10d STRUCTURES AND HOLOGRAPHY

M.P, A.ZAFFARONI, arXiv:1202.5542, S. GIUSTO, L. MARTUCCI, M.P, R. RUSSO, arXiv:1306.1745

- Type II and M-theory flux backgrounds play a central role in
  - compactifications
  - gauge/gravity duality
- A lot of effort has been devoted to the study of such backgrounds
  - compute the back-reaction of the fluxes to determine the full geometry
  - understand the properties of the new spaces
- Considerable progress has been made using techniques of
  - G-structures
  - Generalised Complex Geometry (GCG)
  - Exceptional Generalised Geometry (EGG)

- Major progress has been achieved for SUSY backgrounds with

$$M_{10} = M_4 \times_w M_6 \qquad \longrightarrow \qquad \mathrm{d}s^2_{(10)} = e^{2A(y)}\mathrm{d}s^2_{(4)} + \mathrm{d}s^2_{(6)}(y)$$

describing

  - compactifications to four dimensions

$$M_4 = \{\mathrm{Minkowski}_4, \mathrm{dS}_4?\}$$

  - CFT_3 and deformations (AdS_4)
  - CFT_4 and deformations

$$\mathrm{d}s^2_{AdS_5} = \frac{\mathrm{d}r^2}{r^2} + r^2\mathrm{d}s^2_4 \quad \to \quad e^{-2A}\mathrm{d}r^2 + e^{2A}\mathrm{d}s^2_4$$

- Two main reasons
  - under quite general conditions [lüst, tsimpis 04]

10 dimensional equations of motions	$\Leftrightarrow$	SUSY variations and Bianchi id. for the fluxes

  - use Generalised Complex Geometry [hitchin, gualtieri, grana, minasian, m.p., tomasiello 05]

10 dimensional SUSY variations	$\Leftrightarrow$	differential constraints on 6d forms

- Susy vacua and Generalised Complex Geometry
  - decompose the 10d type II SUSY parameters

$$\epsilon_1 = \zeta_+ \otimes \eta^1_+ + \zeta_- \otimes \eta^1_- \qquad \rightarrow \qquad (\eta^i_+)^* = \eta^i_- \quad \text{6}d \text{ Weyl spinors}$$
$$\epsilon_2 = \zeta_+ \otimes \eta^2_\mp + \zeta_- \otimes \eta^2_\pm \qquad \qquad \zeta^*_+ = \zeta_- \quad \text{4}d \text{ Weyl spinors}$$

  - build polyforms on M_6 as bispinors

$$\Phi_\pm = \eta^1_+ \otimes \bar{\eta}^2_\pm = \frac{1}{8}\sum_p \frac{1}{p!}\left(\bar{\eta}^2_\pm \gamma_{m_1\ldots m_p}\eta^1_+\right) \mathrm{d}x^{m_p\ldots m_1}$$

  - SUGRA equations of motions are equivalent to

10d SUSY variations + Bianchi identities

$$\mathrm{d}(e^{3A}\Phi_1) = 0$$
$$\mathrm{d}(e^{2A}\mathrm{Im}\Phi_2) = 0$$
$$\mathrm{d}(e^{4A}\mathrm{Re}\Phi_2) = e^{4A}e^{-B} * \lambda(F)$$

$$(\mathrm{d} - H)F = \delta(source)$$
$$\mathrm{d}H = 0$$

    - unified description of type IIA and IIB

- Consequences
  - Geometric characterisation of $\mathcal{N}=1$ flux vacua
    - $\Phi_\pm$ are spinors on $T(M_6) \oplus T^*(M_6)$

      positive chirality $\leftrightarrow$ $\Phi_+ \in \Lambda^{even} T^*(M)$ even forms

      negative chirality $\leftrightarrow$ $\Phi_- \in \Lambda^{odd} T^*(M)$ odd forms

      - pure spinors $\rightarrow$ vacuum of Cliff(6,6)
      - define a SU(3) $\times$ SU(3) structure on $T \oplus T^*$

SU(2) on T^*	$\eta_+^1 = a\eta_+$	$\Phi_+ = \frac{a}{8}(\bar{c}_1 e^{-ij} - i\bar{c}_2\omega) \wedge e^{z\wedge\bar{z}/2}$
	$\eta_+^2 = c_1\eta_+ + c_2 z \cdot \eta_-$	$\Phi_- = -i\frac{a}{8}(\bar{c}_2 e^{-ij} + i\bar{c}_1\omega) \wedge z$
SU(3) on T^*	$\eta_+^1 = a\eta_+$	$\Phi_+ = \frac{a\bar{b}}{8} e^{-iJ}$
	$\eta_+^2 = b\eta_+$	$\Phi_- = -i\frac{ab}{8}\Omega$

- Differential conditions
  - one spinor is closed

$$d(e^{3A}\Phi_1) = 0 \to \text{generalised Calabi Yau}$$

  - the RR fields act as torsion

$$d(e^{3A}\Phi_2) = e^{3A} dA \wedge \bar{\Phi}_2 + \frac{i}{8} e^{3A} * \lambda(F)$$

	zero fluxes	fluxes
	T	$T \oplus T^*$
pure spinor	η_0	Φ
integrability	$\nabla_m \eta_0 = 0$	$d\Phi = 0$
	Calabi Yau	Generalised Calabi Yau

- Effective actions on flux backgrounds [grana, louis, waldram 05,06; martucci, koerber 07, 08, ...]
- Moduli counting [tomasiello 07, martucci 09]
- Explicit solutions
  - examples of compactifications on GCY manifolds [grana, minasian, m.p. tomasiello 06]
  - solutions in gauge/gravity duality
    - baryonic branch of Klebanov-Strassler [grana, minasian, m.p. zaffaroni 06]
    - massive deformations of type IIA AdS_4 duals of CFT_3 [m.p, zaffaroni 09, lüst, tsimpis 09]
    - geometry of superconformal $\mathcal{N}=1$ theories [minasian, m.p, zaffaroni 06; gabella, gauntlett, palti, sparks, waldram 09, ...]

- Can we apply similar techniques beyond 4d vacua?
  - product spaces $M_d \times M_{10-d}$
  - black hole solutions
  - non-relativistic geometries
  - non trivial fibrations

- Case by case examples
  - general conditions for $\mathcal{N}=1$ flux backgrounds in $d=3$ and $d=6$ [haack, lüst, martucci, tomasiello 09; lüst, patalong, tsimpis 11]
  - necessary conditions for $\mathcal{N}=1$ flux backgrounds in $d=1$ and even d [koerber, martucci 07; lust, patalong, tsimpis 11]

- 10d Generalised Geometry
  - generalised connection [coimbra, strickland-constable, waldram 11]
  - pure spinor [tomasiello11]

- In this talk
  - description of the 10d pure spinor approach
  - application of the formalism to Lifshitz solutions and D1-D5-P microstates

GENERALISED GEOMETRY AND SUSY IN d=10 [tomasiello 11]

Same logic as in six-dimensions

- build a polyform out of SUSY parameters

$$\Phi = \epsilon_1 \bar{\epsilon}_2$$

- rewrite the SUSY variations as differential eqn on Φ

$\rightarrow$ $e^{-B}\,\Phi$ must contain the same information as metric and B-field

$\rightarrow$ check that this set is equivalent to the SUSY variations

NULL VECTORS

- The SUSY parameters define a pair of null vectors

$$K_{(i)M} = \frac{1}{32}\bar{\epsilon}_i \Gamma_M \epsilon_i \qquad K^M_{(i)} K_{(i)\,M} = 0 \qquad i = 1, 2$$

- A null vector defines a natural splitting in 2 + 8 directions

$$(K_i = e_{-\,i}, e_{+\,i}, e_I) \qquad \Rightarrow \qquad \begin{cases} e_{\pm\,i} \cdot e_{\pm\,i} = 0 \\ e_{\pm\,i} \cdot e_I = 0 \\ e_{-\,i} \cdot e_{+\,i} = 1/2 \end{cases}$$

and the metric

$$\mathrm{d}s^2_{10} = 2 e^+_i e^-_i + \sum_{I=1}^{8} (e^I)^2$$

- Define also

$$K = \frac{1}{2}(K_1 + K_2) \qquad \tilde{K} = \frac{1}{2}(K_1 - K_2)$$

THE POLYFORM

Tensor the 10-d SUSY parameters

$$\Phi = \epsilon_1 \otimes \bar{\epsilon}_2 = \frac{1}{32}\sum_k \frac{1}{k!}\,\bar{\epsilon}_2 \Gamma_{M_k \dots M_1} \epsilon_1 \Gamma^{M_1 \dots M_k}$$

- Spinor on $T(M_{10}) \oplus T^*(M_{10})$ but not pure
  - by construction

$$K_i \epsilon_i = K_{(i)\,M} \Gamma^M \epsilon_i = 0 \qquad \Rightarrow \qquad \overrightarrow{\Gamma}_{-\,i} \epsilon_i = 0$$

  - annihilator of Φ

$$\text{Ann}(\Phi) = \text{span}\{\overrightarrow{\Gamma}_{-1}, \overleftarrow{\Gamma}_{-2}\} \qquad \begin{aligned} \overrightarrow{\Gamma}_M &= \mathrm{d}x^M + \iota_M \\ \overleftarrow{\Gamma}_M &= (-)^{deg}(\mathrm{d}x^M - \iota_M) \end{aligned}$$

- The form of Φ depends on the relation between the null vectors K_1 and K_2
  - K_1 and K_2 proportional

$$K_1 \sim K_2 \sim \overrightarrow{\Gamma}_- = (i\sigma_2 + \sigma_1) \otimes \mathbb{I}_8 \qquad \Rightarrow \qquad \epsilon_i = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \hat{\eta}_i$$

  - K_1 and K_2 not proportional

$$\begin{aligned} K_1 &\sim \Gamma_- = (i\sigma_2 + \sigma_1) \otimes \mathbb{I}_8 \\ K_2 &\sim \Gamma_+ = (i\sigma_2 - \sigma_1) \otimes \mathbb{I}_8 \end{aligned} \qquad \Rightarrow \qquad \epsilon_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \hat{\eta}_1 \qquad \epsilon_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \otimes \hat{\eta}_2$$

	IIA	IIB
$K_1 \sim K_2$	$\hat{\eta}^i$ opposite chirality $\Phi = K \wedge \phi_{G_2}$	$\hat{\eta}^i$ same chirality $\hat{\eta}^1 \sim \hat{\eta}^2 \quad \to \quad \Phi = K \wedge \phi_{SU(4)}$ $\hat{\eta}^1 \perp \hat{\eta}^2 \quad \to \quad \Phi = K \wedge \phi_{Spin(7)}$
$K_1 \perp K_2$	$\hat{\eta}^i$ same chirality $\hat{\eta}^1 \sim \hat{\eta}^2 \quad \to \quad \Phi = e^{-1/2K\wedge\tilde{K}}\phi_{Spin(7)}$ $\hat{\eta}^1 \perp \hat{\eta}^2 \quad \to \quad \Phi = e^{-1/2K\wedge\tilde{K}}\phi_{SU(4)}$	$\hat{\eta}^i$ opposite chirality $\Phi = e^{-1/2K\wedge\tilde{K}}\phi_{G_2}$

SUSY CONDITIONS

- SUSY variations are equivalent to a set of equations on (Φ, e_{+1}, e_{+2})

$$\mathrm{d}_H(e^{-\phi}\,\Phi) = -(\tilde{K}\wedge + \iota_K)F$$
$$\mathrm{d}\tilde{K} = \iota_K H$$
$$(e_{+1}\cdot\Phi\cdot e_{+2}, \Gamma^{MN}[\pm\mathrm{d}_H(e^{-\phi}\Phi\cdot e_{+2}) + e^{\phi}\mathrm{d}^\dagger(e^{-2\phi}e_{+2})\Phi - F])$$
$$(e_{+1}\cdot\Phi\cdot e_{+2}, [\mathrm{d}_H(e^{-\phi}e_{+1}\cdot\Phi) - e^{\phi}\mathrm{d}^\dagger(e^{-2\phi}e_{+2})\Phi - F]\Gamma^{MN})$$

plus $L_K g = 0$ and

$$F = \sum_{k=0}^{5} F_{(2k)} \qquad F = *_{10}\lambda(F)$$

k integer in IIA

k half-integer in IIB

- extra equations (Φ is not pure)
- the vector K is a symmetry of the full solution

LIFSHITZ SOLUTIONS IN TYPE II SUGRA

- Motivation from AdS/CMT
  - study systems (ex strongly correlated electrons) that have critical points with an anisotropic rescaling

$$t \to \lambda^z t \qquad\qquad x^i \to \lambda x^i \quad i = 1 \dots D$$

  - according to the holographic dictionary such behaviour is described by a Lifshitz geometry

$$ds^2 = -r^{2z} dt^2 + r^2 \sum_{i=1}^{D} (dx^i)^2 + \frac{dr^2}{r^2} ,$$

$r \to$ holographic energy direction

- Lifshitz solution in string theory
  - $4d$ models of gravity coupled to a topological term or a massive vector [kachru, liu, mulligan 08; taylor 08, ...]
  - $10d$ solutions : reductions of deformations of AdS solutions [balasubramanian, narayan 10; donos, gauntlett 10, ...]
  - consistent truncations of d=10 and $d = 11$ SUGRA with massive vectors [cassani, faedo 11]
  - solutions of $\mathcal{N} = 2$ SUGRA in $4d$ [halmagyi, m.p. zaffaroni 11]

SUSY LIFSHITZ SOLUTIONS IN IIA THEORY

Look for solutions dual to 3d theories with anisotropic scaling in t and (x, y)

- metric

$$\mathrm{d}s_{10}^2 = -e^{2A_1}\mathrm{d}t^2 + e^{2A_2}(\mathrm{d}x^2 + \mathrm{d}y^2) + (e^1)^2 + \mathrm{d}s_6^2 \qquad qe^1 = \mathrm{d}\phi + \mu$$

  - rotation invariance in (x, y)

- fluxes

$$H^{IIA} = h + \mathrm{d}(e^{01})$$
$$F^{IIA} = -q(e^1 f + e^{0xy} * \lambda(f)) + (1 + e^{01})(w + e^{xy} * \lambda(w))$$

with

$$f = f_1 + f_3 + f_5 \qquad w = w_0 + w_2 + w_4 + w_6$$

SUSY VARIATIONS

- SUSY selects two directions

$$K_1 \cdot \epsilon_1 \;=\; K_2 \cdot \epsilon_2 \;=\; 0$$

- natural choice

$$K \sim e^0 = e^{A_1} \mathrm{d}t \qquad \rightarrow \qquad \text{Killing vector (static solutions)}$$

$$\tilde{K} \sim e^1 = 1/q(\mathrm{d}\varphi + \mu) \qquad\qquad (\mathrm{d}\mu = \alpha)$$

- SUSY conditions

$$\left.\begin{array}{l} (\Gamma^0 + \Gamma^1)\epsilon_1 = 0 \\ (\Gamma^0 - \Gamma^1)\epsilon_2 = 0 \end{array}\right\} \quad \Rightarrow \quad \epsilon_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \hat{\eta}_1 \qquad \epsilon_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \hat{\eta}_2$$

with $\hat{\eta}_{1,2}$ positive chiratlity 8-d spinors

- To construct the spinor Φ

$$\Phi = \epsilon_1 \bar{\epsilon}_2 = -\frac{1}{2}(1 + e^{01})\Phi_{(8)}$$

- further split the spinors

$$\hat{\eta}_i = \frac{2\sqrt{2}}{||\eta_+||} \, e^{A_1/2} \Big[\begin{pmatrix} 1 \\ i \end{pmatrix} \eta^i_+ + \begin{pmatrix} 1 \\ -i \end{pmatrix} \eta^i_- \Big] \qquad i = 1, 2 \,,$$

- define the 6d pure spinors

$$\Phi_\pm = \eta^1_+ \eta^{2\,\dagger}_\pm$$

- so that

$$\Phi_{(8)} = \frac{16}{||\eta_+||^2} \, e^{A_1} \, \{ \mathrm{Im}[(1 + i e^{xy})\Phi_+] - \mathrm{Re}[(e^x - i e^y)\Phi_-] \}$$

- The SUSY conditions and BI reduce to two independent sets of conditions on 6d forms

  - for f, h and $\Phi_\pm$

$$\mathrm{d}_h(q\, e^{A_1-\phi}\frac{1}{||\eta_+||^2}\,\mathrm{Im}\Phi_+) = 0$$
$$\mathrm{d}_h(q\, e^{A_1+2A_2-\phi}\frac{1}{||\eta_+||^2}\,\mathrm{Re}\Phi_+) = \frac{q}{8}\, e^{A_1+2A_2} * \lambda(f)$$
$$\mathrm{d}_h(q\, e^{A_1+A_2-\phi}\frac{1}{||\eta_+||^2}\,\Phi_-) = 0$$

$$\mathrm{d}_h f = \mathrm{d}h = 0$$
$$\mathrm{d}_h(q\, e^{A_1+2A_2} * \lambda(f)) = 0$$
$$\mathrm{d}(qe^{A_1+2A_2-2\phi} * h) = q\, e^{A_1+2A_2} f * f\ |_4\ ,$$

  - for the forms α and w

  - plus a differential equation for q

$$*\mathrm{d}(qe^{2A_2+A_1-2\phi} * \mathrm{d}(qe^{-A_1})) = e^{2A_2}(e^{-2\phi}|\alpha|^2 + |w|^2)\ .$$

From IIB to IIA

- Setting

$$e^{A_1} = \frac{e^{2A_2}}{q}$$

the equations for (f, h, ϕ) become equations for a type IIB SUSY vacuum with 4d Poincaré invariance

$$\begin{aligned} \mathrm{d}(e^{3A}\Phi_-) &= 0 \\ \mathrm{d}(e^{2A}\mathrm{Im}\Phi_+) &= 0 \\ \mathrm{d}(e^{4A}\mathrm{Re}\Phi_+) &= e^{4A}e^{-B} * \lambda(F) \end{aligned}$$

- Main result
  - for a 4d SUSY vacuum in type IIB

$$\mathrm{d}s^2_{10} = e^{2A}(\eta_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu)^2 + \mathrm{d}s^2_6\,, \qquad \mu = 0, \cdots, 3$$
$$H_{IIB} = h \qquad F_{IIB} = f + e^{0xyz} * \lambda(f)$$

  - we can construct a non-relativistic SUSY solution in type IIA

$$\mathrm{d}s^2_{10} = -e^{2A_1}\mathrm{d}t^2 + e^{2A_2}(\mathrm{d}x^2 + \mathrm{d}y^2) + (e^1)^2 + \mathrm{d}s^2_6$$
$$H^{IIA} = h + \mathrm{d}(e^{01})$$
$$F^{IIA} = -q(e^1 f + e^{0xy} * \lambda(f)) + (1 + e^{01})(w + e^{xy} * \lambda(w))$$

with

$$e^{A_1} = \frac{e^{2A}}{q} \qquad e^{A_2} = e^A \qquad e^{\phi_A} = \frac{e^\phi}{q}$$

provided $\exists$ on M_6 α and $w = \sum_{k=0}^{3} w_{2k}$ satisfying the constraints above.

Lif_4 from AdS_5 solutions

- type IIB : conformal Calabi-Yau cone over the SE with

$$\phi = 0 \qquad h = 0 \qquad *f = 4\frac{\mathrm{d}r}{r}$$

- IIA Lif$_4$ solutions on $U(1)$ fibrations over a 5d Sasaki-Einstein Y with [donos, gauntlett 10]

$$\begin{aligned}
B &= \frac{r^2}{q^2}\mathrm{d}t \wedge (\mathrm{d}\varphi + \mu) \\
F_2 &= w_2 \\
F_4 &= -4r^3 \mathrm{d}t \wedge \mathrm{d}x^1 \wedge \mathrm{d}x^2 \wedge \mathrm{d}r + \frac{r^2}{q^2}\mathrm{d}t \wedge (\mathrm{d}\varphi + \mu) \wedge w_2 \\
e^{-2\phi} &= q^2 \qquad\qquad (4q^2 - \Box_Y q^2 = |\alpha|^2 + |w_2|^2)
\end{aligned}$$

with α , $w = w_2$ type $(1,1)$, primitive and harmonic on Y

- Explicit solutions for q for $T^{1,1}$ ($q = cost$) and some $Y^{p,q}$

More general solutions

- Asymptotically Lif_4 solutions
  - Supersymmetric domain walls in type IIB (M_6 is a conformal Calabi-Yau)
    - first example [klebanov, murugan 07]

$$\mathrm{AdS}_5 \times \mathrm{T}^{(1,1)} \quad \to \quad \mathrm{AdS}_5 \times \mathrm{S}^5$$

    Similar solutions for all resolved CY_6 [martelli, sparks 08]
  - non-relativistic type IIA solution solutions interpolating between Lif_4 vacua

- Solutions with hyperscaling violation [dong, harrison, kachru, torroba, wang 12]

$$\mathrm{d}s^2 \to \lambda^{2\theta/D}\mathrm{d}s^2 \qquad t \to \lambda^z t \qquad x^i \to \lambda x^i \qquad u \to \lambda u$$

  - IIB vacuum on a conic Calabi-Yau manifold
  - IIA solution with $z = 3$ $D = 2$ and $\theta = 2$ [narayan 12; dey, roy 12]

$$e^{\phi_A} = r \qquad q = e^{-A} = 1/r$$

$$H^{IIA} = \mathrm{d}(r^4 \mathrm{d}t \wedge \mathrm{d}\varphi)\,, \qquad F_4 = -4r^3 \mathrm{d}t \wedge \mathrm{d}x^1 \wedge \mathrm{d}x^2 \wedge \mathrm{d}r \qquad \alpha = w = 0$$

D1-D5-P MICROSTATES

- Construct regular, horizonless solutions with the same asymptotic as the SV black-hole

	$\mathbb{R}$	S^1	$\mathbb{R}^4$	T^4
P	x	x		
D1	x	x		
D5	x	x		x

- Known regular three charge solutions
  - $5d$ supergravity [bena, warner, 05; berglund, gimon, levi 0]
    - $1/8$ BPS solutions smeared over the compact space $S^1 \times T^4$
  - Minimal $6d$ supergravity plus (at most) one tensor multiplet [bena, giusto, shigemori, warner, 11]
    - no smearing in the S^1

- The generic microstate geometries should be more general
  - no smearing in S^1
  - all type IIB fields are turned on
- hints from
  - entropy counting [bena, wang, warner, 06; de Boer, el-showk, messamah, van den bleeken, 08, 09]
  - worldsheet analysis [giusto, russo, 12]
  - dual CFT [kanitscheider, skenderis, taylor, 07]
- Use GCG to find solutions directly in ten dimensions

P-D1-D5 GEOMETRIES

- Conditions for SUSY bounds states of P-D1-D5
  - existence of a null Killing vector

$$K = \frac{\partial}{\partial u} \qquad u = \frac{1}{\sqrt{2}}(t-y) \qquad v = \frac{1}{\sqrt{2}}(t+y)$$

  - fix the polyform

$$\Phi = \frac{1}{\sqrt{2}}(1 + e^{4G}\mathrm{vol}_4 + e^{4\hat{G}}\hat{\mathrm{vol}}_4 - e^{2G+2\hat{G}}\sum_{A=1}^{3} J_A \wedge \hat{J}_A \,. + e^{4G+4\hat{G}}\mathrm{vol}_4 \wedge \hat{\mathrm{vol}}_4)$$

where

$J_A, \hat{J}_A \rightarrow$ SU(2) structures on Y^4 and T^4

SOLUTION

- Assume that T^4 is rigid and all fields are isotropic along T^4
- General form of the solution
  - metric

$$ds^2_{(10)} = -\frac{2\alpha}{\sqrt{Z\tilde{Z}}}\,(dv+\beta)\left[du+\omega+W(dv+\beta)\right]+\sqrt{Z\tilde{Z}}\,ds_4^2+\sqrt{\frac{Z}{\tilde{Z}}}\,d\hat{s}_4^2\,.$$

  - dilaton

$$e^{2\phi}=\alpha\,\frac{Z}{\tilde{Z}}\,.$$

  - NS and RR fields are completely determined as functions of

$$\omega,\beta,\mathcal{W},Z,\tilde{Z},Z_b,\Theta,\tilde{\Theta},\Theta_b$$

- the functions and forms above must satisfy the constraints

$$*_4\mathcal{D}\beta = \mathcal{D}\beta$$
$$\mathcal{D}J_A - \dot{\beta} \wedge J_A = 0$$

$$\mathcal{D}\omega + *_4\mathcal{D}\omega = Z *_4\Theta + \tilde{Z}\,\tilde{\Theta} - Z_b\,(\Theta_b + *_4\Theta_b) - 2\,W\,\mathcal{D}\beta\ .$$

$$\mathcal{D}\Theta - \dot{\beta} \wedge \Theta = \frac{\mathrm{d}}{\mathrm{d}v} *_4 (\mathcal{D}\tilde{Z} + \tilde{Z}\dot{\beta})$$
$$\mathcal{D}\tilde{\Theta} - \dot{\beta} \wedge \tilde{\Theta} = \frac{\mathrm{d}}{\mathrm{d}v} *_4 (\mathcal{D}Z + Z\dot{\beta})$$
$$\mathcal{D}\Theta_b - \dot{\beta} \wedge \Theta_b = \frac{\mathrm{d}}{\mathrm{d}v} *_4 (\mathcal{D}Z_b + Z_b\dot{\beta})$$

$$\mathcal{D} *_4 (\mathcal{D}Z + \dot{\beta}Z) = -\tilde{\Theta} \wedge \mathcal{D}\beta$$
$$\mathcal{D} *_4 (\mathcal{D}\tilde{Z} + \dot{\beta}\tilde{Z}) = -\Theta \wedge \mathcal{D}\beta$$
$$\mathcal{D} *_4 (\mathcal{D}Z_b + \dot{\beta}Z_b) = -\Theta_b \wedge \mathcal{D}\beta$$

and Einstein equation in vv direction

EXAMPLE

- Start from Mathur, Saxena and Srivastava solution
  - first example of a microstate geometry for the three-charge black hole
  - deformation of the D1-D5 geometry corresponding to a RR state in the dual CFT carrying one unit of momentum
- embedding it in our 10d ansatz
  - determine the non linear completion
  - extend it to the asymptotically flat region

CONCLUSIONS AND OUTLOOK

- GCG can be used to study fully 10d geometries
  - less insight into the geometric structure of the solutions
  - powerful tool to compute explicit solutions
    - non relativistic solutions and black hole microstate
- Study more formal properties

  $\rightarrow$ role of symmetries

  $\rightarrow$ solution generating techniques

  $\rightarrow$ relation to gauge supergravity and effectve actions Extend the analysis to more general backgrounds

SPINOR vs METRIC AND B-FIELD

- G-structures and metric
  - a metric defines an O(d) structure
  - a G-structure determines the metric if $G \subset O(d)$

- Same argument on $T \oplus T^*$
  - the metric plus B-field define

$$O(9,1) \times O(9,1) \quad \text{structure} \quad \rightarrow \quad \{\overrightarrow{\Gamma}_{MN}, \overleftarrow{\Gamma}_{MN}\}$$

  - Φ defines

$$\begin{array}{c}(Spin(7))^2 \times SL(2,\mathbb{R}) \ltimes \mathbb{H}_{33} \\ \text{structure}\end{array} \quad \rightarrow \quad \left\{ \begin{array}{c} \omega_{21}^{I_1 J_1} \overrightarrow{\Gamma}_{I_1 J_1}, \omega_{21}^{I_2 J_2} \overleftarrow{\Gamma}_{I_2 J_2}, \\ \overrightarrow{\Gamma}_{-1 I_1}, \overleftarrow{\Gamma}_{-2 I_2}, \overrightarrow{\Gamma}_{+1-1} + \overleftarrow{\Gamma}_{+2-2}, \\ \overrightarrow{\Gamma}_{-1} \overleftarrow{\Gamma}_{I_2}, \overrightarrow{\Gamma}_{I_1} \overleftarrow{\Gamma}_{-2}, \\ \overrightarrow{\Gamma}_{-1} \overleftarrow{\Gamma}_{+2}, \overrightarrow{\Gamma}_{+1} \overleftarrow{\Gamma}_{-2}, \overrightarrow{\Gamma}_{-1} \overleftarrow{\Gamma}_{-2} \end{array} \right\}$$

- Since $G \supset O(9,1) \times O(9,1)$, we need extra objects not invariant under $\overrightarrow{\Gamma}_M \overleftarrow{\Gamma}_N$

$$(\Phi, \overrightarrow{\Gamma}_{+1}, \overleftarrow{\Gamma}_{+2})$$

Then

$$\mathrm{Ann}(\Phi, \overrightarrow{\Gamma}_{+1}, \overleftarrow{\Gamma}_{+2}) = Spin(7) \times Spin(7) \subset O(9,1) \times O(9,1)$$

- Not such a problem in $d = 6$

$$SU(3) \times SU(3) \subset O(6) \times O(6)$$