Effective Lattice Actions for Finite–Temperature YM Theory

Tom Heinzl

University of Plymouth

Extreme QCD, Swansea
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with: L. Dittmann, T. Kästner and A. Wipf (FSU Jena)

1. Introduction

Definition:

given action $S = S[U]$ for *'microscopic'* d.o.f. U define action for *'macroscopic'* d.o.f. $X \equiv X[U]$ via

$$e^{-S_{\text{eff}}[X]} = \int \mathcal{D}\, U \, \delta(X - X[U]) \, e^{-S[U]}$$

- i.e. 'integrate out' U in favor of X
- S and S_{eff} have same matrix elements for X

typical examples:

- $X \sim$ low–energy d.o.f., e.g. χPT, (p)NRQCD, ...
- $X \sim$ order parameter, cf. Ginzburg–Landau

difficulty:

in general, U–integration cannot be done analytically

way out: 'effective field theory' program

- $S_{\text{eff}}[X]$ should have symmetries of 'parent' action $S[U]$
- make ansatz: $S_{\text{eff}}[X] = \sum_k \lambda_k S_k[X]$
- ideally: systematic expansion in symmetric op.s S_k of increasing mass dimension
- fix parameters λ_k via 'phenomenology'

alternatively:

- 'do U–integration numerically' $\rightarrow$ lattice
- but: can only calculate *expectation values* (EVs) ...

2. Inverse Monte Carlo

schematically:

$$
\begin{tikzcd}
\text{action } S[U] \arrow[r, dashed, "\text{integrate out}", "U"'] \arrow[d, "\text{MC}"'] & \text{eff. action } S_{\text{eff}}[X] \\
\text{config.s } \{U\} \arrow[r, "\text{calculate}", "X = X[U]"'] & \text{eff. config.s } \{X\} \arrow[u, "\text{IMC}"']
\end{tikzcd}
$$

3 basic steps:

1. generate config.s $\left\{U^{(i)}\right\}_{i=1,\ldots,N}$ via standard MC
2. calculate config.s $\left\{X^{(i)}\left[U^{(i)}\right]\right\}$ and compute EVs

$$\langle O[X]\rangle = N^{-1}\sum_{i=1}^{N} O\left[X^{(i)}\right]$$

3. IMC: determine effective couplings λ_k via ...

Schwinger–Dyson equations (Parisi et al. '86)

- target space isometry $\hat{\mathcal{L}}_X \, g^{ab}(X) = 0$ implies
- invariance of measure $\mathcal{D}X \equiv \prod dX \, \sqrt{g(X)} \rightarrow$

$$\int \mathcal{D}X \, \hat{\mathcal{L}}_X \, \{F[X] \, \exp(-S_{\text{eff}}[X])\} = 0$$

with $F[X]$ arbitrary function ($\rightarrow$ fine tuning!)

- ansatz $S_{\text{eff}}[X] = \sum_k \lambda_k \, S_k[X] \rightarrow$

$$\sum_k \langle F \hat{\mathcal{L}}_X S_k \rangle \lambda_k = \langle \hat{\mathcal{L}}_X F \rangle \qquad \text{SDE}$$

- (overdetermined) linear system for the λ_k
- (expensive !) check: $\langle O \rangle_S \stackrel{?}{=} \langle O \rangle_{S_{\text{eff}}}$

3. Polyakov–Loop Dynamics

3.1 Generalities

Definition:

gauge invariant order parameter for finite–T YM =
Polyakov loop = traced holonomy (or Polyakov line)

$$L_{\mathbf{x}}[U] \equiv \frac{1}{N_C} \mathrm{tr}_F \, \mathfrak{P}_{\mathbf{x}}[U] \, , \quad \mathfrak{P}_{\mathbf{x}}[U] \equiv \prod_{t=1}^{N_t} U_{t,\mathbf{x};0}$$

with Wilson coupling $\beta = 2N_C/g^2$:

- $\beta < \beta_c$: $\langle L \rangle = 0$ — confinement phase
- $\beta > \beta_c$: $\langle L \rangle \neq 0$ — deconfinement phase

critical behaviour:

- phase transition $\longrightarrow$ spontaneously broken symmetry: group centre transformations under which

$$L_{\mathbf{x}} \to \frac{1}{N_C} \mathrm{tr}\, \Omega_{\mathbf{x},N_t}\, \mathfrak{P}_{\mathbf{x}}\, \Omega_{\mathbf{x},0} = z\, L_{\mathbf{x}}$$

  with $\Omega_{\mathbf{x},t}$ periodic up to centre element $z \in \mathbb{Z}(N_C)$.

- Svetitsky–Yaffe conjecture ('82): the effective theory describing finite–T $SU(N_C)$ Yang–Mills theory in $d+1$ dimensions is a $\mathbb{Z}(N_C)$ spin model in d dimensions with short–range interactions
- for $SU(2)$ well established on lattice via comparison of critical exponents (Karsch et al., de Forcrand et al. '01)

Schwinger–Dyson Equations:

- target space: $\mathfrak{P} \equiv P^\mu \sigma_\mu \in SU(2) \cong S^3 \rightarrow$
- $O(4)$ symmetry generated by angular momenta $M^{\mu\nu}$
- gauge invariance: $S_{\text{eff}}[\mathfrak{P}] \equiv S_{\text{eff}}[P^0] \equiv S_{\text{eff}}[L]$
- SDE for ansatz $S_{\text{eff}}[L] = \sum_k \lambda_k S_k$:

$$\sum_k \langle (1 - L_{\mathbf{x}}^2) G S_{k,\mathbf{x}} \rangle \lambda_k = \langle (1 - L_{\mathbf{x}}^2) G_{,\mathbf{x}} - 3 L_{\mathbf{x}} G \rangle$$

  where $G_{,\mathbf{x}} \equiv \partial G / \partial L_{\mathbf{x}}$ etc.
- optimal choice: $G \in \{S_{l,\mathbf{y}}\}$ = op.s in EOM
  $\rightarrow$ SDEs = relations between two–point functions
- exact, overdetermined linear system for the λ_k

3.2 Effective Action from Character Expansion

- any (gauge invariant) function on $SU(N_C)$ can be expanded in terms of adapted 'Fourier' basis: group characters

$$\chi_R[U] \equiv \mathrm{tr}_R\, U\ , \quad U \in SU(N_C)\ , \quad R \quad \text{irrep}$$

- $SU(2)$: write χ_p with $p = 2j$ ($j = \frac{1}{2}, 1, \frac{3}{2}, \ldots$,, 'color spin')

$$\chi_1 = 2L\ , \quad \chi_2 = 4L^2 - 1\ , \quad \chi_3 = 8L^3 - 4L\ , \ldots$$

- impose $\mathbb{Z}(2)$ on characters using $\chi_p \to (-1)^p\, \chi_p$
- and reducibility

$$\chi_p \chi_q = \sum_r n_{pqr} \chi_r\ , \quad n_{pqr} \text{ integer}$$

result (cf. Dumitru et al. '03):

$$S_{\text{eff}}[\chi] = \sum_{\langle \mathbf{xy} \rangle} \sum_{\substack{pq \\ p-q \text{ even}}} \lambda_{pq} \, \chi_p[L_{\mathbf{x}}] \, \chi_q[L_{\mathbf{y}}] + \sum_{\mathbf{x}} \sum_{p \text{ even}} \lambda_{p0} \, \chi_p[L_{\mathbf{x}}]$$

- $\mathbb{Z}(2)$–symmetric sum over higher and higher $SU(2)$ rep.s
- both NN (!) hopping and potential terms

3.3 Mean–Field Approximation (MFA)

consider fundamental plus adjoint rep.s: $p = 1, 2$

- effective action:

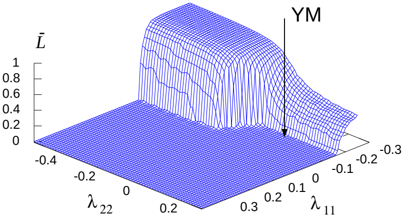
$$S_{\text{eff}}[\chi] = \sum_{\substack{\langle \mathbf{xy} \rangle \\ p=1,2}} \lambda_{pp}\, \chi_p[L_{\mathbf{x}}]\, \chi_p[L_{\mathbf{y}}] + \sum_{\mathbf{x}} \lambda_{20}\, \chi_2[L_{\mathbf{x}}]$$

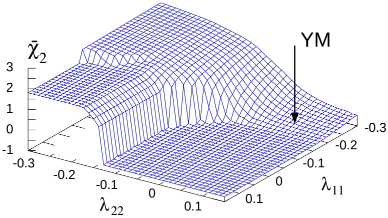
- MF potential:

$$V_{\text{MF}}(\lambda, \chi) \equiv -d \sum_{p=1,2} \lambda_{pp} {\chi_p}^2 - \log z(\lambda, \chi)$$

- MF (gap) equations: $\partial V_{\text{MF}} / \partial \chi_p = 0$, $p = 1, 2$ yield VEVs $\bar{\chi}_p$

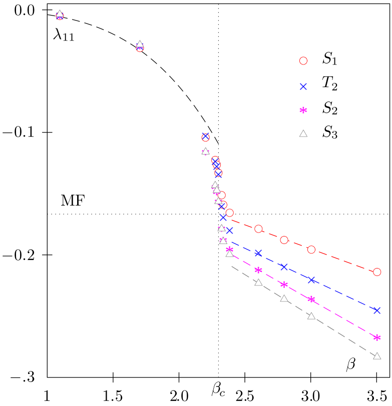
results for $\bar{\chi}_p$:



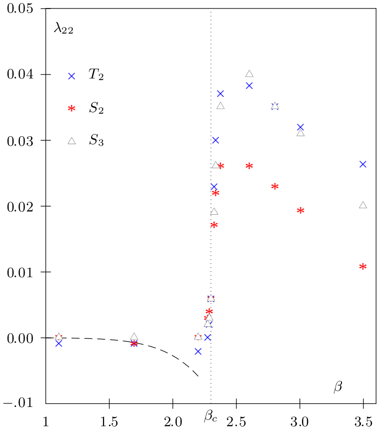


3.4 Inverse Monte Carlo ($\Omega = 4 \times 20^3$, $\beta_c = 2.30$)

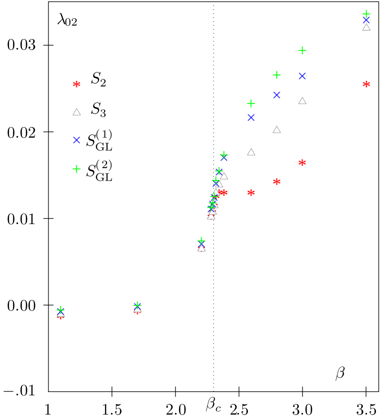
LO hopping term: coupling λ_{11}



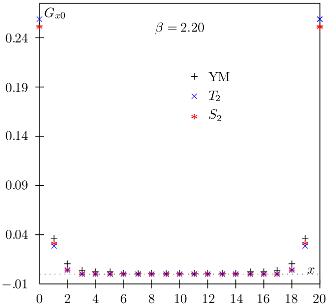
NLO hopping term: adjoint coupling λ_{22}

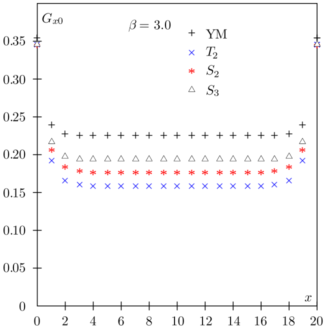


NLO potential term: coupling λ_{20}



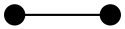
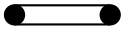
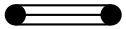
observables: two–point functions



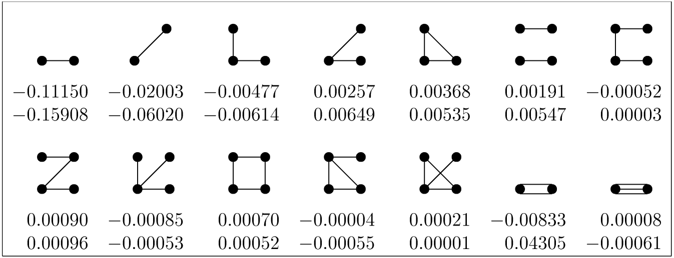


improvement: NNN interactions

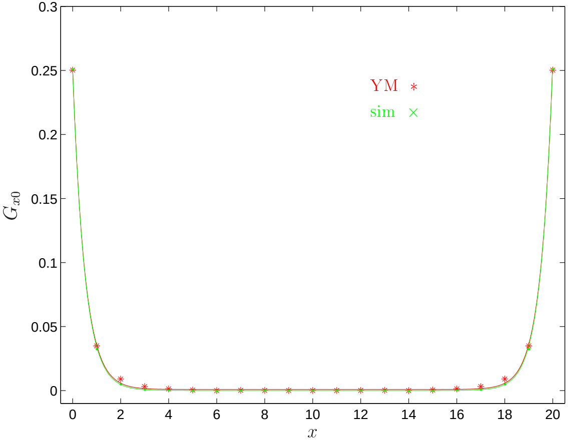
- define basic 'link' $X_{p;\,\mathbf{xy}} \equiv \chi_p[L_\mathbf{x}]\chi_p[L_\mathbf{y}]$ and graphical notation

$X_{1;\,\mathbf{xy}} \equiv$  , $X_{2;\,\mathbf{xy}} \equiv$  , $X_{3;\,\mathbf{xy}} \equiv$  , $\ldots$

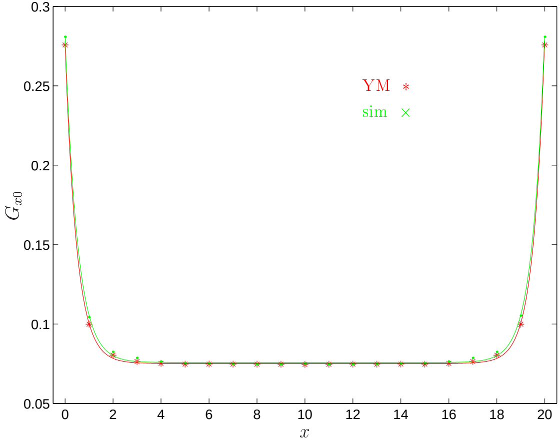
- include 14 operators built form $X_{p;\,\mathbf{xy}}$
- couplings decay with number of 'links' and rep label p:



two–point functions: $\beta = 2.20 < \beta_c$



two–point functions: $\beta = 2.40 > \beta_c$



4. Summary and Outlook

Summary:

- IMC based on SDEs = powerful method to numerically determine effective actions
- application: Polyakov loop dynamics in $SU(2)$
- Z(2)–symmetric effective actions $S_{\text{eff}}[L]$ describe confinement–deconfinement phase transition
  - NN characters: reasonable
  - NNN characters: quite good

Outlook:

- try alternative variables, e.g. $\mathbb{Z}(2)$ spins
- explicit symmetry breaking ('fermions')
- $SU(3)$: in progress
- large N_C
- renormalise Polyakov loop
- study continuum limit