Aspects of universality in gauge/gravity duality

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Based on joint work with
M. Ammon, Y.Y. Bu, S. Halter, P. Kerner, S. Klug (Müller), J. Shock, S. Steinfurt, M. Strydom, H. Zeller

Universal result: Shear viscosity/Entropy density

Kovtun, Son, Starinets

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- Deviations
- Further examples

Outline

- Fermionic analogue of shear viscosity
- Anisotropic holographic superfluids
- Universality in condensed matter
- Dynamically generated lattice in background magnetic field
- Slow-walking inflation

Fermionic analogues of the shear viscosity

J.E., Steinfurt 1302.1869

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Phonino mode: Pole in supercurrent correlator

Supersymmetry spontaneously broken at nonzero temperature

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$$\partial^\mu \left\langle T S_\mu(x) \bar{S}_\nu(0) \right\rangle = \delta^{(4)}(x)\, 2\, \gamma^\mu \left\langle T_{\mu\nu} \right\rangle$$

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Diffusion constant: $\omega = v_s k - i D_s k^2$

Holographic computation of diffusion constant in d=4:

$$2\pi T D_s = \frac{4\sqrt{2}}{9}$$

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Finite chemical potential: Gauntlett, Sonner, Waldram 1106.4694, 1108.1205

Reminder: Holographic proof of universality of eta/s

Kovtun, Son, Starinets 2004

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Absorption cross section:

$$\sigma_{\text{abs},0}(\omega) = -\frac{2\kappa^2}{\omega} \, \text{Im} \, G^R(\omega) = \frac{\kappa^2}{\omega} \int d^d x \, e^{i\omega t} \, \langle [T_{xy}(x), T_{xy}(0)] \rangle$$

(Klebanov '97; Gubser, Klebanov, Tseytlin '97)

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Gibbons, Das, Mathur 1996

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$$\sigma_{\mathrm{abs},0}(0) = a$$

Gibbons, Das, Mathur 1996

Area a related to entropy ➡ $\eta / s = 1/4\pi$

Supersymmetric hydrodynamics

- Describe the IR of a supersymmetric theory with SUSY breaking by temperature ("supersymmetric hydrodynamics") as the effective theory of the phonino and the normal fluid

(Hoyos, Keren-Zur, Oz '12)

- No classical fermionic charges!
- The constitutive relation (Kovtun, Yaffe '03) with $\rho = S^0$ (first order in the derivative expansion) is not changed by this interpretation!

$$S^i_{\text{diss}} = -D_s \nabla^i \rho - D_\sigma \sigma^{ij} \nabla_j \rho$$

- conformal: $T^\mu_\mu = 0 \leftrightarrow \gamma^\mu S_\mu = 0 \leftrightarrow D_s = D_\sigma$
- However it has to be seen as a quantum-mechanical relation where ρ is the quantum phonino field!

Constitutive relation

- In arbitrary space-time dimension d, reorder the constitutive relation according to representations of $O(d-1)$:

$$S^i_{\text{diss}} = -D_{3/2} \underbrace{\left(\delta^i_j - \frac{1}{d-1}\gamma^i\gamma_j\right)}_{\gamma^i \text{ irreducible} \leftrightarrow \text{spin } 3/2} \nabla^j \rho - D_{1/2} \underbrace{\gamma^i \nabla\!\!\!/ \rho}_{\gamma^i \text{ ``trace''}}$$

- completely analogous to

$$T^{ij}_{\text{diss}} = -\eta \underbrace{\left(\delta^{ik}\delta^{jl} + \delta^{jk}\delta^{il} - \frac{2}{d-1}\delta^{ij}\delta^{kl}\right)}_{\text{symmetric traceless} \leftrightarrow \text{spin } 2} \nabla^k u^l - \zeta\delta^{ij} \underbrace{(\nabla \cdot u)}_{\text{trace}}$$

- conformal: $T^\mu_\mu = 0 \leftrightarrow \zeta = 0 \quad \& \quad \gamma^\mu S_\mu = 0 \leftrightarrow D_{1/2} = 0$
- expectation: $D_{3/2}$, rather than D_s, universal as η ?!

Universal relation for supercurrent diffusion

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Dissipative contribution to supersymmetry current:

$$S^i_{\rm diss} = -D_{3/2}\left(\delta^i_j - \frac{1}{d-1}\gamma^i\gamma_j\right)\nabla^j\rho - D_{1/2}\gamma^i\not{\nabla}\rho$$

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$$\epsilon D_{3/2} = \frac{1}{4\pi G}\,\sigma_{\rm abs,1/2}(0)$$

Also: $$2\,\pi T D_s = \frac{2^{2/d}d(d-2)}{2(d-1)^2}$$

Universal absorption cross sections

Das, Gibbons, Mathur 1996

- Take a spherically symmetric, asymptotically flat, non-extremal black hole background:

$$ds^2 = -f(r)dt^2 + g(r)\left(dr^2 + r^2 d\Omega_p^2\right)$$

- Note that at the horizon $f(r_H) = 0$ but $g(r_H) \neq 0$.
- Then for minimally coupled massless s-wave scalars in the low-energy limit $\omega \to 0$:

$$\sigma_0 = A$$

- Similarly, for minimally coupled massless Dirac fermions:

$$\sigma_{1/2} = 2g(r_H)^{-p/2}A$$

- This is twice the area of the horizon in a conformally related spatially flat space-time $ds^2 = dr^2 + r^2 d\Omega_p^2$.

Fermionic absorption cross section

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horizon area evaluated in conformally related flat space

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horizon area evaluated in conformally related flat space

$$\sigma_{\mathrm{abs},1/2}(\omega) = \frac{\kappa_{d+1}^2}{2\ \mathrm{Tr}\left(-\gamma^0\gamma^0\right)}\ \mathrm{Tr}\left(-\gamma^0\ \mathrm{Im}\int d^d x\, e^{i\omega t}\ \langle\, S(x)\bar{S}(0)\,\rangle\right)$$

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Constitutive relation, Kubo formula ➡

$$\epsilon D_{3/2} = \frac{1}{\mathrm{Tr}\left(-\gamma^0\gamma^0\right)}\left(\frac{1}{d-2}\right)\lim_{\omega,k\to 0}\ \mathrm{Tr}\left(-\gamma^0\ \mathrm{Im}\int d^dx\, e^{i\omega t}\ \langle\, S_T^i(x)\bar{S}_T^i(0)\,\rangle\right)$$

J.E., Steinfurt 2013

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Holographic proof of eta/s universality relies on space-time isotropy

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Holographic p-wave superfluids/superconductors

Holographic p-wave superfluid with backreaction

Ammon, J.E., Grass, Kerner, O'Bannon 2009

$SU(2)$ Einstein-Yang-Mills theory in (4+1)-dimensional asymptotically AdS Space

$$S = \frac{1}{2\kappa_5{}^2}\int \mathrm{d}^5x\sqrt{-g}\left[R - \Lambda - \frac{\alpha^2}{2}F^a_{MN}F^{aMN}\right] + S_{\mathrm{bdy}}$$

with

$$\alpha \equiv \frac{\kappa_5}{g_{\mathrm{YM}}}$$

gauge field ansatz

$$A = \phi(r)\tau^3\mathrm{d}t + w(r)\tau^1\mathrm{d}x$$

Field Theory	$\Leftrightarrow$	Gravity
chemical potential μ $SU(2) \to U(1)_3$		$A_t^3 = \phi(r) \neq 0$ $SU(2) \to U(1)_3$
$\langle \mathcal{J}_1^x \rangle \neq 0$ $U(1)_3 \to \mathbb{Z}_2,\ SO(3) \to SO(2)$		$A_x^1 = w(r) \neq 0$ $U(1)_3 \to \mathbb{Z}_2,\ SO(3) \to SO(2)$

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Fluctuations:

J.E., Kerner, Zeller 1011.5912, 1110.0007

	dynamical fields	constraints	# physical modes
helicity 2	$h_{yz}, h_{yy} - h_{zz}$	none	2
helicity 1	$h_{ty}, h_{xy}; a_y^a$	h_{yr}	4
	$h_{tz}, h_{xz}; a_z^a$	h_{zr}	4
helicity 0	$h_{tt}, h_{xx}, h_{yy} + h_{zz}, h_{xt};$ a_t^a, a_x^a	$h_{tr}, h_{xr}, h_{rr}; a_r^a$	4

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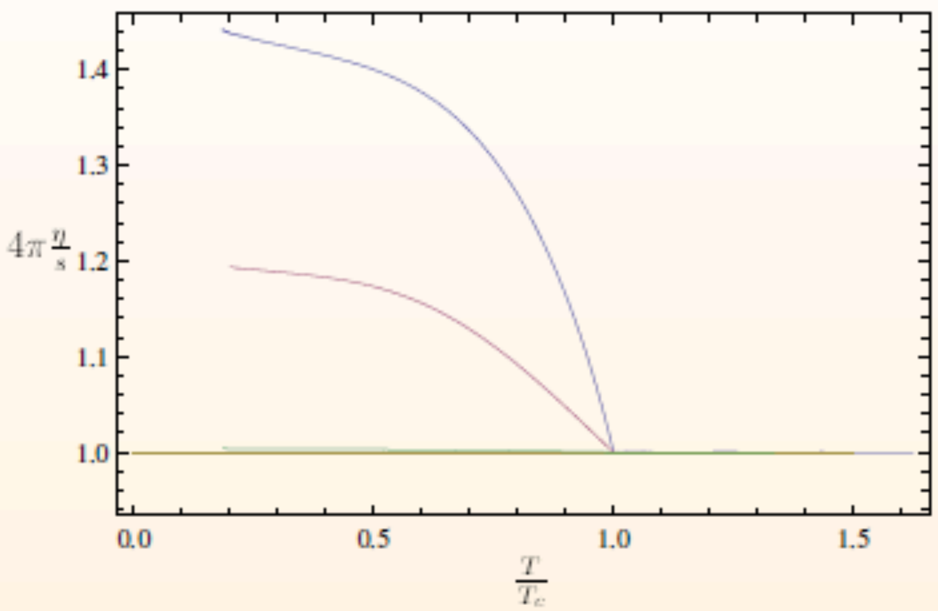
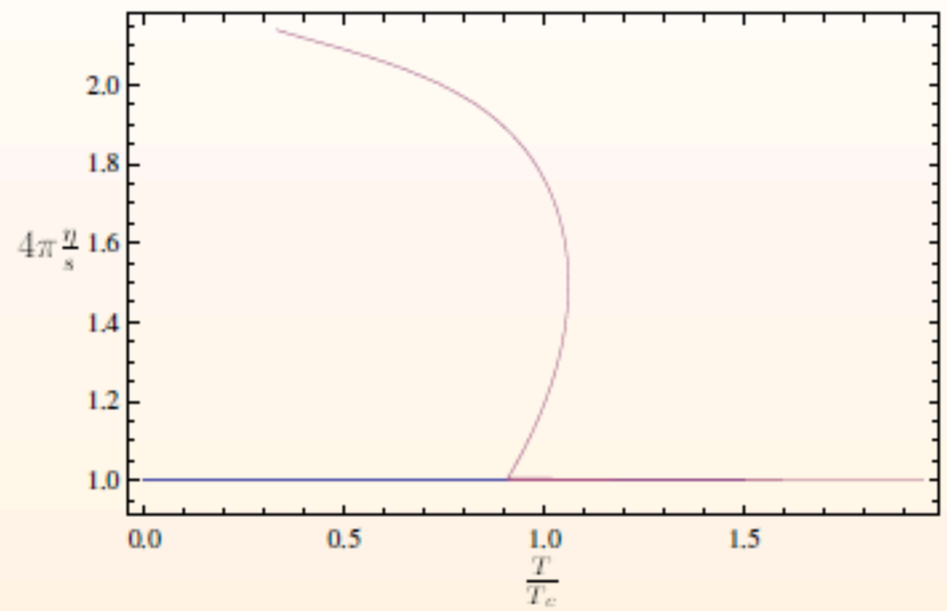
helicity 1 modes decouple in 2 blocks:

even parity: $\{\Psi_t = g^{yy} h_{t\perp},\ a_\perp^3,\ h_{r\perp}\}$

odd parity: $\{\Psi_x = g^{yy} h_{x\perp},\ a_\perp^1,\ a_\perp^2\}$

Anisotropic shear viscosity



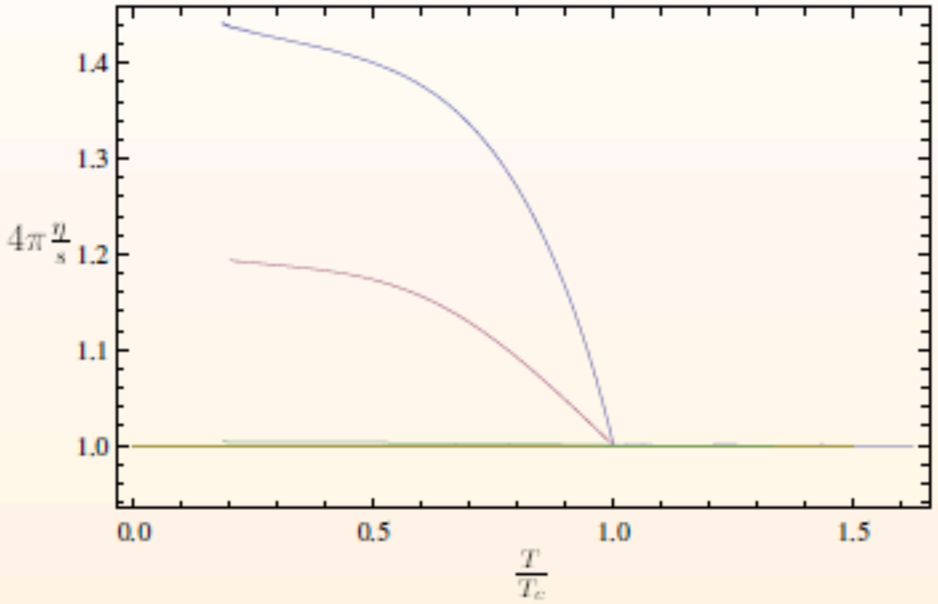


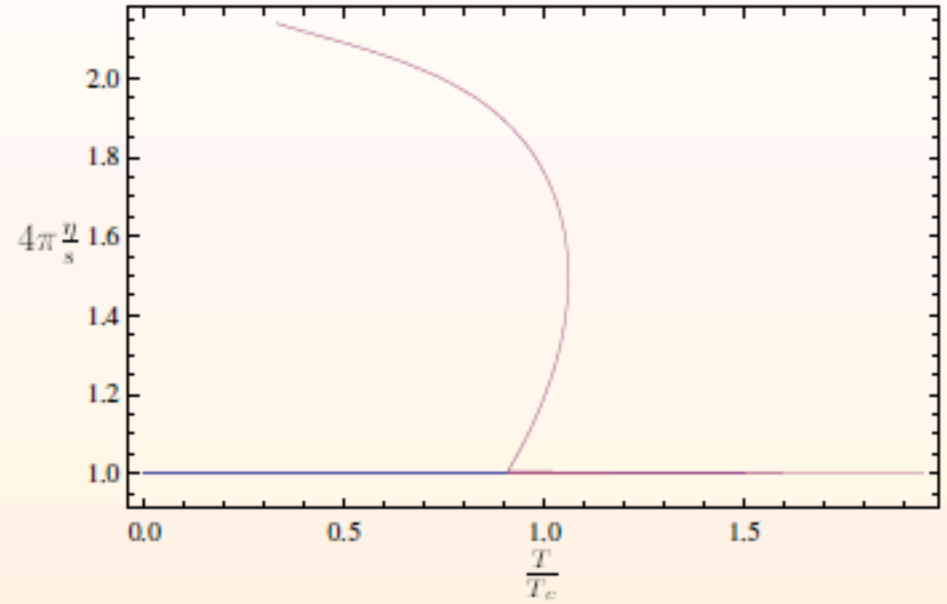
$\eta_{yz}/s = 1/4\pi$; η_{xy}/s dependent on T and on α

Critical behaviour: $1 - 4\pi\frac{\eta_{xy}}{s} \propto \left(1 - \frac{T}{T_c}\right)^{\beta}$ with $\beta = 1.00 \pm 3\%$, α-independent

Non-universal behaviour at leading order in λ and N

Anisotropic shear viscosity





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Non-universal behaviour at leading order in λ and N

Critical exponent confirmed analytically in Basu, Oh 1109.4592

Helicity zero

J.E., Fernandez, Zeller 1212.4838

Transport coefficient λ associated to $h_{xx} - h_{yy}$

In unbroken phase: $\frac{\lambda}{s} = \frac{1}{72\pi}$ is universal

Normal stress difference induced by anisotropic strain

Of relevance for nematic crystals

Piezoelectric effect: Strain causes current

Is there a similar universal result as $\frac{\eta}{s} = \frac{1}{4\pi}\frac{\hbar}{k_B}$ within condensed matter applications of holography?

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Candidate: Homes' Law

Homes' Law

Homes' Law $\rho_s = C\sigma(T_c)T_c$

Shown to hold experimentally to great accuracy (Homes et al, Nature 2004)

Zaanen (Nature, 2004): $\tau(T_c) = \frac{\hbar}{k_B T_c}$

Planckian dissipation: Shortest possible dissipation timescale

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Planckian dissipation: Shortest possible dissipation timescale

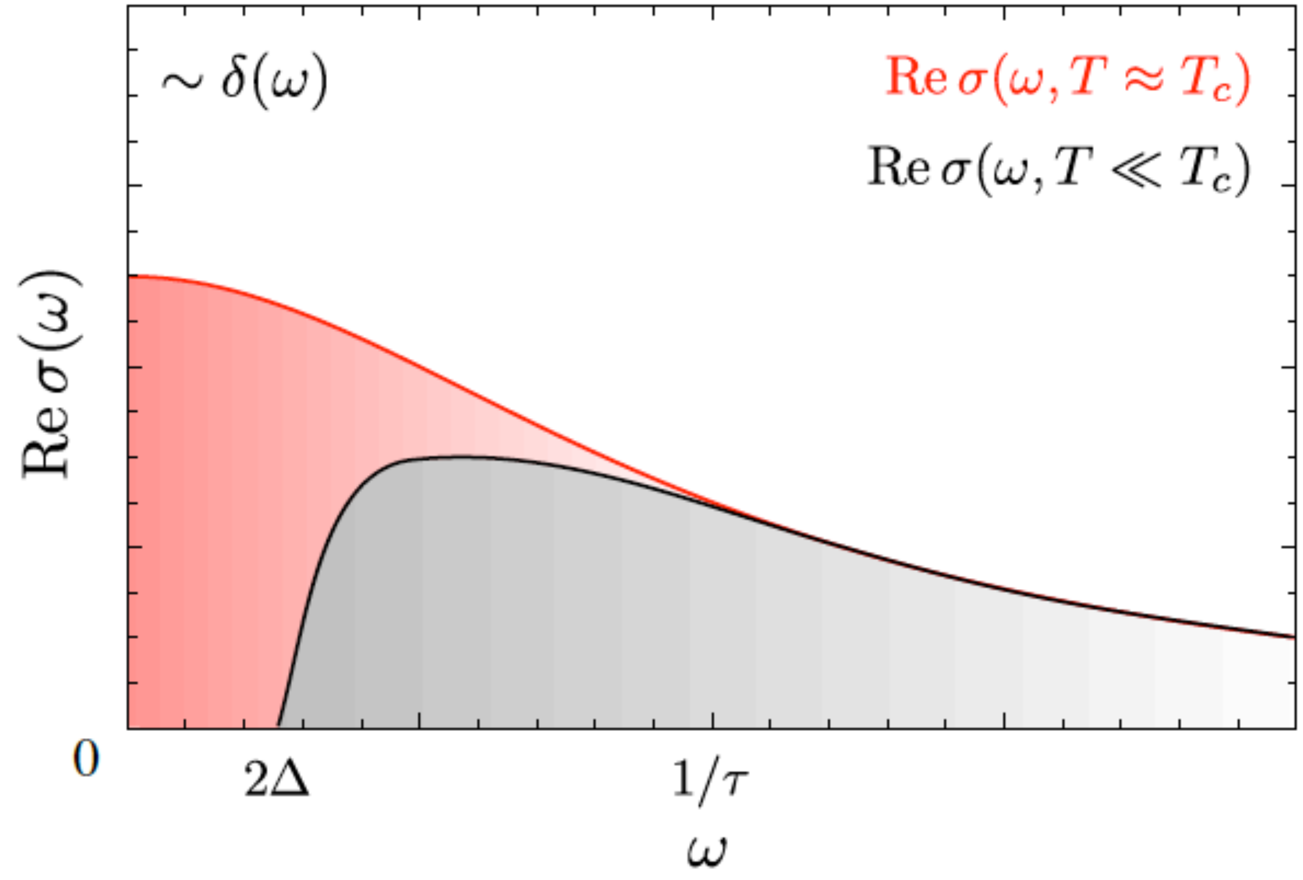
Holographic version:

Preliminary results in J.E., Kerner, Müller 1206.5305

See also Horowitz, Santos 1302.6586

Not possible to calculate superconducting density ρ_s holographically in translation invariant systems

$$Re\ \sigma(\omega) = \rho_s \delta(\omega)$$



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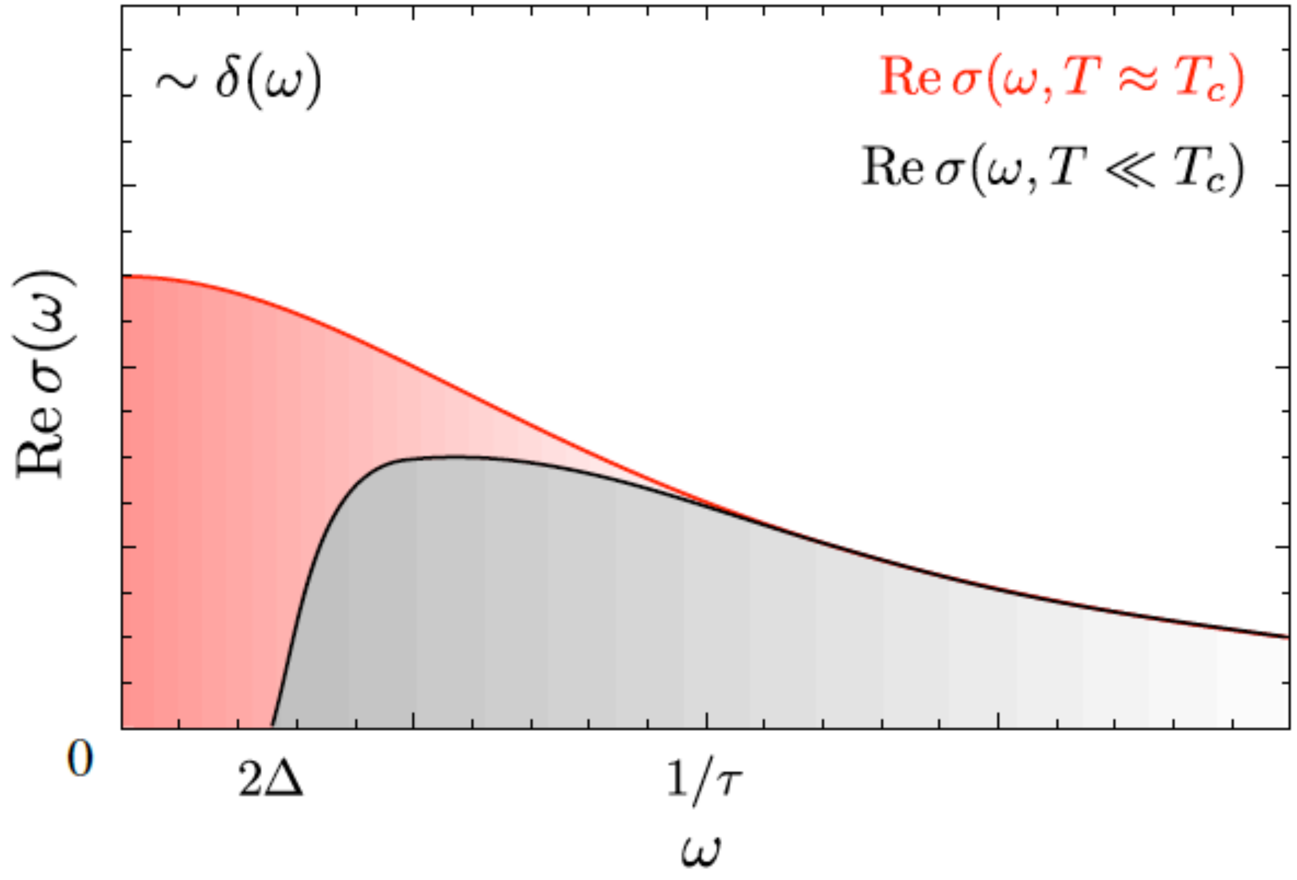

Idea: Rewrite Homes' Law using sum rules

Homes' Law

Homes' Law: $\rho_s = C\sigma(T_c)T_c$

Sum rule: ${\omega_P}^2(T=0) = {\omega_P}^2(T=T_c)$

$$\frac{\omega_{\mathrm{P}}^2}{8} = \int_0^\infty \mathrm{d}\omega \, \mathrm{Re}\, \sigma(\omega)$$

$\rho_s \propto \omega_P^2(T=0)$

Drude law: $\sigma = \frac{ne^2\tau}{m}, \quad {\omega_P}^2 = \frac{4\pi ne^2}{m}$

$\Rightarrow \quad 4\pi\sigma(T_c) = {\omega_P}^2(T_c)\tau(T_c)$

$\Rightarrow$ Homes' Law equivalent to

$$\tau(T_c)T_c = const$$

Homes' Law

Assume diffusion can be used to determine the timescale

$\Rightarrow D(T_c)T_c = const$

Holography in the probe limit without backreaction (Einstein-Maxwell theory):

$$D = \frac{1}{4\pi}\frac{d}{d-2}\frac{1}{T}$$

Homes' Law

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Holography in the probe limit without backreaction
(Einstein-Maxwell theory):

$$D = \frac{1}{4\pi}\frac{d}{d-2}\frac{1}{T}$$

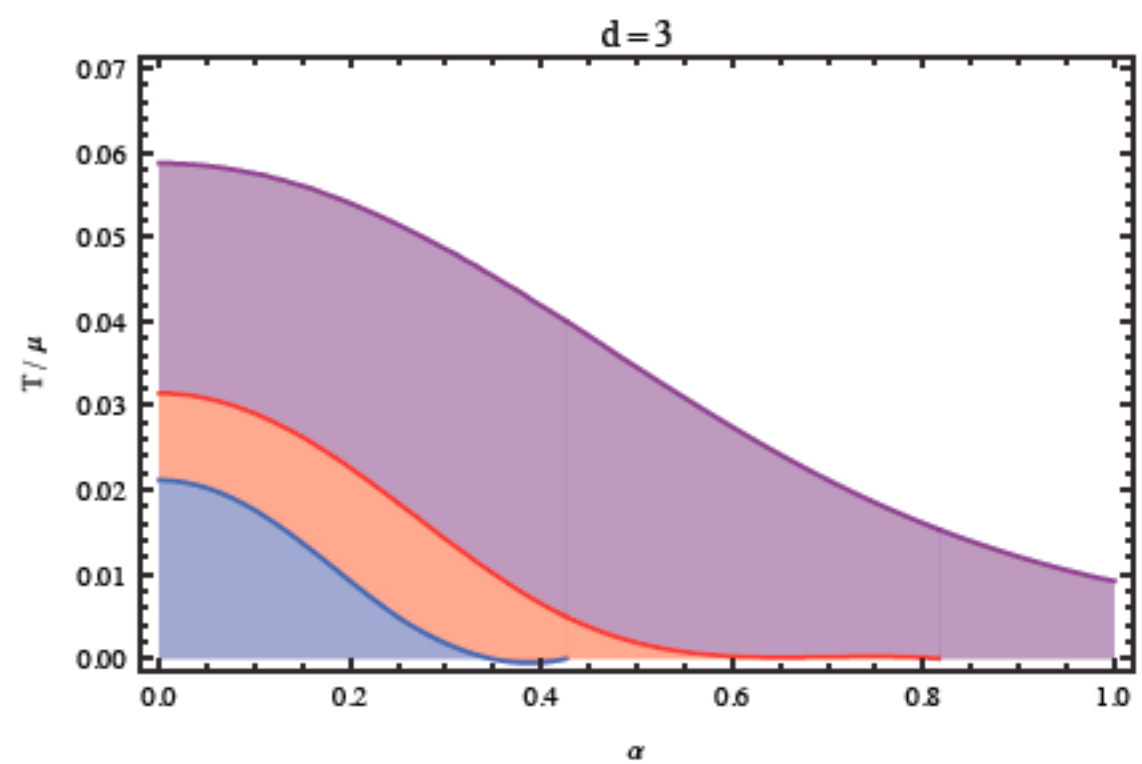
Including the backreaction we expect $D = \frac{1}{4\pi}\frac{d}{d-2}\frac{1}{T}f(\frac{T}{\mu})$

Including the backreaction

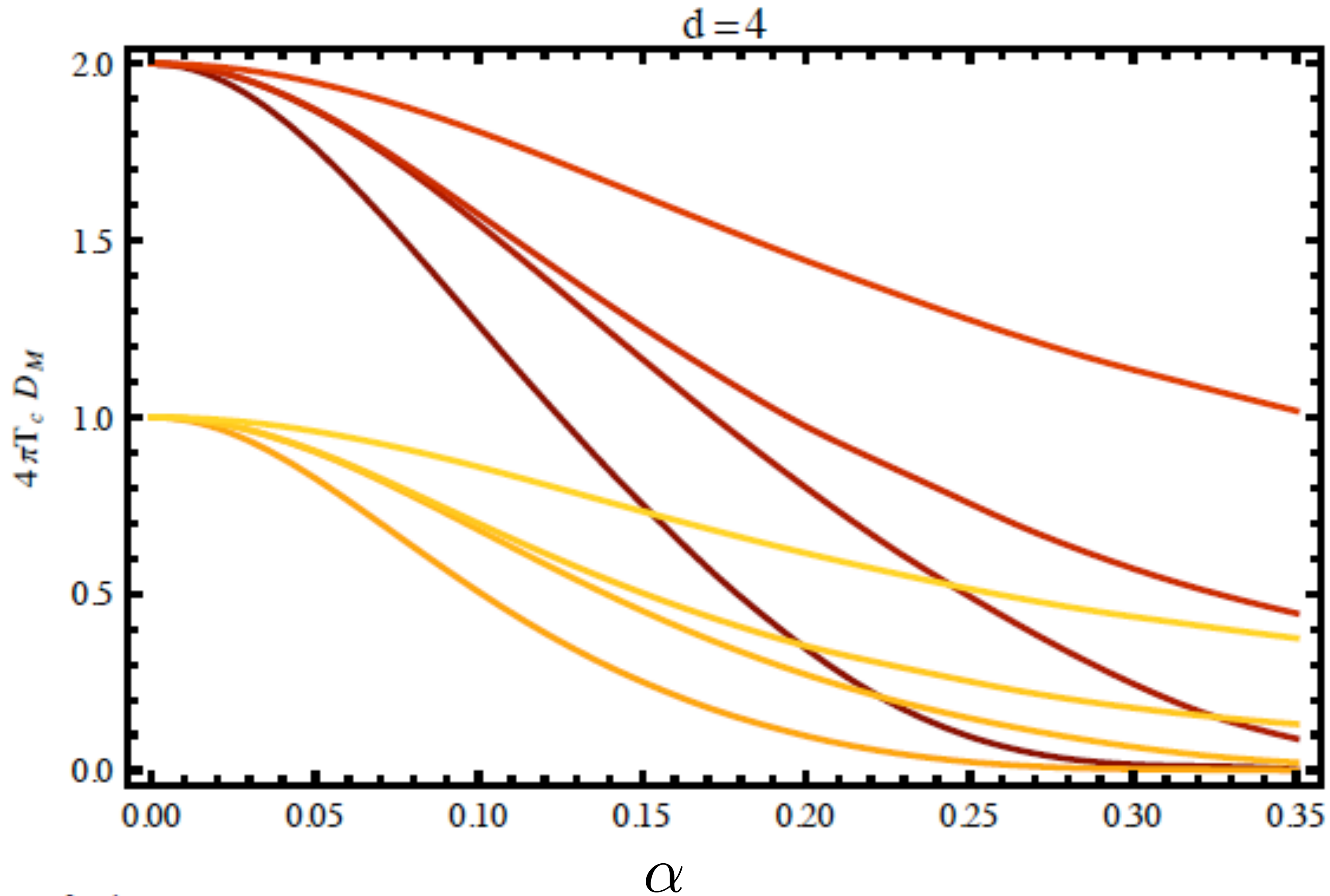
$$S = \frac{1}{2\kappa^2}\int \mathrm{d}^{d+1}x\,\sqrt{-g}\left[R - 2\Lambda - \frac{2\kappa^2}{e^2}\left(\frac{1}{4}F_{ab}F^{ab} - |\nabla\Phi - \mathrm{i}A\Phi|^2 - V(|\Phi|)\right)\right]$$

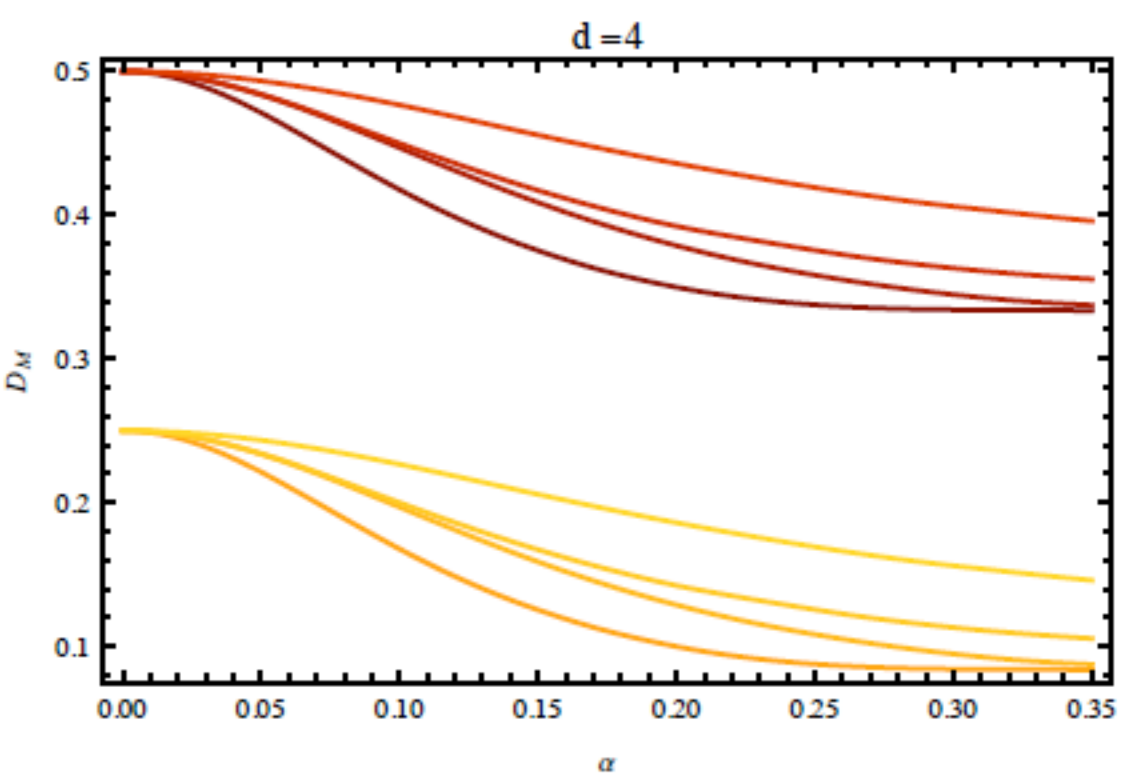
Backreaction parameter $\alpha^2 L^2 = \frac{\kappa^2}{e^2}$

Phase diagram



R charge and momentum diffusion times T_c vs. α

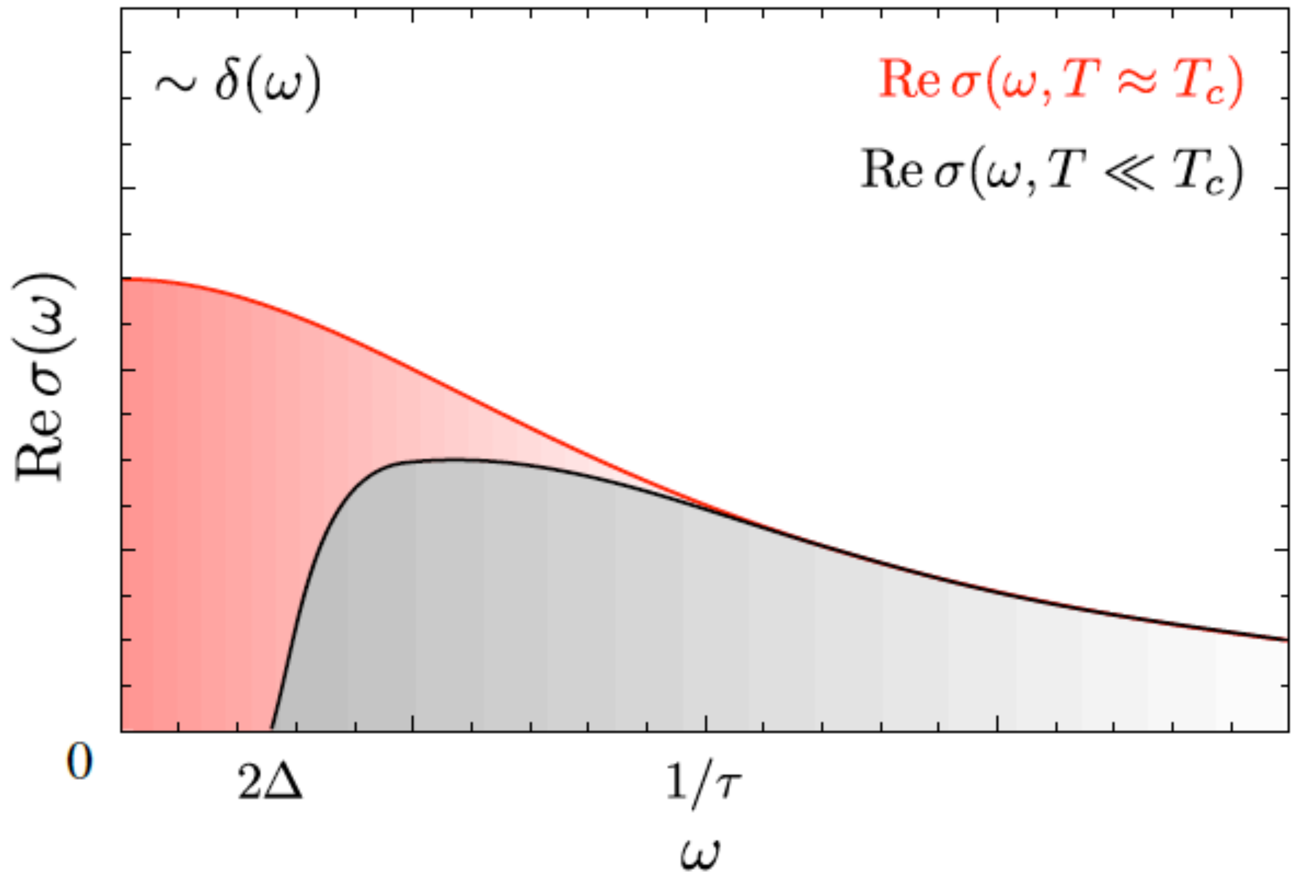




Reasons for decrease

Decrease may originate from pseudogap states whose number increases with backreaction

$$1 - \frac{8N_{\rm s}}{\omega_{{\rm P}n}^2} = \frac{C}{4\pi}\tau_c T_c$$



Sum rule $\quad \frac{\omega_{\rm P}^2}{8} = \int_0^\infty {\rm d}\omega\, {\rm Re}\,\sigma(\omega)$

$$N_{\rm n} = \int_0^\infty {\rm d}\omega\, {\rm Re}\,\sigma(\omega)\Big|_{T>T_c} = \frac{\omega_{\rm Pn}^2}{8}, \qquad N_{\rm s} = \int_{0+}^\infty {\rm d}\omega\, {\rm Re}\,\sigma(\omega)\Big|_{T<T_c}$$

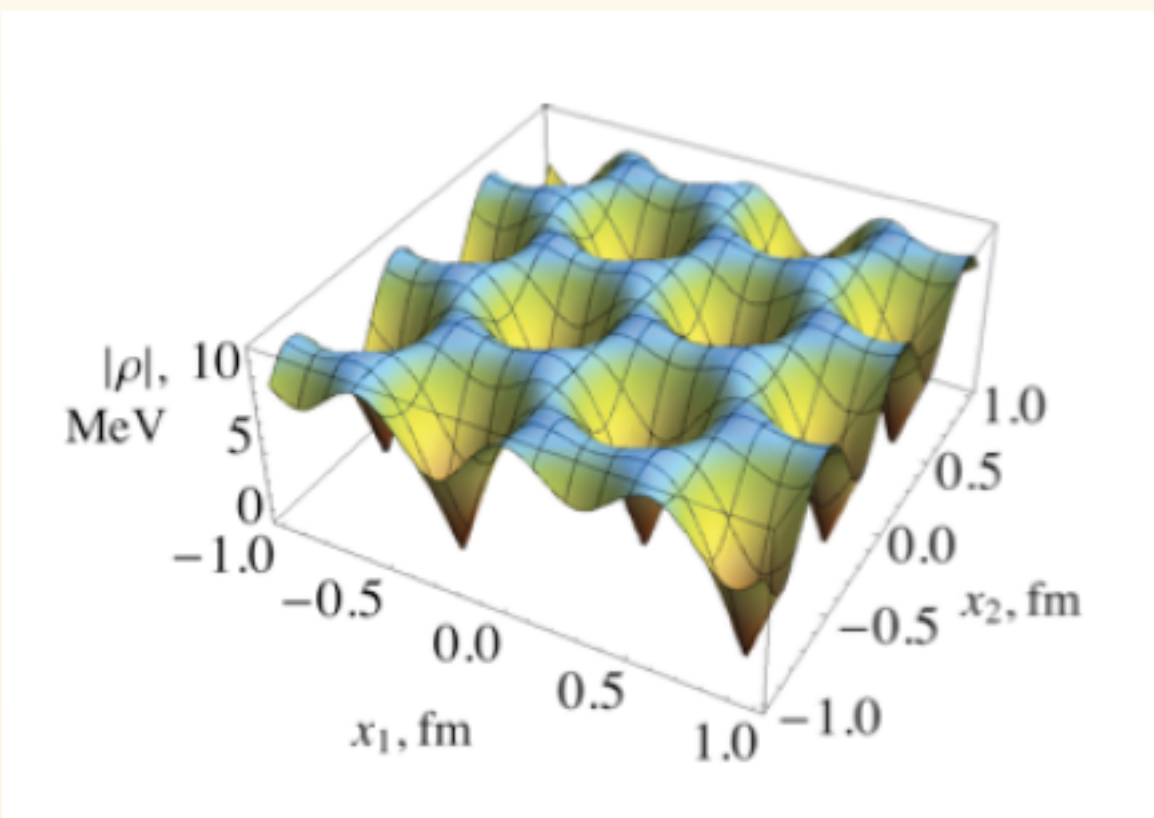
$$\rho_{\rm s} \equiv \omega_{\rm Ps}^2 = 8\,(N_{\rm n} - N_{\rm s}) = \omega_{\rm Pn}^2 - 8N_{\rm s}$$

B field induced rho meson condensation

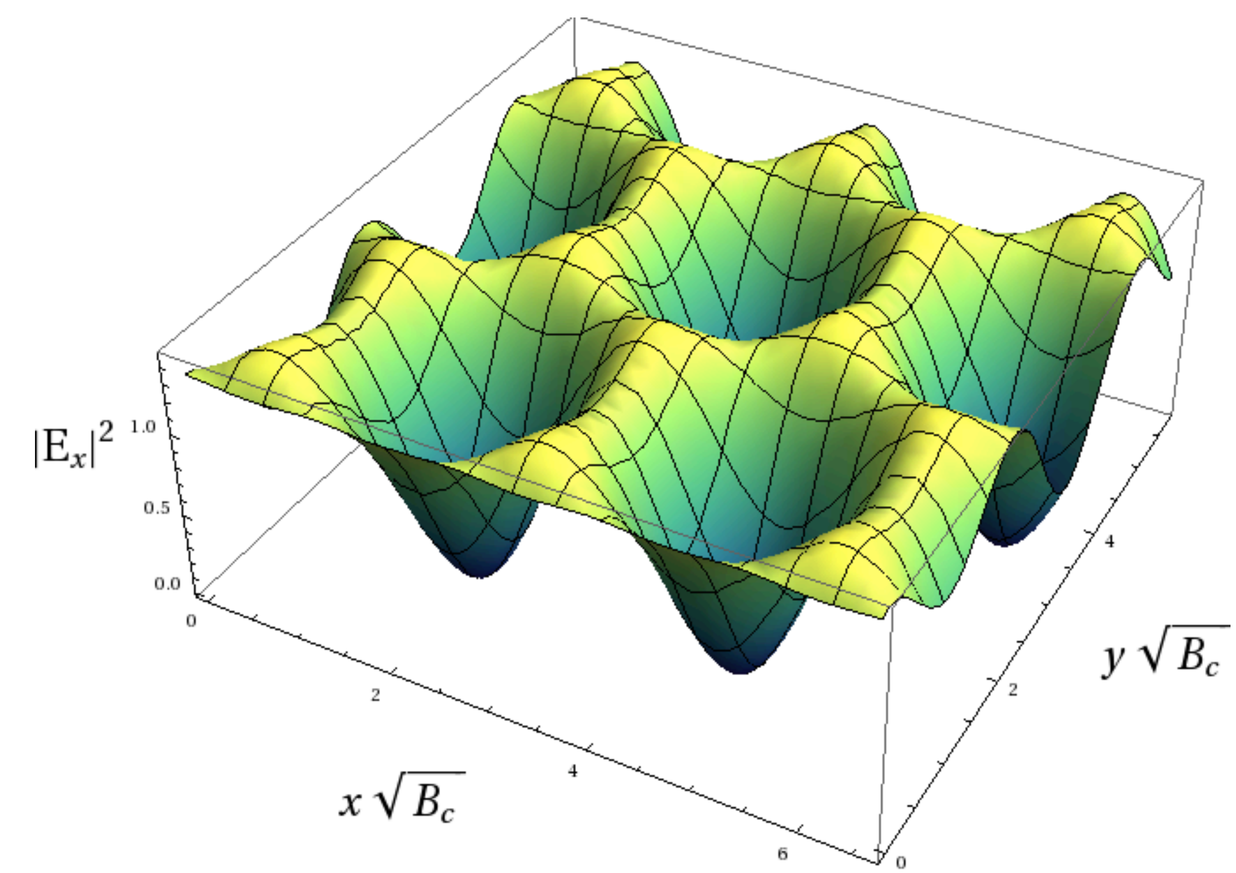
Isospin condensate is easily replaced by B field:

$$A_t \to A_x$$

Effective field theory:
(Chernodub)



Gauge/gravity duality
magnetic field in black hole supergravity background Ammon, J.E., Kerner, Strydom 2011
Bu, J.E., Shock, Strydom 2012



External B-field leads to rho meson condensation in the QCD vacuum

Condensation in magnetic field

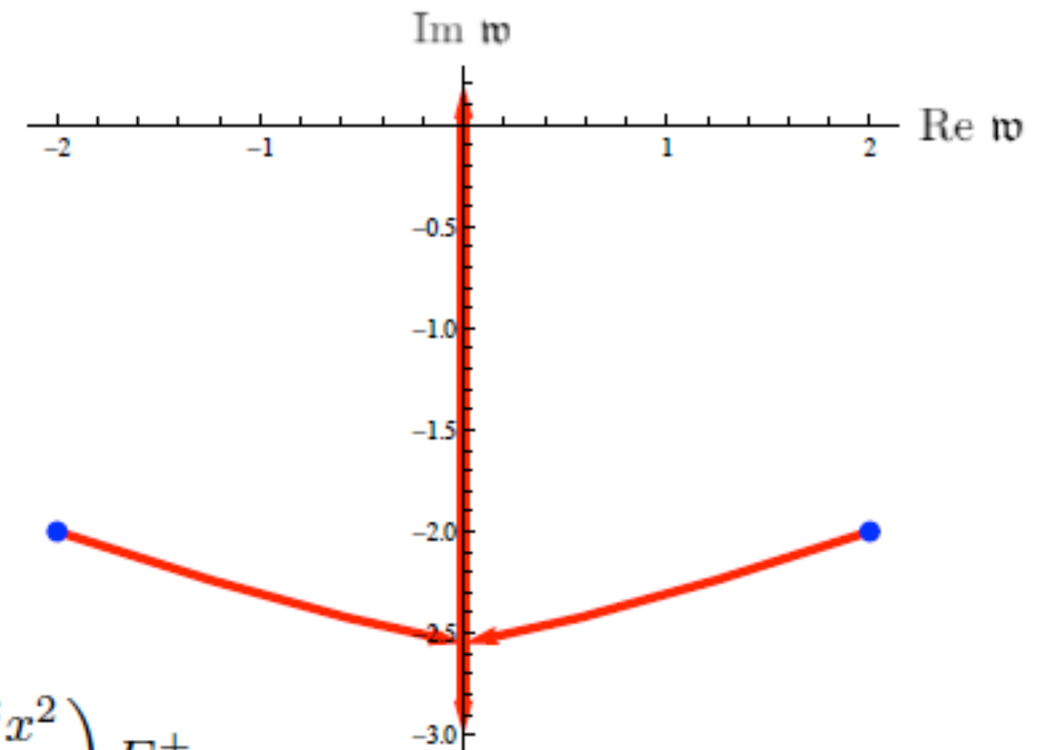
$$S = \int d^5x\sqrt{-g}\left[\frac{1}{2\kappa_5^2}(R-2\Lambda) - \frac{1}{4\hat{g}^2}F^a_{\mu\nu}F^{a\mu\nu}\right] + S_{\rm bdy}$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + \epsilon^{abc}A^b_\mu A^c_\nu$$

$$A^3_y = xB$$

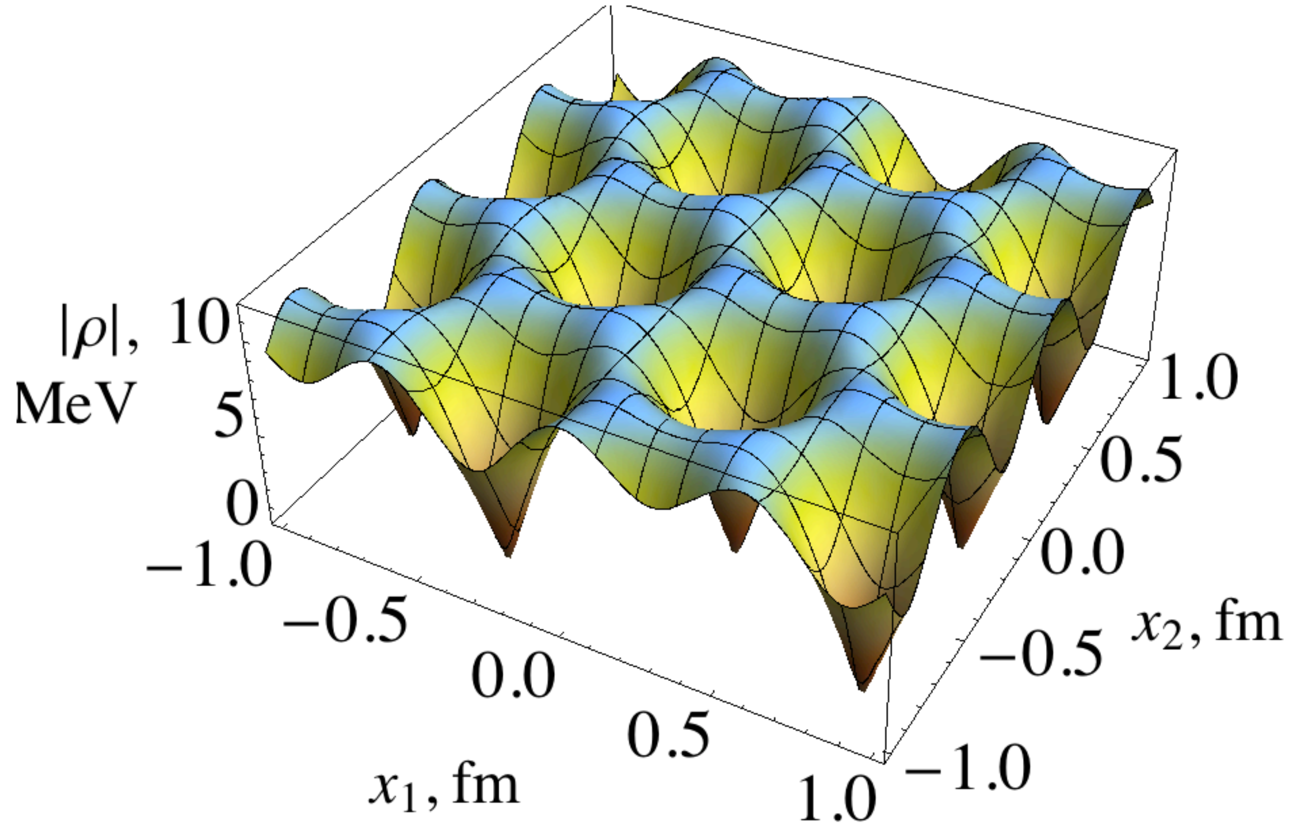
Fluctuations

$$0 = \partial_u^2 E_x^+ + \frac{1}{f}\partial_x^2 E_x^+ + \left(\frac{f'}{f} - \frac{1}{u}\right)\partial_u E_x^+ - \frac{2}{xf}\partial_x E_x^+ + \left(\frac{\omega^2}{f^2} - \frac{B^2x^2}{f}\right)E_x^+$$

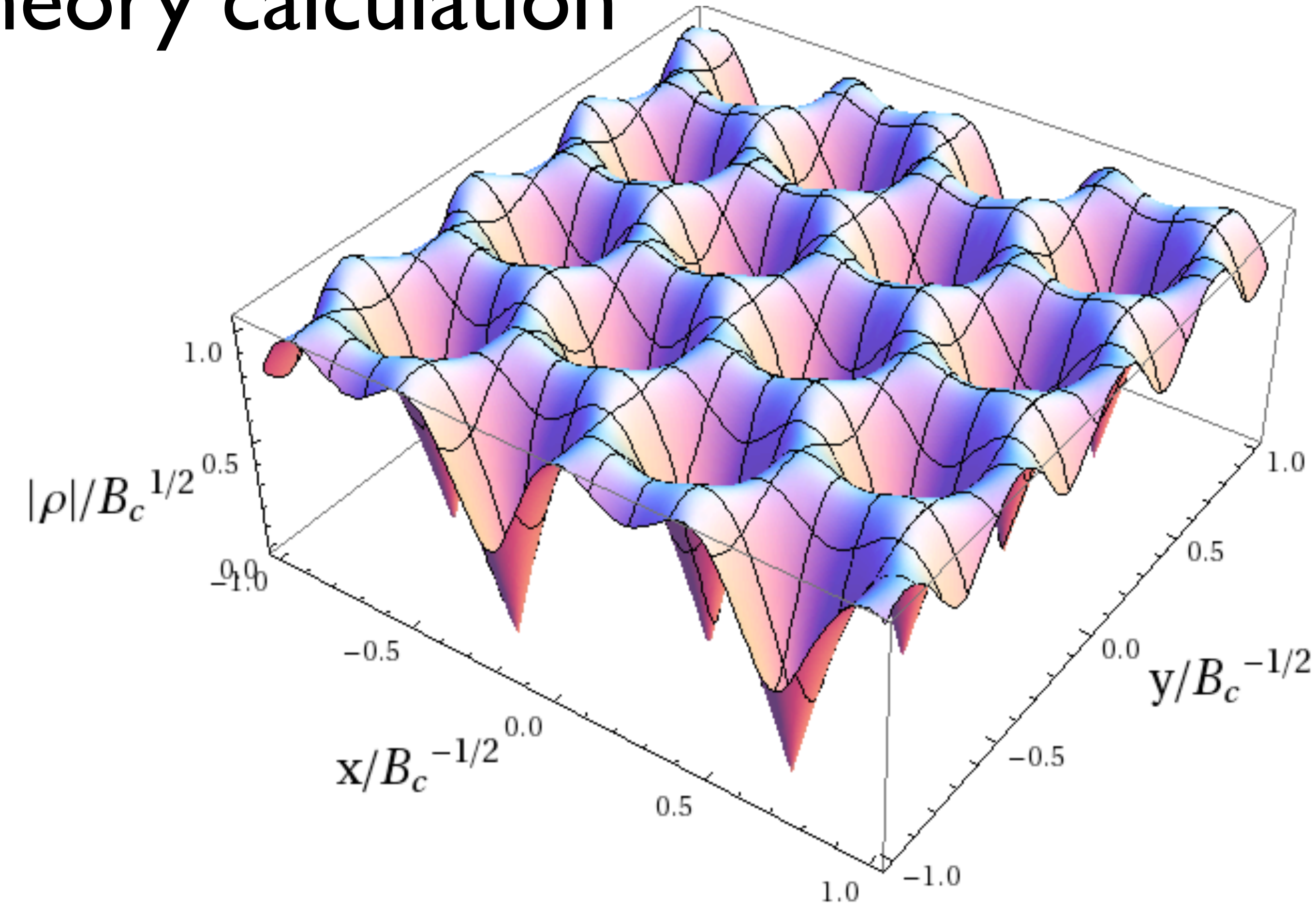


cf. Chernodub;
Callebaut, Dudas, Verschelde;
Donos, Gauntlett, Pantelidou

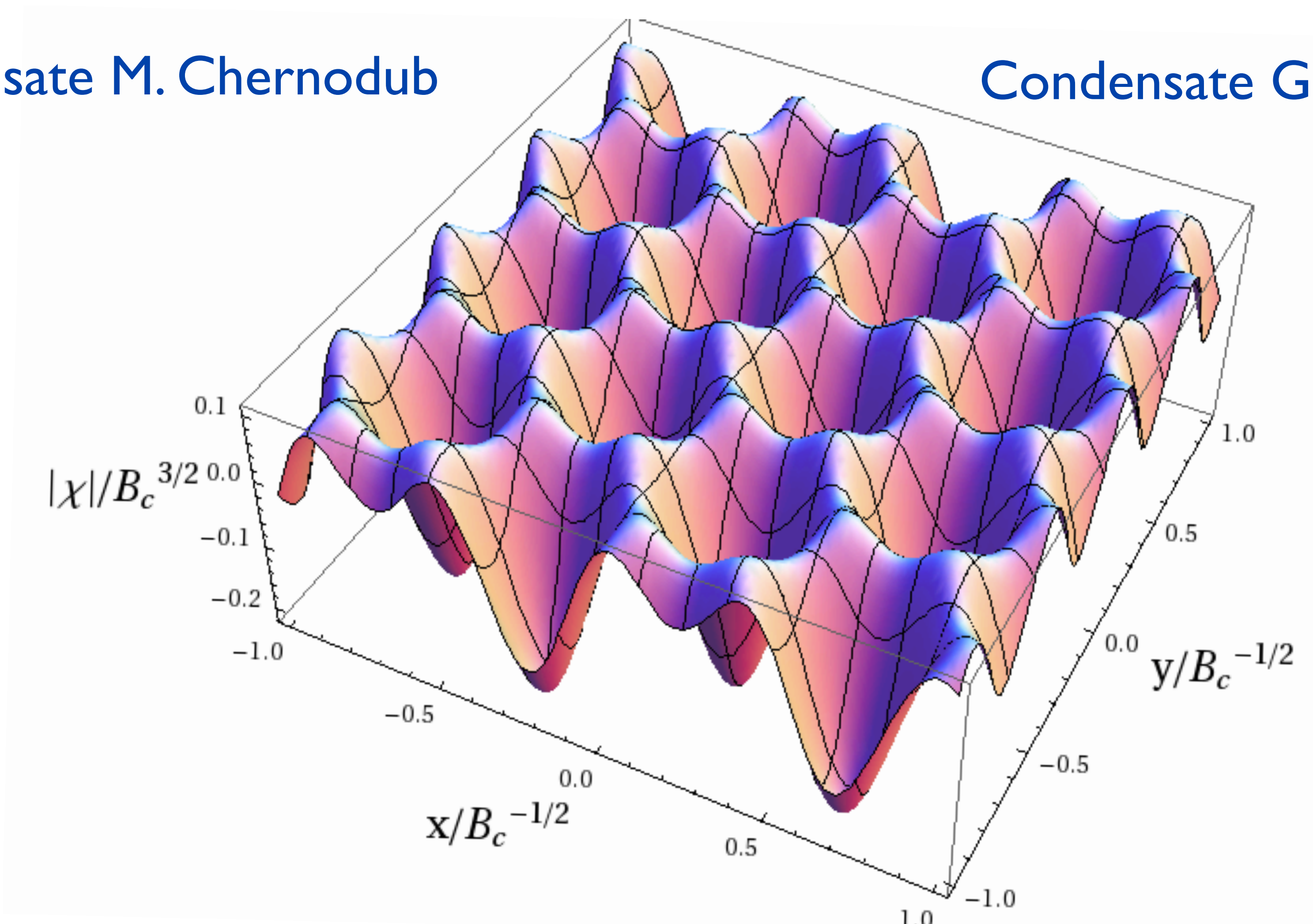
Comparison to field theory calculation



Condensate M. Chernodub



Condensate Gauge/Gravity Duality



Magnetization Gauge/Gravity Duality

Bu, J.E., J. Shock, M. Strydom, 2012

Holographic Set-up

The model:

$$S = \int d^5x\sqrt{-g}\left\{\frac{1}{16\pi G_N}\left(R+\frac{12}{L^2}\right)-\frac{1}{4\hat{g}^2}\mathrm{tr}\left(F_{\mu\nu}F^{\mu\nu}\right)\right\}+S_{\mathrm{bdy}}$$

Assume the probe limit. The metric in 5 dimensions, working in Poincaré coordinates with the boundary at $u=0$:

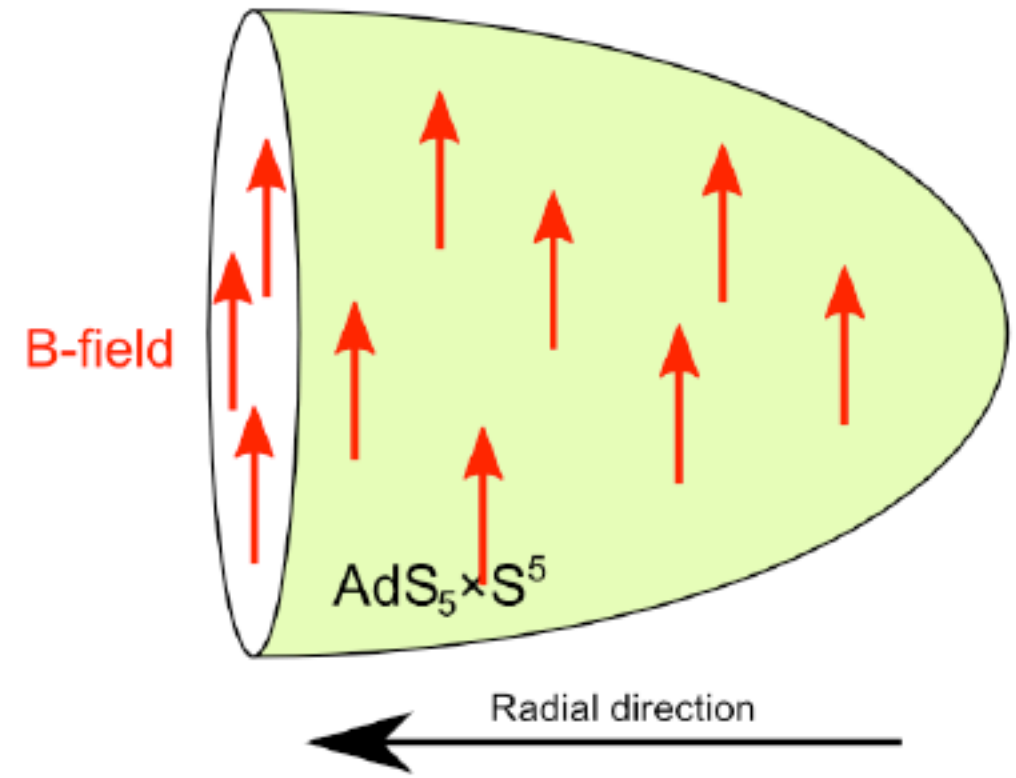
$$ds^2 = \frac{L^2}{u^2}\left(-f(u)dt^2+dx^2+dy^2+dz^2+\frac{du^2}{f(u)}\right)$$

AdS-Schwarzschild: $f(u)=1-u^4/u_H^4$.

Magnetic field

- $SU(2)$ flavour field

Choose $F^3_{xy} = B$, $F^a_{\mu\nu} = 0$ otherwise.
Also fix $\mathcal{A}^3_y = xB$ and other components so that only $U(1)$ gauge symmetry remains.



$$\text{Isospin chemical potential} \Leftrightarrow \mathcal{A}^3_t \text{ non-zero at boundary.}$$
$$\text{Magnetic field} \Leftrightarrow \mathcal{A}^3_y \text{ non-zero at boundary.}$$

B-field induced condensation

- The remaining components of A_μ^a act as an order parameter. Boundary expansion:

$$\mathcal{A}_\mu^a \approx u^2 \langle J_\mu^a \rangle + \mathcal{O}(u^4)$$

- It is consistent to switch on only $\mathcal{A}_{x,y}^{1,2}(x, y, u)$.
- When $B < B_c$, these components are zero.
- When $B > B_c$, some of these components become nonzero.

Expansion about critical point

We look at $B \approx B_c$. Then we can focus on a small condensate and look at fluctuations of $\mathcal{A} \approx \varepsilon A + \varepsilon^3 a + \dots$.
Defining $E_{x,y} = A^1_{x,y} + iA^2_{x,y}$, we can focus on fields charged under the $U(1)$:

$$\begin{aligned} E_L &= x^2 B E_x - i(x\partial_x E_y - E_y) \\ &\to e^{-i\Lambda^3} E_L \end{aligned}$$

These source a vector condensate when they condense.

Perturbative strategy for finding ground state

$$\nabla^\mu F^a_{\mu\nu} + \epsilon^{abc}\mathcal{A}^{b\mu}F^c_{\mu\nu} = 0.$$

$$\mathcal{E}_{x,y} = \mathcal{A}^1_{x,y} + i\mathcal{A}^2_{x,y}$$

$$\begin{aligned}
\mathcal{E}_{x,y} &= \quad \boxed{\varepsilon E_{x,y}} + \boxed{\varepsilon^3 e_{x,y}} + \mathcal{O}(\varepsilon^5) \\
\mathcal{A}^3_y &= xB_c + \boxed{\varepsilon^2 a^3_y} + \mathcal{O}(\varepsilon^4) \\
\mathcal{A}^3_x &= \quad \boxed{\varepsilon^2 a^3_x} + \mathcal{O}(\varepsilon^4)
\end{aligned}$$

9 coupled equations for 3 gauge components in x, y and radial directions. $\varepsilon \sim \langle J \rangle$. We need to go to $3^{\rm rd}$ order.

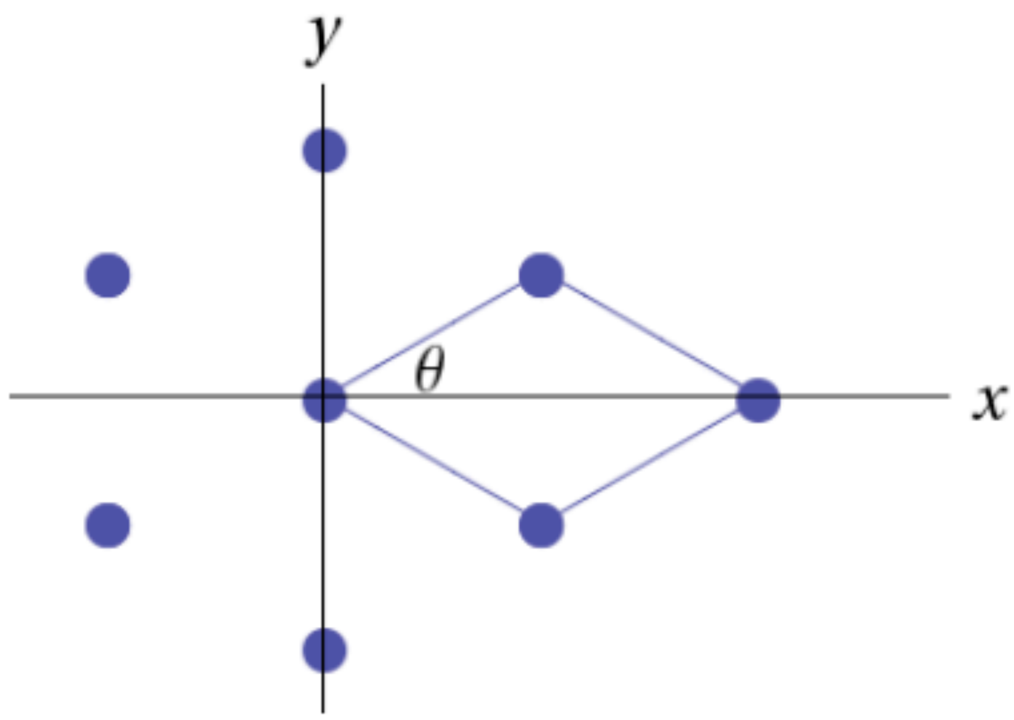
Linear order solution

$$E_x = \sum_{n=-\infty}^{\infty} C_n e^{-inky - \frac{1}{2}B_c\left(x - \frac{nk}{B_c}\right)^2} U(u)$$

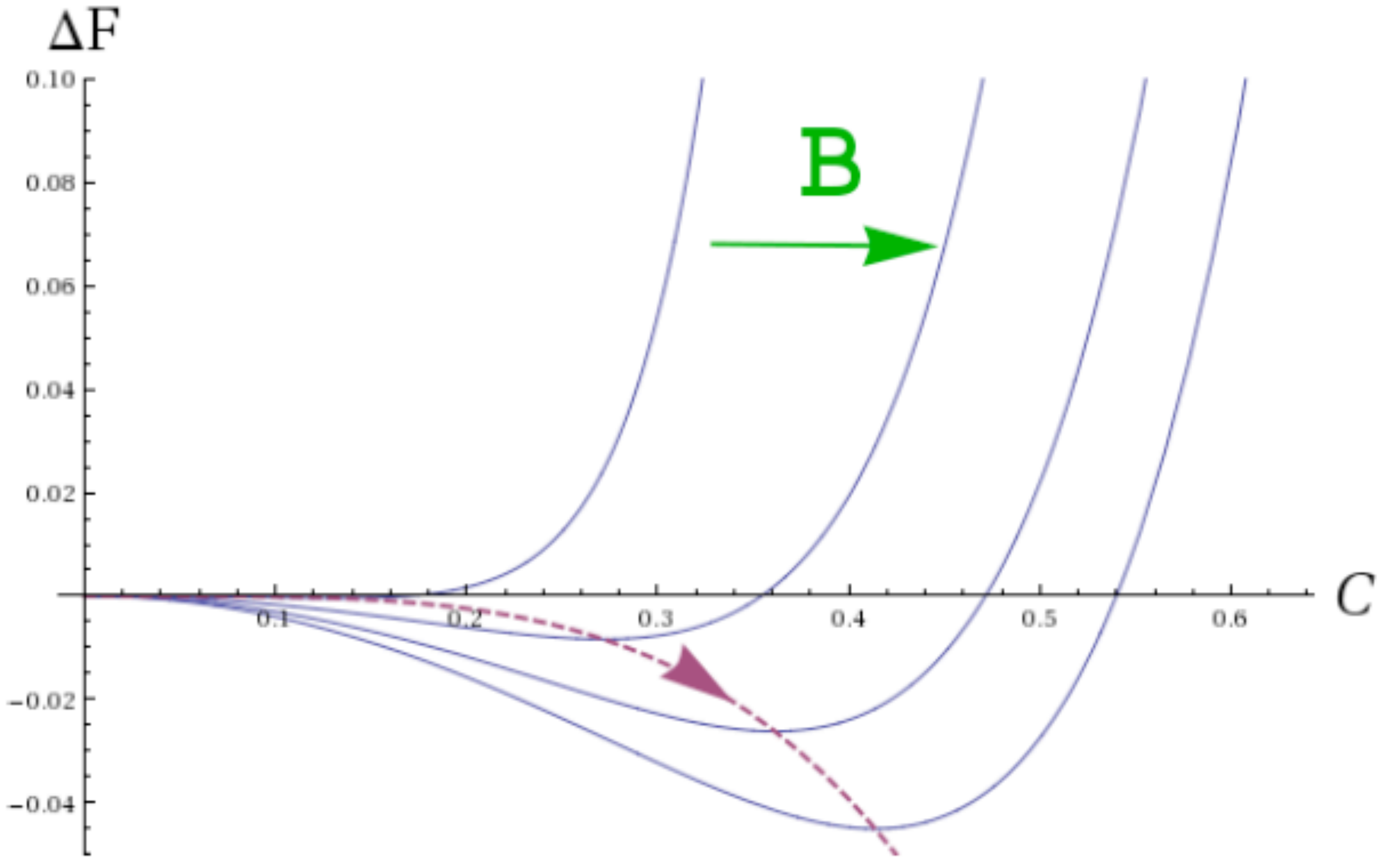
- This is the Abrikosov solution.
- $U(u)$ is the radial factor.
- Free parameters: k, C_n. We need to go to higher order to fix these.
- For $B \approx B_c$, C_n is small.

Fixing C_n

- We expect a lattice. This requires $C_n = C_{n+N}$ for some N.
- Choosing $N = 1$, $C_n = C$, and $k = \sqrt{2\pi B_c}$ gives a square lattice.
- Choosing $N = 2$, $C_1 = iC_0$, and $k = 3^{\frac{1}{4}}\sqrt{\pi B_c}$ gives a triangular lattice.
- Choosing $N = 2$, $C_1 = iC_0$, and varying k can give a rhombic lattice.

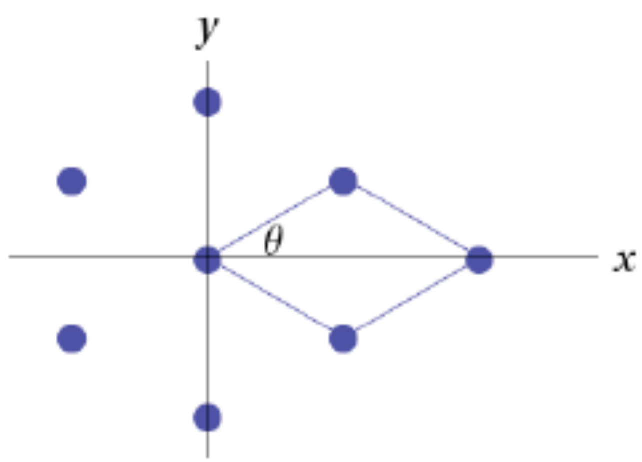


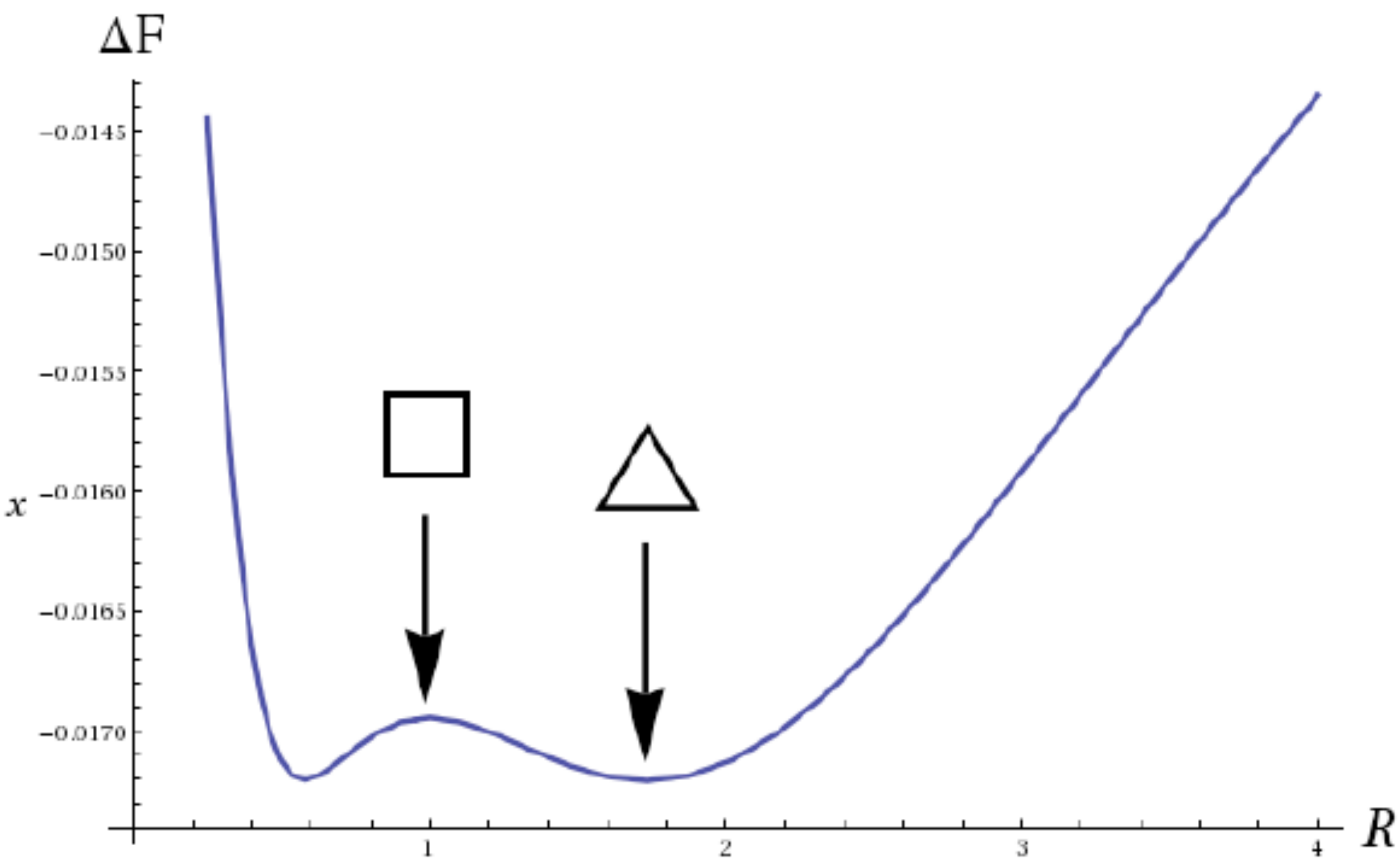
Free energy



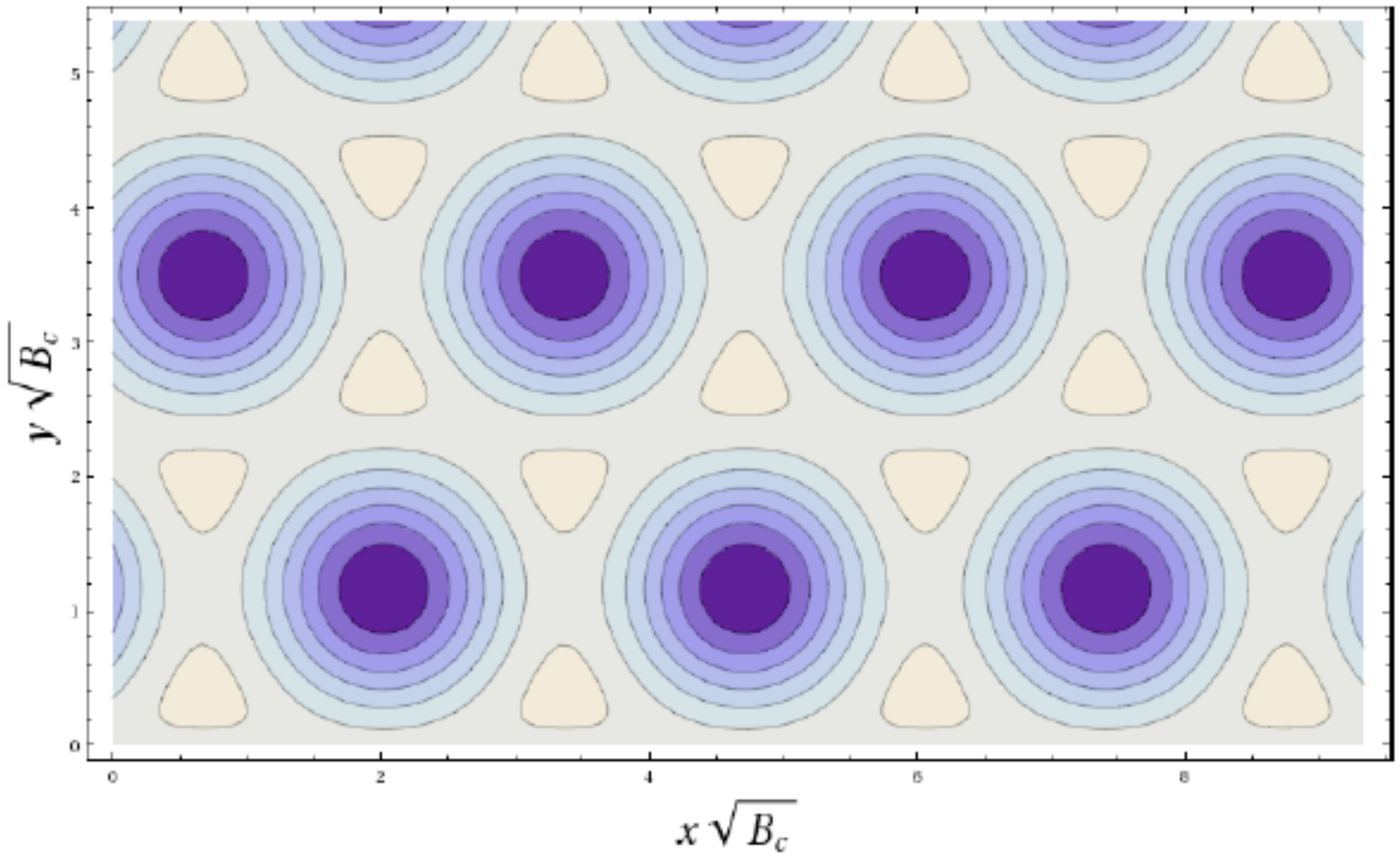
Free energy

free energy as a function of $R = \frac{L_x}{L_y}$





Triangular ground state lattice



Bu, J.E., Shock, Strydom 1210.6669

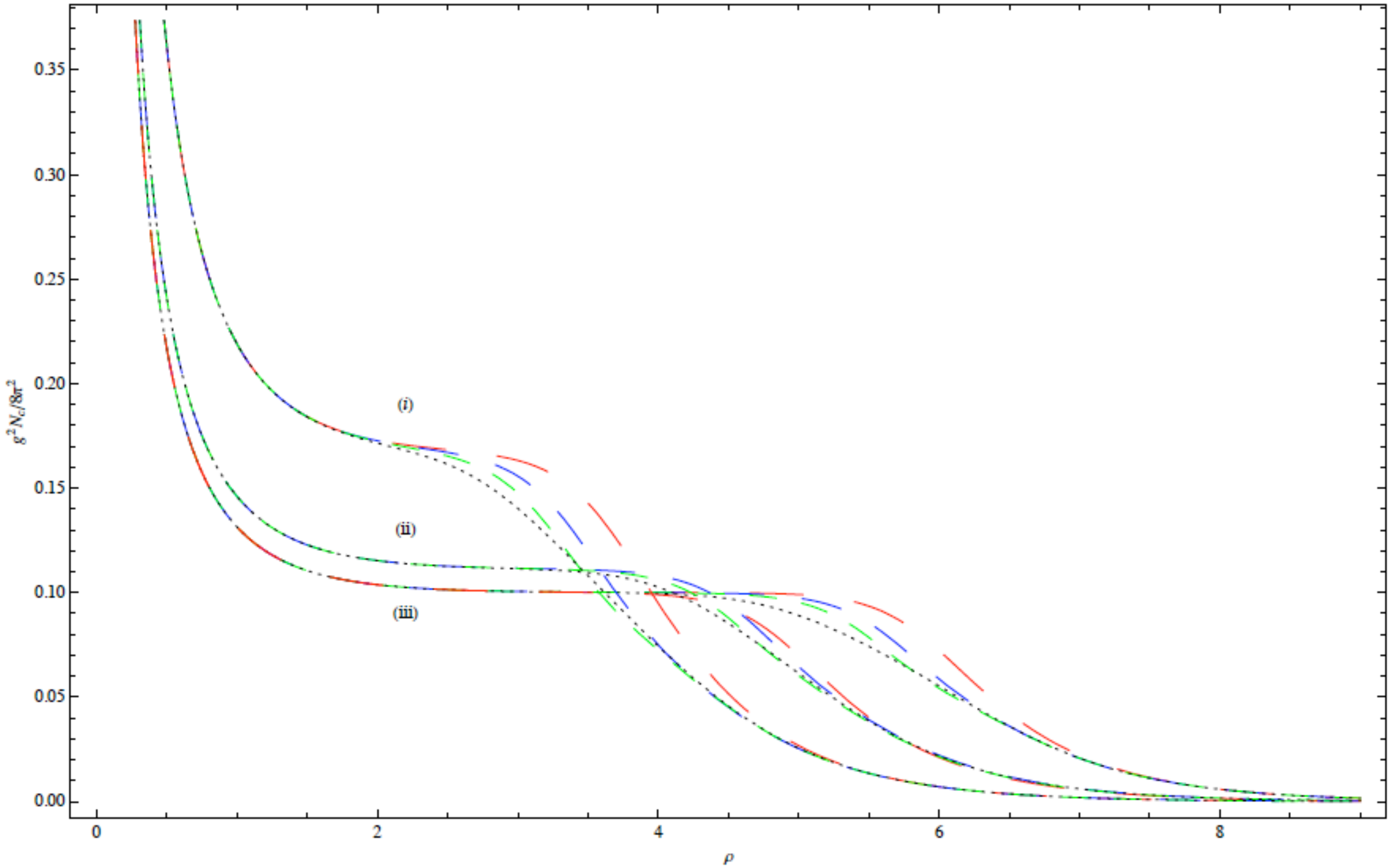
Slow-walking inflation

J.E., Halter, Núñez, Tasinato 1210.4179

- Warped throat supergravity solutions displaying `walking behaviour'
- Consider motion of D3-brane in this geometry
- Brane potential gives predictions in agreement with observations

Walking solutions

Nunez, Papadimitriou, Piai 2008,
Elander, Nunez, Piai 2009



Walking solutions

Nunez, Papadimitriou, Piai 2008,
Elander, Nunez, Piai 2009

$$ds_E^2 = e^{\Phi(\rho)/2}\Big[dx_{1,3}^2 + e^{2k(\rho)}d\rho^2 + e^{2q(\rho)}(d\theta^2 + \sin^2\theta d\phi^2)$$
$$+ \frac{e^{2g(\rho)}}{4}\left[(\tilde{\omega}_1 + a(\rho)d\theta)^2 + (\tilde{\omega}_2 - a(\rho)\sin\theta d\phi)^2\right] + \frac{e^{2k(\rho)}}{4}(\tilde{\omega}_3 + \cos\theta d\phi)^2\Big]$$

N_c D5-branes wrapping an S^2 inside CY cone (conifold)
Geometry asymptotes to Klebanov-Strassler in UV

N=1 supersymmetry

D3-brane inflation

$$\Sigma_4 = [t, x_1, x_2, x_3]\,, \quad \rho = \rho(t)$$

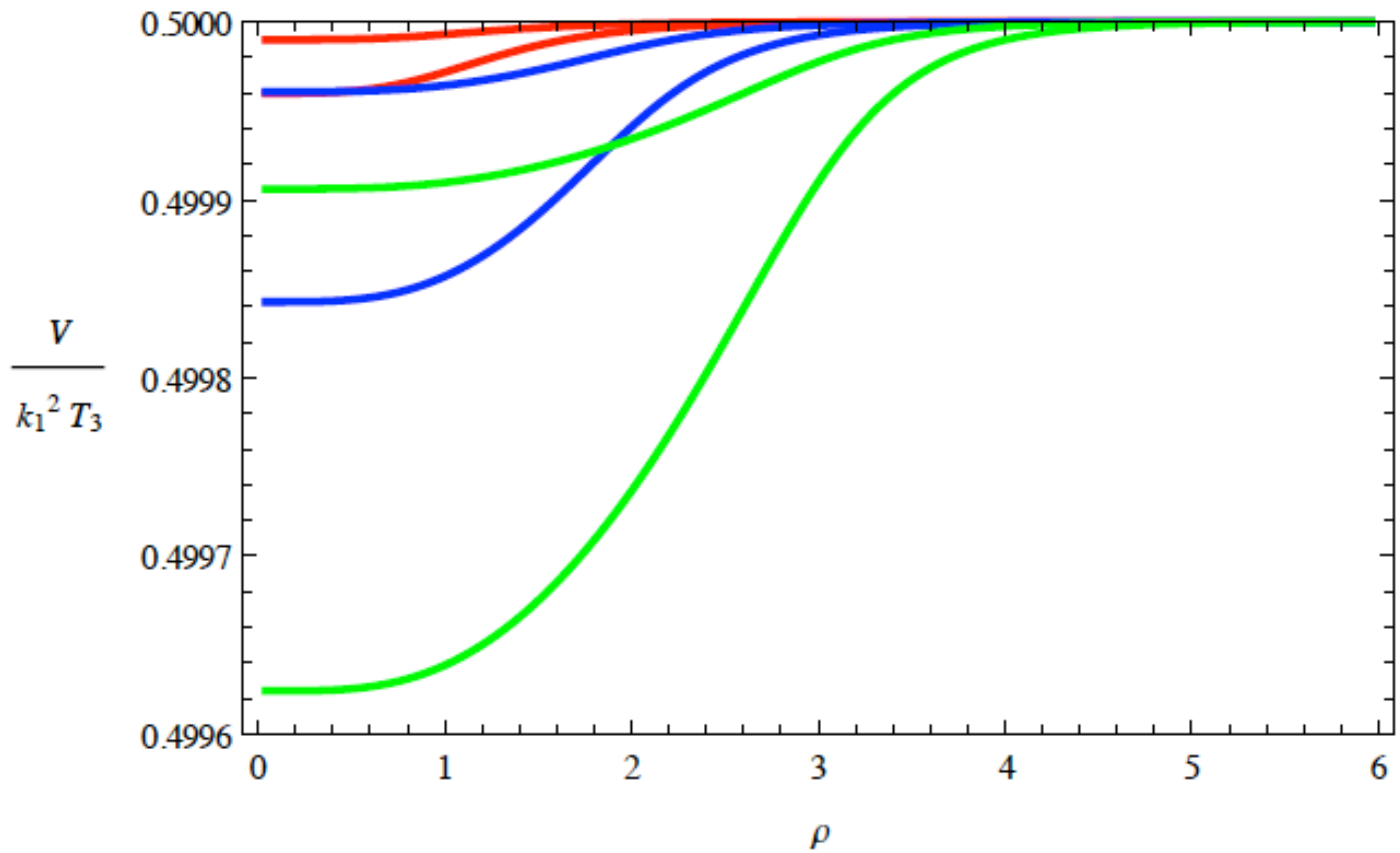
Induced metric: $$ds^2_{ind} = H_1\,(dx_1^2 + dx_2^2 + dx_3^2) + (H_2\,\dot{\rho}^2 - H_1)\,dt^2$$

DBI action: $$S_{BIWZ} = -T_3 \int d^4x \left(e^{-\Phi}\sqrt{-\det[g_{ind}]} - \mathcal{C}_4 \right)$$

Effective potential: $$V = T_3\, e^{-\Phi}\, H_1^2 \left(1 - \frac{e^{\Phi}\,\mathcal{C}_4}{H_1^2} \right)$$

Slow roll parameters: $$\epsilon = \frac{M_P^2}{2V^2}\left(\frac{\partial V}{\partial r}\right)^2\,, \quad \eta = \frac{M_P^2}{V}\frac{\partial^2 V}{\partial r^2}$$

Effective potential



D5 in IR create force on D3 probe in IR
Force becomes weaker in UV KS region

→ Inflection point

Implications for cosmological parameters

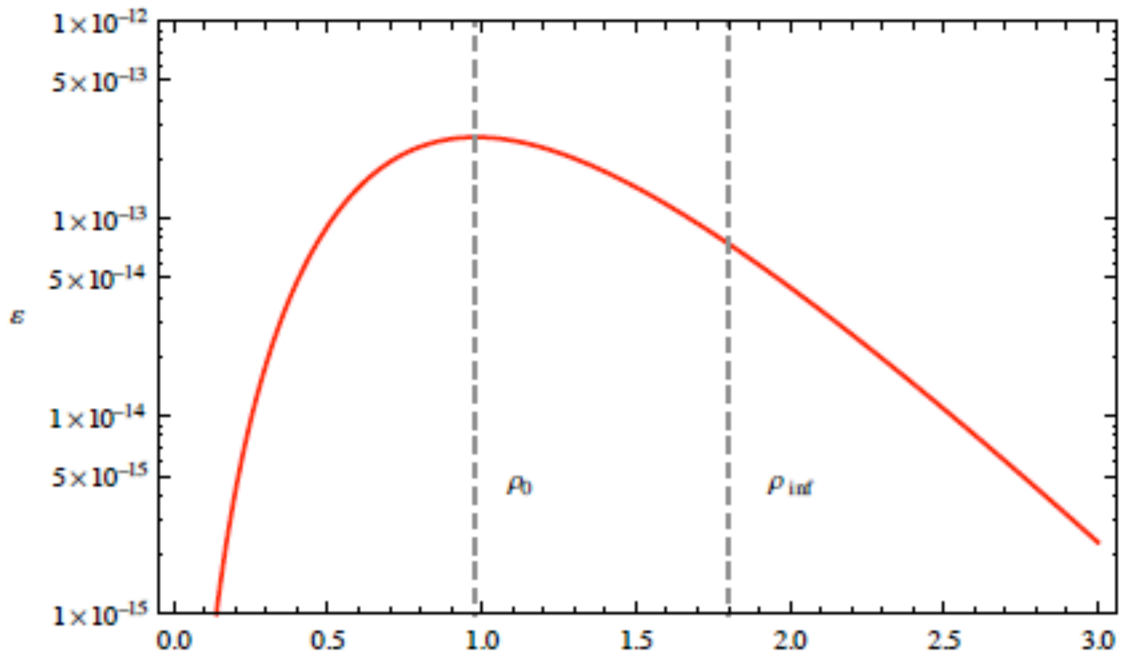
Inflection point ensures small η

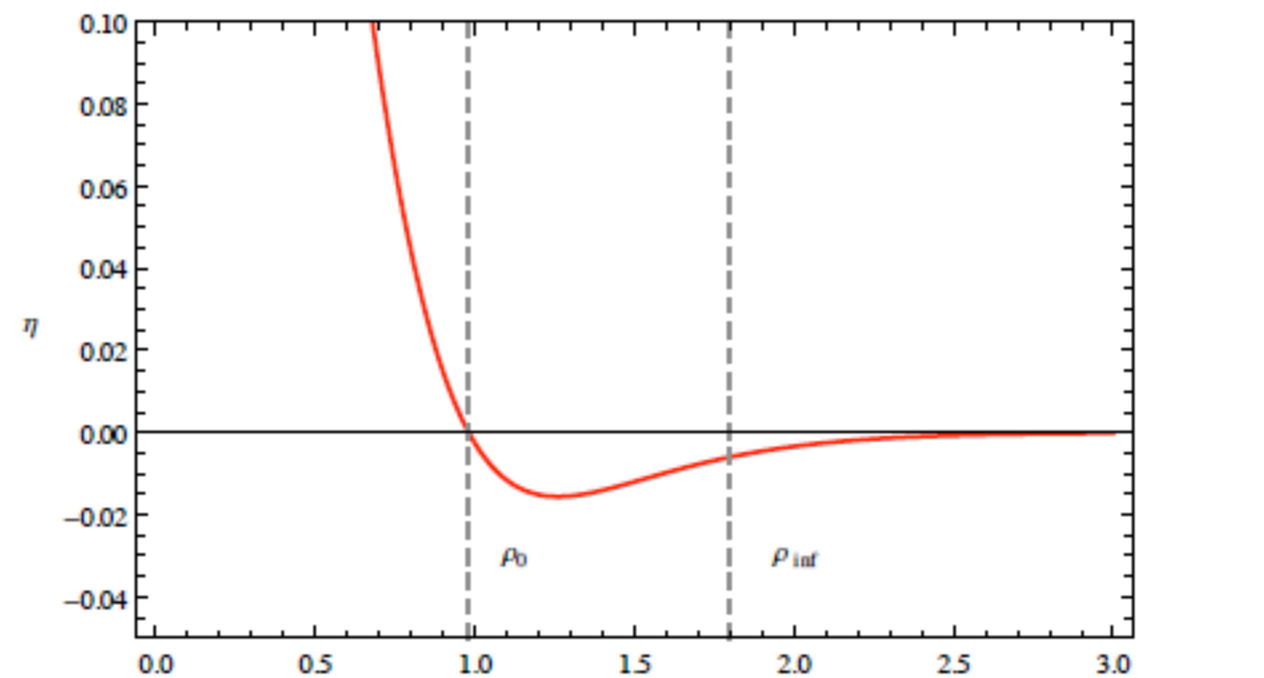
ϵ suppressed by parameter c_+

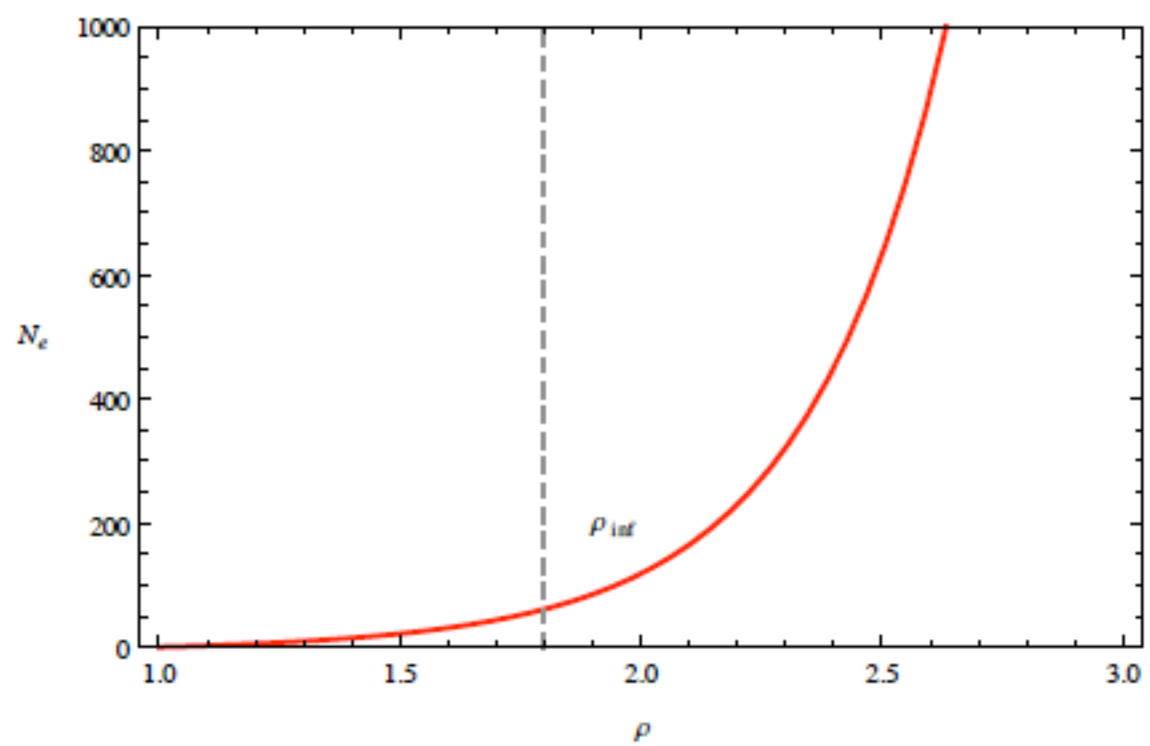
$$e^{4\Phi-4\Phi_0} \simeq \frac{3}{8c_+^3}\left(1 - \frac{3\,N_c^2\,e^{-8\rho/3}\,(8\rho-1)}{4\,c_+^2} + \dots\right)$$

Number of e-folds $N_e = \int H dt \simeq \frac{1}{M_P^2}\int \frac{V}{\partial_r V}\left(\frac{dr}{d\rho}\right) d\rho$

Implications for cosmological parameters





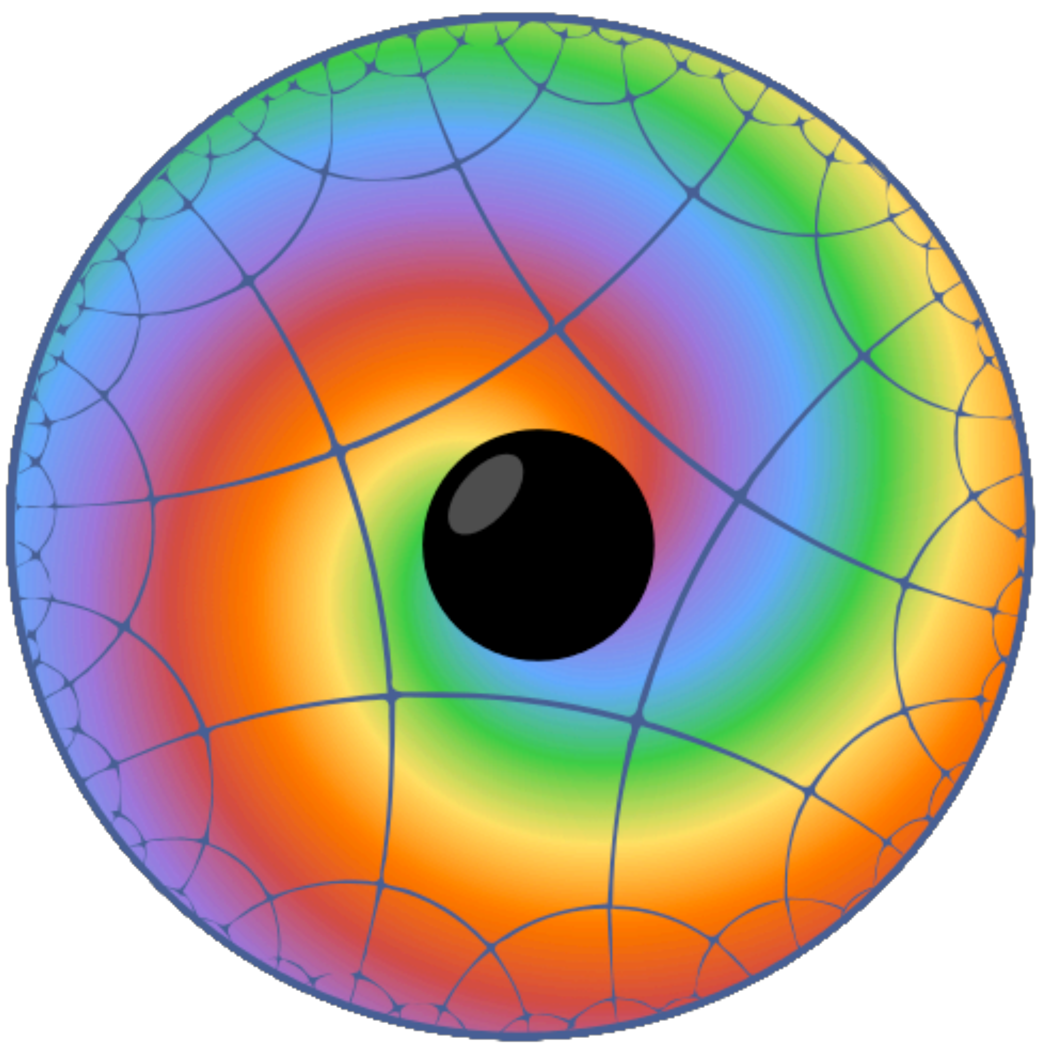


Conclusion

- Phonino related to supersymmetry current absorption cross section
- Anisotropic shear viscosity:
  Non-universal contribution at leading order in N and λ
- Homes' Law: new candidate for universal quantity
- B-field leads to condensation with triangular lattice ground state
- Gauge/gravity duality provides new approaches to universality

Gauge/Gravity Duality 2013
Max-Planck-Institut für Physik

29 July 2013 to 2 August 2013



http://ggd2013.mpp.mpg.de/