Supersymmetric gauge theories on curved manifolds and their gravity duals

Dario Martelli

King's College London

Based on work with: Davide Cassani [4d]; Achilleas Passias; James Sparks [3d].

Strong Fields, Strings and Holography

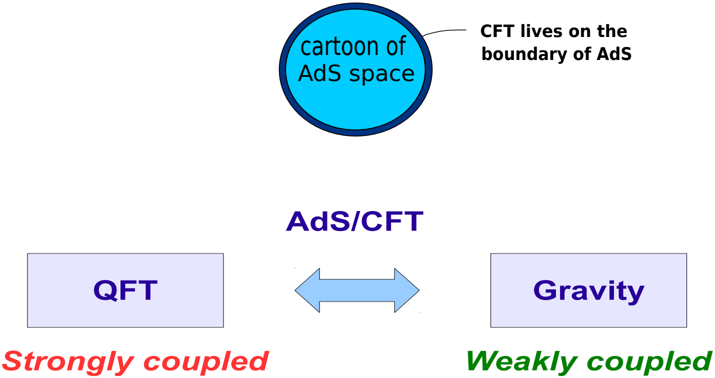
Swansea University, Swansea 16 - 19 July 2013

Outline

1. Introduction
2. Part I: 3d field theories
   1. 3d supersymmetric gauge theories on curved manifolds
   2. Bi-axially squashed three-sphere with Taub-NUT-AdS dual
   3. Two-parameter deformed three-sphere with gravity dual
3. Part II: 4d field theories
   1. 4d supersymmetric gauge theories on curved manifolds
   2. Supersymmetry and superconformal anomalies [To appear]
   3. A new supersymmetric deformation of AdS_5 [To appear]

Gauge/Gravity duality

Conjectured equivalence between (quantum) **gravity** in "bulk" space-times and **quantum field theories** on their boundaries



Supersymmetry

- When bulk and boundary are supersymmetric we can perform detailed computations on both sides and (in certain limits) compare them
- Supersymmetry *in the bulk* $\Rightarrow$ **supersymmetric solutions of supergravity equations**
- There exist Killing spinors obeying first order equations (KSE)
- Supersymmetry *on the boundary* $\Rightarrow$ **"rigid" KSE on curved space**

3d supersymmetric field theories from M2-branes

[BL/G], [ABJM]

- Worldvolume theory on $\mathbf{N}$ M2-branes in flat $\mathbb{R}^{1,2}$ space-time
- $\mathbf{N}$ M2-branes on $\mathbb{R}^{1,2} \times \mathbb{R}^8/\mathbb{Z}_k$, where the $\mathbb{Z}_k$ quotient leaves $\mathcal{N} = 6 \subset \mathcal{N} = 8$ supersymmetry unbroken
- Low-energy theory is an $\mathcal{N} = 6$ superconformal $\mathbf{U(N)_k \times U(N)_{-k}}$ Chern-Simons theory coupled to bi-fundamental matter, with $\mathbf{k} \in \mathbb{N}$ a Chern-Simons coupling:

$$S = S_{CS} + S_{\mathrm{matter}} + S_{\mathrm{potential}}$$

$$S_{CS} = \frac{k}{4\pi} \int \mathrm{Tr} \left(\mathcal{A} \wedge \mathrm{d}\mathcal{A} + \frac{2}{3} \mathcal{A}^3 \right) + \text{supersymmetry completion}$$

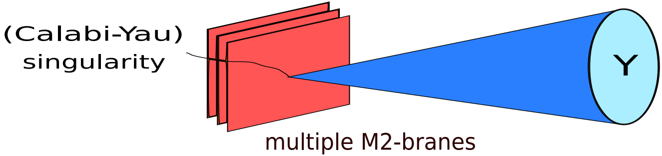
M-theory dual of ABJM

- The supergravity dual is the $AdS_4 \times S^7/\mathbb{Z}_k$ solution to $d = 11$ supergravity with quantized flux of G:

$$N = \frac{1}{(2\pi\ell_p)^6} \int_{S^7/\mathbb{Z}_k} *G$$

- $3/4$ unbroken supersymmetry
- N is the number of M2 branes $=$ N in $U(N)$
- k is the Chern-Simons level

Generalisations with less supersymmetry



- M2-branes at other isolated singularities in 8 dimensions: $\mathbb{R}^{1,2} \times \mathbf{X_8}$ with $\mathbf{X_8}$ Calabi-Yau
- Conical metric $\mathbf{ds^2_{X_8} = dr^2 + r^2 ds^2_{Y_7}}$: in the near-horizon leads to supergravity solution $\mathrm{AdS}_4 \times \mathbf{Y_7}$, with $\mathbf{Y_7}$ a Sasaki-Einstein manifold
- Field theories are $\mathcal{N} = \mathbf{2}$ quiver gauge theories with Chern-Simons terms

The boundary of Euclidean AdS_4

- Conformal boundary of Euclidean-AdS_4 is $\mathbf{S}^3$ with "round" (Einstein) metric

- One can put an arbitrary $\mathbf{d = 3}$, $\mathcal{N} = \mathbf{2}$ gauge theory on the round $\mathbf{S}^3$, preserving supersymmetry [Kapustin-Willet-Yaakov, Jafferis, Hama-Hosomichi-Lee]

- Key ingredient: on the round $\mathbf{S}^3$ there exist Killing spinors ϵ

$$\text{flat space} \quad \partial_\mu \epsilon = 0 \quad \longrightarrow \quad \text{sphere} \quad \nabla_\mu \epsilon = \frac{i}{2} \gamma_\mu \epsilon$$

- Supersymmetric Lagrangian can be obtained taking $\mathbf{m_{pl}} \to \infty$ limit of a suitable supergravity (in the same dimension) to obtain a rigid supersymmetric theory [Festuccia-Seiberg]

Exact free energy

- Using localisation, the exact path integral $\mathbf{Z}$ of an $\mathcal{N} = 2$ gauge theory on the three-sphere is reduced to a matrix integral, containing the "double sine" function

$$s_\beta(x) = \prod_{m,n\geq 0} \frac{m\beta + n\beta^{-1} + (\beta + \beta^{-1})/2 - ix}{m\beta + n\beta^{-1} + (\beta + \beta^{-1})/2 + ix}\,, \qquad \beta = 1$$

- For the ABJM model [Drukker-Marino-Putrov]:

$$-\log Z_{\text{field theory}} = \frac{\pi\sqrt{2}}{3} k^{1/2} N^{3/2} + O(N^{1/2})$$

- This agrees (including numerical factors!) with the holographic free energy of AdS_4 (holographically renormalized action of AdS_4), reproducing the famous $\mathbf{N}^{3/2}$ scaling

Large N free energy

- For more general $\mathcal{N} = 2$ SCFTs, similar results have been obtained by extracting the large $\mathbf{N}$ limit of the corresponding matrix integrals:

$$-\log Z_{\text{field theory}} = \sqrt{\frac{2\pi^6}{27\mathrm{Vol}(Y_7)}} N^{3/2} + O(N^{1/2})$$

(at least when the matter representation of the gauge group is *real*)

- This agrees with the holographic free energy computed from the (Euclidean) M-theory solutions $AdS_4 \times Y_7$, with generic Sasaki-Einstein manifold Y_7 [DM-Sparks,Cheon-Kim-Kim,Jafferis-Klebanov-Pufu-Safdi]

More general three-manifolds

One can put $\mathcal{N} = 2$ theories on 3-manifolds more general than the round $\mathbf{S^3}$, still preserving supersymmetry. General rigid KSE for 3-manifolds:

$$\left[\nabla_\alpha - i A^{(3)}_\alpha - i V^{(3)}_\alpha + \frac{H}{2}\gamma_\alpha + \epsilon_{\alpha\beta\rho} V^{(3)\beta}\gamma^\rho\right]\chi = 0$$

χ is the supersymmetry parameter. $A^{(3)}_\alpha, V^{(3)}_\alpha, H$ are fixed background fields [Klare-Tomasiello-Zaffaroni,Closset-Dumitrescu-Festuccia-Komardgodski]

Results about supersymmetry, localization, and reduction to matrix integrals go through if we replace the round $\mathbf{S^3}$ by the bi-axially squashed $\mathbf{S^3}$, with metric

$$ds_3^2 = d\theta^2 + \sin^2\theta d\phi^2 + 4s^2\left(d\psi + \cos\theta d\phi\right)^2$$

and specific background fields $A^{(3)}, V^{(3)}, H$

flat space $\partial_\alpha - iq\mathcal{A}_\alpha \longrightarrow$ curved space $\nabla_\alpha - iq\mathcal{A}_\alpha - iR \cdot A^{(3)}{}_\alpha$

The two supersymmetric biaxially squashed three-spheres

Supersymmetry can be preserved in two cases, adding slightly different background gauge fields:

$$1/4\text{ BPS:}\quad A^{(3)} = -\frac{1}{2}(4s^2-1)\,(\mathrm{d}\psi + \cos\theta\,\mathrm{d}\phi) \qquad \text{[Hama-Hosomichi-Lee]}$$

$$1/2\text{ BPS:}\quad A^{(3)} = -s\sqrt{4s^2-1}\,(\mathrm{d}\psi + \cos\theta\,\mathrm{d}\phi) \qquad \text{[Imamura-Yokoyama]}$$

Here $0 < s =$ squashing parameter, with the round metric on S^3 being $s = \frac{1}{2}$

In the 1/2 BPS case the partition function involves $s_b(x)$, where $4s = b + \frac{1}{b}$

The large N limit of the partition function for $d = 3$, $\mathcal{N} = 2$ theories can be computed from the matrix models and to leading order in N is:

$$\log Z_{\text{field theory}}[s] = \log Z_{\text{round } S^3} \times \begin{cases} 1 & 1/4\text{ BPS} \\ 4s^2 & 1/2\text{ BPS} \end{cases}$$

Gravity duals

Idea: find a supersymmetric filling $\mathbf{M_4}$ of the squashed $\mathbf{S^3}$ in $\mathbf{d=4}$, $\mathcal{N}=\mathbf{2}$ gauged supergravity (Einstein-Maxwell theory), and use the fact that any[1] such solution uplifts to a supersymmetric solution $\mathbf{M_4}\times\mathbf{Y_7}$ of $\mathbf{d=11}$ supergravity

Action: $\mathbf{S}=-\frac{\mathbf{1}}{\mathbf{16\pi G_4}}\int \mathrm{d}^4\mathbf{x}\sqrt{\mathbf{g}}\left(\mathbf{R}+\mathbf{6}-\mathbf{F}^2\right)$

Killing Spinor Equation: $\left(\nabla_\mu-\mathbf{i}\mathbf{A}_\mu+\frac{1}{2}\Gamma_\mu+\frac{\mathbf{i}}{4}\mathbf{F}_{\nu\rho}\Gamma^{\nu\rho}\Gamma_\mu\right)\epsilon=0$

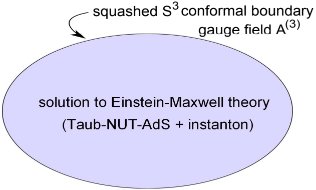
Where $\Gamma_\mu\in\mathrm{Cliff}(4,0)$, so $\{\Gamma_\mu,\Gamma_\nu\}=2\mathbf{g}_{\mu\nu}$

Dirichlet problem: find an $(\mathbf{M_4},\mathbf{g}_{\mu\nu})$ and gauge field $\mathbf{A}$ such that

- The conformal boundary of $\mathbf{M_4}$ is the squashed $\mathbf{S^3}$
- The $\mathbf{d=4}$ gauge field $\mathbf{A}$ restricts to $\mathbf{A^{(3)}}$ on the conformal boundary
- The $\mathbf{d=4}$ Killing spinor ϵ restricts to the $\mathbf{d=3}$ Killing spinor χ

[1] Locally.

Gravity duals



$\mathbf{M_4}$ = Taub-NUT-AdS

A = self-dual gauge field (*F=F)

The gauge fields and Killing spinors are different for the 1/4 BPS and 1/2 BPS solutions

Taub-NUT-AdS is an asymptotically locally AdS Einstein metric (with self-dual Weyl tensor) on $\mathbb{R}^4$:

$$\mathrm{d}s_4^2 = \frac{r^2 - s^2}{\Omega(r)}\mathrm{d}r^2 + (r^2 - s^2)(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2) + \frac{4s^2\Omega(r)}{(r^2 - s^2)}(\mathrm{d}\psi + \cos\theta\mathrm{d}\phi)^2$$

where $\Omega(r) = (r - s)^2\,[1 + (r - s)(r + 3s)]$

$$A = f(r, s)(\mathrm{d}\psi + \cos\theta\mathrm{d}\phi)$$

Holographic free energy

The holographic free energy is

$$-\log Z_{\text{gravity}} = S_{\text{Einstein-Maxwell}} + S_{\text{Gibbons-Hawking}} + S_{\text{counterterm}}$$

Remarkably, we find

$$\log Z_{\text{gravity}}[s] = \log Z_{AdS_4} \times \begin{cases} 1 & 1/4 \text{ BPS} \\ 4s^2 & 1/2 \text{ BPS} \end{cases}$$

agreeing exactly with the leading large **N** matrix model results!

For the 1/4 BPS case the independence of **s** is non-trivial: each term in the action has a complicated **s**-dependence, which cancels only when all are summed

The other one-parameter deformation of the three-sphere

- There is another known one-parameter deformation of $\mathbf{S^3}$, preserving $\mathbf{U(1)} \times \mathbf{U(1)}$ symmetry – the "ellipsoid" [Hama-Hosomichi-Lee] (this was in fact the first non-trivial example)

$$\mathrm{d}s_3^2 = \mathsf{f}^2(\vartheta)\mathrm{d}\vartheta^2 + \cos^2\vartheta \mathrm{d}\varphi_1^2 + \frac{1}{\mathsf{b}^4}\sin^2\vartheta \mathrm{d}\varphi_2^2$$

$$\mathsf{A}^{(3)} = \frac{1}{2\mathsf{f}(\vartheta)}\left(\mathrm{d}\varphi_1 - \frac{1}{\mathsf{b}^2}\mathrm{d}\varphi_2\right) , \qquad \mathsf{V}^{(3)} = 0 , \quad \mathsf{H} = -\frac{\mathrm{i}}{\mathsf{f}(\vartheta)}$$

where

$$\mathsf{f}^{-2}(\vartheta) = \sin^2\vartheta + \mathsf{b}^4\cos^2\vartheta$$

- The original $\mathbf{f(\vartheta)}$ in HHL is slightly different, but we [DM-Passias-Sparks] showed that it can be an arbitrary function, provided it gives a smooth metric with the topology of the three-sphere

A two-parameter squashed three-sphere

[DM-Passias]

- New family of metrics on a deformed three-sphere, depending on two non-trivial parameters

- A possible way of writing the metric:

$$\mathbf{ds_3^2 = \frac{d\theta^2}{f(\theta)} + f(\theta)\sin^2\theta\, d\hat{\phi}^2 + (d\hat{\psi} + (\cos\theta + a\sin^2\theta)d\hat{\phi})^2}$$

  where

$$\mathbf{f(\theta) = v^2 - a^2\sin^2\theta - 2a\cos\theta}$$

- The parameters are **a** $\in \mathbb{R}$ and **v** $\in \mathbb{R}$

- This looks like a deformation of the Hopf fibration over (a deformed) $\mathbf{S^2}$. However, these coordinates are only local (cf. *irregular* Sasaki-Einstein manifolds looking like a "fibration" over a Kähler-Einstein "manifold")

Two-parameter deformations

- Global regularity of the metric can be checked introducing two different angular coordinates as

$$\hat{\psi} = \frac{1}{v^2 - 2a}\varphi_1 + \frac{1}{v^2 + 2a}\varphi_2$$

$$\hat{\phi} = -\frac{1}{v^2 - 2a}\varphi_1 + \frac{1}{v^2 + 2a}\varphi_2$$

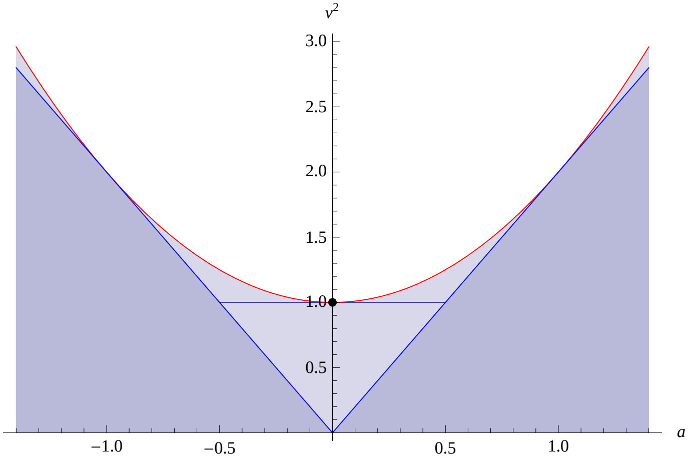
- $\varphi_1, \varphi_2 \in [0, 2\pi]$ parameterise a torus and S^3 is realized as a T^2 fibration over an interval (parameterized by $\theta \in [0, \pi]$)

- The other background fields are all non-trivial

$$A^{(3)} = QA_i(\theta)d\varphi_i\ , \quad V^{(3)} = \frac{v^2 - 1}{Q}\sum_i V_i(\theta)d\varphi_i\ , \quad H = i(\tfrac{1}{2} - a\cos\theta)$$

- $A^{(3)}$ and $V^{(3)}$ can be real, imaginary, or complex, depending on $Q = Q(v, a)$

Parameter space



Plot of the moduli space of solutions in the $(\mathbf{a}, \mathbf{v}^2)$ plane

The special one-parameter families

$$Q = \begin{cases} \pm\frac{1}{2}(a + \sqrt{1 - v^2 + a^2}) \\ \pm\frac{1}{2}(a - \sqrt{1 - v^2 + a^2}) \\ \pm\frac{v^2-1}{2} \end{cases}$$

- When $1 - v^2 + a^2 < 0$ there are two complex conjugate configurations. NB: the metric is always real, $\mathbf{H}$ is always pure imaginary

- The two known cases are recovered from the one-parameter sub-families defined by $a = 0$ or $v^2 = 1$

- Setting $a = 0$, and defining $s = \frac{1}{2v}$ gives the biaxially squashed metric, with the two distinct background fields

- Setting $v^2 = 1$, and defining $a = \frac{1}{2}\frac{b^2-1}{b^2+1}$ gives the ellipsoid metric, with the unique background field

Gravity duals

- Four-dimensional supersymmetric gravity dual solution constructed (as before) in minimal gauged supergravity
- Originates from the class of Plebanski-Demianski solutions of Maxwell-Einstein supergravity
- Solution comprises an ALEAdS self-dual metric on the ball (with topology of $\mathbb{R}^4$ $\Rightarrow$ upliftable to M-theory) and different instantons
- The metric is real, but the three (generically) different values of $\mathbf{Q}$ correspond to a generically complex instanton field
- Includes all previous solutions (with $\mathbb{R}^4$ topology) as special cases

Holographic free energy

- The holographic free energies in the three cases read

$$\mathcal{F} = \frac{\pi}{2G_4} \begin{cases} \dfrac{1}{1-4Q^2} \\ 1 \end{cases}$$

- Remarkably, when it's non-trivial, it depends only on one parameter Q
- In general Q is complex, therefore $\mathcal{F}$ is complex. In the cases $a = 0$ or $v^2 = 1$ one recovers the expressions of the previous holographic free energies
- Setting $Q = \frac{1}{2}\frac{\beta^2-1}{\beta^2+1}$ gives the following expression for the (large N) free energy

$$\mathcal{F} = \frac{\pi}{8G_4}\left(\beta + \frac{1}{\beta}\right)^2$$

- We conjectured that the full localised partition function on this background will be given by a matrix integral involving $s_\beta(x)$

Four-dimensional rigid supersymmetry

- General rigid ("new minimal") KSE for $d = 4, \mathcal{N} = 1$ gauge theories:

$$\left[\nabla_m - ia_m + iv_m + \frac{i}{2}v^n\gamma_{mn}\right]\zeta = 0$$

- ζ is a chiral supersymmetry parameter and a_m, v_m are background fields
- The combination $A_m = a_m - \frac{3}{2}v_m$ couples to the R-symmetry current J^m
- 4d field theories on supersymmetric curved backgrounds:
  1. Localization computations not yet as developed as in 3d but certainly will appear soon
  2. Putting 4d SCFTs on curved backgrounds is necessary for detecting superconformal anomalies

Charged conformal Killing spinors (CKS)

- An essentially equivalent supersymmetry equation obeyed by ζ is

$$\nabla^A_m \zeta = \frac{1}{4}\gamma_m \gamma^n \nabla^A_n \zeta$$

  where $\nabla^A_m = \nabla_m - iA_m$

- This has the same form in Lorentzian and Euclidean signature. The main difference is that A_m is real in the first case, and complex in the second case

- In Euclidean signature: equivalent to Hermitian metric [Klare-Tomasiello-Zaffaroni,Festuccia-Seiberg]

- In Lorentzian signature: equivalent to existence of null conformal Killing vector [Cassani-Klare-DM-Tomasiello-Zaffaroni]

Extracting information on the geometry

- In the references above it was shown that the geometry determines (not very explicitly) the field $\mathbf{A_m}$
- By using a different method, we have obtained useful relations between the geometry and the gauge field $\mathbf{A_m}$
- The starting point is the integrability condition of the CKS equation

$$\Big(\frac{1}{4}\mathbf{C_{mnpq}} - \frac{\mathbf{i}}{3}\mathbf{g_{p[m}F_{n]q}}\Big)\gamma^{\mathbf{pq}}\zeta - \frac{\mathbf{i}}{3}\Big(\mathbf{F_{mn}} - \frac{1}{2}\gamma_{\mathbf{mnpq}}\mathbf{F^{pq}}\Big)\zeta \;=\; \mathbf{0}$$

where

$$\mathbf{C_{mnpq}} \;=\; \mathbf{R_{mnpq}} - \frac{1}{2}\left(\mathbf{g_{m[p}R_{q]n}} - \mathbf{g_{n[p}R_{q]m}}\right) + \frac{1}{3}\mathbf{R}\,\mathbf{g_{m[p}g_{q]n}}$$

is the Weyl tensor of the metric $\mathbf{g_{mn}}$ and $\mathbf{F_{mn}} = \partial_\mathbf{m}\mathbf{A_n} - \partial_\mathbf{n}\mathbf{A_m}$

Implications of integrability of the CKS equation

- Idea: given a metric $\mathbf{g_{mn}}$, we can express $\mathbf{F_{mn}}$ in terms of the Weyl tensor
- Strategy: decompose $\mathbf{C_{mnpq}}$ and $\mathbf{F_{mn}}$ in a basis of two-forms, *a la* Newman-Penrose, and then use the integrability to relate the coefficients of the expansions (Weyl scalars)
- In Lorentzian signature we obtain:

$$\mathbf{C_{mnpq}C^{mnpq}} = \frac{8}{3}\mathbf{F_{mn}F^{mn}}\,, \qquad \mathbf{C_{mnpq}\widetilde{C}^{mnpq}} = \frac{8}{3}\mathbf{F_{mn}\widetilde{F}^{mn}}$$

- In Euclidean signature we obtain:

$$\mathbf{C_{mnpq}C^{mnpq}} - \frac{8}{3}\mathbf{F_{mn}F^{mn}} = -\mathbf{C_{mnpq}\widetilde{C}^{mnpq}} + \frac{8}{3}\mathbf{F_{mn}\widetilde{F}^{mn}}$$

where $\mathbf{\widetilde{C}_{mnpq}} = \frac{1}{2}\epsilon_{\mathbf{mn}}{}^{\mathbf{rs}}\mathbf{C_{rspq}}$ and $\mathbf{\widetilde{F}_{mn}} = \frac{1}{2}\epsilon_{\mathbf{mn}}{}^{\mathbf{rs}}\mathbf{F_{rs}}$

Superconformal anomalies

- The trace and R-symmetry anomalies of $\mathcal{N} = 1$ SCFT [Anselmi *et al*][2] read

$$\langle T^m_m \rangle = \frac{c}{16\pi^2}\mathscr{C}^2 - \frac{a}{16\pi^2}\mathscr{E} - \frac{c}{6\pi^2} F_{mn}F^{mn}$$

$$\langle \nabla_m J^m \rangle = \frac{c-a}{24\pi^2} R_{mnpq}\widetilde{R}^{mnpq} + \frac{5a-3c}{27\pi^2} F_{mn}\widetilde{F}^{mn}$$

  where **a** and **c** are the central charges and

$$\mathscr{C}^2 \equiv C_{mnpq}C^{mnpq} = R_{mnpq}R^{mnpq} - 2R_{mn}R^{mn} + \frac{1}{3}R^2$$

$$\mathscr{E} \equiv \frac{1}{4}\epsilon^{mnpq}\epsilon^{rsuv}R_{mnrs}R_{pquv} = R_{mnpq}R^{mnpq} - 4R_{mn}R^{mn} + R^2$$

$$\mathscr{P} \equiv \frac{1}{2}\epsilon^{mnpq}R_{mnrs}R_{pq}{}^{rs} = \frac{1}{2}\epsilon^{mnpq}C_{mnrs}C_{pq}{}^{rs}$$

[2] After correcting some errors in this reference

Taming the anomalies

- Using the identities implied by supersymmetry we find that the anomalies become topological
- In Euclidean signature:

$$\langle T^m_m \rangle = -\frac{c}{16\pi^2}\left(\mathscr{P} - \frac{8}{3}\mathrm{Re}F\widetilde{F}\right) - \frac{a}{16\pi^2}\mathscr{E} + i\,\frac{c}{6\pi^2}\mathrm{Im}F\widetilde{F}$$

$$\langle \nabla_m J^m \rangle = \frac{c-a}{24\pi^2}\mathscr{P} + \frac{5a-3c}{27\pi^2}\mathrm{Re}F\widetilde{F} + i\,\frac{5a-3c}{27\pi^2}\mathrm{Im}F\widetilde{F}$$

- In Lorentzian signature (and Euclidean, assuming two CKS of opposite chiralities):

$$\langle T^m_m \rangle \;=\; -\frac{a}{16\pi^2}\mathscr{E}$$

$$\langle \nabla_m J^m \rangle \;=\; \frac{a}{9\pi^2}\mathscr{P} \;=\; a\frac{8}{27\pi^2}F\widetilde{F}$$

Topological formulas for the integrated anomalies

- When the 4d Euclidean manifold is compact we can integrate the anomalies on $\mathbf{M}$, obtaining the following relations

$$\int_M d^4x\sqrt{g}\,\langle T^m_m\rangle = -3c\sigma(M) + \frac{c}{3}\nu(M) - a2\chi(M)$$

$$\int_M d^4x\sqrt{g}\,\nabla_m J^m = 2(c-a)\sigma(M) + (5a-3c)\frac{2}{27}\nu(M)$$

where

$$\mathbb{Z} \ni \chi(M) = \frac{1}{32\pi^2}\int_M d^4x\sqrt{g}\,\mathscr{E}$$

$$\mathbb{Z} \ni \sigma(M) = \frac{1}{3}\int_M p_1(M) = \frac{1}{48\pi^2}\int_M d^4x\sqrt{g}\,\mathscr{P}$$

$$\mathbb{N} \ni \nu(M) = \int_M c_1(M)\wedge c_1(M)$$

- With two solutions ζ_+ and ζ_- with opposite charge, we conclude

$$\nu(M) = \sigma(M) = \chi(M) = 0$$

Searching a 5d gravity dual to 4d SCFT on a supersymmetric curved manifold

- [Klare-Tomasiello-Zaffaroni]/[KTZ+Cassani+DM] showed that *locally* $\mathbf{d=4}$ rigid susy arises at the boundary of supersymmetric Euclidean/Lorentzian AlAdS solutions of minimal gauged supergravity in $\mathbf{d=5}$
- Examples of 5d sugra solutions with non-trivial boundary? Very few!
- A deformation of AdS_5 [Gauntlett-Gutowski], with boundary $\mathbb{R}\times \mathbf{S^3}$ preserving $\mathbf{SU(2)}\times\mathbf{U(1)}$ symmetry. Impossible to Euclideanize & compactify
- A magnetic string [Klemm-Sabra] with boundary $\mathbb{R}^{1,1}\times\mathbf{H^2}$ (or $\mathbb{T}^2\times\mathbf{H^2}$) and $\mathbf{F}\propto \mathrm{vol}(\mathbf{H^2})$
- Would like a non conformally flat, compact and Euclidean boundary
- A priori endless possibilities (i.e. take any compact complex manifold). However $\boldsymbol{\sigma}(\mathbf{M})=\mathbf{0}$ gives a first restriction: e.g. for del Pezzo surfaces $\mathbf{dP_k}$, only $\mathbf{dP_1}$ has vanishing signature. In particular $\mathbb{C}\mathbf{P^2}$ it's not allowed

A new supersymmetric deformation of AdS_5 [Cassani-DM]

1. "Uplift" known 3d supersymmetric backgrounds to 4d
2. Require large symmetry
3. Solve both Euclidean and Lorentzian rigid KSE
4. $\boldsymbol{\sigma = 0}$, $\boldsymbol{\chi = 0}$ mod 2

- This singles out $\mathbf{S}^3_{\textbf{squashed}} \times \mathbf{S}^1$ with $\mathbf{SU(2)} \times \mathbf{U(1)} \times \mathbf{U(1)}$ symmetry
- We looked for a supersymmetric "filling" of this boundary, in minimal gauged supergravity in $\mathbf{d = 5}$, which is topologically global AdS_5

A new supersymmetric deformation of AdS_5 [Cassani-DM]

1. "Uplift" known 3d supersymmetric backgrounds to 4d
2. Require large symmetry
3. Solve both Euclidean and Lorentzian rigid KSE
4. $\sigma = 0$, $\chi = 0$ mod 2

- This singles out $S^3_{\text{squashed}} \times S^1$ with $SU(2) \times U(1) \times U(1)$ symmetry
- We looked for a supersymmetric "filling" of this boundary, in minimal gauged supergravity in $d = 5$, which is topologically global AdS_5
- We found a new one-parameter supersymmetric deformation of AdS_5 with the above rigid susy boundary!
- We found the solution numerically, and analytically at first order in the deformation parameter ξ

Some properties of the solution

- The holographic anomaly vanishes $\langle \mathbf{T}^{\mathbf{i}}_{\mathbf{i}} \rangle = \mathbf{0}$, in agreement with our general results about anomalies in supersymmetric backgrounds

- The Casimir energy on the deformed $\mathbf{S}^3_\xi$ may be computed from the renormalised holographic energy-momentum tensor (up to ambiguities)

$$\mathbf{E}(\xi) \ = \ \int_{\mathbf{S}^3_\xi} \langle \mathbf{T}_{tt} \rangle \, \mathrm{vol}(\mathbf{S}^3_\xi) \ = \ \frac{\pi \ell^2}{32 \mathbf{G}_5} \left[3 + \xi(2 - \log 2) + \mathcal{O}(\xi^2) \right]$$

- Euclidean version of solution is obtained by $\mathbf{t} \rightarrow \mathbf{it}$ ($\mathbf{t}$ is global time in AdS). Boundary metric is real (gauge field is complex), but bulk 5d metric is complex!

- Would be interesting to compute Casimir energy exactly using localisation

THE END