings models from precision CMB and inflation ?

- This talk is about Inflation, CMB and the moduli fields



How does this picture change for string models ?



How does this picture change for string models ?

We will have $M_{\rm pl}$ in the numerator.

Introduction

- From the very early days of supergravity/string model building is was realised that a generic implication of having moduli fields is a non-standard cosmological timeline.
- Modular Cosmology -

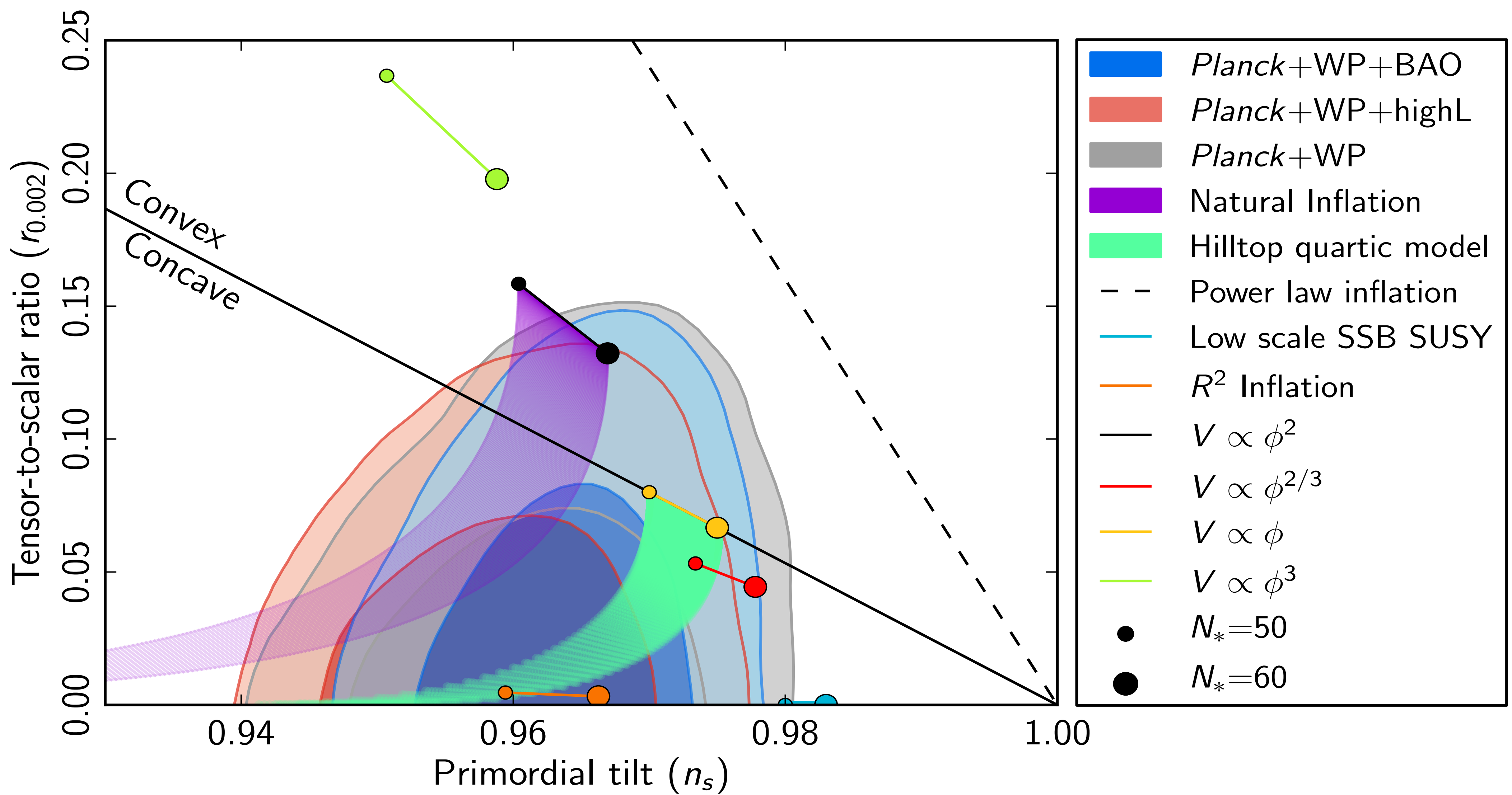
  Moduli particles produced during inflation.
  The post-inflationary (late time) cosmological history has an epoch in which the energy density is dominated by cold moduli particles. Thermal history after decay of moduli particles.
- With the developments in moduli stabilisation this picture has been validated in large class of string models.

- Confront Modular cosmology with precision CMB data.

- A statement often found in textbooks on inflation:

"The number of e-foldings between horizon exit of the modes relevant for CMB observations and the end of inflation ($N_{\rm infl}$) is 55 ± 5."

- Used in computing the inflationary predictions



- A statement often found in textbooks on inflation:

"The number of e-foldings between horizon exit of the modes relevant for CMB observations and the end of inflation ($N_{\rm infl}$) is 55 ± 5."

- What are the assumptions ?

Basic form of the standard cosmological timeline.

→ Inflation → Reheating → Radiation Domination →

→ Matter Domination ⋯⋯▸ Today

is used as a theoretical prior. Sensitive to post-inflationary history.

- What is the analogue of

  "The number of e-foldings between horizon exit of the modes relevant for CMB observations and the end of inflation $(N_{\rm infl})$ is 55 ± 5."

  for modular cosmology ?

- Answer : The preferred central value of $(N_{\rm infl})$ depends on moduli masses.

- Apart from direct implications for inflationary model building

PLAN

I. Review of Modular Cosmology.

II. Number of e-foldings of inflation in Modular Cosmology.

III. Phenomenological Implications.

Cosmology and Moduli

G. D. Coughlan, W. Fischler, E. W. Kolb, S. Raby and G. G. Ross 1984
Banks, Kaplan, Nelson 93
DeCarlos, Casas, Quevedo, Roulet 93
Banks Berkooz Steinhart 95
Banks, Berkooz, Shenker, Moore, Steinhart 95
Dine, Randall, Thomas 95
Linde 96
Acharya, Kane and Kumar 12
Douglas 12
R. Easther, R. Galvez, O. Ozsoy and S. Watson 13

Cosmology and Moduli

- The arena for our analysis - models where moduli have been stabilised. For the purposes of this talk - moduli will be massive scalars which interact solely via Planck suppressed interactions.

- The moduli acquire masses from sub-leading effects in the in effective action; their masses well below the string scale.

- It is quite common to have the present day mass of moduli to be below Hubble during inflation.

Cosmology and Moduli

- The arena for our analysis - models where moduli have been stabilised. For the purposes of this talk - moduli will be massive scalars which interact solely via Planck suppressed interactions.

- The moduli acquire masses from sub-leading effects in the in effective action; their masses well below the string scale.

- It is quite common to have the present day mass of moduli to be below Hubble during inflation. ⟶ non-standard cosmological timeline

  MODULAR COSMOLOGY

Cosmology and Moduli

- Starting point of the analysis moduli dynamics during inflation.

Goncharov, Linde, Vysotsky 1984; Dine, Fischler, Nemeschansky 1984; Coughlan, Holman, Ramond, Ross 1984; Dine, Randall, Thomas 1995; Linde 1996.

- An illustrative model (one modulus)

$$\mathcal{L} \supset -\frac{1}{2} m_{\varphi}^2 \varphi^2 - \frac{C}{2} H^2 (\varphi - \hat{\varphi})^2 - V_{\text{infl}}(\chi)$$

- Mass term, curvature coupling, inflaton potential.
- m_{φ} is the post inflationary mass.

Cosmology and Moduli

- Potential for φ having two parts

$$V(\varphi) = V_{\mathrm{postinfl}}(\varphi) + V_{\mathrm{curv}}(\varphi)$$

Post-inflationary part and Curvature coupling part do not have the same minimum.

- Reflected by the fact that the curvature coupling is to $(\varphi - \hat{\varphi})$

$$\mathcal{L} \supset -\frac{1}{2} m_\varphi^2 \varphi^2 - \frac{C}{2} H^2 (\varphi - \hat{\varphi})^2 - V_{\mathrm{infl}}(\chi)$$

- The quantity $\hat{\varphi}$ is model dependent. From very general principles

$$Y = \frac{\hat{\varphi}}{M_{\mathrm{pl}}} \approx 1$$

Dine, Randall, Thomas 1995

Cosmology and Moduli

- Starting point of the analysis moduli dynamics during inflation.

Goncharov, Linde, Vysotsky 1984; Dine, Fischler, Nemeschansky 1984; Coughlan, Holman, Ramond, Ross 1984; Dine, Randall, Thomas 1995; Linde 1996.

- Analysis of dynamics during inflation gives, for $m_\varphi \lesssim H_{\rm infl}$

$$\mathcal{L} \supset -\frac{1}{2} m_\varphi^2 \varphi^2 - \frac{C}{2} H^2 (\varphi - \hat{\varphi})^2 - V_{\rm infl}(\chi)$$

At the end of inflation the modulus φ has VEV $\hat{\varphi}$, i.e. it is displaced from its post inflationary minimum.

- Single modulus approximation is often good as from then on dynamics of the lightest most relevant.

Cosmology and Moduli

Thus just after reheating, energy density has two components

- Radiation: To which the inflaton has dumped its energy density.

- Modulus: Potential energy due to displacement.

- If $m_\varphi < H_{\rm infl}$ then the former dominates.

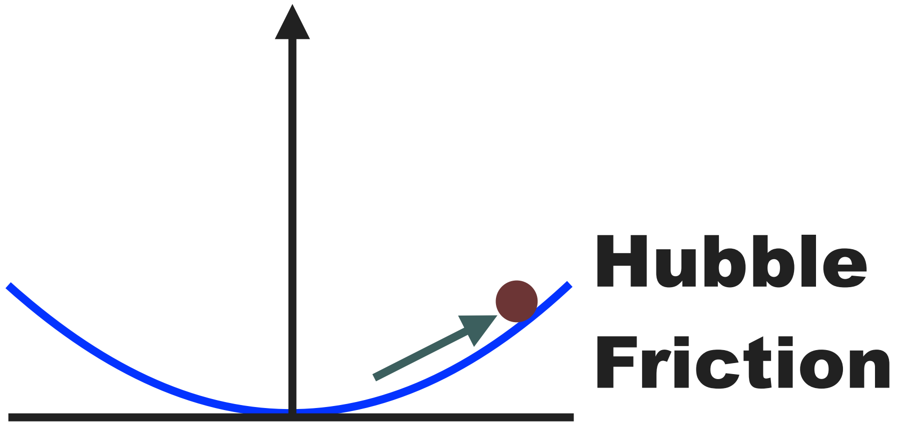
Cosmology and Moduli

- The energy density associated with radiation falls off as

$$\rho_{\text{rad}}(t) \propto \frac{1}{a^4(t)}$$

- On the other hand, for the modulus

Initially, high value of Hubble friction keeps it pinned to its expectation value.



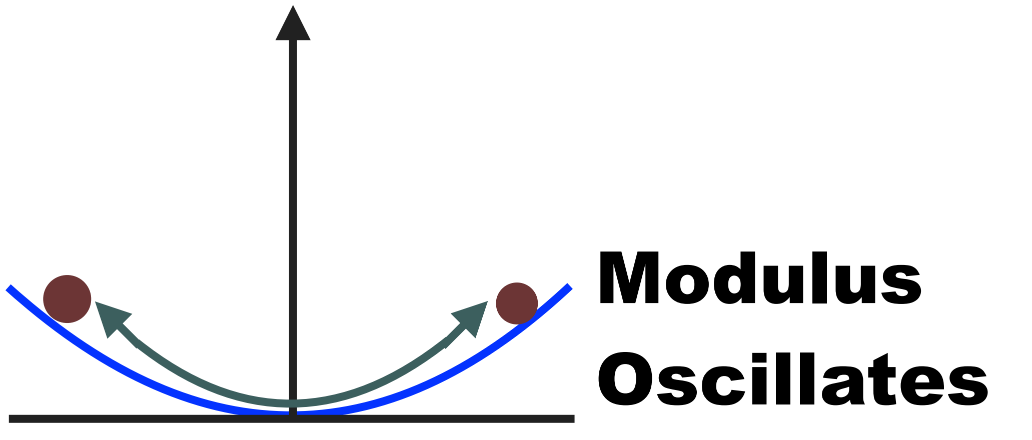
Cosmology and Moduli

- The energy density associated with radiation falls off as

$$\rho_{\text{rad}}(t) \propto \frac{1}{a^4(t)}$$

- As the universe expands, Hubble falls

When $H \lesssim m_\varphi$ the modulus begins to oscillate.



- Time average of energy density falls off as

$$\rho_{\text{modulus}}(t) \propto \frac{1}{a^3(t)}$$

Quickly dominates over Radiation.

Cosmological evolution of cold moduli particles.

Modulus Domination

- A modification the standard cosmological history

Inflation → Reheating → Radiation domination

→ Modulus domination ....

- Modulus domination continues until decay of modulus at

$$\tau_{\rm mod} \approx \frac{16\pi M_{\rm pl}^2}{m_\varphi^3}$$

the characteristic lifetime for decay via their Planck suppressed interactions.

Modulus decays ... Universe Reheats ...Thermal History

Modulus Domination

- A modification the standard cosmological history

Inflation → Reheating → Radiation domination

→ Modulus domination ....

- Modulus domination continues until decay of modulus at

$$\tau_{\rm mod} \approx \frac{16\pi M_{\rm pl}^2}{m_\varphi^3}$$

← Lifetimes are long

the characteristic lifetime for decay via their Planck suppressed interactions.

Modulus decays ... Universe Reheats ...Thermal History

Modular Cosmology

Inflation

↓

Reheating

↓

Radiation Domination

↓

Modulus Domination

↓

Reheating (after modulus decay)

↓

Radiation Domination

⋮

Today

Conventional Cosmology

Inflation

↓

Reheating

↓

Radiation Domination

⋮

Today

Any Signatures ?

A Bound from Nucleosynthesis

G. D. Coughlan, W. Fischler, E. W. Kolb, S. Raby and G. G. Ross 1984

- To account for the success of big bang nucleosynthesis, the reheat temperature after modulus decay has to be at least as large as the binding of energy of light elements.

$$T_{\text{reheat}} \gtrsim 1 \text{ MeV}$$

- Reheat temperature in terms of width

$$T_{\text{reheat}} \approx \sqrt{\Gamma M_{\text{pl}}} \qquad \Gamma \approx \frac{m_\varphi^3}{16\pi M_{\text{pl}}^2}$$

- Lighter the modulus lower the reheat temperature. Lower bound on reheat temperature translates to a lower bound for the modulus mass $m_\varphi \gtrsim 30 \text{ TeV}$.

Modular Cosmology, Inflation & CMB

- Confront modular cosmology
  - Inflation (moduli displaced from minimum at end)
  - Reheating
  - Radiation domination
  - Modulus domination
  - Decay
  - Reheat
  - Thermal history … Today

  the latest data in cosmology - Precision CMB.

- Inflation, Moduli and Precision CMB data

  ArXiv: 1409.7037 with KOUSHIK DUTTA

- We will also take more input from the inflationary paradigm than before.

Inflation and Inhomogeneities

- Additional input from inflation is from its success. That is inflation is theory for inhomogeneities. This gives

Observed Inhomogeneties ⟷ INFLATION ⟷ Energy Density at an early time

- More precisely,

$$A_s = \frac{2}{3\pi^2 r}\left(\frac{\rho}{M_{\rm pl}^4}\right)$$

  - ρ - Energy density of universe at the time of horizon exit of pivot mode.
  - r - Strength of gravity waves.

Inflation and Inhomogeneities

$$A_s = \frac{2}{3\pi^2 r}\left(\frac{\rho}{M_{\rm pl}^4}\right)$$

- Observationally, inhomogeneities in the CMB well charted. For e.g. Planck collaboration release gives $A_s = 2.2 \times 10^{-9}$ @ $k = 0.05 \ \text{Mpc}^{-1}$.

- Because of the freezing of modes after horizon exit; the formula is insensitive to the details of post inflationary physics.

Inflation, Inhomogeneities and Energy Densities

$$A_s = \frac{2}{3\pi^2 r}\left(\frac{\rho}{M_{\rm pl}^4}\right)$$

- Thus, if we treat the time of horizon exit of the pivot mode as $t = 0$; then by determining A_s and r we have an initial condition for the energy density of the universe.

- CMB data also gives us the energy density today, by determining the Hubble constant today.

Inflation, Inhomogeneities and Energy Densities

- An early time and today's energy densities known. This implies a consistency condition

Any history we ascribe must be such that the early time energy density evolves to the energy density today.

Energy density at horizon exit, ρ_k (from inhomogeneities).

Energy density today, ρ_0

The above gives $50 \lesssim N_{\rm infl} \lesssim 60$ for conventional cosmologies

What does this imply for modular cosmology ?

$t_1 \longrightarrow t_{\rm int} \longrightarrow t_2$

matter dom. radiation dom.

ρ_1 ρ_2

$$K = \ln\left(\frac{\rho(t_1)}{\rho(t_2)}\right) = \ln\left(\frac{\rho(t_1)}{\rho(t_{\rm int})}\right) + \ln\left(\frac{\rho(t_{\rm int})}{\rho(t_2)}\right)$$

$$= 3\ln\left(\frac{a(t_{\rm int})}{a(t_1)}\right) + 4\ln\left(\frac{a(t_2)}{a(t_{\rm int})}\right) = 3N_{\rm mat} + 4N_{\rm rad}$$

- From the knowledge of the energy densities ρ_1 and ρ_2 a linear combination of the number of e-foldings in the epochs is fixed.

A Relation Between the e-foldings

- Recall the cosmological timeline

$$N = \ln\left(\frac{a(t_{\rm end})}{a(t_{\rm begin})}\right)$$

→ Inflation → Reheating → Radiation Domination →

$N_{\rm infl}$ $N_{\rm rh1}$ $N_{\rm rad}$

→ Modulus Domination → Reheating (after decay) →

$N_{\rm modulus}$ $N_{\rm rh2}$

⋯⋯▸ Today

- For instance $N_{\rm infl}$ is the number of e-foldings between horizon exit of the pivot mode and the end of inflation.

- Follow the same philosophy as before
  - Condition for horizon exit.
  - No entropy production after modulus decay.

We obtain

$$\frac{1}{4}N_{\mathrm{modulus}} + \frac{1}{4}(1-3w_{\mathrm{rh1}})N_{\mathrm{rh1}} + \frac{1}{4}(1-3w_{\mathrm{rh2}})N_{\mathrm{rh2}} + N_{\mathrm{infl}} = 55.43 + \frac{1}{4}\ln r + \frac{1}{4}\ln\left(\frac{\rho_{\mathrm{k}}}{\rho_{\mathrm{end}}}\right).$$

- $N_{\mathrm{rh1}}, N_{\mathrm{rh2}}$ the number of e-foldings during the two reheating epochs.

- Reheating can occur via various mechanisms. We do not commit to any particular mechanism. $w_{\mathrm{rh1}}, w_{\mathrm{rh2}}$ are the (averaged) effective equation of state during the epochs.

- ρ_k - Energy density at the time of horizon exit of pivot mode. ρ_{end} - Energy density at the end of inflation.

Ratio depends on broad characteristic of inflationary potential

We obtain

$$\mathrm{N_{infl}} + \frac{1}{4}\mathrm{N_{modulus}} + \frac{1}{4}(1-3\mathbf{w_{rh1}})\mathrm{N_{rh1}} + \frac{1}{4}(1-3\mathbf{w_{rh2}})\mathrm{N_{rh2}} \approx \mathbf{55} + \frac{1}{4}\ln\mathbf{r} + \frac{1}{4}\ln\left(\frac{\rho_{\mathbf{k}}}{\rho_{\mathrm{end}}}\right)$$

Planck 2013 results. XXII Constraints on Inflation

$$N_* \approx 71.21 - \ln\left(\frac{k_*}{a_0 H_0}\right) + \frac{1}{4}\ln\left(\frac{V_{\mathrm{hor}}}{M_{\mathrm{pl}}^4}\right) + \frac{1}{4}\ln\left(\frac{V_{\mathrm{hor}}}{\rho_{\mathrm{end}}}\right) + \frac{1-3w_{\mathrm{int}}}{12(1+w_{\mathrm{int}})}\ln\left(\frac{\rho_{\mathrm{th}}}{\rho_{\mathrm{end}}}\right), \quad (24)$$

- Translating to our notation and plugging in for the knowns

$$\mathrm{N_{infl}} + \frac{1}{4}(1-3\mathbf{w_{rh}})\mathrm{N_{rh}} \approx \mathbf{55} + \frac{1}{4}\ln\mathbf{r} + \frac{1}{4}\ln\left(\frac{\rho_{\mathbf{k}}}{\rho_{\mathrm{end}}}\right)$$

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$$\mathbf{N_{infl}} + \frac{1}{4}(1 - \mathbf{3w_{rh}})\mathbf{N_{rh}} \approx \mathbf{55} + \frac{1}{4}\ln \mathbf{r} + \frac{1}{4}\ln\left(\frac{\rho_{\mathbf{k}}}{\rho_{\mathrm{end}}}\right)$$

We obtain

$$\mathbf{N_{infl}} + \frac{1}{4}\mathbf{N_{modulus}} + \frac{1}{4}(1 - \mathbf{3w_{rh1}})\mathbf{N_{rh1}} + \frac{1}{4}(1 - \mathbf{3w_{rh2}})\mathbf{N_{rh2}} \approx \mathbf{55} + \frac{1}{4}\ln \mathbf{r} + \frac{1}{4}\ln\left(\frac{\rho_{\mathbf{k}}}{\rho_{\mathrm{end}}}\right)$$

Key difference for modular cosmology

The Modulus Domination Epoch

- Modulus begins to oscillate when

$$H \lesssim m_\varphi$$

- Characteristic life time for decay via Planck suppressed interactions

$$\tau_{\rm mod} \approx \frac{16\pi M_{\rm pl}^2}{m_\varphi^3}$$

- By explicitly tracking the FRW cosmology

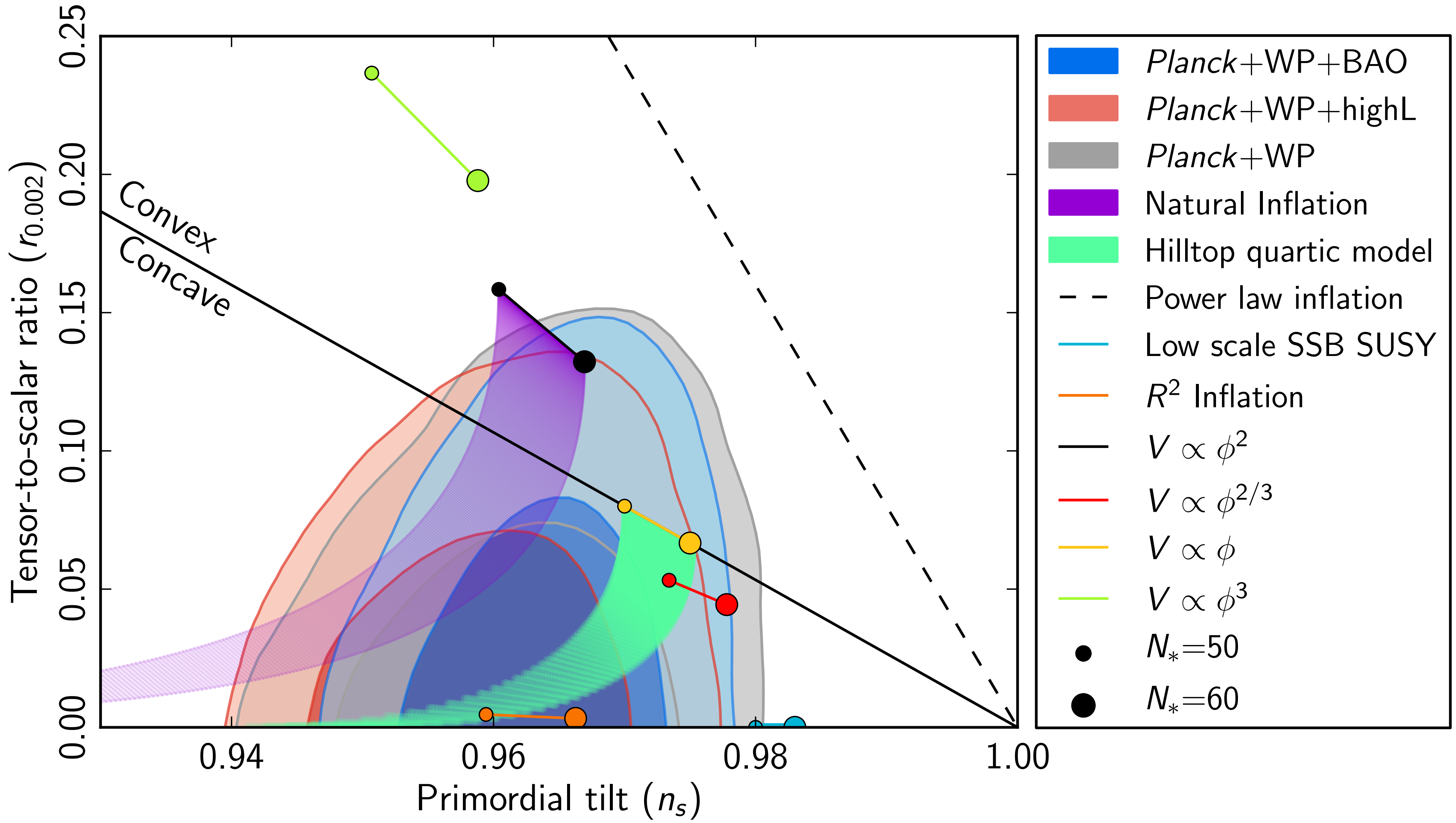
$$N_{\rm modulus} \approx \frac{4}{3} \ln\left(\frac{\sqrt{16\pi} M_{\rm pl} Y^2}{m_\varphi}\right)$$

Y is the initial field displacement in Planck units

Finally we have

$$N_{\rm infl} + \frac{1}{4}(1-3w_{\rm rh1})N_{\rm rh1} + \frac{1}{4}(1-3w_{\rm rh2})N_{\rm rh2}$$
$$\approx 55 - \frac{1}{3}\ln\left(\frac{\sqrt{16\pi}M_{\rm pl}Y^2}{m_\varphi}\right) + \frac{1}{4}\ln r + \frac{1}{4}\ln\left(\frac{\rho_k}{\rho_{\rm end}}\right)$$

Next, how does this affect inflationary predictions ?



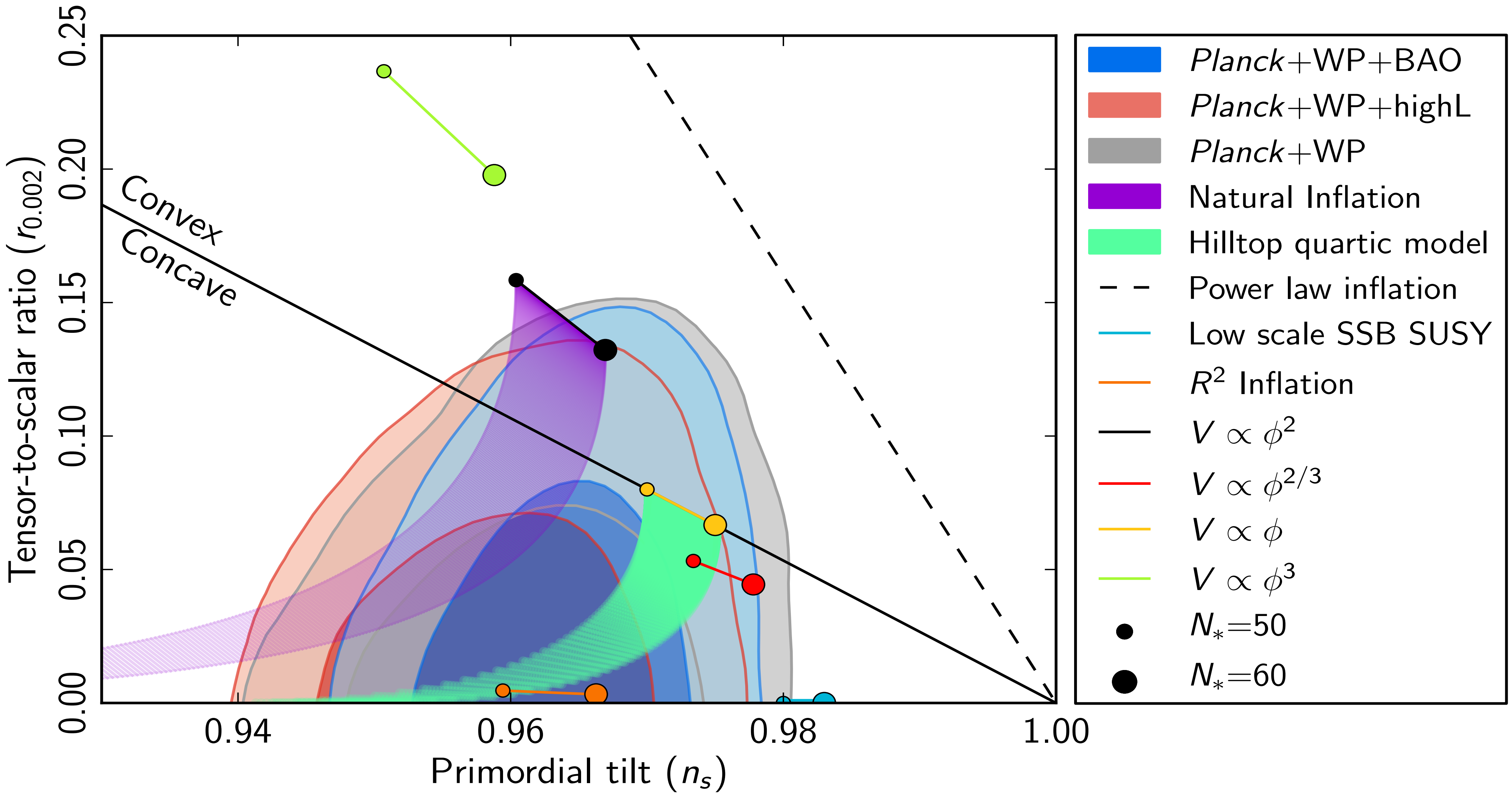
We confront a model of inflation with data by

- Computing n_s and r as a function of $N_{\rm infl}$. For e.g. For $m^2\chi^2$; $n_s = 1 - 2/N$ $r = 8/N$

- Motivated by

$$\mathbf{N_{infl} + \frac{1}{4}(1 - 3w_{rh})N_{rh} \approx 55 + \frac{1}{4}\ln r + \frac{1}{4}\ln\left(\frac{\rho_k}{\rho_{end}}\right)}$$

Do the predictions match with observations for $N_{\rm infl} = 55 \pm 5$?



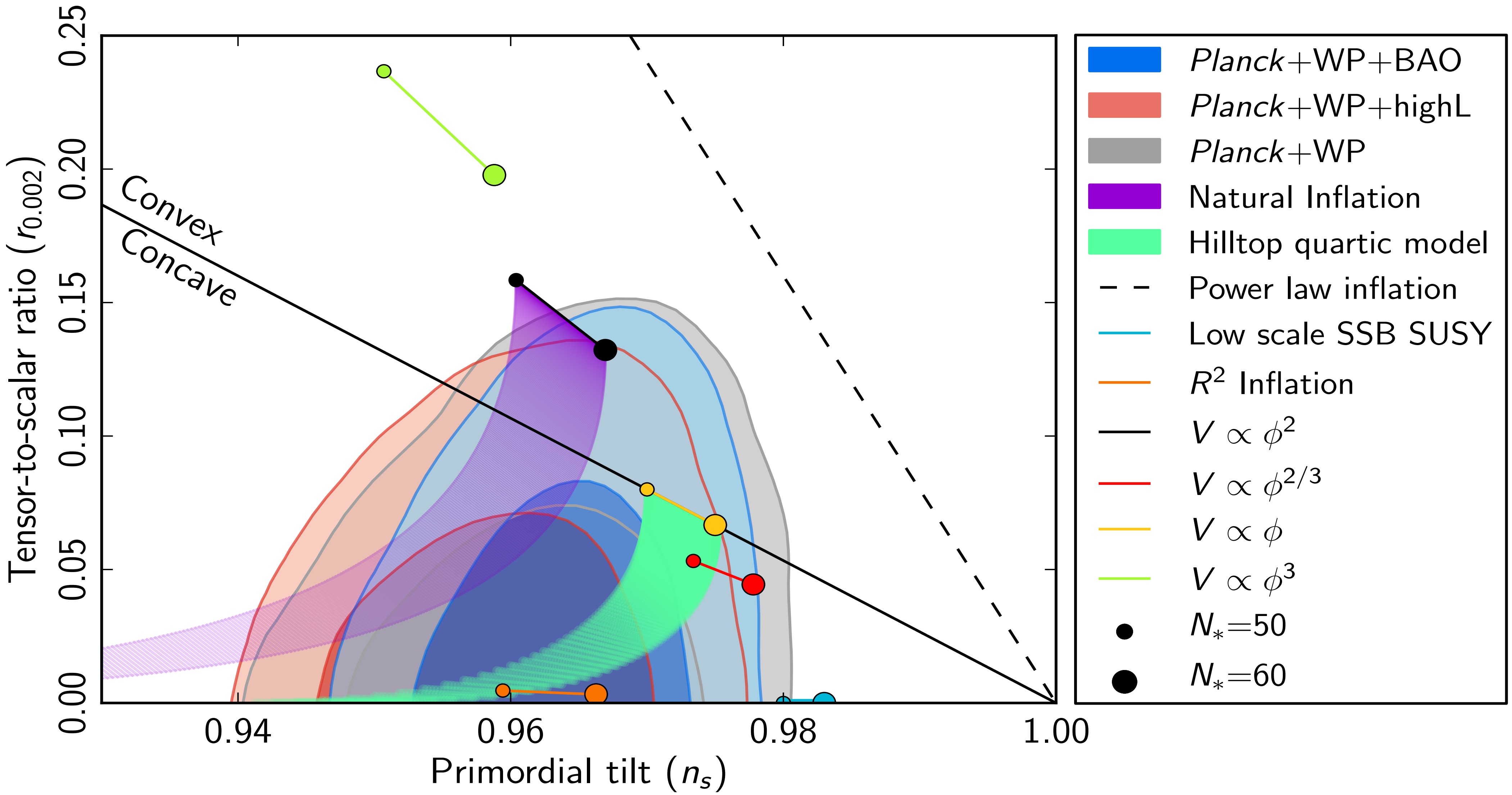
For Modular Cosmology

$$N_{\rm infl} + \frac{1}{4}(1-3w_{\rm rh1})N_{\rm rh1} + \frac{1}{4}(1-3w_{\rm rh2})N_{\rm rh2}$$

$$\approx 55 - \frac{1}{3}\ln\left(\frac{\sqrt{16\pi}M_{\rm pl}Y^2}{m_\varphi}\right) + \frac{1}{4}\ln r + \frac{1}{4}\ln\left(\frac{\rho_k}{\rho_{\rm end}}\right)$$

The central value of $N_{\rm infl}$ shifts

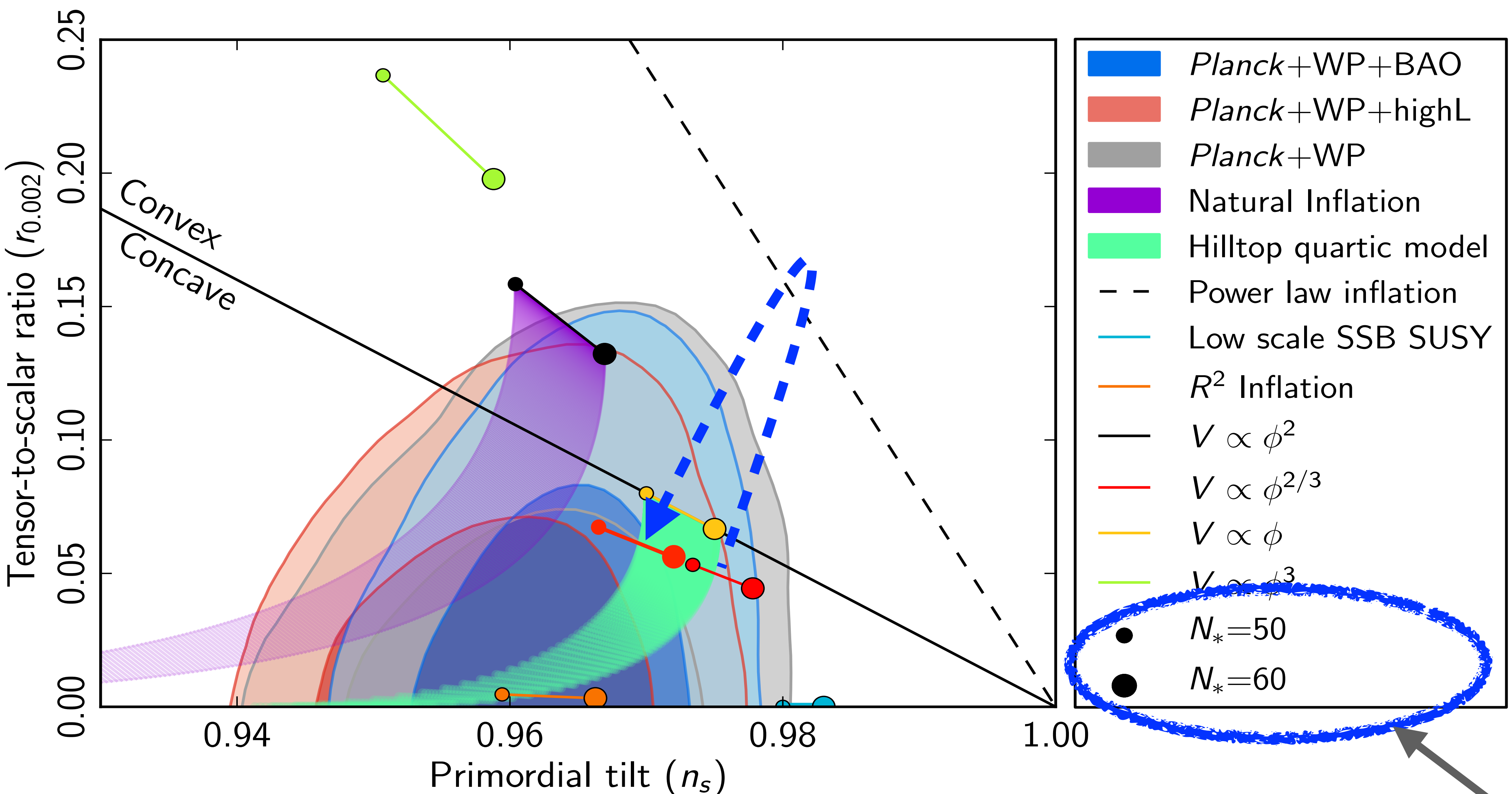
$$55 \to 55 - \frac{1}{3}\ln\left(\frac{\sqrt{16\pi}M_{\rm pl}Y^2}{m_\varphi}\right)$$



The central value of $N_{\rm infl}$ shifts

$$55 \to 55 - \frac{1}{3} \ln \left(\frac{\sqrt{16\pi} M_{\rm pl} Y^2}{m_\varphi} \right)$$

We will see that the shift is signifiant in a class of models; with phenomenologically interesting implications. Cannot be accounted for in usual leeway of ± 5 .

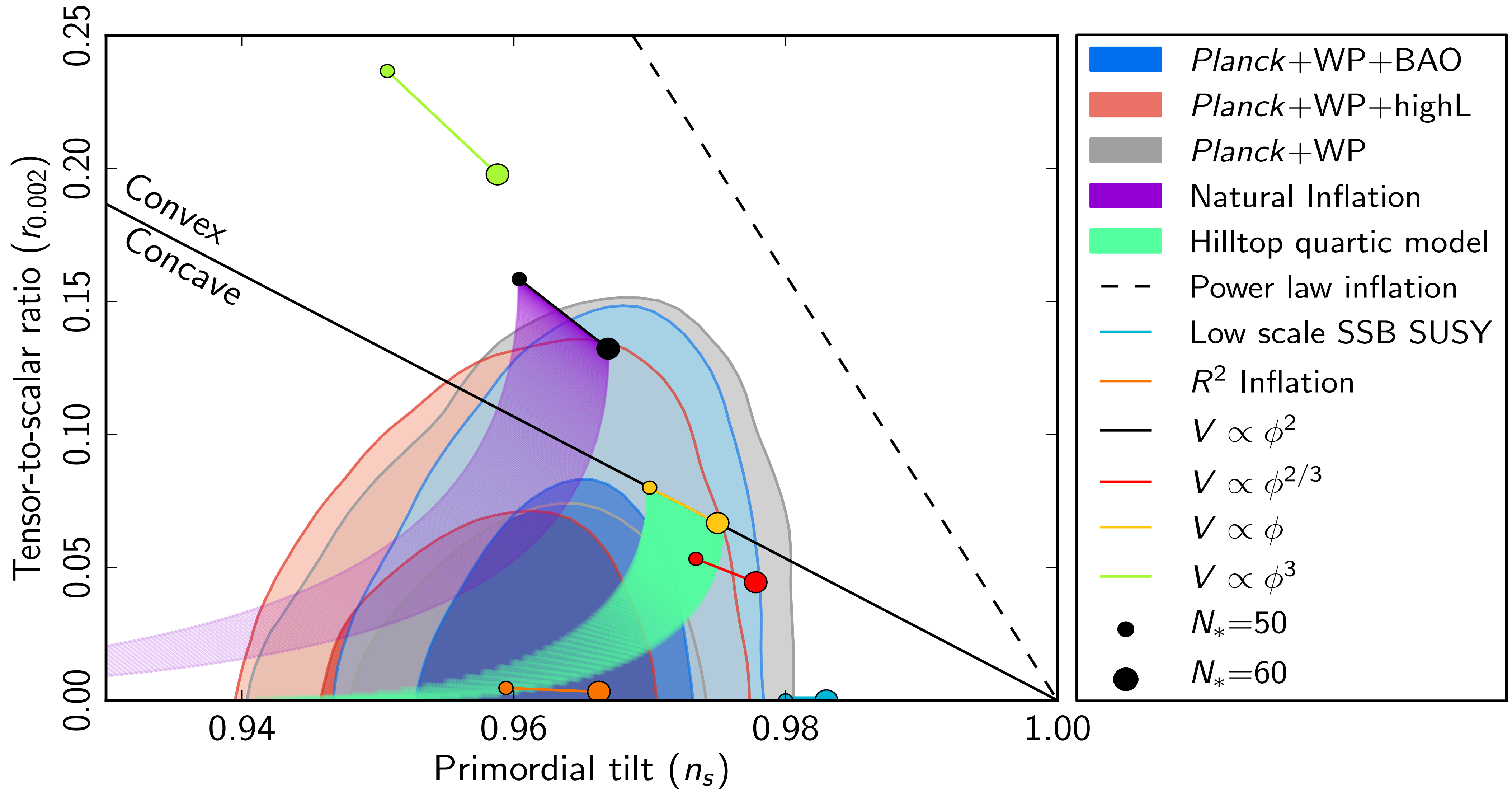


Change the 50 - 60 range

The central value of $N_{\rm infl}$ shifts

$$55 \to 55 - \frac{1}{3} \ln \left(\frac{\sqrt{16\pi} M_{\rm pl} Y^2}{m_\varphi} \right)$$

We will see that the shift is signifiant in a class of models; will have important implications. Cannot be accounted for in usual leeway of ± 5.



Modulus mass input for inflationary predictions

The central value of $N_{\rm infl}$ shifts

$$55 \to 55 - \frac{1}{3} \ln \left(\frac{\sqrt{16\pi} M_{\rm pl} Y^2}{m_\varphi} \right)$$

We will see that the shift is signifiant in a class of models; will have important implications. Cannot be accounted for in usual leeway of ± 5.

Gravity Mediated Models and Inflation

- Gravity mediated models where moduli masses are tied to the soft masses. For typical values
    - $m_\varphi \approx 100 - 1000 \;\; \text{TeV}$
    - $Y \approx 1/10$
- The preferred central value

$$\hat{N}_{\text{infl}} = 55 - \frac{1}{3} \ln \left(\frac{\sqrt{16\pi} M_{\text{pl}} Y^2}{m_\varphi} \right) \approx 45$$

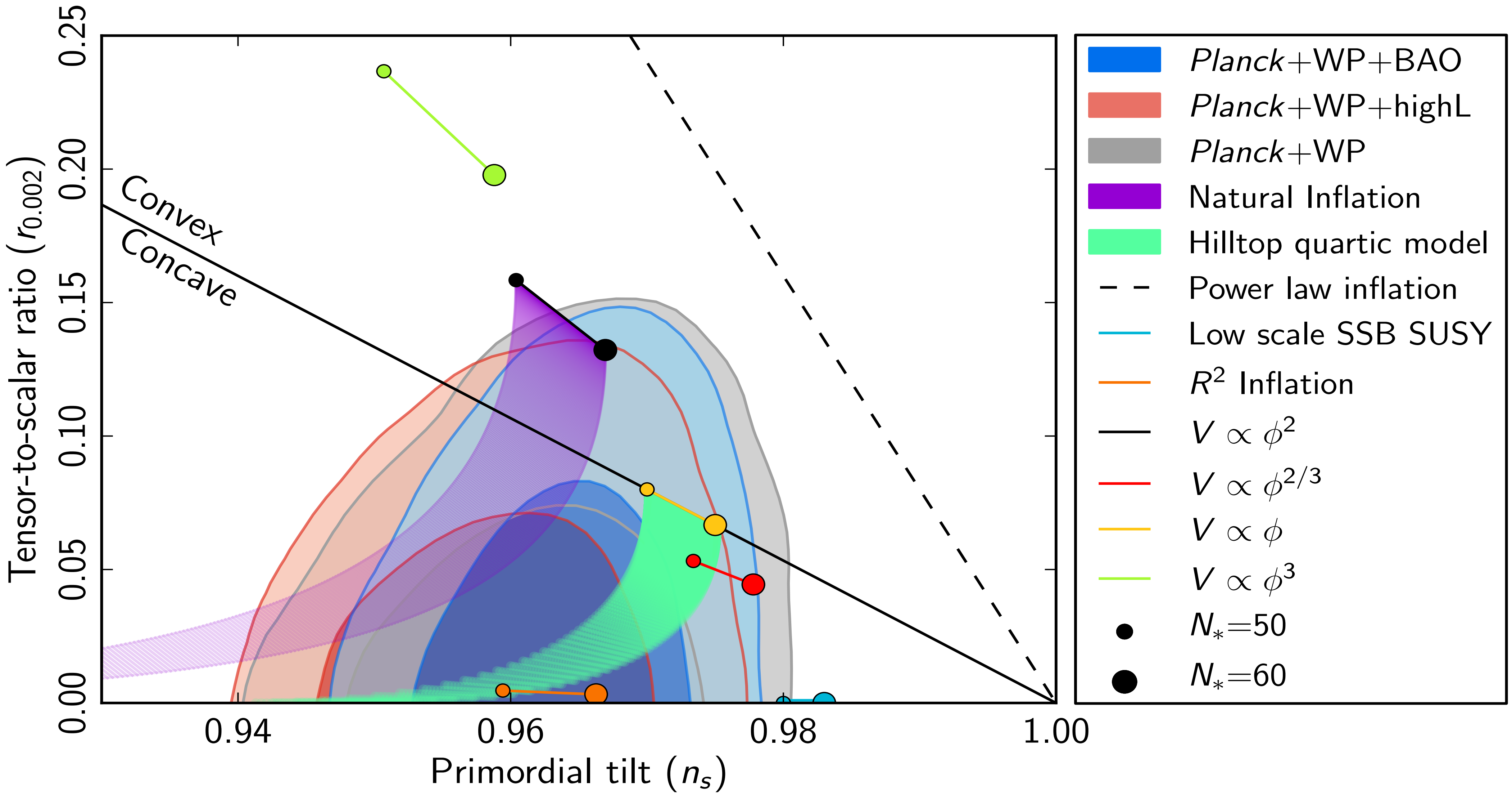
Successful models of inflation very different from the ones we are used to.

- The preferred central value

$$\hat{N}_{\rm infl} = 55 - \frac{1}{3}\ln\left(\frac{\sqrt{16\pi}M_{\rm pl}Y^2}{m_\varphi}\right) \approx 45$$

- Changing the mass of the modulus by one order of magnitude changes by less than one.

Supersymmetry breaking scale ⟷ Number of e-foldings during inflation



- The central value $\hat{N}_{\rm infl}$ hits 50 for $m_\varphi \approx 10^{10}$ GeV .

One should explicitly include the effect of the modulus domination epoch if $m_\varphi \lesssim 10^{10}$ GeV.

A Bound on Moduli masses from Inflationary Sector

- So far, we have been using $N_{\rm infl}$ as an input for inflationary predictions. But if we have a preferred model of inflation

$$N_{\rm infl} \approx \frac{\beta}{1-n_s}$$

Model dependent constant

- Precision measurement of n_s determines $N_{\rm infl}$.

$$\frac{1}{3}\ln\left(\frac{\sqrt{16\pi}M_{\rm pl}Y^2}{m_\varphi}\right) + \frac{1}{4}(1-3w_{\rm rh1})N_{\rm rh1} + \frac{1}{4}(1-3w_{\rm rh2})N_{\rm rh2}$$
$$\approx 55 - N_{\rm infl} + \frac{1}{4}\ln r + \frac{1}{4}\ln\left(\frac{\rho_k}{\rho_{\rm end}}\right)$$

- LHS post-inflanationary, RHS inflationary.

Positive

$$\frac{1}{3}\ln\left(\frac{\sqrt{16\pi}M_{\rm pl}Y^2}{m_\varphi}\right) + \frac{1}{4}(1-3w_{\rm rh1})N_{\rm rh1} + \frac{1}{4}(1-3w_{\rm rh2})N_{\rm rh2}$$
$$\approx 55.43 - N_{\rm infl} + \frac{1}{4}\ln r + \frac{1}{4}\ln\left(\frac{\rho_k}{\rho_{\rm end}}\right)$$

- Reheating dependence ?? Reheating can occur by various mechanisms. Explicit numerical and analytic studies strongly suggest the averaged effective equation of state

$$w_{\rm rh} < 1/3$$

- The terms associated with reheating are positive definite; the maximum value of the term containing the modulus mass is given by the RHS. We have a bound

$$m_\varphi \gtrsim \sqrt{16\pi}M_{\rm pl}Y^2\ e^{-3\left(55.43-N_{\rm infl}+\frac{1}{4}\ln(\frac{\rho_{\rm k}}{\rho_{\rm end}})+\frac{1}{4}\ln r\right)}$$

Bound on Modulus Mass

$$m_\varphi \gtrsim \sqrt{16\pi} M_{\rm pl} Y^2 \ e^{-3\left(55.43 - N_{\rm infl} + \frac{1}{4}\ln(\frac{\rho_{\rm k}}{\rho_{\rm end}}) + \frac{1}{4}\ln r\right)}$$

- Larger the number of e-foldings stronger the bound

- Lower the value of r, stronger the bound.

- The second term's magnitude depends on the broad characteristics of the inflationary potential (small field/large field).

Small Field Models

$$m_\varphi \gtrsim \sqrt{16\pi} M_{\rm pl} Y^2 \; e^{-3\left(55.43 - N_{\rm infl} + \frac{1}{4}\ln(\frac{\rho_{\rm k}}{\rho_{\rm end}}) + \frac{1}{4}\ln r\right)}$$

- Since lower the value of r, stronger the bound. For a conservative estimate of the bound we take $r = 0.01$.
- The form of the potential in these models is typically plateau-like. The term involving the energy densities makes a negligible contribution.
- For the initial field displacement, take a conservative value

$$Y = \varphi_{\rm in}/M_{\rm pl} = 1/100$$

- Then for $N_{\rm infl} = 50$; $m_\varphi \gtrsim 4.5 \times 10^6$ TeV ; much stronger than the bound based on nucleosynthesis considerations.

Large Field Models

$$m_\varphi \gtrsim \sqrt{16\pi} M_{\rm pl} Y^2 \; e^{-3\left(55.43 - N_{\rm infl} + \frac{1}{4}\ln(\frac{\rho_{\rm k}}{\rho_{\rm end}}) + \frac{1}{4}\ln r\right)}$$

- The basic shape of the potentials can be parametrised by a one parameter family $M^{4-P}\chi^P$ of power law potentials.

- One can compute the relevant terms the relevant terms in the exponent. Again, the scale is set by the number of e-folding.

- Bound only one order of magnitude less, still strong.

Conclusions

- Modular cosmology very well motivated from the point of view of string/supergravity models.

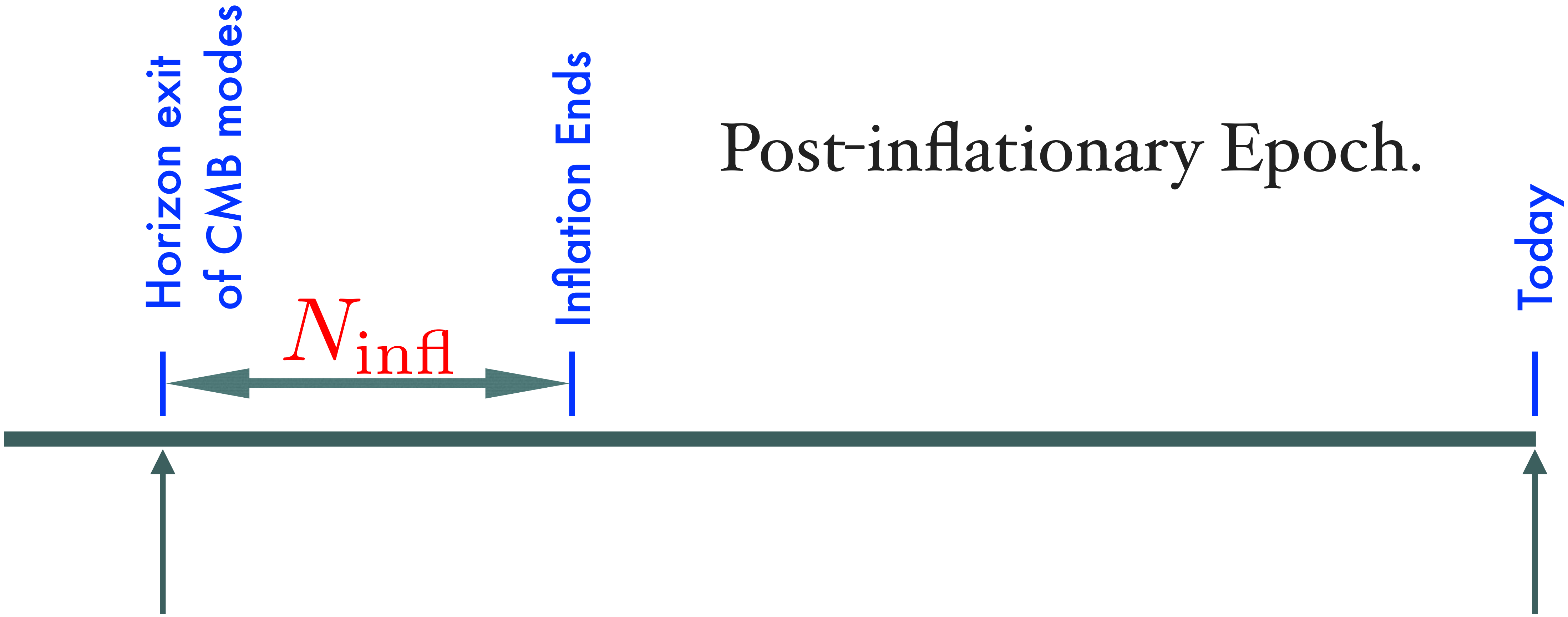
  -moduli particles produced during inflation

  -long epoch of modulus domination

- We have examined modular cosmology in the light of CMB data and inflation.

Preferred central value of the number e-foldings inflation

$$55 \to 55 - \frac{1}{3} \ln \left(\frac{\sqrt{16\pi} M_{\rm pl} Y^2}{m_\varphi} \right)$$

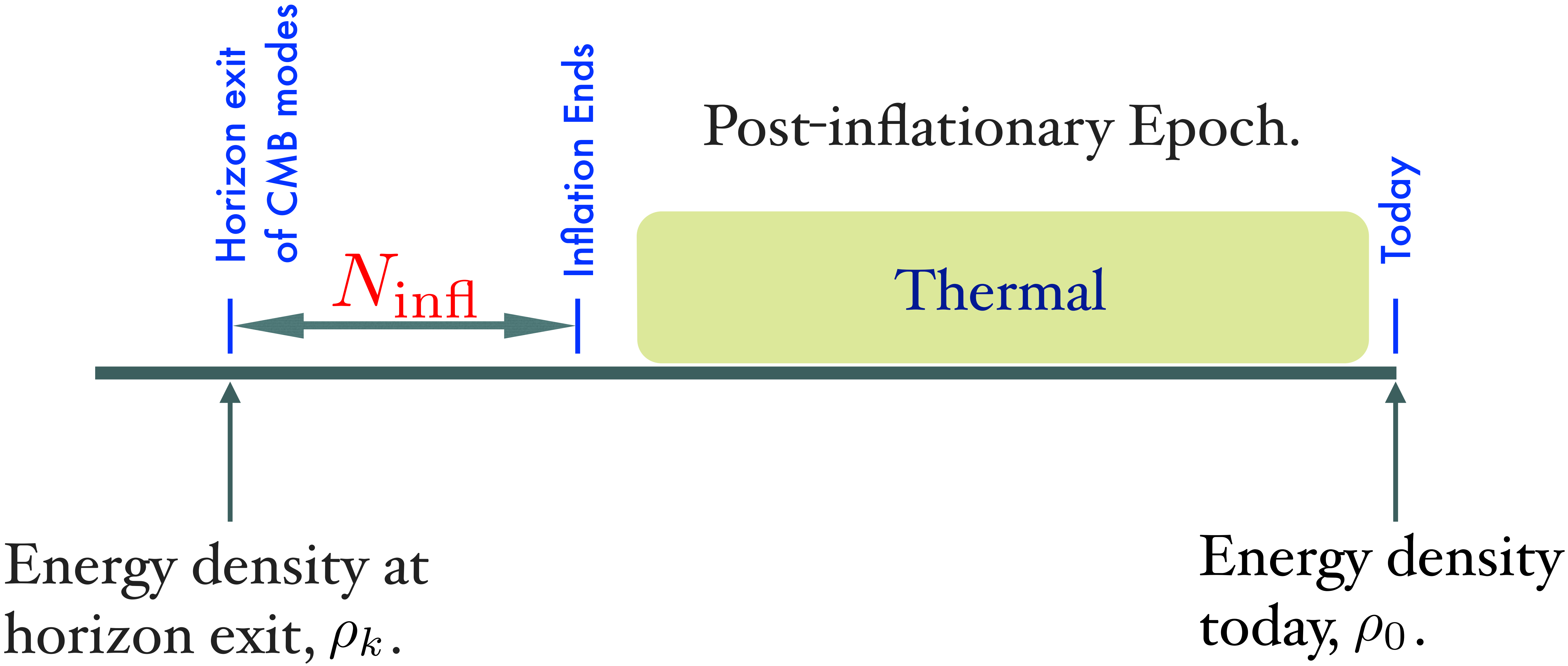


Energy density at horizon exit, ρ_k (from inhomogeneities).

Energy density today, ρ_0.

N_{infl} determined using

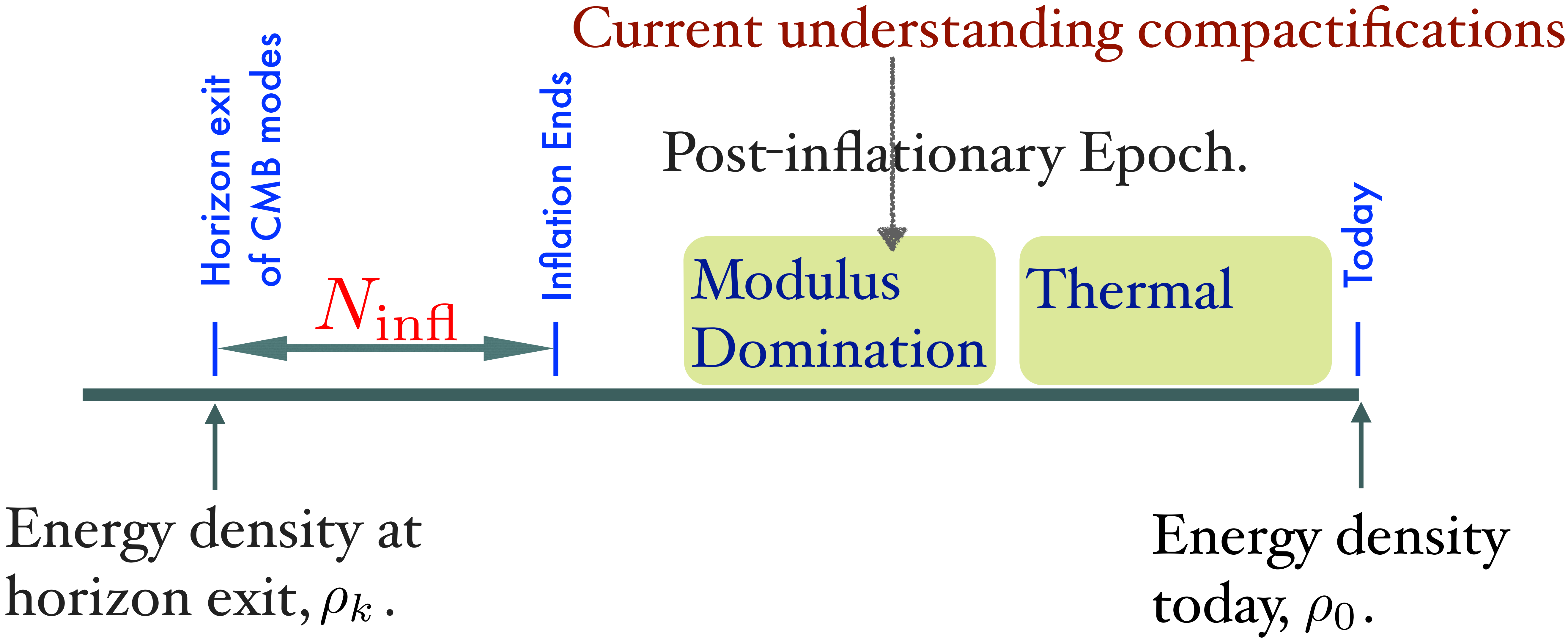
- Consistent evolution of ρ_k to ρ_0
- Prior on Post-inflationary cosmology



N_{infl} determined using

- Consistent evolution of ρ_k to ρ_0
- Prior on Post-inflationary cosmology

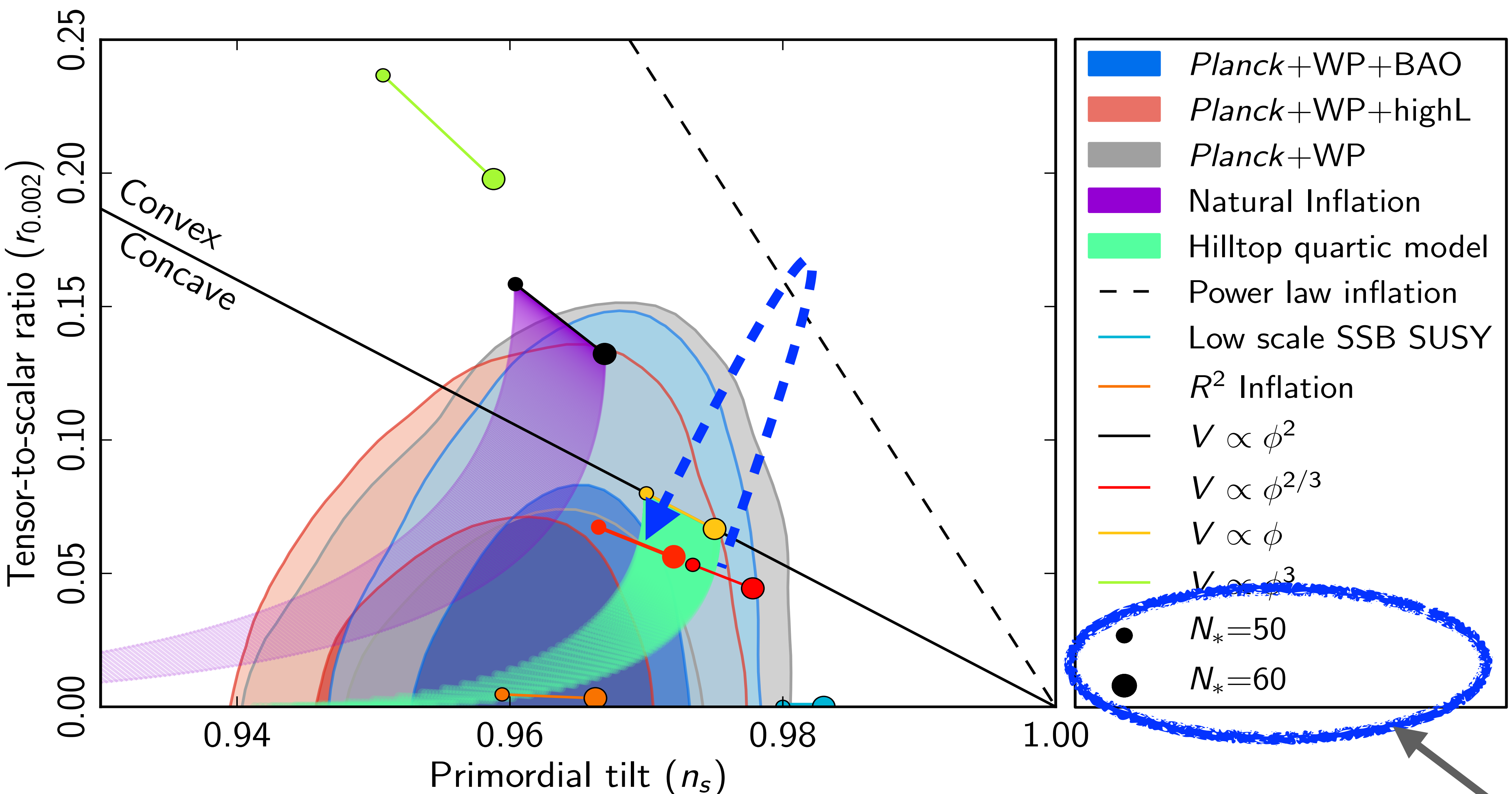
$$N_{\text{infl}} = 55 \pm 5$$



$N_{\rm infl}$ determined using

- Consistent evolution of ρ_k to ρ_0
- Prior on Post-inflationary cosmology

$$N_{\rm infl} = \left(55 - \frac{1}{3}\ln\left(\frac{\sqrt{16\pi}M_{\rm pl}Y^2}{m_\varphi}\right)\right) \pm 5$$



Change the 50 - 60 range

$$55 \to 55 - \frac{1}{3} \ln \left(\frac{\sqrt{16\pi} M_{\rm pl} Y^2}{m_\varphi} \right)$$

- Modulus mass needed to make inflationary predictions

$$m_\varphi \lesssim 10^{10} \ \ \mathrm{GeV}$$

- For Gravity mediated models of SUSY breaking

Hierarchy

$$\hat{N}_{\rm infl} = 55 - \frac{1}{3} \ln \left(\frac{\sqrt{16\pi} M_{\rm pl} Y^2}{m_\varphi} \right)$$

Supersymmetry breaking scale ⟷ Number of e-foldings during inflation

- With a preferred model of inflation and bound on moduli masses, potential equality with understanding of reheating.