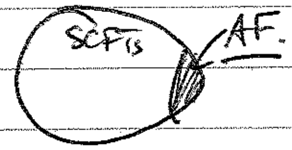
Exploring The 4d Superconformal Zoo | BW, QMUL
w/ JH, YT, CV, BB, OB, NB, FB, YT

Everybody loves 4d SCFTs. The usual way we engineer them is $\overset{UV \leftarrow AF}{\circ} \rightsquigarrow \circ$ IR ← interacting. But this is weird! CFTs = Ops & OPEs.

So maybe we're missing a huge part of the picture!



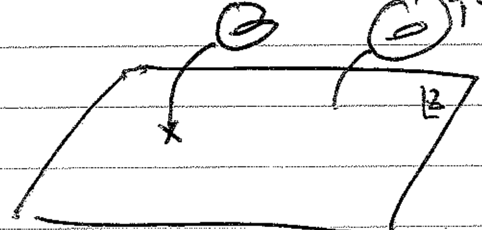
Goal today: Explore the rest of the space, using string/M-theory techniques.

Outline (1) F
(2) M.

(1) F | F-Theory = IIB.

Recall that there's an axiodilaton $\tau_{IIB} = C_0 + ie^{-\phi} = C_0 + \frac{i}{g_s}$.

IIB has an $SL(2,\mathbb{Z})$, where $\tau \to \frac{a\tau+b}{c\tau+d}$, $ad-bc=1$, $\in \mathbb{Z}$.
⇒ think of τ like CS of T^2.



$y^2 = x^3 + f(z)x + g(z)$. As move around, τ changes. What can happen?

When $\Delta = 4f^3 + 27g^2 = 0$, cycles pinch
⇒ T^2 degenerates ⇒ monodromy.

Thanks to Kodaira, we know that the degenerations of a T^2 fall into an ADE correspondence, depending on f, g.

$\boxed{\text{Ex}}$ $y^2 = x^3 + z^4 \Rightarrow E_6 \Rightarrow \begin{pmatrix} -1 & -1 \\ -1 & 0 \end{pmatrix}$

So here's a way of engineering $N=2$ thys:

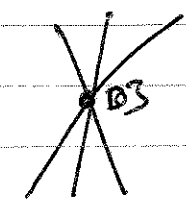
ADE = gauge symmetry 7-brane,
global symm on D3.

$\Rightarrow$ Can get $E_{6,7,8}$ MN thys from F-thy (old story)

	0	1	2	3	4 (z^1)	5	6 (z^2)	7	8 (z)	9
7	x	x	x	x	x	x	x	x		
3	x	x	x	x						

<u>What about $N=1$?</u>

Can take $f \to f + \delta f(z, z^1, z^2)$ $\quad g \to g + \delta g(z^1, z^2, z)$



This gives a WV superpotential $W = \mathrm{Tr}\left[\phi(z, z^1, z^2)\cdot \mathcal{O}\right]$ $\quad \mathcal{O} \in \mathcal{G}$, Adj.

This will break $N=2$ if $[\phi, \phi^\dagger] \neq 0$.

<u>Ex</u> $D_4 \Rightarrow SO(2)$ w/ $N_f = 4$.

$$W = \sum_{i=1}^{4} Q_i \Phi \widetilde{Q}_i + m Q_1 \widetilde{Q}_2.$$

$$\Rightarrow \sum_{i=3}^{4} Q_i \Phi \widetilde{Q}_i + \frac{1}{m} Q_2 \Phi^2 \widetilde{Q}_1$$

$R(Q_3) = R(Q_4) = \hat{} = \tilde{}$ $\qquad \Rightarrow 2R(Q_3) + R(\Phi) = 2$

$R(\Phi)$, $R(Q_2) = R(\widetilde{Q}_1)$. $\qquad R(Q_1) + R(\Phi) = 1$

anom: $4 + \frac{4}{2}(R(\Phi) - 1) + \frac{4}{2}(R(Q_3) - 1) + 2(R(Q_2) - 1) = 0$

$$2R(\Phi) + 2R(Q_3) + R(Q_2) = 3.$$

$\Rightarrow$ need a-max $\Rightarrow$ get consistent SCFT.

Similarly, can do this for general $\mathcal{O}$, $W = \mathrm{Tr}(\phi \cdot \mathcal{O})$

In general, U(1)'s are $U(1) \times SU(2) \times G$

$$\downarrow$$

$$R_{N=2} + I_3 + \sum F_i$$

Why can you do a-max, $3\,\mathrm{Tr}\,R^3 - \mathrm{Tr}\,R$? Answer: because we know 't Hooft anomalies even in weird thys!

e.g. $\mathrm{Tr}\,R_{N=2}^3 = \mathrm{Tr}\,R_{N=2} = 48(a-c)$

$\mathrm{Tr}\,R_{N=2} I_3^2 = 4a - 2c$

$\Rightarrow$ Can do a-max even w/o $\mathcal{L}$! If you know info about global symms, you know a lot.

<u>Interesting flows</u> $\quad N=2 \qquad N=1$

$E_8 \searrow$

$\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \subset SU(2)$, E_7

$\downarrow$

$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \subset SU(3)$, E_6

$+\begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \sim \begin{pmatrix} \sqrt{2} & & \\ & -\sqrt{2} & \\ & & 0 \end{pmatrix}$

$\swarrow$

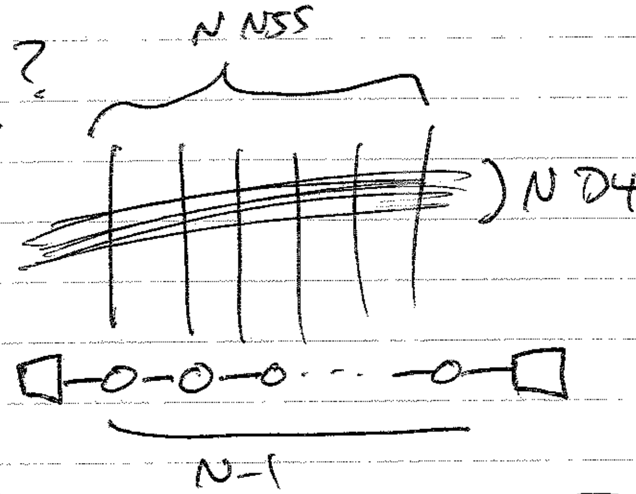
E_7 & G goes up!

② M | M-theory is also a natural place to look for new $N=1$ theories, thanks to the M5-brane.

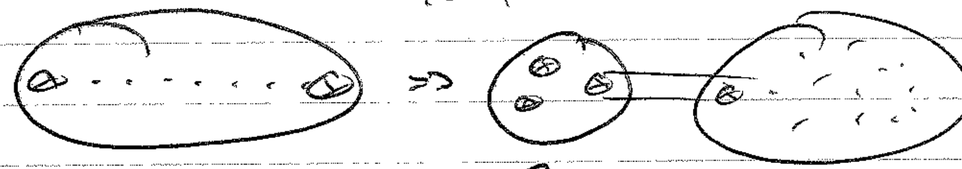
Flat stack of M5's:
- (2,0) in 6d
- N^3 d.o.f.
- $\cancel{\mathcal{L}}$.

But what about 4d theories?

Old idea: NS5's & D4's



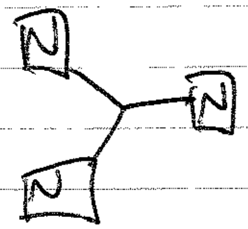
$\Rightarrow$ N M5's on



new $\mathcal{N}=2$ SCFT.

This new theory is called "T_N" and has lots of interesting properties:

- $G = SU(2) \times U(1) \times SU(N)^3$
- know SW curve, dims of ops
- know 't Hooft anomalies,
- No $\cancel{\mathcal{L}}$.



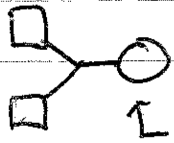
So how can we get new SCFTs? Easy:
<u>GAUGE GLOBAL SYMMETRIES!</u>

$\Rightarrow \int d^4\theta\, J^A V^A$

ordinary Vector mult

Current, whose expansion in terms of free fields is unknown.

The big question is: When are such theories conformal?



$\beta \sim \mathrm{Tr}_R T^A T^B \sim \langle J^A J^B \rangle \sim k_G$

"central charge"

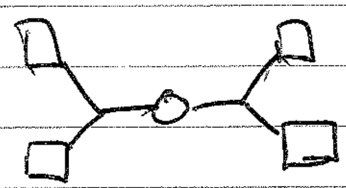
④

For example, for free fields in $\oplus r_i$, $k_G = 2\sum_i T(r_i)$.

$$\beta = 3T(G) - \sum_i T(r_i) \Rightarrow 3T(G) - \sum_i T(r_i) - \frac{k_G}{2}.$$

It's known that k_G for $SU(N)$ in T_N is $2N$.

$\therefore$



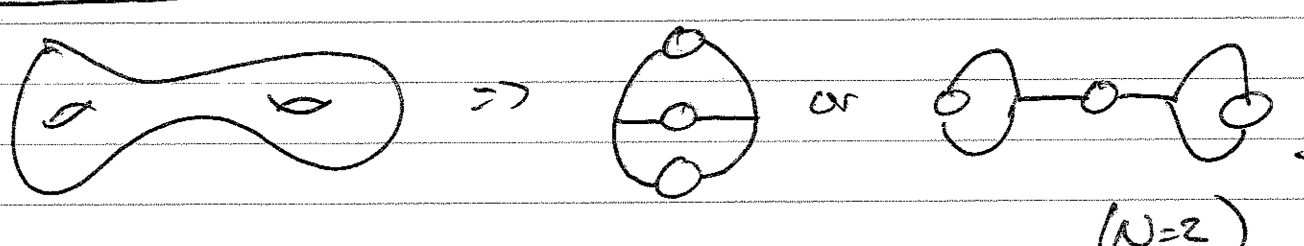
$$\beta \sim 3T(G) - T(G) - \cancel{2}\frac{k_G}{\cancel{2}} = 2N - 2N = 0.$$

$\Rightarrow$ Conformal.

Now, for $\mathcal{N}=1$ thys, the strategy is largely the same. The 1-loop β-function doesn't vanish, but $\mathrm{Tr}\, R T^a T^b$ would.

Once again — as long as you know global symms, you can do detailed calculations.

Examples of known SCFTs | M5's on Σ_g |



$(\mathcal{N}=2)$.

Integrate out Φ $(W = \mathrm{Tr}\,\Phi^2)$. $\Rightarrow$ $\mathcal{N}=1$

These thys are the dual CFTs to the MN solns — connected by flow here, not obv. there.

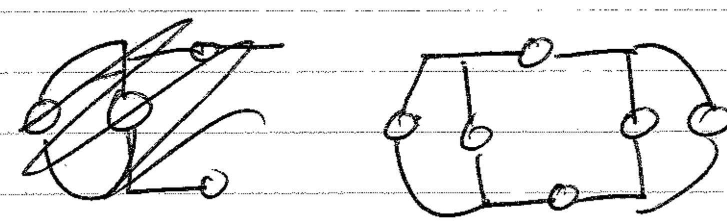
In general, can ~~cons~~ consider solns. of form $AdS_5 \times \Sigma_g \times \hat{S}^4$ $\leftarrow U(1)^2$

s.t. $\boxed{p+q = 2g-2}$

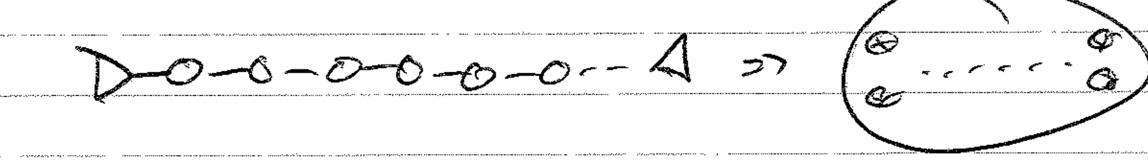
$q=0,\ p=2g-2 \Rightarrow \mathbb{C}\times T^*\Sigma_g \Rightarrow \mathcal{N}=2$ MN

$p=q=g-1 \Rightarrow \mathcal{N}=1$ MN.

Any $p,q \in \mathbb{Z} \Rightarrow$ New $\mathcal{N}=1$ solns. **All** new $\mathcal{N}=1$ SCFTs (for $p,q \geq 0$).



distribute $\mathcal{N}=1, 2$ VM's appropriately.

"S_ℓ" thys 

etc.

The point is, we have a new building block: T_N. What thys can we engineer from this?

$T_N \downarrow$ More! $\Rightarrow$ diff. punctures.

So let's build some damn theories!