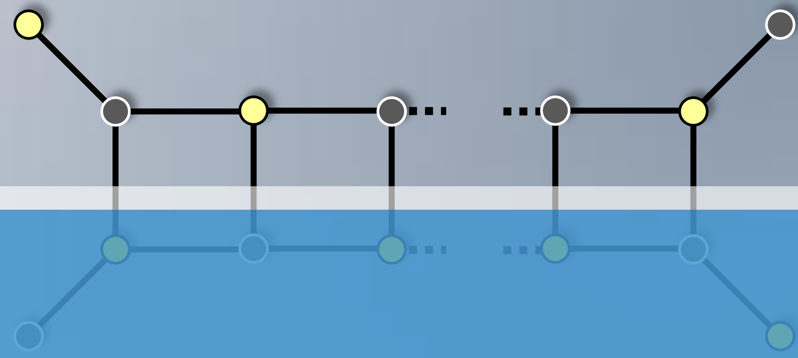
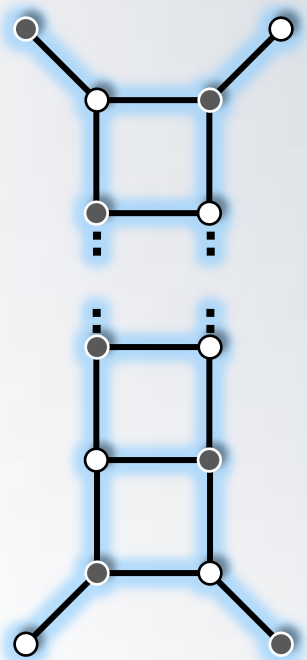
A New Class of QFTs



From D-branes to on-shell diagrams



Sebastián Franco

IPPP Durham University

Based on: **arXiv:1207.0807**, S. Franco
arXiv:1211.5139, S.Franco, D. Galloni and R.-K. Seong
arXiv:1301.0316, S.Franco
arXiv:1306.6331, S.Franco and A. Uranga
S. Franco, D. Galloni, A Mariotti (in progress)

Outline

- Introduction and Motivation
- A new class of gauge theories: Bipartite Field Theories (BFTs)
- BFTs Everywhere
  - D3-Branes over CY 3-folds
  - Cluster Integrable Systems
  - On-Shell Diagrams
- BFTs and Calabi-Yau Manifolds
- String Theory Embedding
- Conclusions and Future Directions

Introduction and Motivation

- Over the last decade, we have witnessed remarkable progress in our understanding of Quantum Field Theory in various dimensions

- Most celebrated example: **Gauge/Gravity Correspondence**

QFT ⇄ **String Theory**

$SCFT_d$ $AdS_{d+1} \times X^{9-d}$

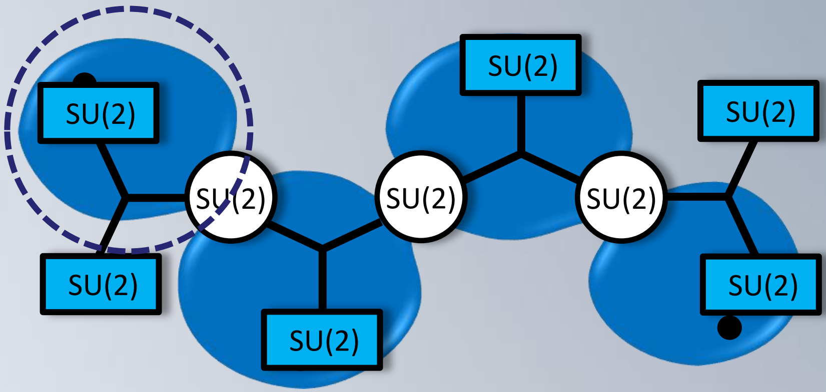
More generally, SUSY gauge theories and geometry are intimately related (e.g. moduli spaces, Seiberg-Witten theory, etc)

Many other directions:

- Duality
- Integrability
- Scattering amplitudes
- RG flows and degrees of freedom
- Superconformal index
- Conformal bootstrap

- **General trend:** defining SUSY gauge theories in terms of geometric or combinatorial objects such as bipartite graphs on a 2-torus, Riemann surfaces (Gaiotto and Sicilian theories) or 3-manifolds

  - ✱ Complicated theories are engineered by sewing or gluing elementary building blocks
  - ✱ QFT dualities correspond to rearrangements of the underlying geometric object



- Today we will discuss a new class of quiver gauge theories, whose Lagrangian are specified by bipartite graphs on bordered Riemann surfaces

- These theories are related to a variety of interesting physical systems, such as D3-branes on CY3-folds, cluster integrable systems and scattering amplitudes

- Furthermore, they combine several interesting ideas in the modern approach to QFTs

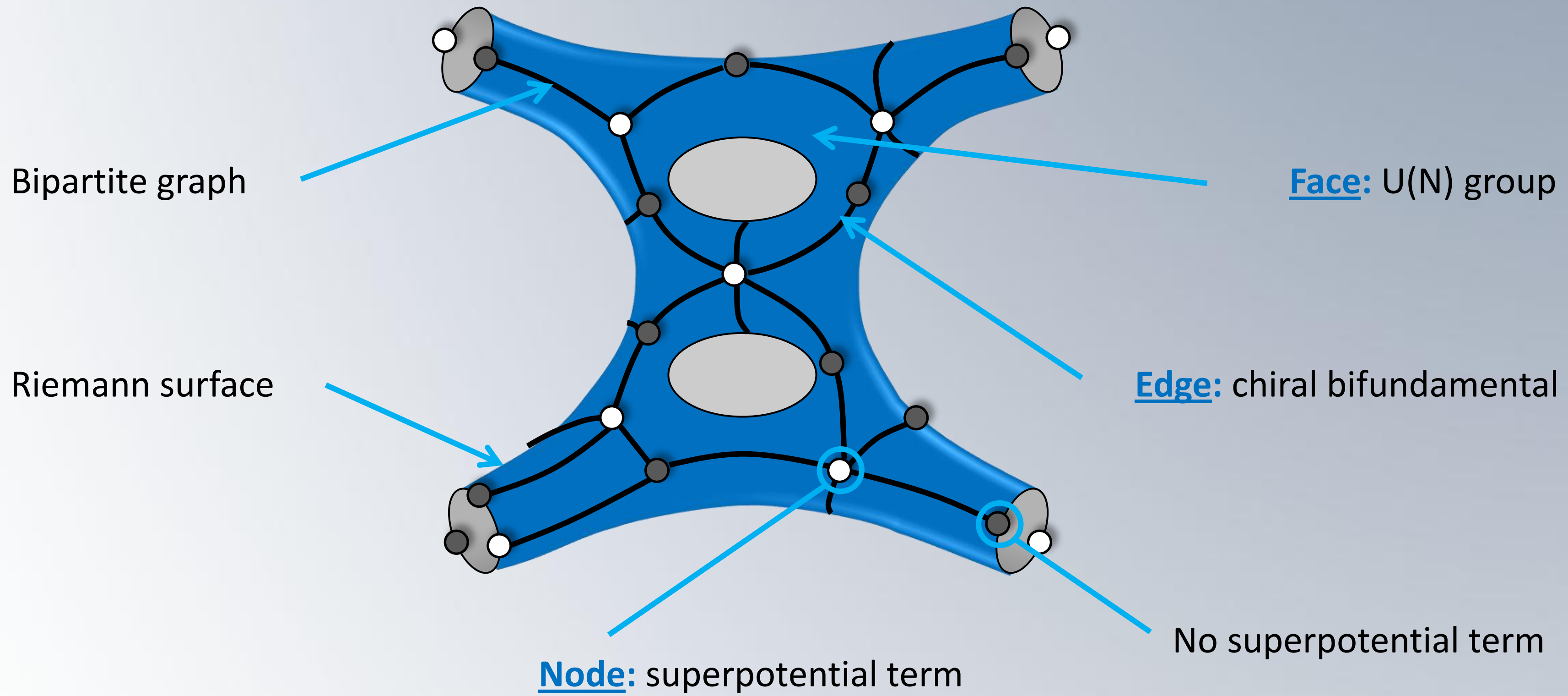
Bipartite Field Theories

Franco

See also: Yamazaki, Xie

Bipartite Field Theory (BFT) ⇨ a 4d N=1 gauge theory whose Lagrangian is defined by a bipartite graph on a Riemann surface (with boundaries)

- **Bipartite Graph:**
  - Every edge connects nodes of different color
  - Every boundary node is connected to a single edge



- **Node color:**
  - Sign of corresponding superpotential term
  - Chirality of bifundamental fields

Gauge and Global Symmetries

- There are two types of faces in the graph:

  - **Internal faces:** automatically anomaly free → **Gauged**
  - **External faces** → **Global**

- This is a rather natural choice in cases in which the graph has a brane interpretation

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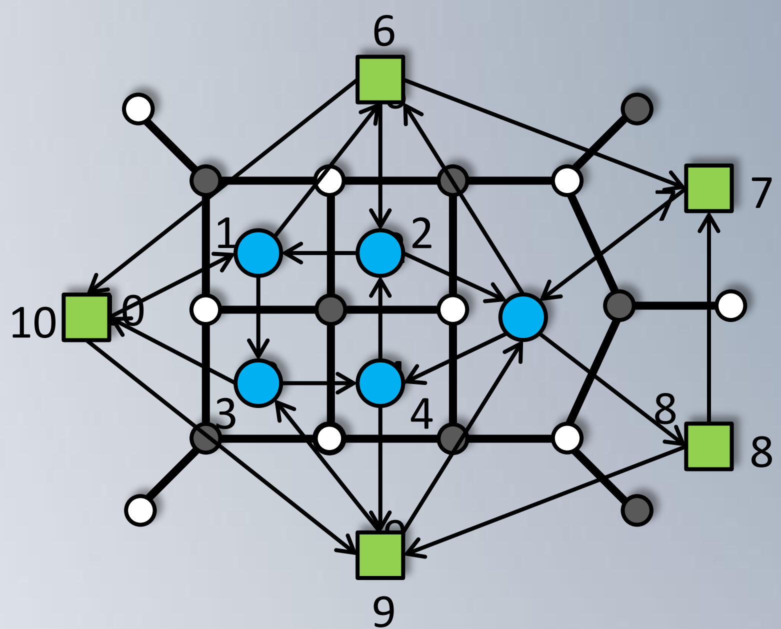
- The BFT is given by a quiver dual to the bipartite graph



Graph	BFT
Internal face (2n-sided)	Gauge group with n flavors
External face	Global symmetry group
Edge between two faces	Chiral multiplet in the bifundamental representation
k-valent node	Monomial in the superpotential involving k chiral multiplets, with (+/-) for (white/black) nodes

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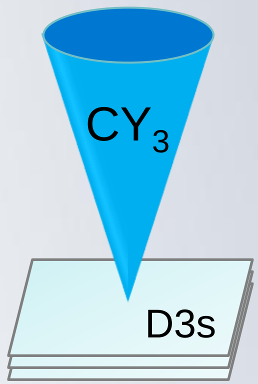
BFTs Everywhere

BFTs Everywhere

1. D3-Branes over CY 3-folds

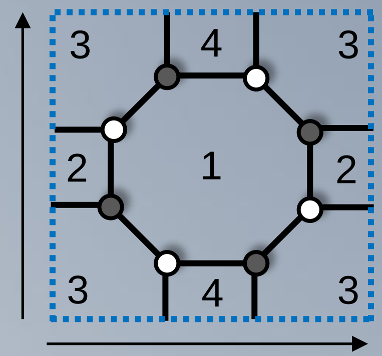
- The 4d, N = 1 SCFT on a stack of D3-branes probing a toric CY_3 is a BFT on a 2-torus

Franco, Hanany, Kennaway, Vegh, Wecht



Example: cone over F_0

$U(1)^2$ global symmetry



- Cluster Algebras and CYs — **Eager, SF**
- Mirror Symmetry — **Feng, He, Kennaway, Vafa**
- Toric/Seiberg Duality — **SF, Hanany, Kennaway, Vegh, Wecht**
- D-brane Instantons — **SF, Hanany, Krefl, Park, Uranga**
- AdS/CFT Correspondence in 3+1 and 2+1 dimensions — **SF, Hanany, Martelli, Sparks, Vegh, Wecht**; **SF, Hanany, Park, Rodriguez-Gomez**; **SF, Klebanov, Rodriguez-Gomez**
- Dynamical SUSY Breaking — **SF, Uranga**
- Local constructions of MSSM + CKM — **Quevedo et. al.**

- Mirror symmetry relates this configuration to a system of D6-branes that is encoded by another BFT on an higher genus Riemann surface Σ

Feng, He, Kennaway, Vafa

2. Integrable Systems

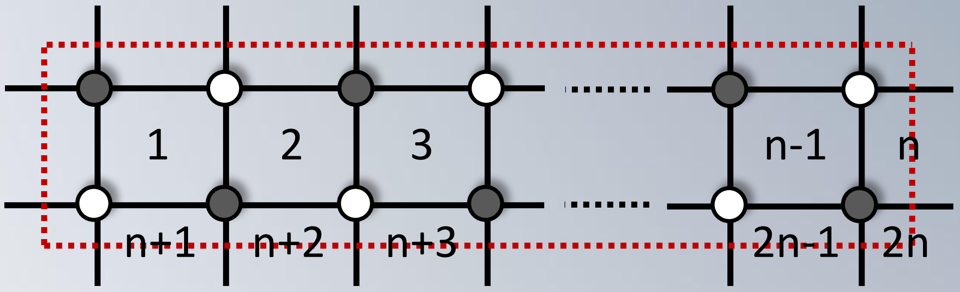
- Bipartite graphs on a 2-torus are also in one-to-one correspondence with an infinite class of integrable systems in (0+1) dimensions: Cluster Integrable Systems

Goncharov, Kenyon

Franco

Eager, Franco, Schaeffer

Franco, Galloni, He



E.g.: n-particle, relativistic, periodic Toda chain

- Constructing all integrals of motion is straightforward and combinatorial

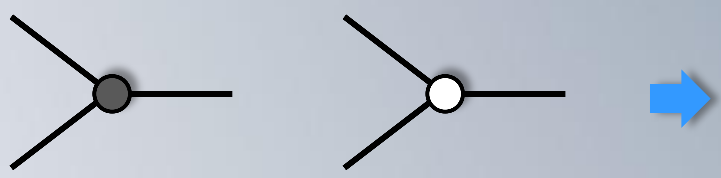
- Rich connections to other scenarios in which these integrable systems appear, such as 5d N=1 (on S^1) and 4d N=2 gauge theories → avatars of the spectral curve Σ

3. Scattering Amplitudes

- Recently, a connection between scattering amplitudes in planar N = 4 SYM, the Grassmannian and bipartite graphs has been established

  Arkani-Hamed, Bourjaily, Cachazo, Goncharov, Postnikov, Trnka

- The central idea is to focus on on-shell diagrams. They are constructed by combining 3-point MHV and $\overline{\text{MHV}}$ amplitudes



Bipartite graphs

- **<u>The Grassmannian G(k, n)</u>:** space of k-dimensional planes in n dimensions

Points in G(k,n):

$$C = \underbrace{\begin{pmatrix} * & * & * & * \\ * & * & * & * \\ * & * & * & * \end{pmatrix}}_{\leftarrow n \rightarrow} \; \updownarrow k$$

Rows: n-dimensional vectors spanning the planes

n = # scattered particles   k = # negative helicity

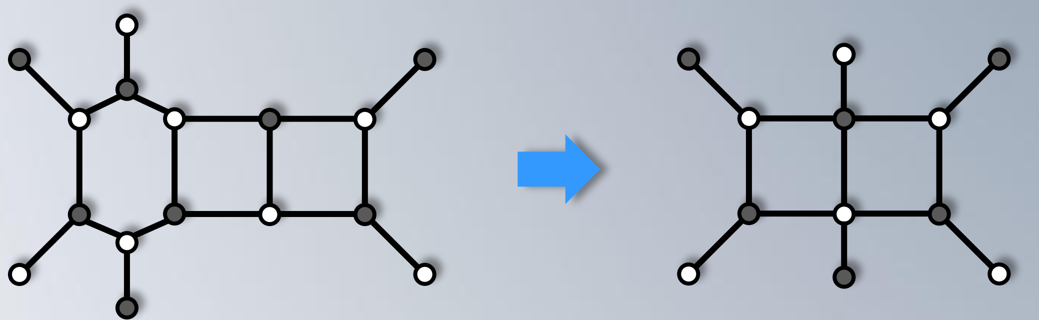
- Leading singularities are in one-to-one correspondence with certain subspaces, also denoted cells, of G(k,n) parametrized by a constrained matrix C

On-shell diagrams ⟷ Cells in G(k,n) ⟷ Bipartite graphs

Postnikov

- Scattering amplitudes can be determined in terms of on-shell diagrams

- All necessary information for determining leading singularities (equivalently cells in the Grassmannian) is contained in certain minimal or reduced graphs



The additional data in reducible graphs is necessary for determining the loop integrand

$$\text{Loop Integrand} = \prod d\log f_i \times (\text{Reduced Graph})$$

- The on-shell approach is equivalent to a U(1) gauge theory on the graph which, in turn, is equivalent to an Abelian BFT

Arkani-Hamed, Bourjaily, Cachazo, Goncharov, Postnikov, Trnka

BFT Dynamics

Graphical Gauge Theory Dynamics

Franco

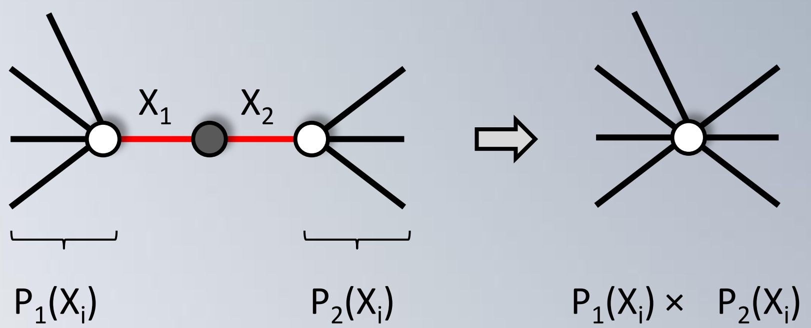
Yamazaki, Xie

Gauge Theory Dynamics ⟺ Graph Transformations

- Let us first focus on IR equivalences

Massive Fields

- 2-valent nodes map to mass terms in the BFT. Integrating out the corresponding massive fields results in the condensation of the two nearest nodes



$$W = X_1 P_1(X_i) + X_2 P_2(X_i) - X_1 X_2 + \cdots$$

The equations of motion of the massive fields become:

$$\left.\begin{array}{l} \partial_{X_1} W = 0 \;\Leftrightarrow\; X_2 = P_1(X_i) \\ \partial_{X_2} W = 0 \;\Leftrightarrow\; X_1 = P_2(X_i) \end{array}\right\} \Rightarrow \quad W = P_1(X_i) P_2(X_i) + \cdots$$

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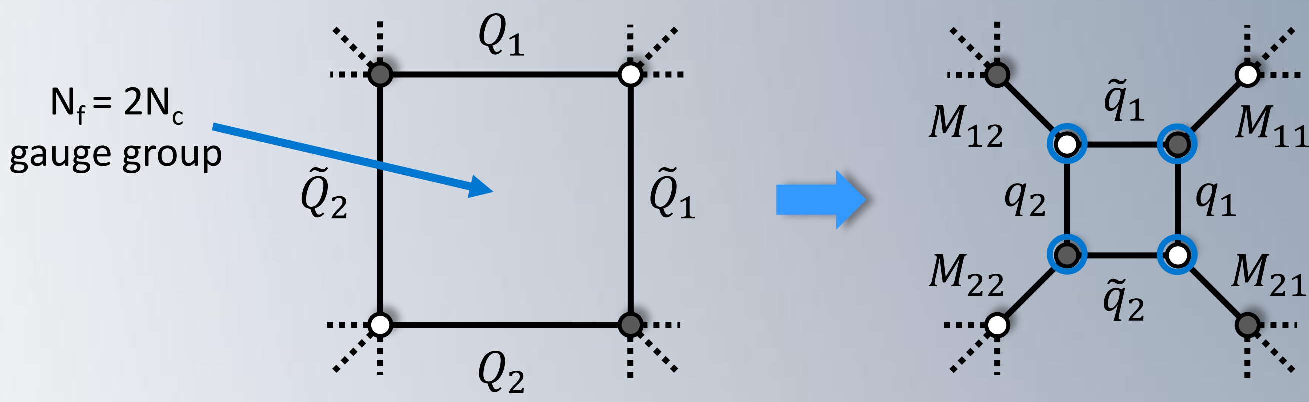
Seiberg Duality

- Seiberg duality of an $N_f = 2N_c$ gauge group translated into a "square move"

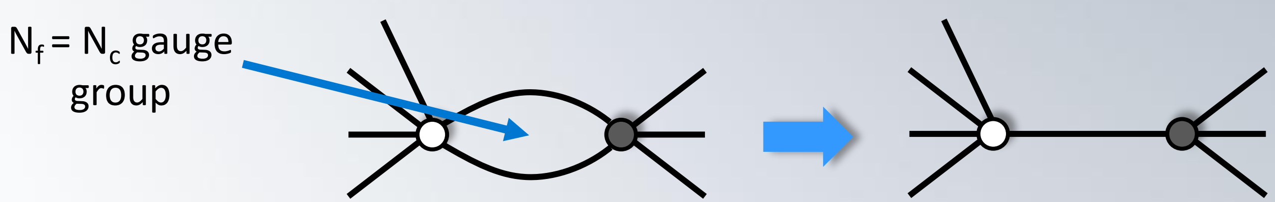


This transformation:

1) Replaces electric quarks by magnetic quarks
2) Introduces mesons: $M_{ij} = \tilde{Q}_i Q_j$
3) Cubic superpotential couplings: $\Delta W = \sum_{ij} \; q_i M_{ij} \tilde{q}_j$

Confinement

- When an $N_f = N_c$ gauge group confines, the corresponding face is eliminated



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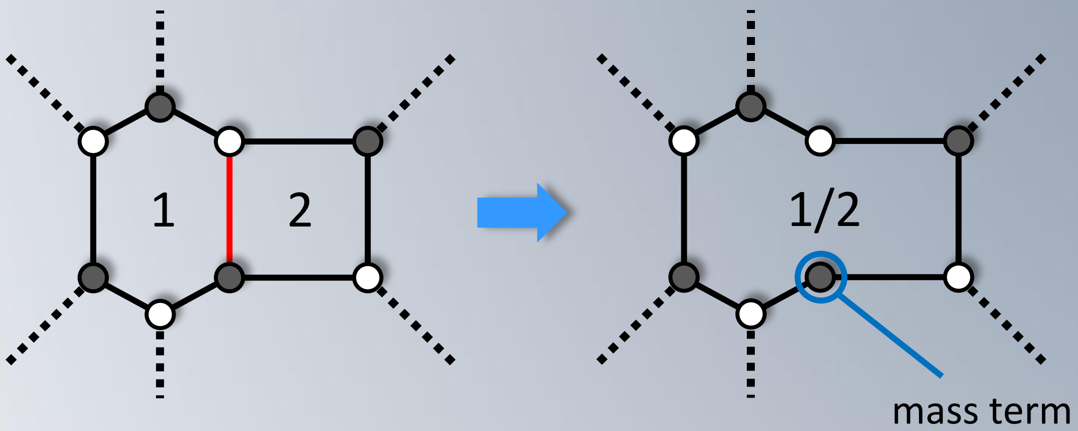
* Other gauge theory transformations also have a natural graphic implementation

Higgsing

* Removing an edge and combining two faces into a single one

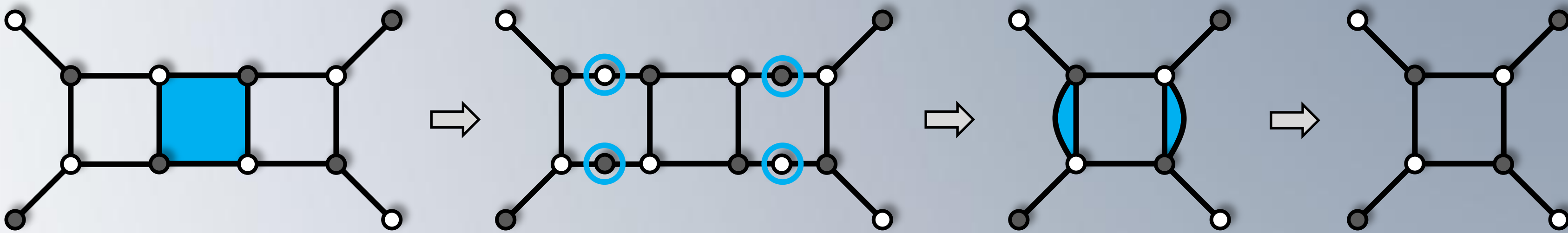


$U(N) \times U(N) \longrightarrow U(N)_d$

* This corresponds to a non-zero VEV for the scalar component of the removed edge
* More generally, Higgsing translates into:
  * The boundary operator on cells in the positive Grassmannian — **Franco**
  * BCF bridges in on-shell diagrams — **Arkani-Hamed et. al.**

Reduced Graphs

- **Reduced Graph:** a BFT with minimal gauge symmetry within an IR equivalence class / a graph with the minimal number of internal faces within an IR equivalence class connected by moves and reductions
- Only defined up to equivalence moves ⟶ not unique



Example:

1) Seiberg dualize an $N_f = 2N_c$ gauge group

2) Integrate out massive fields

3) Confine $N_f = N_c$ gauge groups

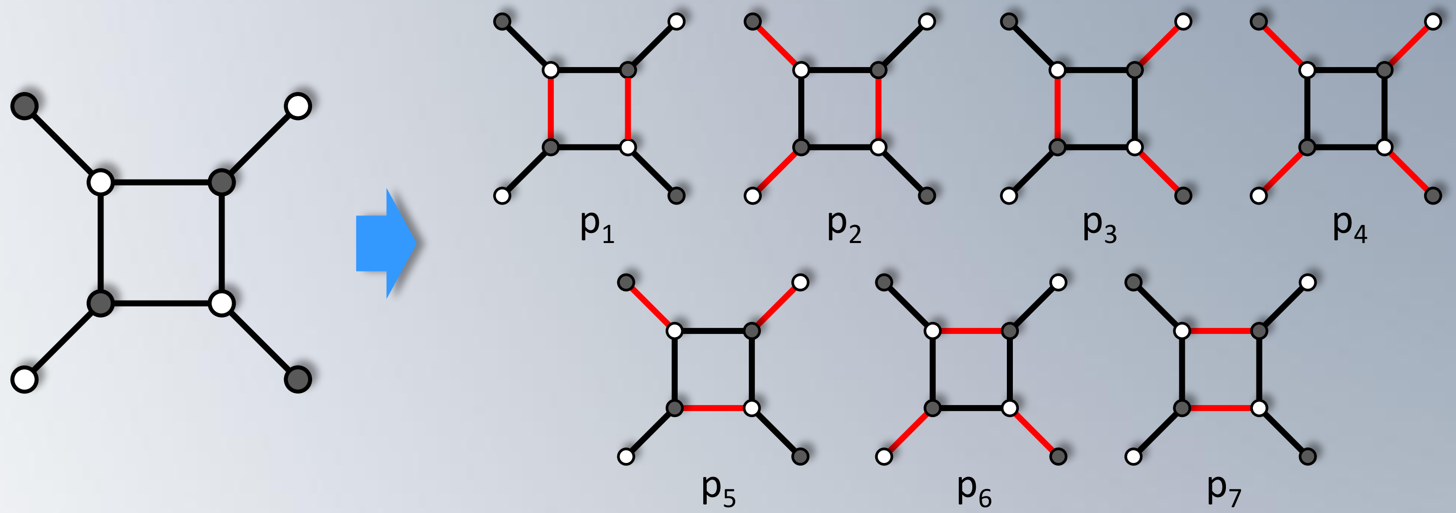
- Reduced graphs play a central role in scattering amplitudes ⟷ Leading Singularities

Arkani-Hamed, Bourjaily, Cachazo, Goncharov, Postnikov, Trnka

- Two important questions:
  - How to determine whether two graphs are connected by mergers and moves? And reductions?
  - How to identify reduced graphs

Perfect Matchings

- Perfect matchings play a central role in connecting BFTs to geometry
- **(Almost) Perfect Matching:** p is a subset of the edges in the graph such that:
  - Every internal node is the endpoint of exactly on edge in p
  - Every external node belongs to either one or zero edges in p



- Finding the perfect matchings reduces to calculating the determinant of an adjacency matrix of the graph (Kasteleyn matrix), and some generalizations

Franco

- Map between chiral fields in the quiver X_i and perfect matchings p_m:

$$X_i = \prod_{\mu} p_\mu^{P_{i\mu}} \qquad P_{i\mu} = \begin{cases} 1 & \text{if } X_i \in p_\mu \\ 0 & \text{if } X_i \notin p_\mu \end{cases}$$

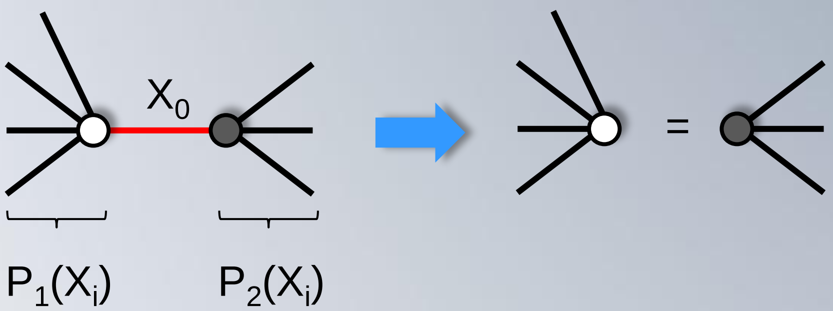
- BFTs are naturally associated to certain geometries via their moduli spaces
- The moduli space of any BFT is automatically a toric CY and perfect matchings simplify its computation. We can identify them with GLSM fields in their toric description

F-Flatness and Perfect Matchings

- For any bifundamental field X_0 associated to an internal edge:

$$W = X_0 P_1(X_i) - X_0 P_2(X_i) + \cdots \longrightarrow \partial_{X_0} W = 0 \Leftrightarrow P_1(X_i) = P_2(X_i)$$

Graphically:



- The parametrization of bifundamental fields in terms of perfect matchings given by the matrix P, automatically satisfies F-term equations of internal edges

$$\prod_{i\in P_1}\prod_{\mu} p_\mu^{P_{i\mu}} = \prod_{i\in P_2}\prod_{\mu} p_\mu^{P_{i\mu}}$$

$$P_{i\mu} = \begin{cases} 1 & \text{if } X_i \in p_\mu \\ 0 & \text{if } X_i \notin p_\mu \end{cases}$$

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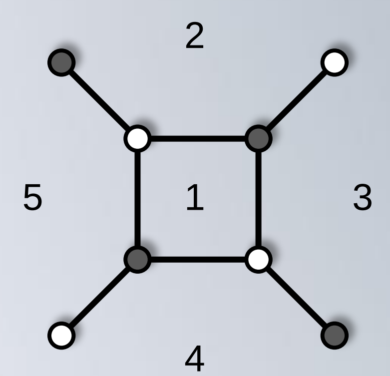
The Master Space

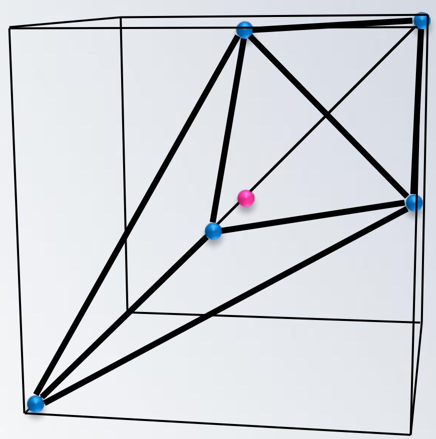
Master space
- space of solutions to F-term equations — Forcella, Hanany, He, Zaffaroni
- parametrized in terms of perfect matchings (GLSM fields) and toric

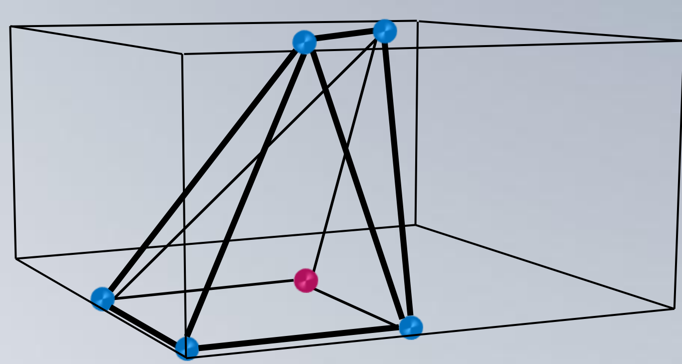
Relations ⟷ GLSM charges **Toric Diagram**

Example:



- 7 perfect matchings
- 5d toric CY





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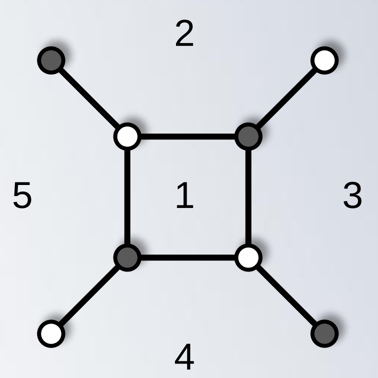
Moduli Space

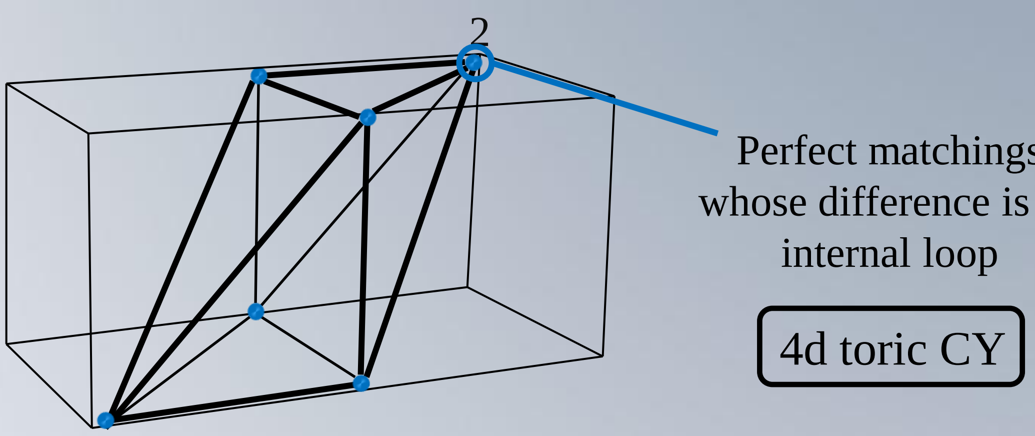
- Space of solutions to vanishing F and D-terms
- Projection of the Master Space onto vanishing D-terms

- One D-term contribution for every gauge group ⟷ internal face of the graph

- Example:

$$Q_1 = \left(\begin{array}{ccccccc} p_1 & p_2 & p_3 & p_4 & p_5 & p_6 & p_7 \\ \hline 0 & 1 & 1 & 0 & -1 & -1 & 0 \end{array}\right)$$





- The moduli space is invariant under all equivalence moves (integrating out massive fields, Seiberg duality, etc) → Ideal diagnostic for identifying graphs related by them

- It applies to completely general BFTs. Other methods, e.g. permutations, exist for planar graphs

	Master Space	Moduli Space
Useful for:	**Higgsing** (Edge removal)	**IR Equivalences** (Moves and reductions)
Grassmanian:	Matching Polytope	Matroid Polytope

- Remarkably, when restricting to planar graphs, precisely these two structures arise in the classification of cells in the positive Grassmannian

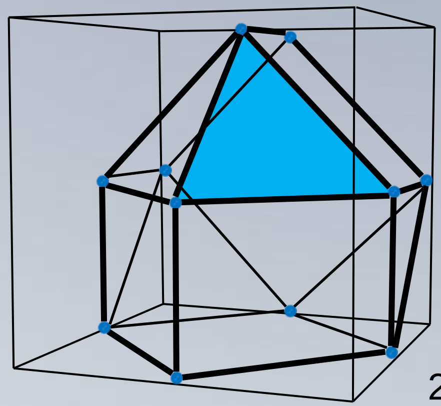
Franco
Franco, Galloni, Seong

Postnikov, Speyer, Williams
Arkani-Hamed, Bourjaily, Cachazo, Goncharov, Postnikov, Trnka

> BFTs thus provide natural generalizations of these objects beyond planar graphs

IR fixed points ⟷ Cells in the Grassmannian
Leading Singularities

Higgsing ⟷ Boundary operator
BCF bridges



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String Theory Embedding

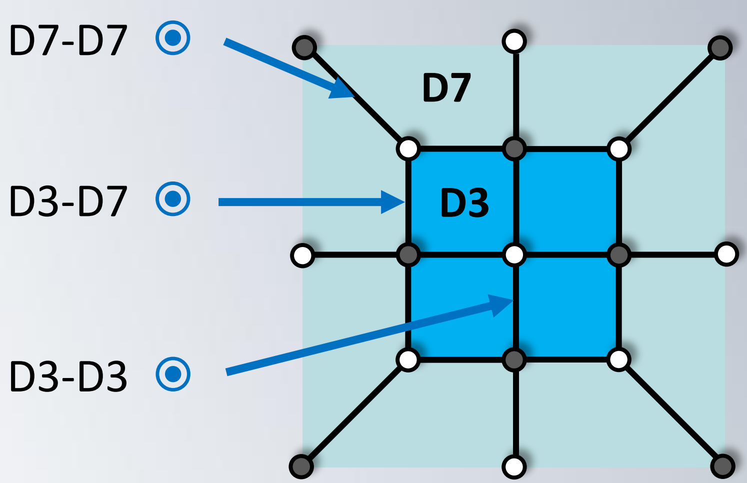
Stringy Embedding of BFTs

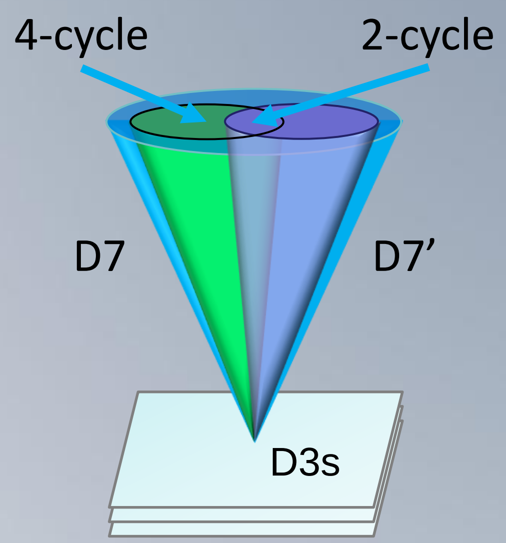
- It is possible to engineer generic planar BFTs using D-branes over toric CY 3-folds

Franco, Uranga

Heckman, Vafa, Yamazaki, Xie *(sub-classes)*

- Internal faces ⟷ Fractional D3-branes
- External faces ⟷ Flavor D7-branes





- D3-branes on toric CY 3-folds correspond to bipartite graphs on T^2

Franco, Hanany, Kennaway, Vegh, Wecht

- We developed a general framework for computing the spectrum and superpotential interactions for a general D-brane configuration over toric singularities

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- **Mirror Geometry:**

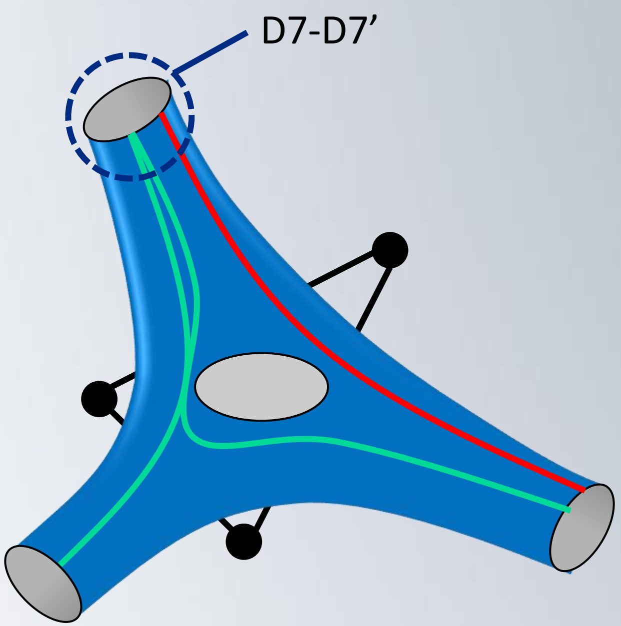
$$\begin{cases} P(x,y) = w \\ u\,v = w \end{cases}$$

$$\Sigma: \quad P(x,y) = 0$$

- The bipartite graph is mapped from T^2 to the Riemann surface Σ, which controls the physics

Feng, He, Kennaway, Vafa

Franco, Uranga



Example: C^3/Z_3

	D6-branes over 3-cycles (1-cycles on Σ)
D3-branes	compact
D7-branes	non-compact

pair of punctures ⟷ 4-cycle

full trajectory ⟷ "Chan-Paton" degrees of freedom

- The spectrum and interactions can be straightforwardly determined from the mirror configuration. E.g.: non-compact 1-cycles sharing a puncture lead to D7-D7' states

- Infinite families of planar and non-planar BFTs can be explicitly engineered in terms of D-branes using these tools

Conclusions

- We introduced BFTs, a new class of 4d, N = 1 gauge theories defined by bipartite graphs on Riemann surfaces. We also developed efficient tools for studying them.

  - Gauge theory dynamics is captured by simple graph transformations
  - CY manifolds emerge as moduli spaces
  - BFTs provide natural generalizations, based on standard N=1 gauge theory knowledge, of Grassmannian objects (e.g. matching and matroid polytopes) beyond the planar case

    Franco **Postnikov, Speyer, Williams**

    Franco, Galloni, Seong **Arkani-Hamed, Bourjaily, Cachazo, Goncharov, Postnikov, Trnka**
  - For planar graphs, global classification of IR fixed points by cells in the Grassmannian

- We developed a full understanding of D3-D7 systems on toric CYs. This provides a D-brane embedding of BFTs but has many other applications

- BFTs provide an alternative perspective on various equivalent systems: D-brane probes, integrable systems and on-shell diagrams

Other Topics

- Field theory interpretation of cluster transformations **Franco**

- Non-planar → planar reductions **Franco, Galloni, Seong**

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The Future

- Explore the role of moduli space CYs for scattering amplitudes beyond the planar case

- BFTs generate ideal triangulations of Riemann surfaces (Seiberg-Witten and Gaiotto curves of 4d, N=2 theories) → N=2 BPS quivers

  Franco

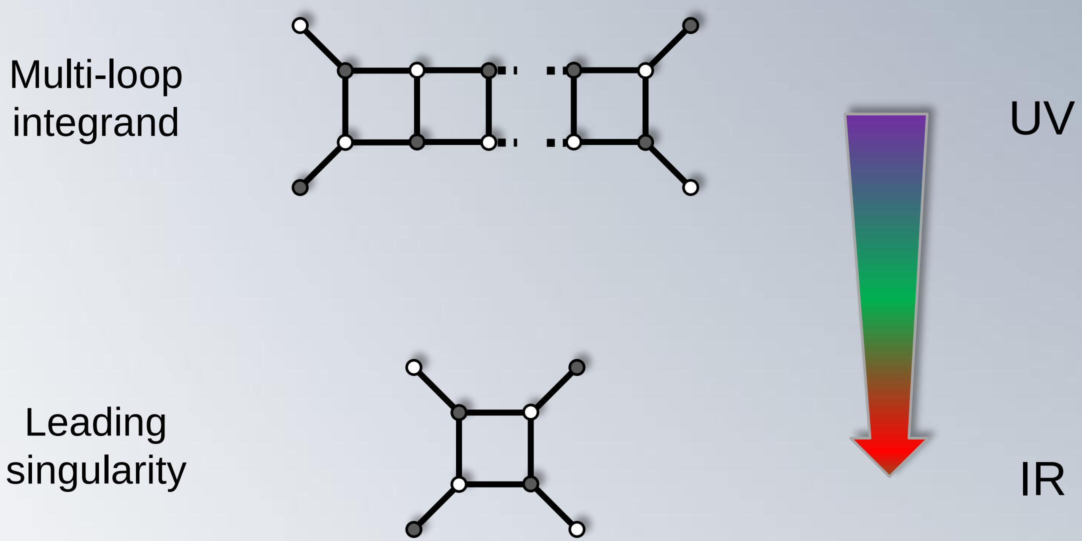
  Alim, Cecotti, Cordova, Espahbodi, Rastogi, Vafa

  Heckman, Vafa, Yamazaki, Xie

- New approaches to the positroid and matroid stratification of the Grassmannian

  Franco, Galloni, Mariotti *(in progress)*

- **Reducibility and Gauge Theory Dynamics**

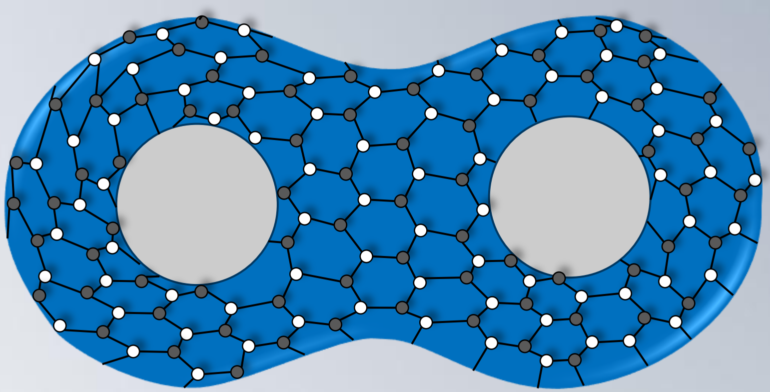


- RG flow interpretation of graph reductions?
- Field theoretic criterion for graph reducibility?
- If so, can we map the classification of leading singularities to a classification of IR fixed points?

Deconstruction

BFTs might provide the natural framework for studying 6d gauge theories via deconstruction. This could result in a more physical understanding of the emergence of certain mathematical structures such as the Grassmannian and cluster algebras

Arkani-Hamed, Cohen, Georgi



Two data points: the 6d (2,0) and little string theories on T^2 are deconstructed by BFTs on T^2

Arkani-Hamed, Cohen, Kaplan, Karch, Motl

Perhaps theories by Bah, Beem, Bobev, Wecht

THANK YOU!