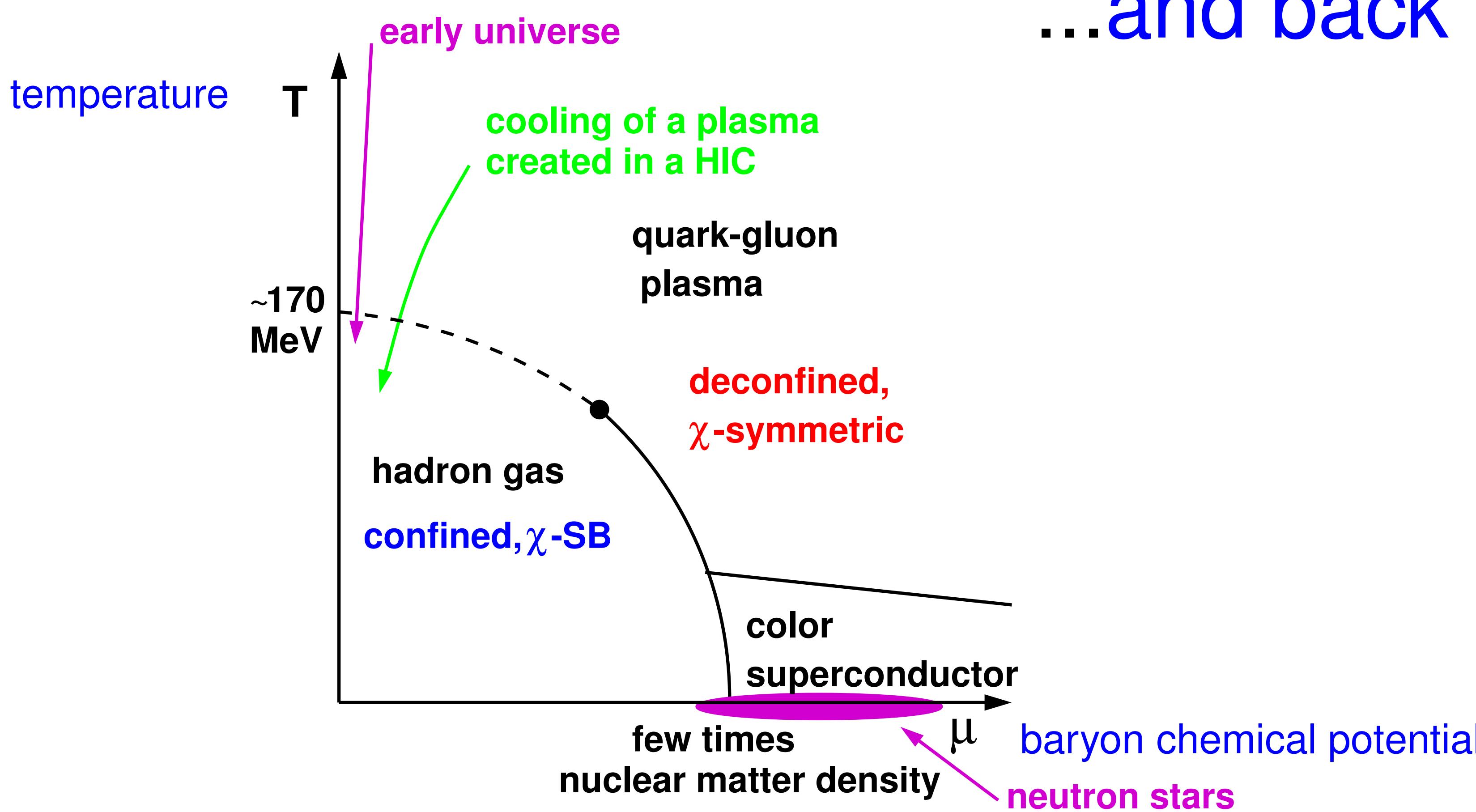
Thermodynamics of strongly interacting matter from Lattice QCD

Frithjof Karsch, BNL

Phase diagram of strongly interacting matter

From Hadron Gas to Quark Gluon Plasma

...and back



Strongly interacting (coupled) QGP

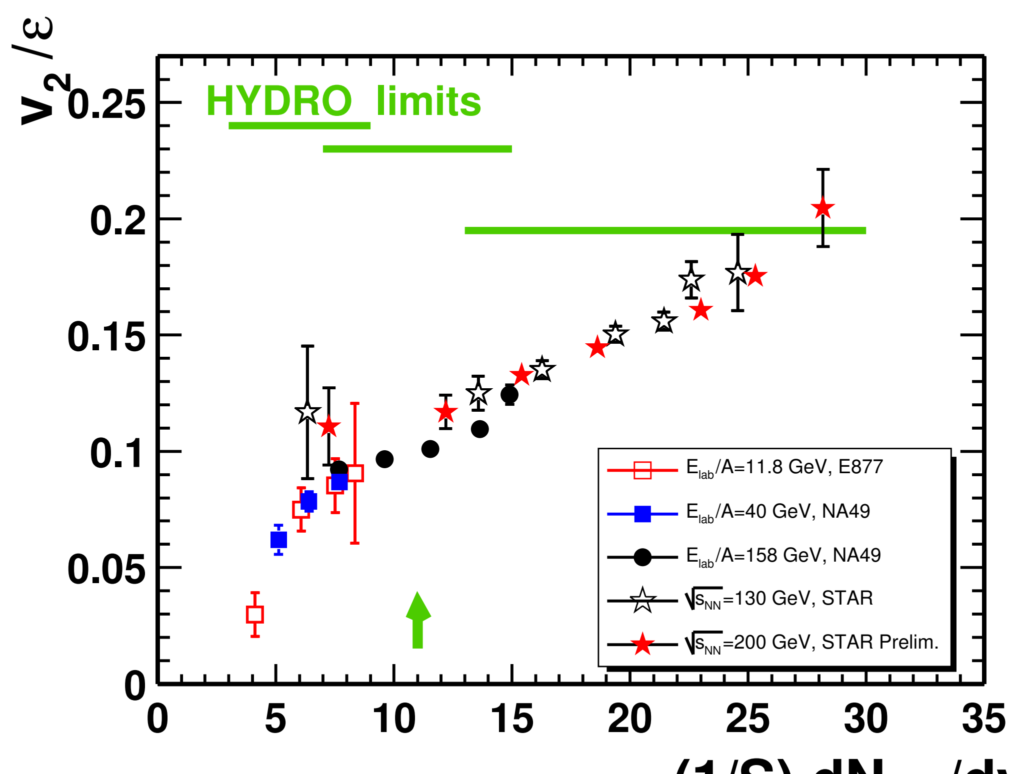
- experimental findings at RHIC:
  - rapid equilibration
  - large elliptic flow

$\Downarrow$

strongly interacting

strongly coupled

$\Uparrow$



- ..one attempt to explain these findings:
  - large number of heavy colored bound states that could exist for $T_c \leq T \lesssim (2-3)T_c$ due to a strong (Coulomb) interaction among quarks above T_c; E. Shuryak, I. Zahed, 2004

...a contribution to the discussion of the strongly coupled QGP (sQGP) scenario:

- Deconfinement:
  Heavy quark free energies; asymptotic freedom & screening; the running of the QCD coupling constant at short and large distances
- Equation of State:
  the QCD transition; non-perturbative structure of the EoS at high temperature;
- finite density QCD and fluctuations in the QGP:
  Do they rule out the sQGP model?

Deconfinement and asymptotic freedom

asymptotic freedom $\Rightarrow$ deconfinement (the original concept):

- N. Cabibbo, G. Parisi, Exponential Hadronic Spectrum and Quark Liberation, PL B59 (1975) 67;
- J.C. Collins, M.J. Perry, Superdense Matter: Neutrons and asymptotically free quarks? PRL 34 (1975) 1353

- deconfinement is a consequence of asymptotic freedom
- deconfinement $\Leftrightarrow$ liberation of many new degrees of freedom, asymptotically free $q\bar{q} + g$ gas
- deconfinement is density driven

$\Uparrow$ evidence from LGT

Confinement and deconfinement

confinement

- stick together, find a comfortable separation
- controlled by confinement potential

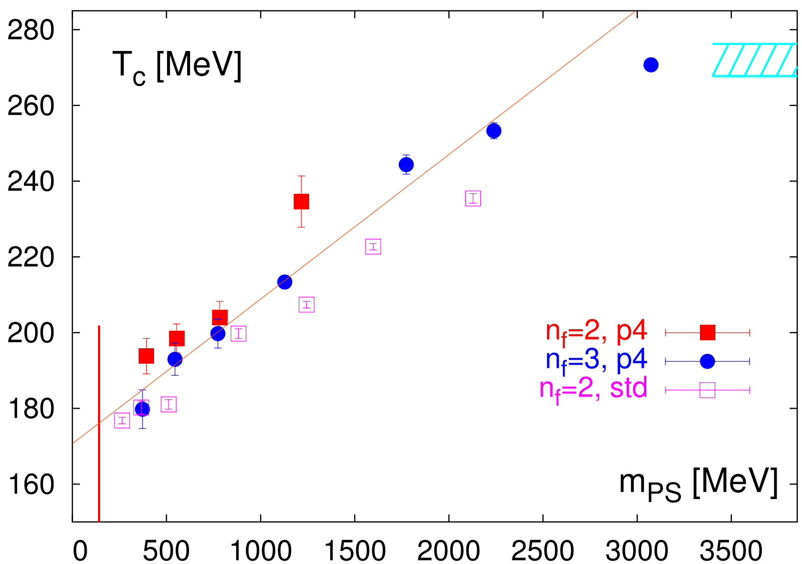
$$V(r) = -\frac{4}{3}\frac{\alpha(r)}{r} + \sigma r$$

deconfinement

- free floating in the croud
- average distance always smaller than r_{af}:

$$r_{af} = \sqrt{\frac{4}{3}\frac{\alpha(r)}{\sigma}} \simeq 0.25\,\mathrm{fm}$$

Density driven transition: Critical temperature & EoS



$m_{PS} \simeq 140\ MeV:\ \ T_c \simeq 175\ MeV$

$m_{GB} \simeq 1.5\ GeV:\ \ T_c \simeq 265\ MeV$
$(m_{PS} = \infty)$

lightest masses apparently do

not control the transition

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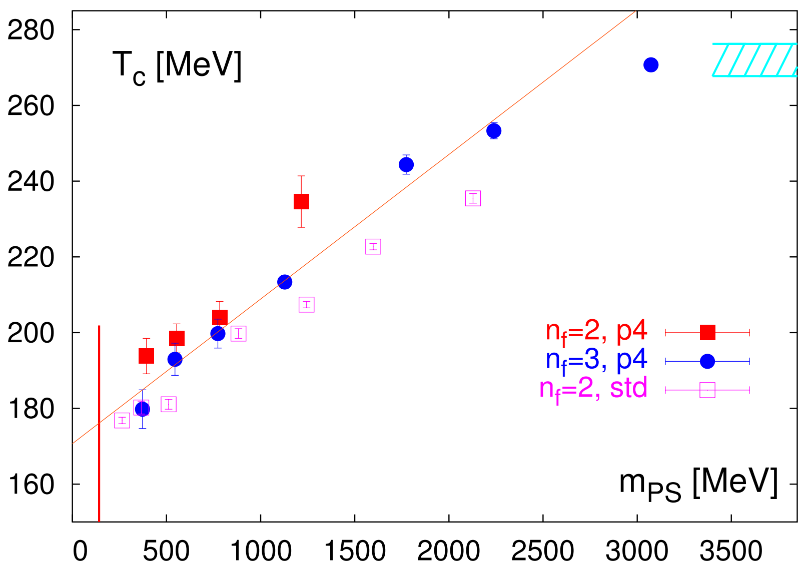

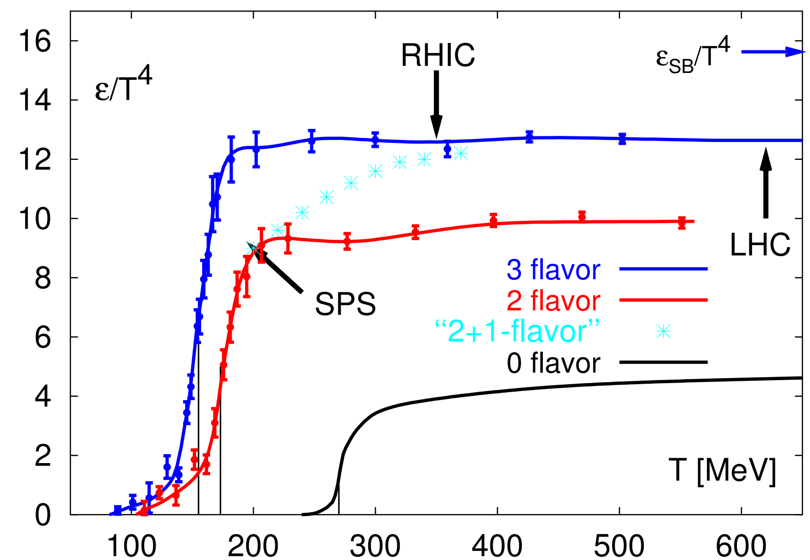

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lightest masses apparently do not control the transition

$$\boldsymbol{n_f = 2: \quad \epsilon_c \simeq (6 \pm 2)\ T_c^4}$$
$$\boldsymbol{\simeq (0.3 - 1.3)\ GeV/fm^3}$$
$$\boldsymbol{n_f = 0: \quad \epsilon_c \simeq (0.5 - 1)\ T_c^4}$$
$$\boldsymbol{\simeq (0.3 - 0.7)\ GeV/fm^3}$$

change in ϵ_c/T_c^4 compensated by shift in T_c
transition sets in at similar energy (or parton) densities $\Rightarrow$ percolation

Heavy quark free energies: Testing $\bar{q}q$ interactions in matter

- Static quark and anti-quark sources in a thermal heat bath

  ⟹ change in free energy due to presence of external sources

  L.G. McLerran, B. Svetitsky, Phys. Rev. D24 (1981) 450

$$e^{-F_{\bar{q}q}(r,T)/T} = \frac{1}{9}\langle \mathrm{Tr} L_{\vec{x}} \mathrm{Tr} L^{\dagger}_{\vec{0}} \rangle$$

- asymptotic freedom, screening, string breaking

singlet free energy

in 2-flavor QCD

$(m_q/T = 0.4)$

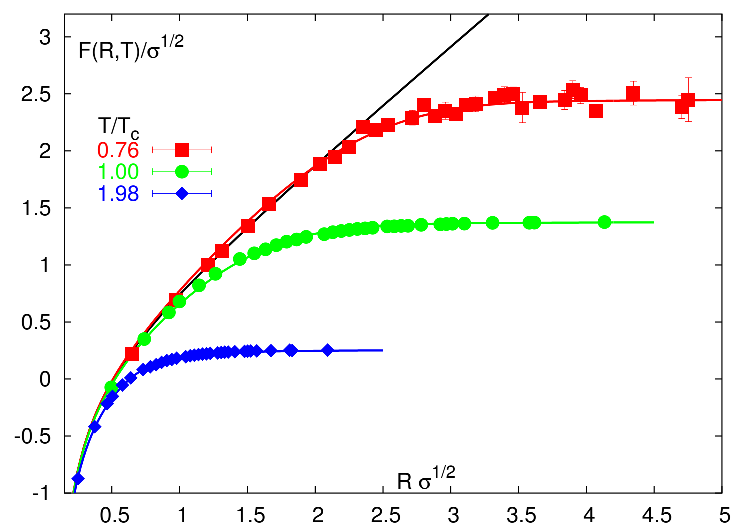
O.Kaczmarek, F. Zantow;

hep-lat/0503017

similar:

P.Petreczky, K. Petrov

hep-lat/0405009



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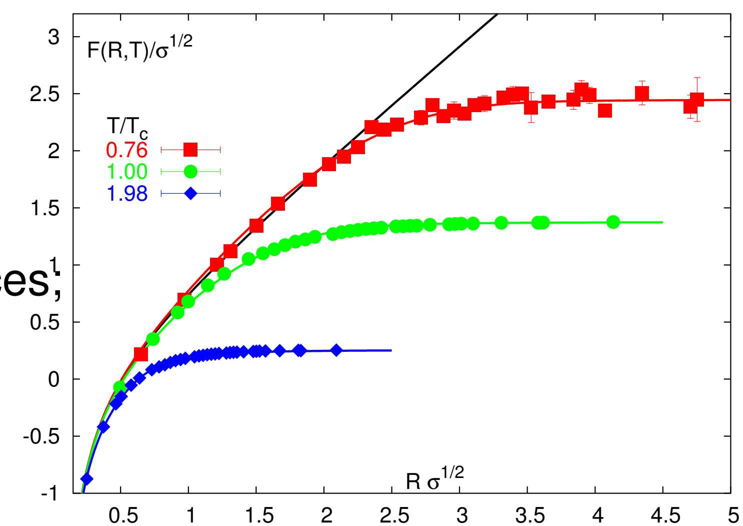
asymptotic freedom; screening at short distances; $F(r,T) \sim const.$ for $r \lesssim r_{af}$



$T \lesssim 0.75\ T_c$:
string breaking
$F(r,T) \simeq V(r,T=0)$

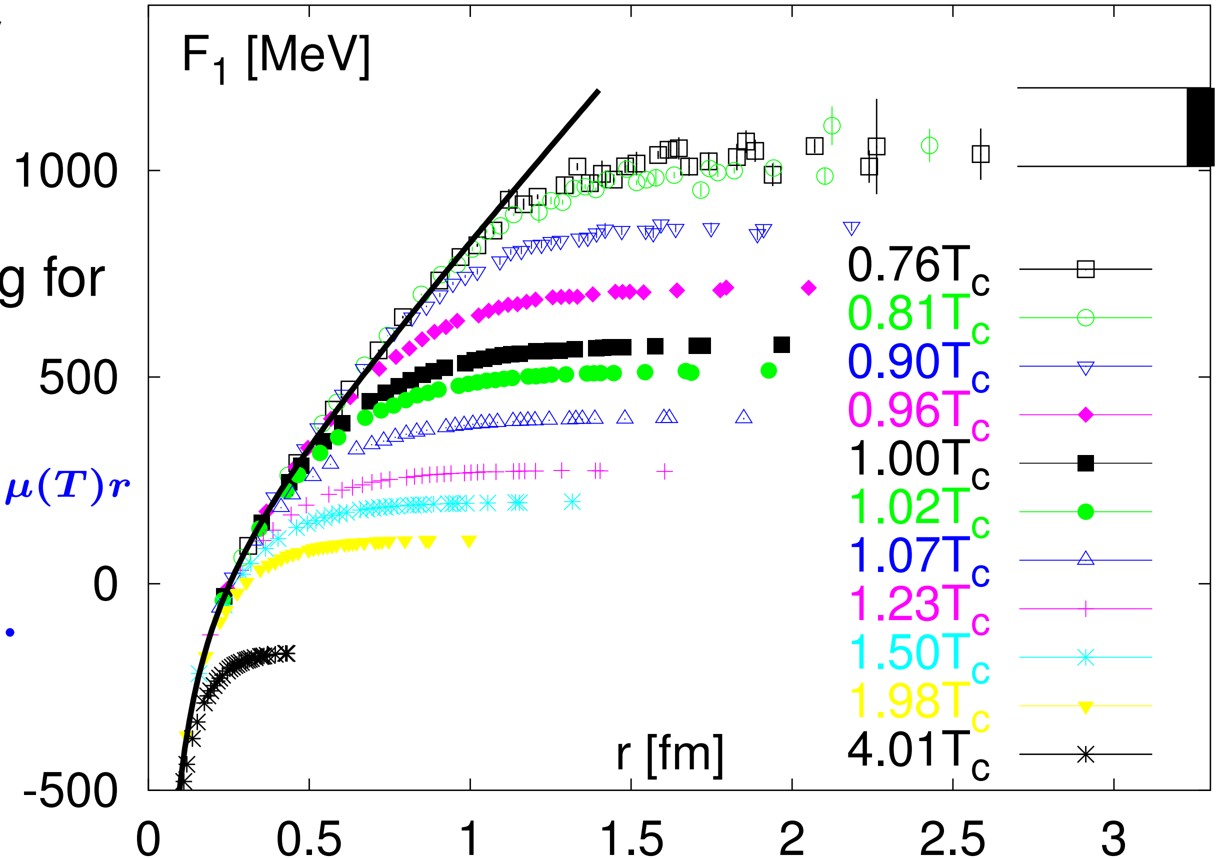
$T \simeq T_c$:
screening sets in at $r \simeq 0.3$ fm;
significant r-dep. upto $r \simeq 1$ fm $T \gtrsim 2\ T_c$:

Singlet free energy
– remnant of confinement vs. sQCD –

pure gauge: O.Kaczmarek, FK, P. Petreczky, F. Zantow, hep-lat/0406036
2-flavor QCD: O.Kaczmarek, F. Zantow, hep-lat/0503017

- singlet free energy
- $T \simeq T_c$: screening for $r \gtrsim 0.5$fm

$$F_1(r,T) \sim \frac{\alpha(T)}{r} e^{-\mu(T)r} + const.$$



- $F_1(r,T)$ follows linear rise of $V_{\bar{q}q}(r,T=0) = -\frac{4\alpha(r,T=0)}{3r} + \sigma r$ for $T \lesssim 1.5 T_c$, $r \lesssim 0.3$ fm

Singlet free energy and asymptotic freedom

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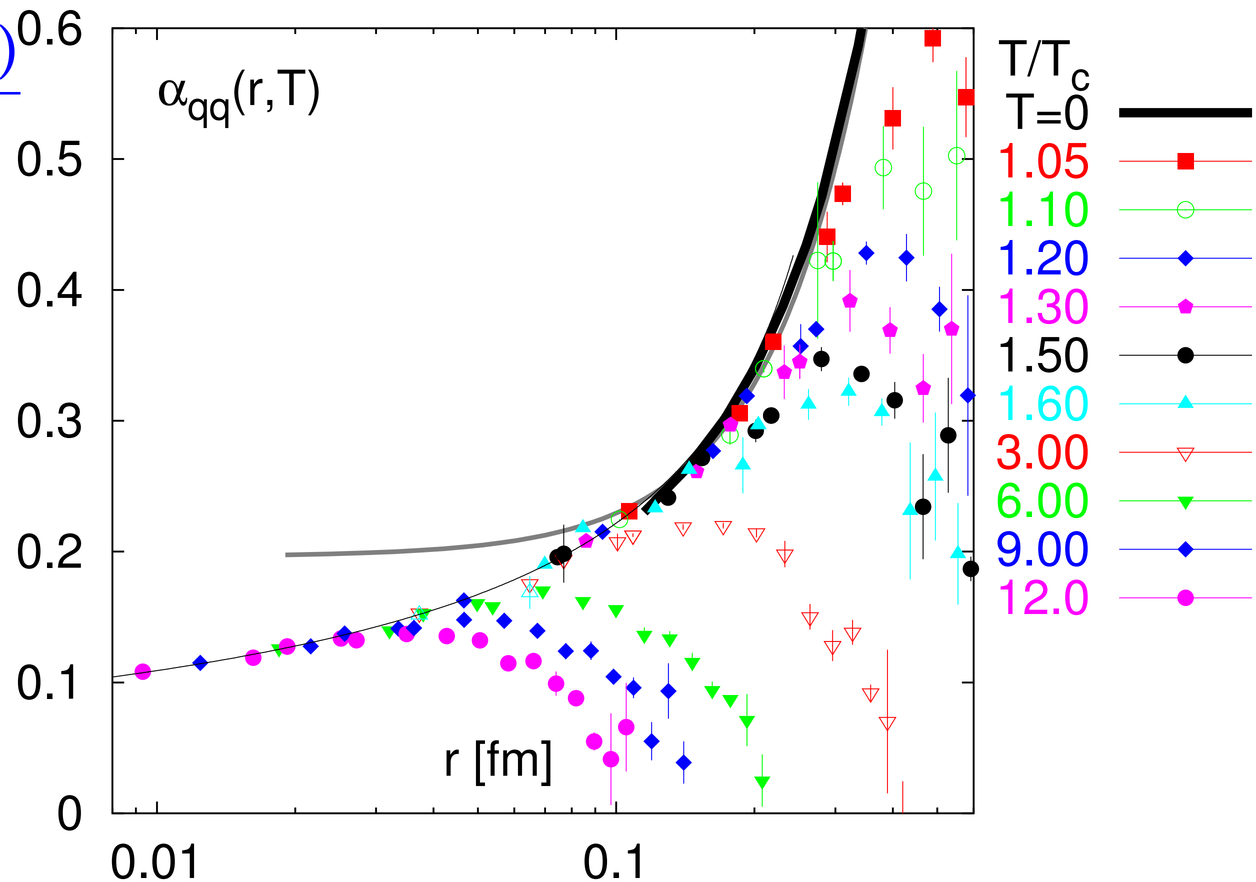
2-flavor QCD: O.Kaczmarek, F. Zantow, hep-lat/0503017

- singlet free energy defines a running coupling:

$$\alpha_{\mathrm{eff}} = \frac{3r^2}{4}\,\frac{\mathrm{d}F_1(r,T)}{\mathrm{d}r}$$



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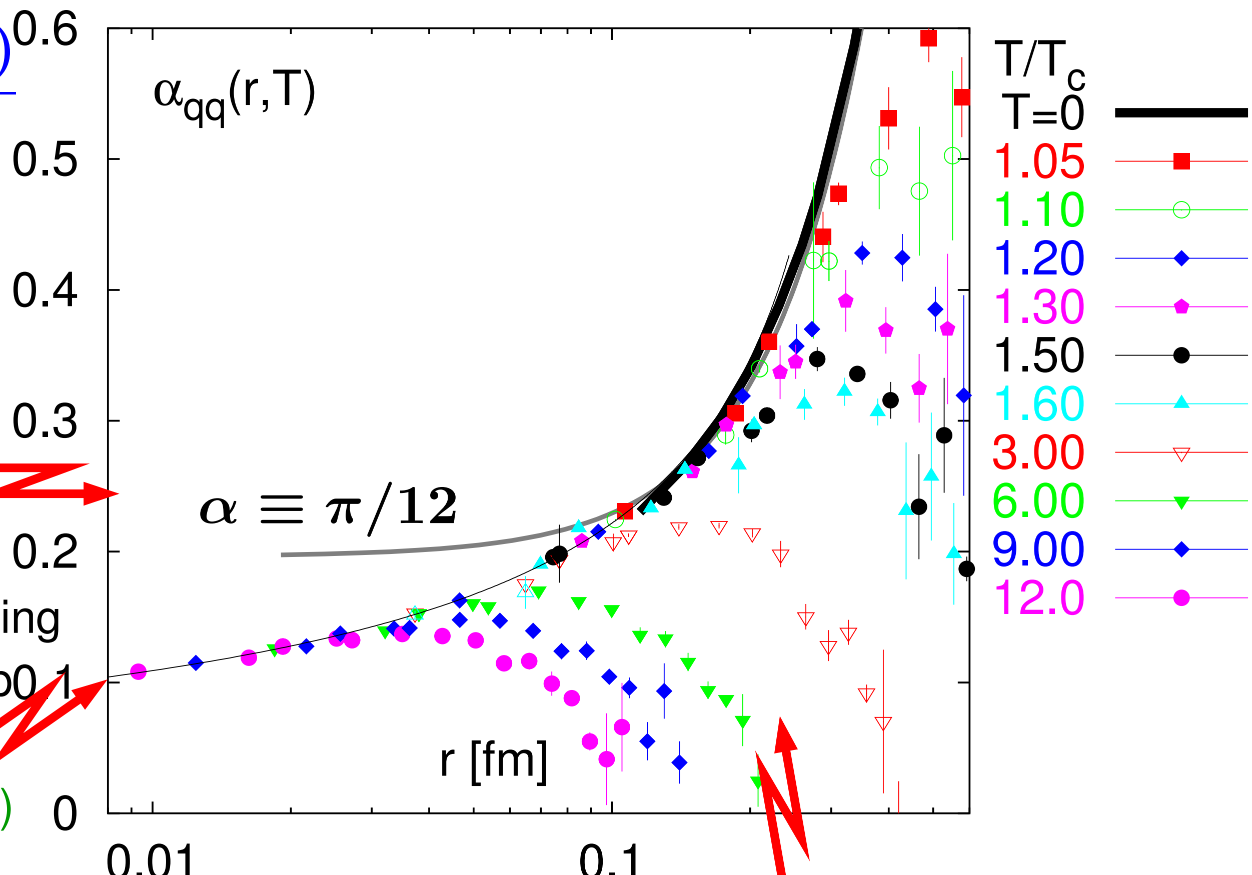

large distance: constant
Coulomb term (string model)

short distance: running coupling
$\alpha(r)$ from $(T=0)$, 3-loop
(S. Necco, R. Sommer,
Nucl. Phys. B622 (2002) 328)

T-dependence starts in non-perturbative
regime for $T \lesssim 3\ T_c$

- short distance physics $\Leftrightarrow$ vacuum physics

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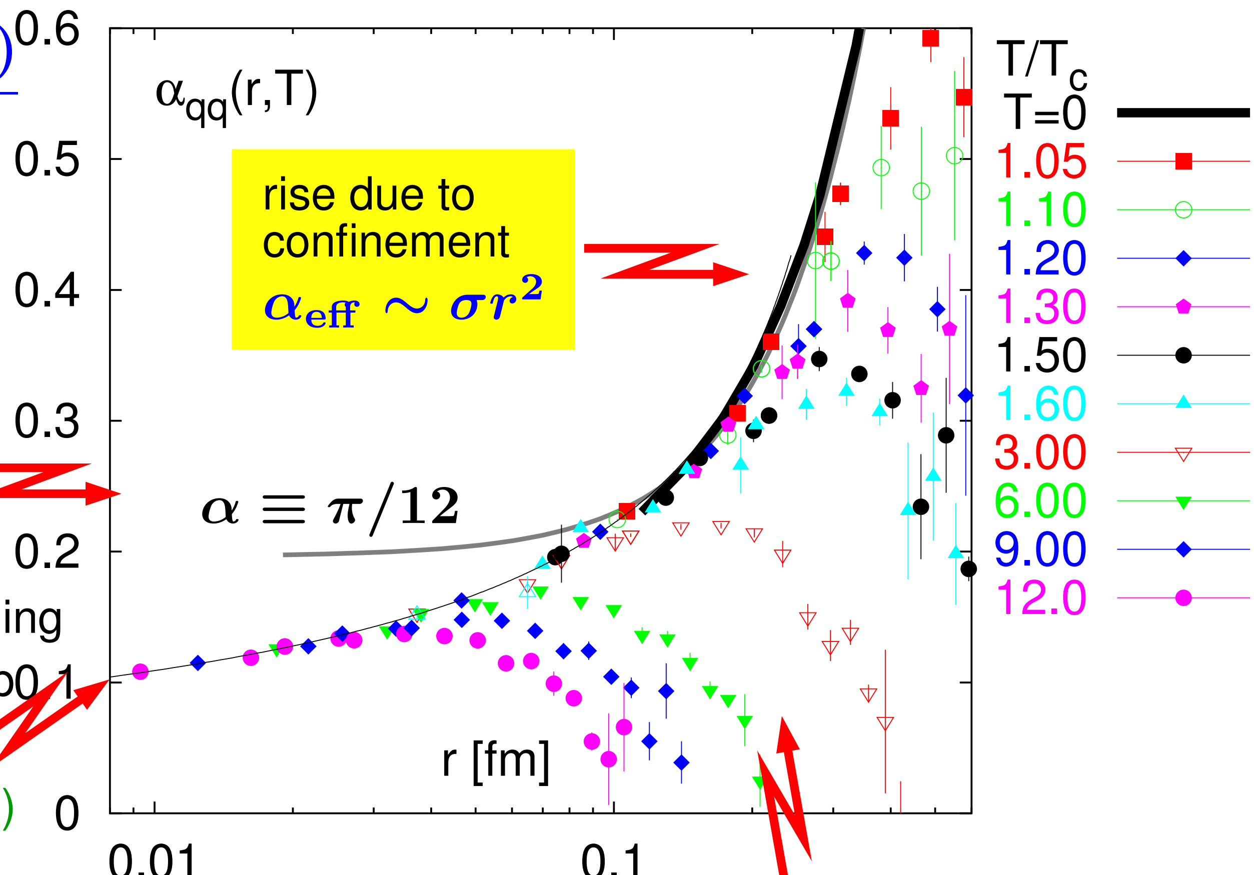

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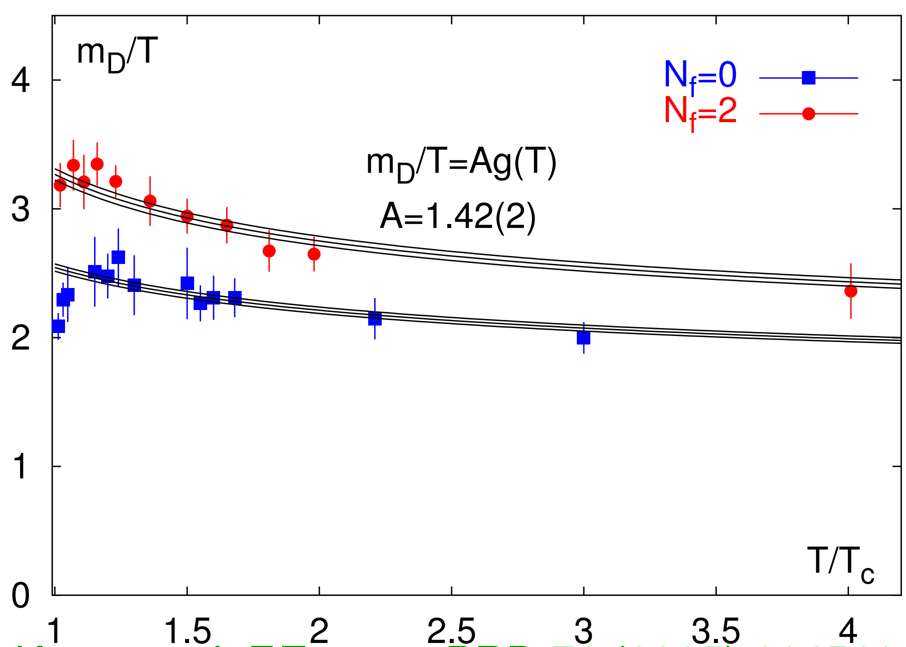
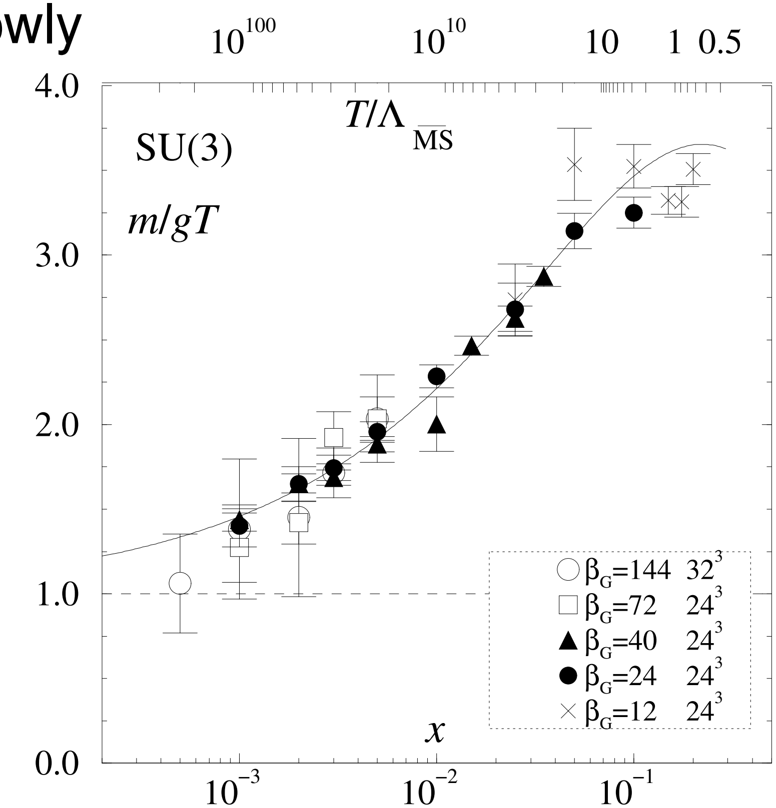
- short distance physics ⇔ vacuum physics

Non-perturbative Debye screening

- leading order perturbation theory: $m_D = g(T)T\sqrt{1 + \frac{n_f}{6}}$
- $T_c < T \lesssim 10T_c$: non-perturbative effects are well represented by an "A-factor": $m_D \equiv Ag(T)T,\ A \simeq 1.5$
- perturbative limit is reached very slowly (logarithms at work!!)



O.Kaczmarek,F.Zantow, PRD 71 (2005) 114510



K.Kajantie et al, PRL 79 (1997) 3130

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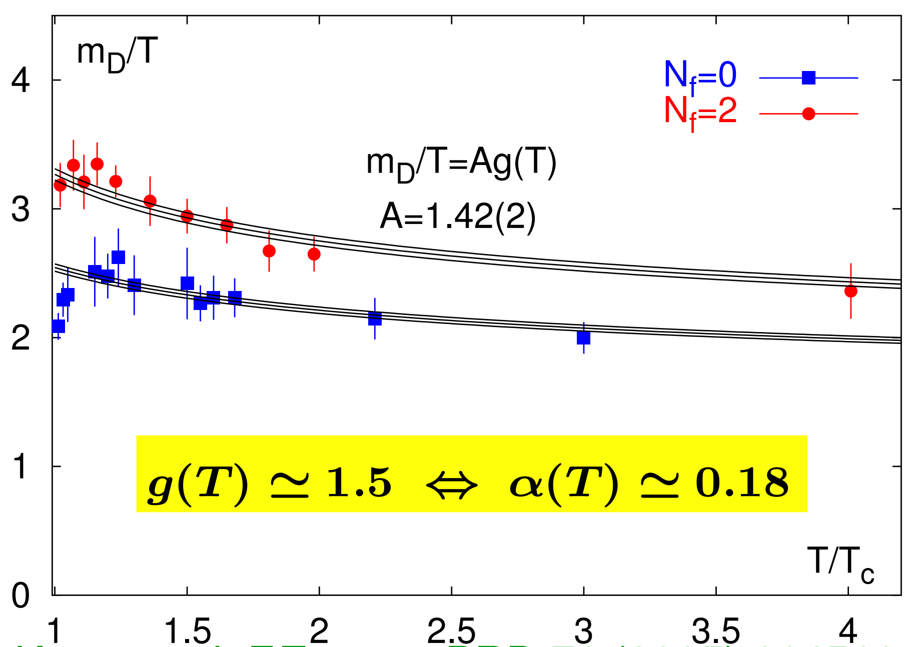

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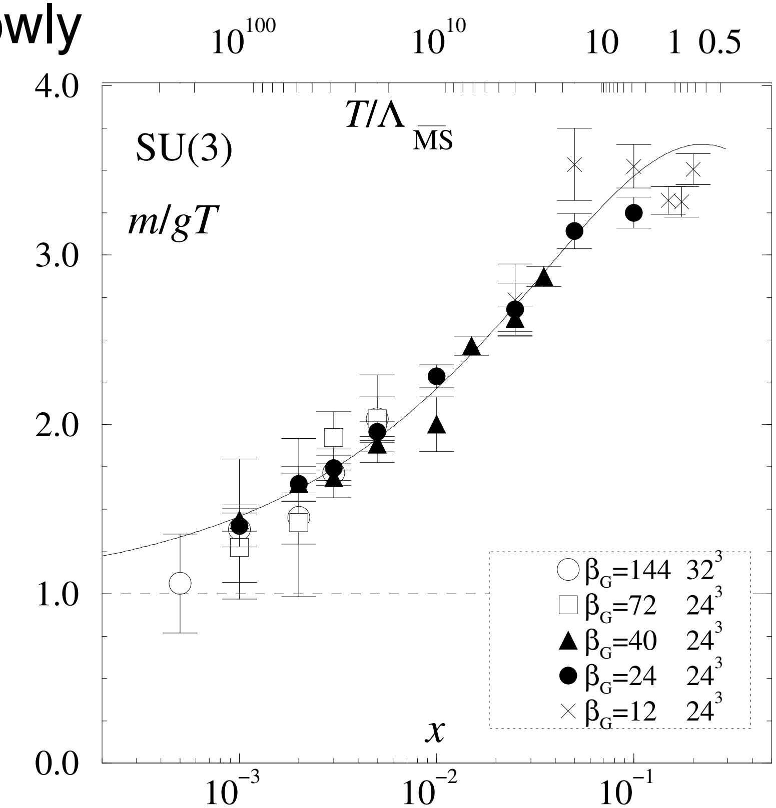
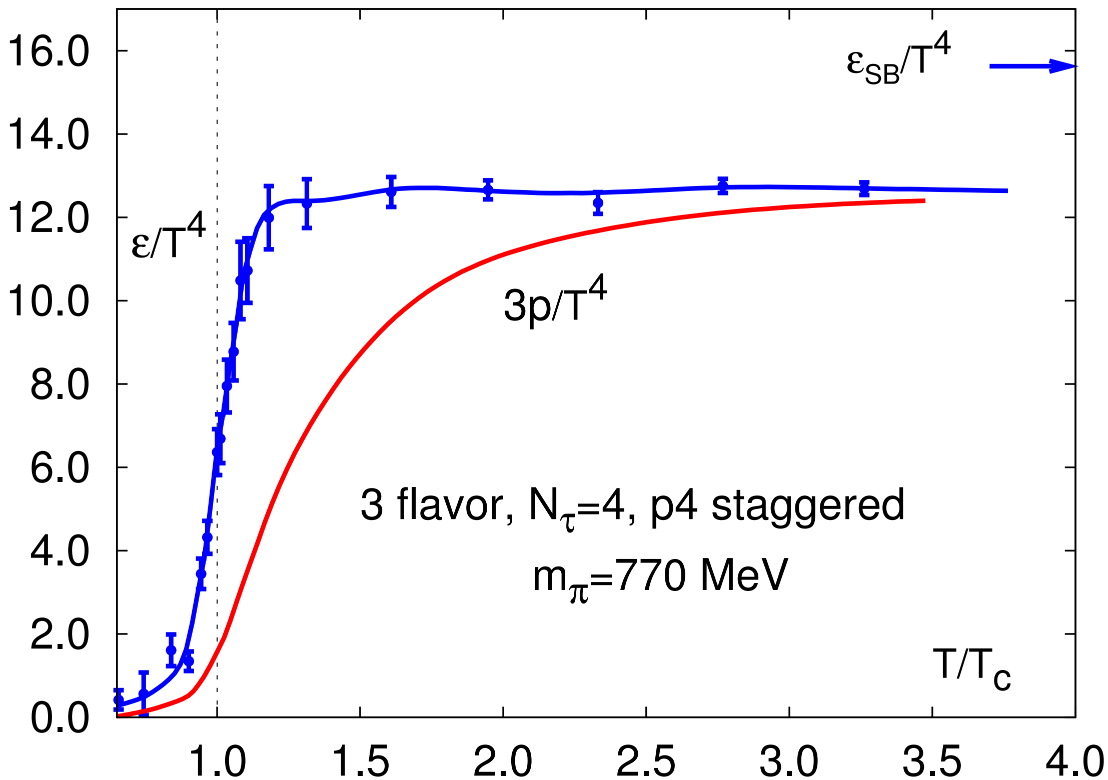


K.Kajantie et al, PRL 79 (1997) 3130

QCD equation of state

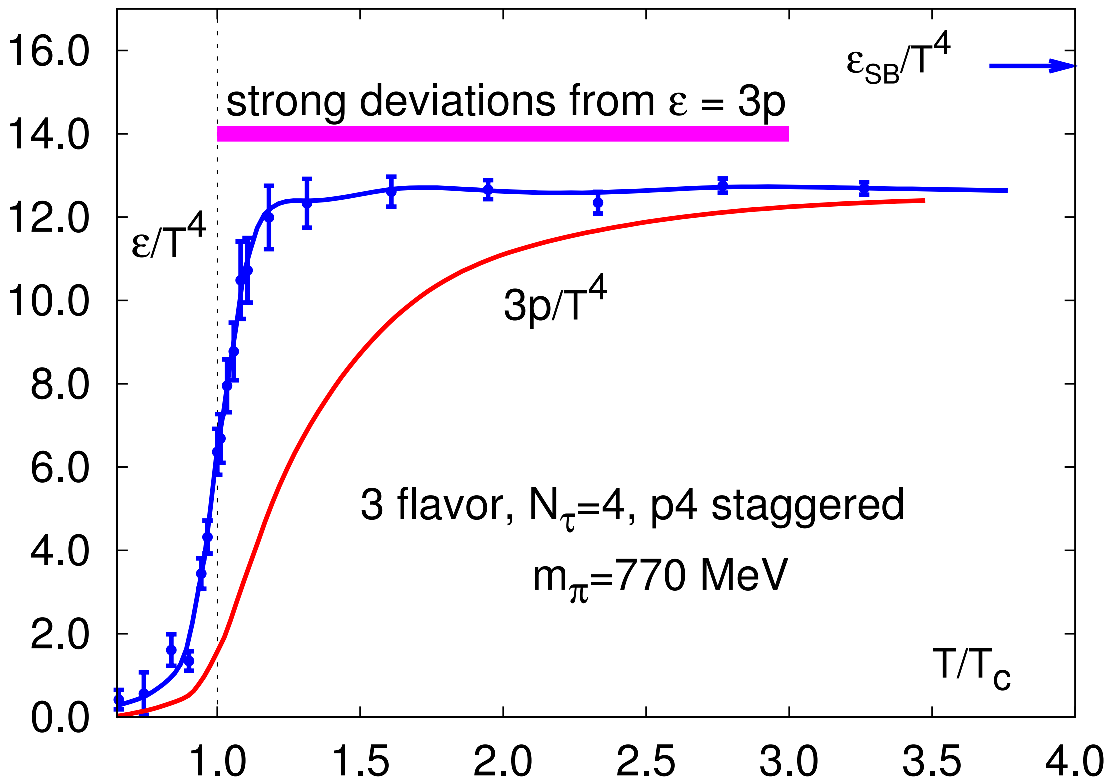
- lattice calculations established basic results on the transition at $T \neq 0$, the EoS and properties of the high temperature phase
  - $T_c = (175 \pm 8 \pm sys)\ MeV$ (for $n_f = 2$)
    $\epsilon_c = (6 \pm 2) T_c^4 \simeq (0.3 - 1.3)\ GeV/fm^3$



FK, E. Laermann and A. Peikert, Phys. Lett. B478 (2000) 447.

QCD equation of state

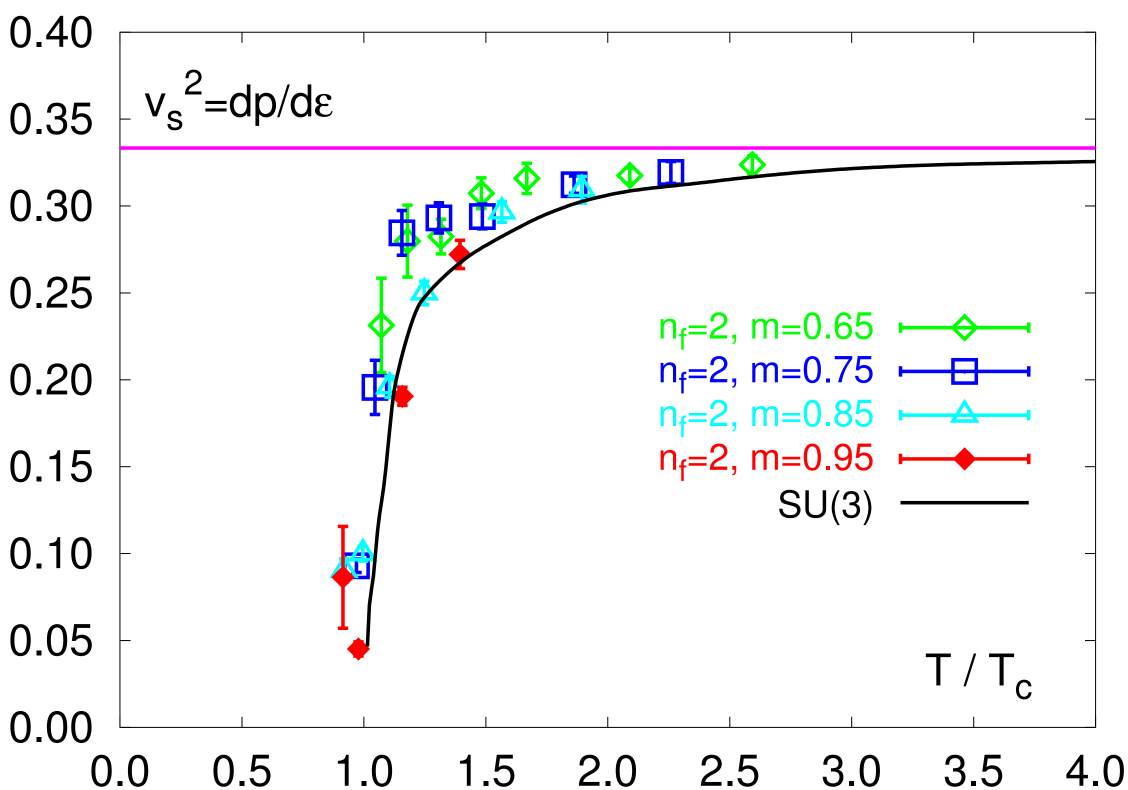
- two features of EoS are central in the ongoing discussion of a strongly coupled QGP (sQGP)
  - strong deviations from ideal gas behavior ($\epsilon = 3p$) for $T_c \leq T \sim 3T_c$



strong influence on
- velocity of sound (+)
- large screening masses

Velocity of sound

- steep EoS:
  rapid change of energy density; slow change of pressure
  $\Rightarrow$ reduced velocity of sound $\Rightarrow$ more time for equilibration

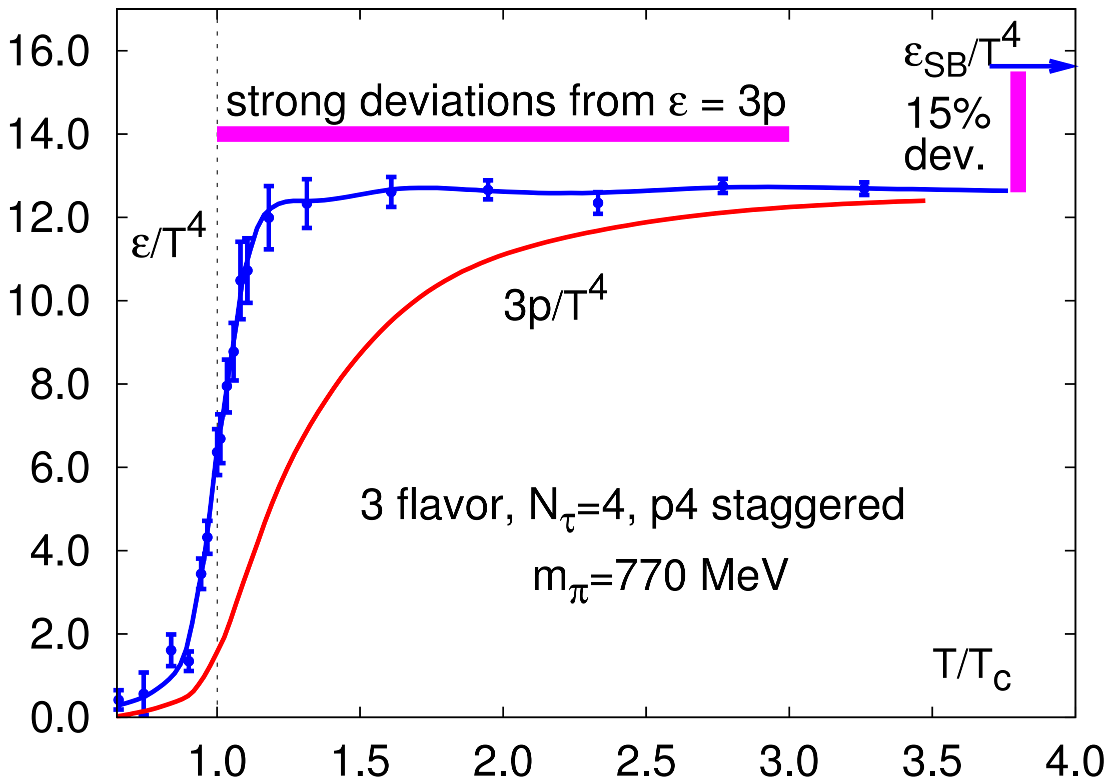


$n_f = 2$:
A. Ali Khan et al., PR D64 2001

pure gauge theory:
G. Boyd et al., NP B469 1996

QCD equation of state

- two features of EoS are central in the ongoing discussion of strongly coupled QGP (sQGP)
  - deviations from Stefan-Boltzmann limit persist even at high temperature



EoS non-perturbative even at high T:
- thermal parton masses (+)
- bound states (?)

High temperature perturbation theory: complete result up to $\mathcal{O}(g^5)$

free energy density in the high temperature limit ($\alpha_s \equiv g^2/4\pi$):

$$\frac{f}{T^4} = -\frac{1}{V}\ln Z = -\frac{d_q \pi^2}{90}\left[1 + f_2\alpha_s + f_3\alpha_s^{3/2} + + f_4\alpha_s^2 + f_5\alpha_s^{5/2} + O(\alpha_s^3)\right]$$

P. Arnold and C. Zhai, Phys. Rev. D50 (1994) 7603; [SU(3)]

C. Zhai and B. Kastening, Phys. Rev. D52 (1995) 7232; [QCD, $\mu = 0$]

A. Vuorinen, hep-ph/0305183; [QCD, $\mu > 0$]

High temperature perturbation theory: complete result up to $\mathcal{O}(g^5)$

free energy density in the high temperature limit:

resummed self-energy;

electric screening; $m_e(T) = \sqrt{\frac{N_c}{3} + \frac{n_f}{6}}\, g(T)\; T$

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loops at all orders contribute

non-perturbative magnetic mass

$m_m(T) \sim \mathcal{O}(g^2\, T)$,

(A.D. Linde, 1980)

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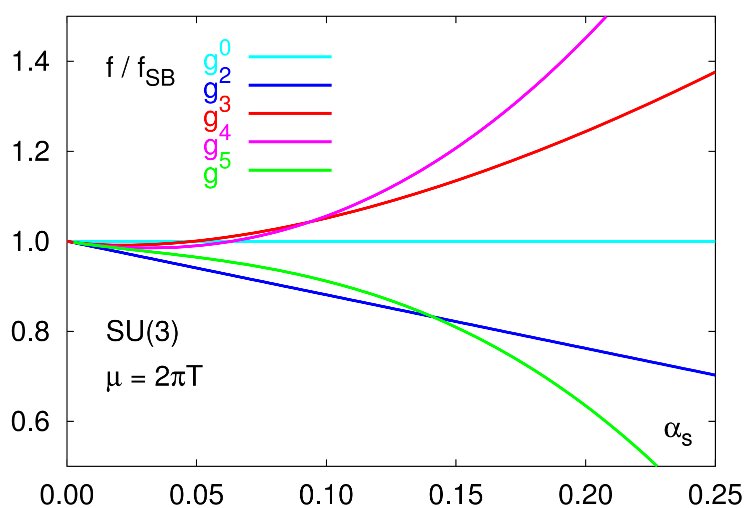


loops at all orders contribute

non-perturbative magnetic mass

$m_m(T) \sim \mathcal{O}(g^2\, T)$,

(A.D. Linde, 1980)

poor convergence properties; $g(T) < 1$ needed;

thermodynamics non-perturbative even at

$T >> \Lambda_{QCD} \simeq 200$ MeV

HTL quark propagator

E. Braaten, R.D. Pisarski, T.C. Yuan, PRL 64 (1990) 2242

- ϕ^4: screened perturbation theory shows better convergence

  FK, A. Patkos, P. Petreczky, PL B401 (1997) 69

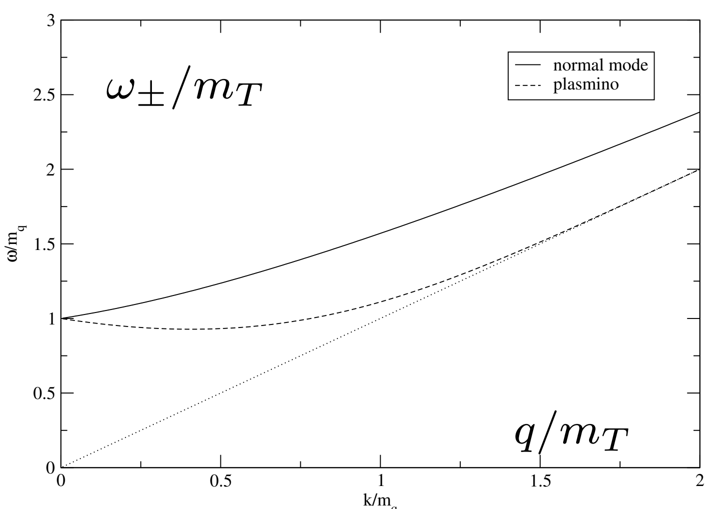
- HTL-resummed perturbation theory for pressure takes into account
  thermal masses of quark and gluon propagators

$$\rho_{\mathrm{HTL}}(\omega,\vec{q}) = \frac{1}{2}\rho_+(\omega,q)(\gamma_0 - i\,\hat{q}\cdot\vec{\gamma}) + \frac{1}{2}\rho_-(\omega,q)(\gamma_0 + i\,\hat{q}\cdot\vec{\gamma})$$

with

$$\rho_\pm(\omega,q) \;=\; \frac{\omega^2-q^2}{2m_T^2}\left[\delta(\omega-\omega_\pm)+\delta(\omega+\omega_\mp)\right] + \beta_\pm(\omega,q)\Theta(q^2-\omega^2)$$

pole cut



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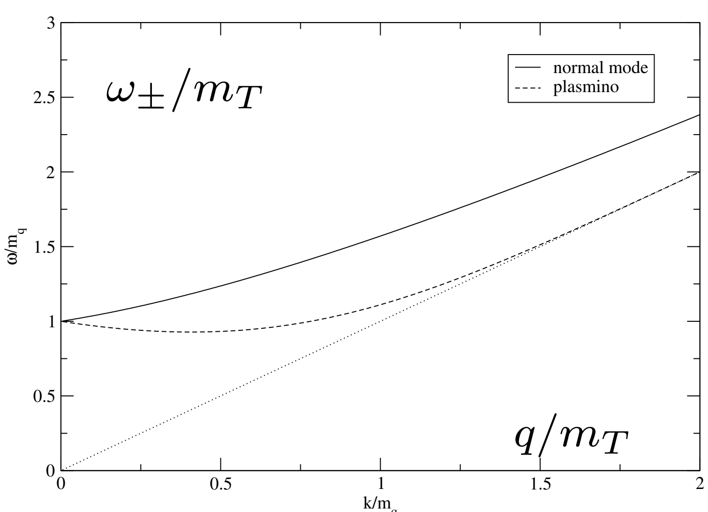
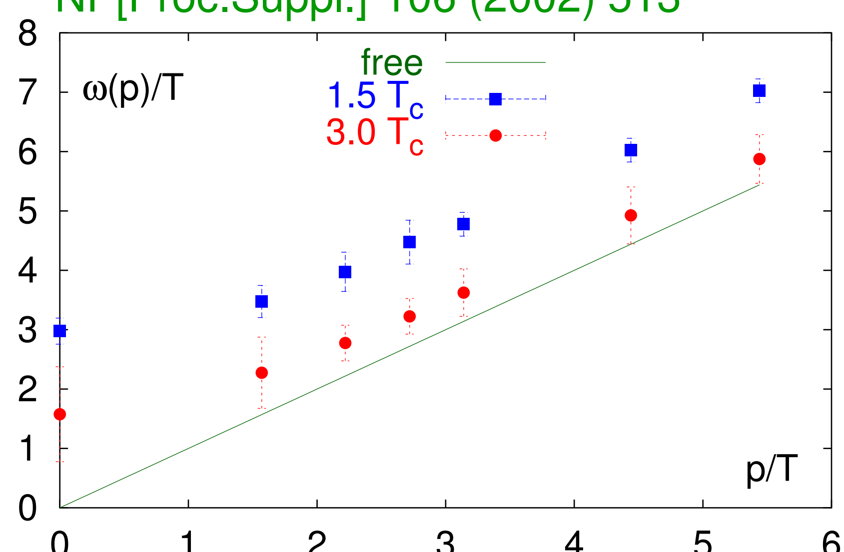
lattice quark dispersion relation:

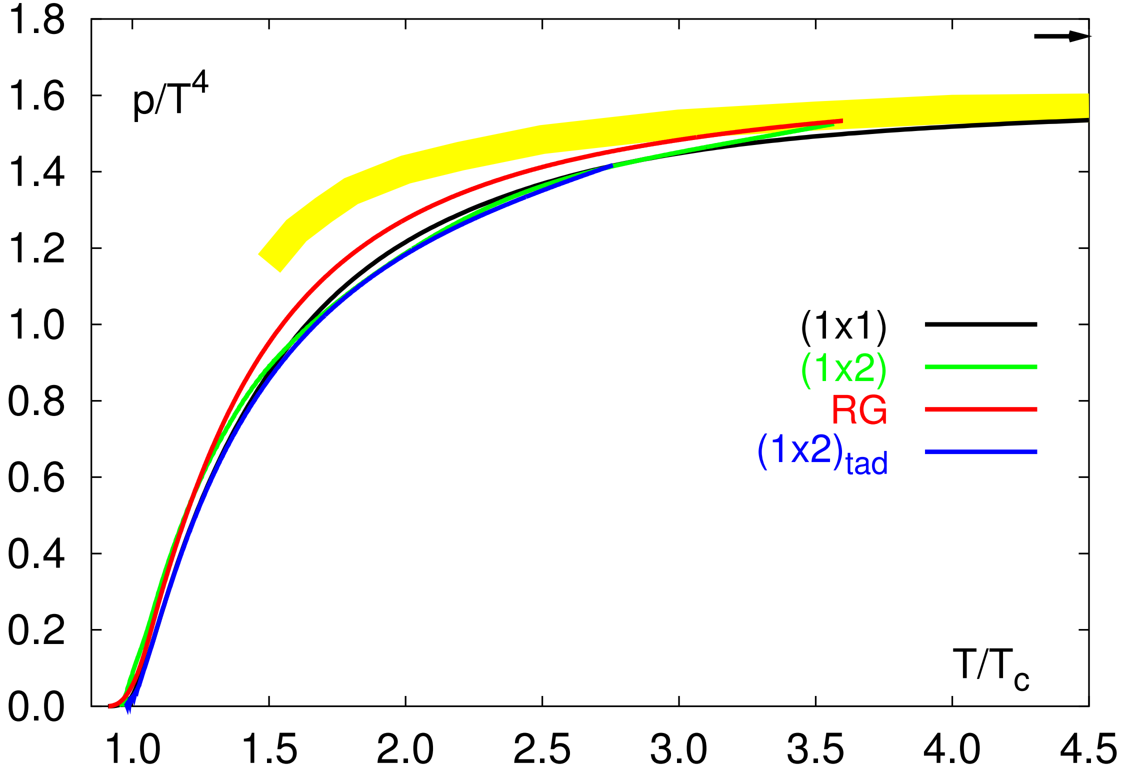
P. Petreczky et al., NP[Proc.Suppl.] 106 (2002) 513





SU(3) Equation of State pressure: LGT vs. HTL

high T part of the pressure calculated on the lattice is in good agreement with HTL-resummed perturbation theory for $T \gtrsim 3T_c$



HTL: J.P. Blaizot, E. Iancu, A. Rebhan, PL B470 (99) 181

supports quasi-particle picture/models: A. Peshier et al, PRD54 (96) 2399 P. Lévai and U. Heinz, PR C57 (98); R.A. Schneider and W. Weise, PR C64 (01)

The sQGP model

E.V. Shuryak, I. Zahed, PR D 70 (2004) 054507

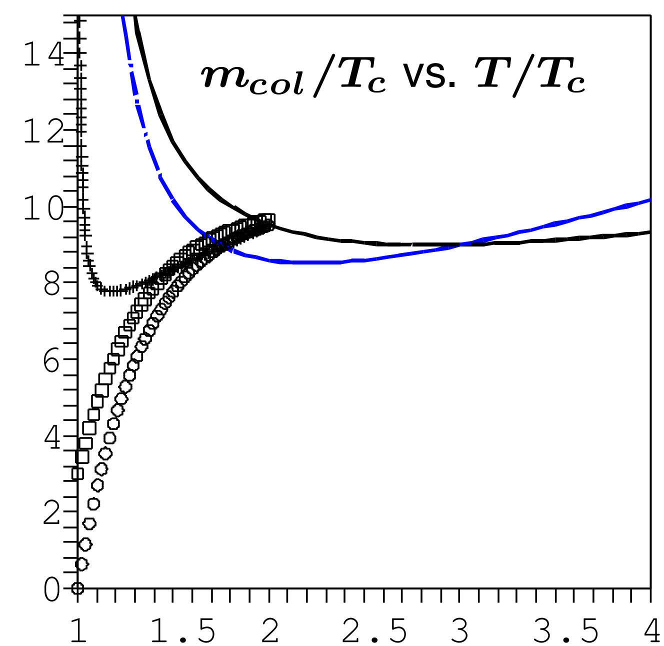
- heavy colored bound states for $T_c \le T \lesssim 2T_c$:

  $$m_{\rm col} \approx 11.5\, T_c \left((T/3T_c)^{0.5} + 0.1T_c/(T - T_c)\right)$$

- $\sim$ 200 qg and $\bar{q}g$ states; $\sim$ 100 qq and $\bar{q}\bar{q}$ states ($n_f = 2$)
- contribute to the pressure like in a resonance gas (Boltzmann approximation) with reduced weight factor,

  $$R(T) = \frac{1}{1 + \exp[2(T - 2T_c)/T_c]}$$

  to mimic "melting" of the states



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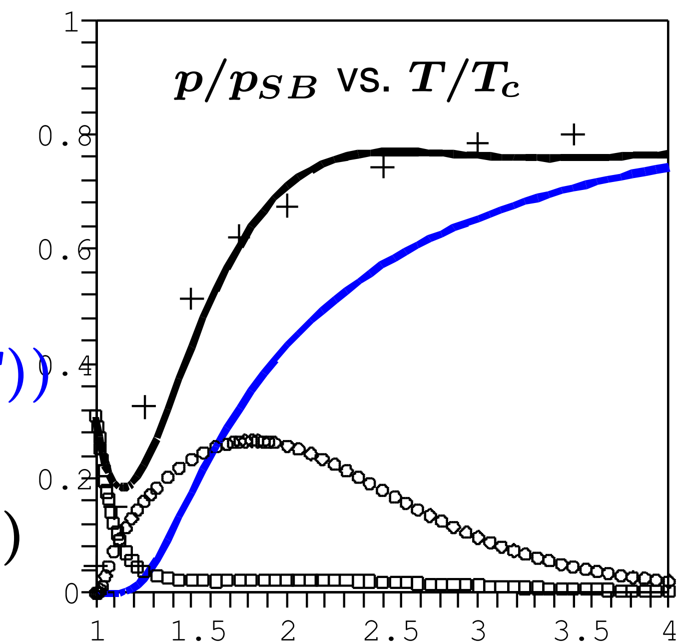

$$p/T^4 \simeq F_q(T) + R(T)\left(F_{qg}(T) + F_{qq}(T)\right)$$

$$F_x(T) \sim (d.o.f.)_x \left(\frac{m_{col}}{T}\right)^2 K_2(m_{col}/T)$$

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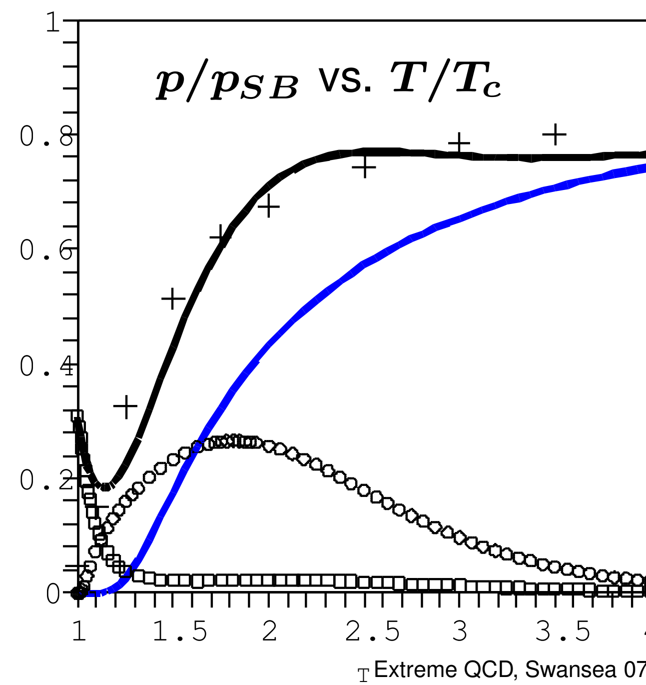

$\mu_q > 0$: $p/T^4 = F_q(T)\cosh(\mu_q/T)$

$$+R(T)\Big(F_{qg}(T)\cosh(\mu_q/T)$$

$$+F_{qq}(T)\cosh(2\mu_q/T)\Big)$$

Bulk thermodynamics with non-vanishing chemical potential

$$Z(V,T,\mu) = \int \mathcal{D}\mathcal{A}\mathcal{D}\psi\mathcal{D}\bar{\psi}\ \mathrm{e}^{-S_E(V,T,\mu)}$$
$$= \int \mathcal{D}\mathcal{A}\left[det\ M(\mu)\right]^f\ \mathrm{e}^{-S_G(V,T)}$$

$\Uparrow$complex fermion determinant;

Bulk thermodynamics with non-vanishing chemical potential

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$$= \int \mathcal{D}\mathcal{A}\ \left[det\ M(\mu)\right]^f\ \mathrm{e}^{-S_G(V,T)}$$

⇑complex fermion determinant;

⇓Taylor expansion;

$$\frac{p}{T^4} = \frac{1}{VT^3}\ln Z(V,T,\mu)$$

$$\equiv \sum_{n=0}^{\infty} c_n(T)\left(\frac{\mu}{T}\right)^n$$

$$= c_0 + c_2\left(\frac{\mu}{T}\right)^2 + c_4\left(\frac{\mu}{T}\right)^4 + c_6\left(\frac{\mu}{T}\right)^6 + \mathcal{O}((\mu/T)^8)$$

$$\mu = 0 \quad \Rightarrow \quad \frac{p}{T^4} \equiv c_0(T)$$

The pressure for $\mu_q/T > 0$

C.R. Allton et al. (Bielefeld-Swansea), PRD68 (2003) 014507

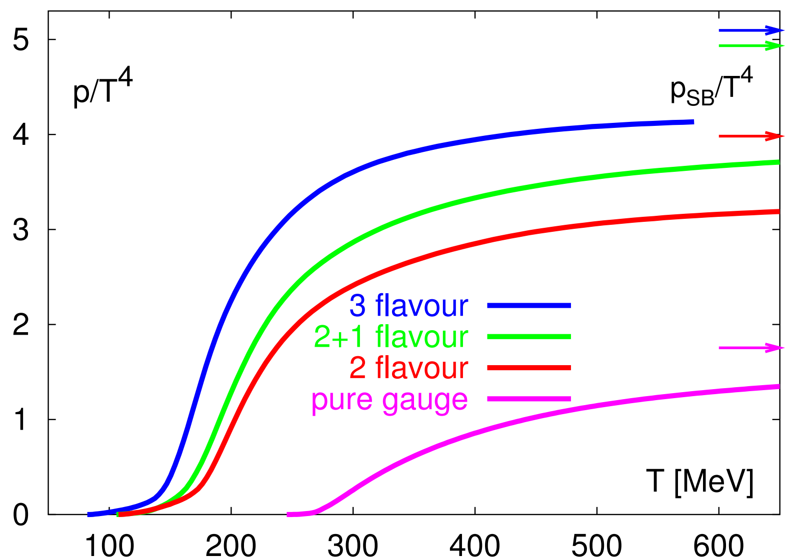
$\mu_q = 0, \quad 16^3 \times 4$ lattice

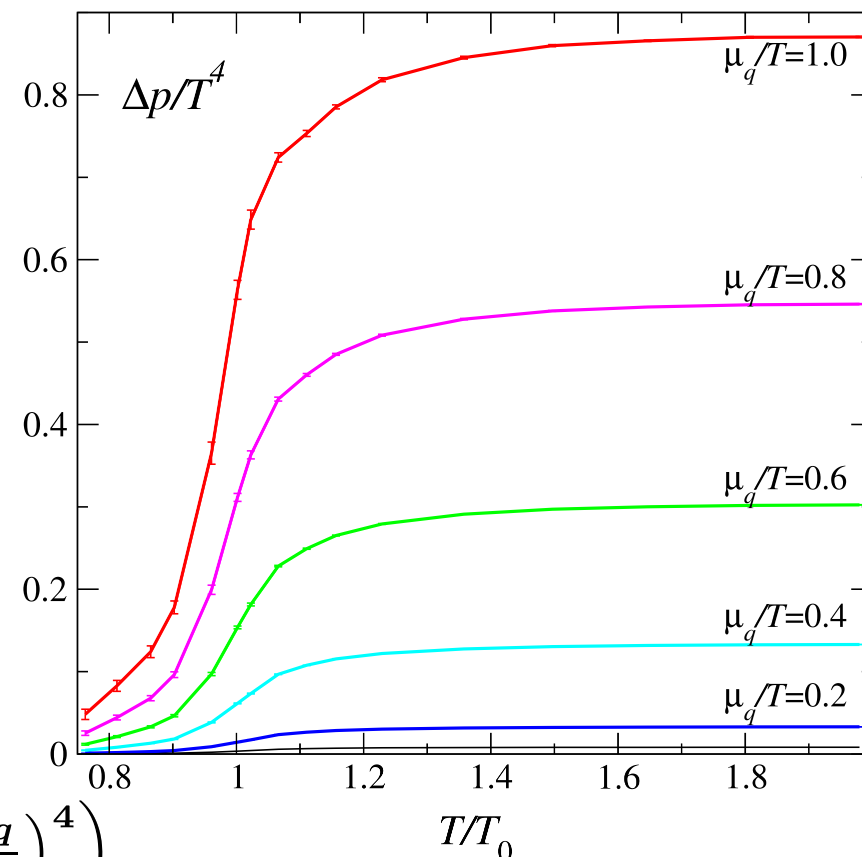
improved staggered fermions;

$n_f = 2,\ m_\pi \simeq 770\ MeV$

contribution from $\mu_q/T > 0$

Taylor expansion, $\mathcal{O}((\mu/T)^4)$





high-T, ideal gas limit

$$\frac{p}{T^4}\Big|_\infty = n_f \left(\frac{7\pi^2}{60} + \frac{1}{2}\left(\frac{\mu_q}{T}\right)^2 + \frac{1}{4\pi^2}\left(\frac{\mu_q}{T}\right)^4 \right)$$

The pressure for $\mu_q/T > 0$

C.R. Allton et al. (Bielefeld-Swansea), PRD68 (2003) 014507

$\mu_q = 0, \quad 16^3 \times 4$ lattice

improved staggered fermions;

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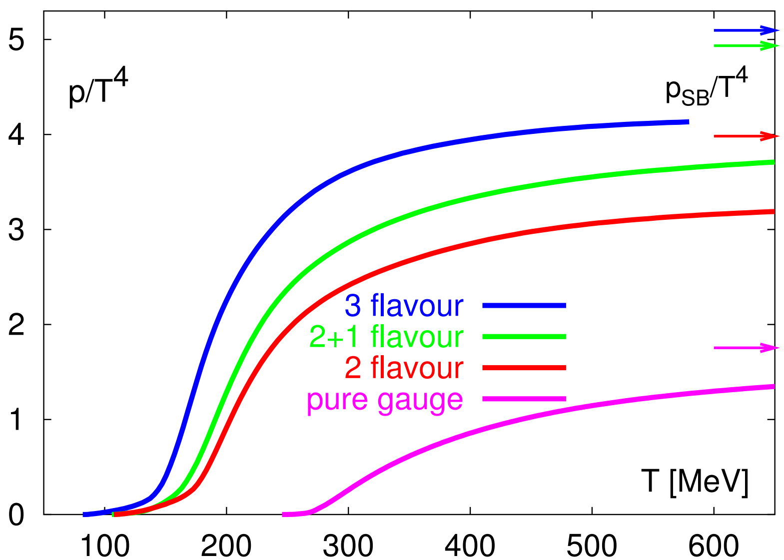
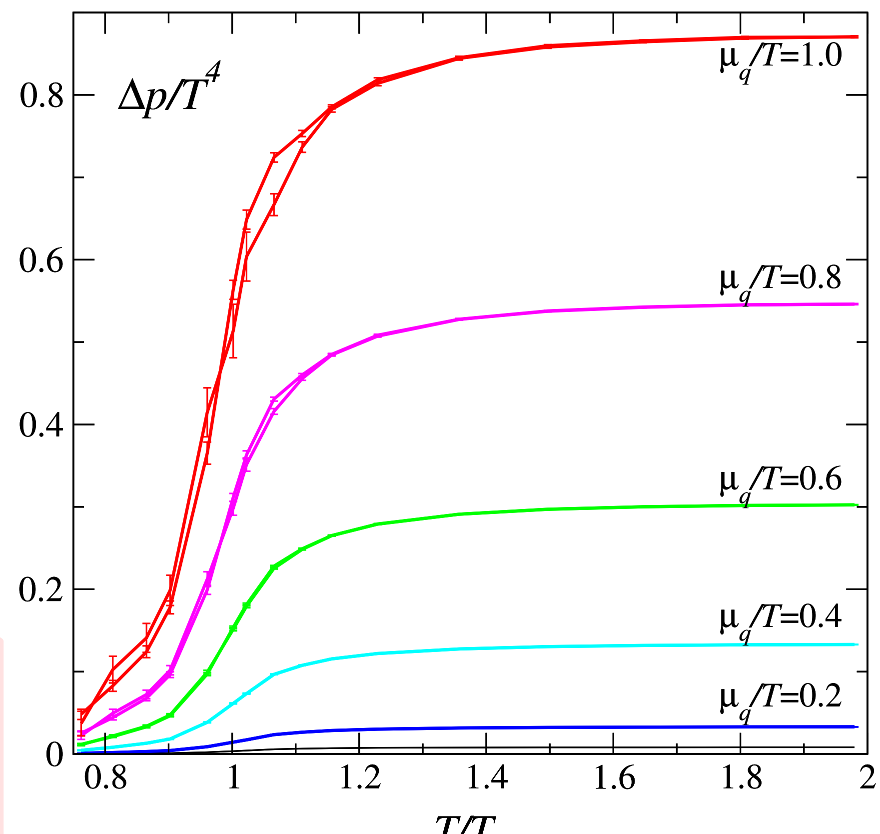


PRD71 (2005) 054508

contribution from $\mu_q/T > 0$

NEW: Taylor expansion, $\mathcal{O}((\mu/T)^6)$



pattern for $\mu_q = 0$ and $\mu_q > 0$ similar;
quite large contribution in hadronic phase;
$\mathcal{O}((\mu/T)^6)$ correction small for $\mu_q/T \lesssim 1$

RHIC: $\mu_q/T \lesssim 0.1$

Hadronic fluctuations at $\mu_q = 0$ from Taylor expansion coefficients for $\mu_q > 0$

S. Ejiri, FK, K.Redlich, in preparation

- quark number and isospin chemical potentials:

$$\mu_q = \frac{1}{2}(\mu_u + \mu_d), \quad \mu_I = \frac{1}{2}(\mu_u - \mu_d)$$

- expansion coefficients evaluated at $\mu_{q,I} = 0$ are related to hadronic fluctuations at $\mu = 0$:

  $\Uparrow$ baryon number, isospin, charge

  event-by-event fluctuations at RHIC and LHC

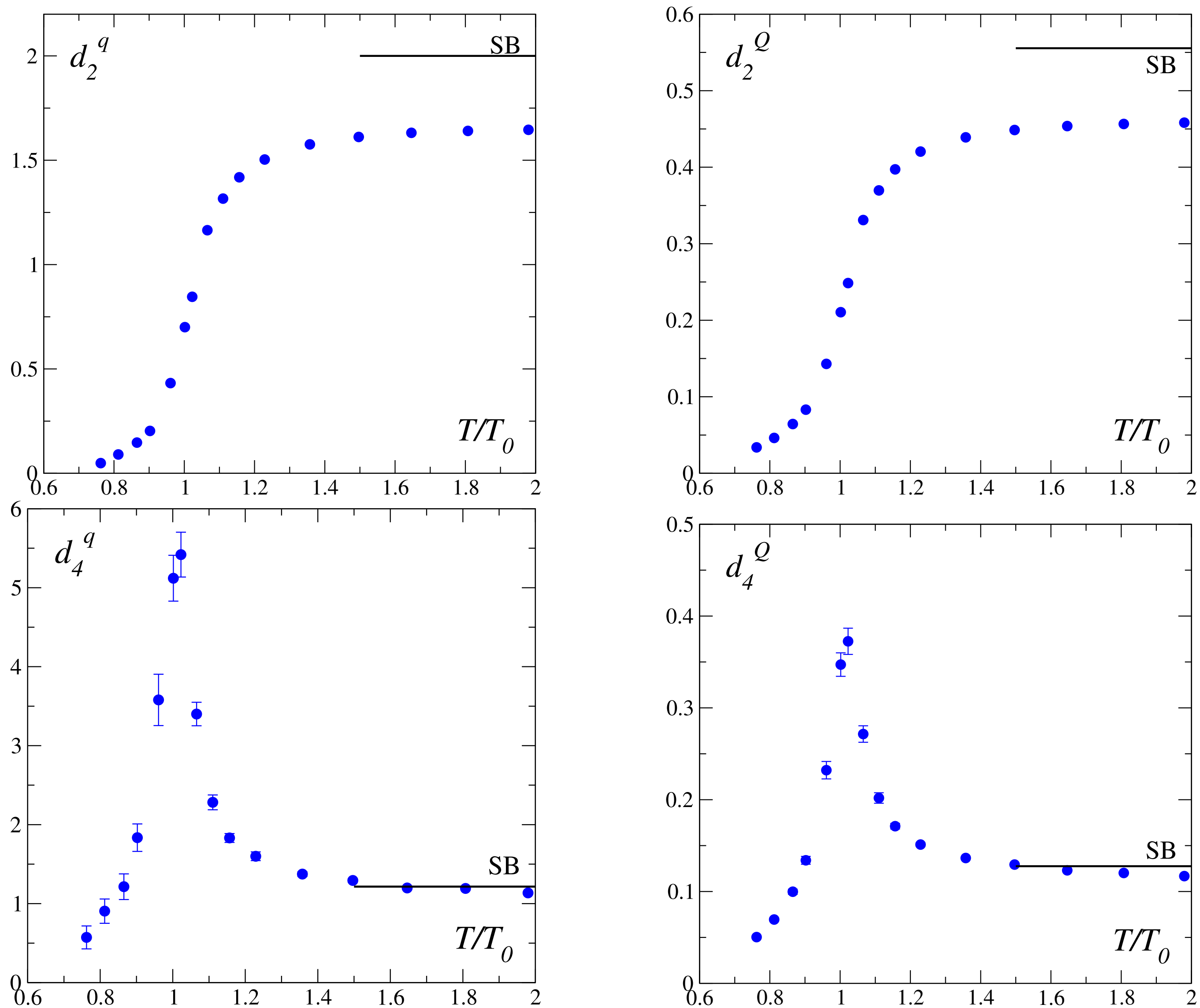
$$d_2^x = \frac{\partial^2 \ln \mathcal{Z}}{\partial(\mu_x/T)^2} = \frac{1}{VT^3}\langle(\delta N_x)^2\rangle_{\mu=0} = \frac{1}{VT^3}\langle N_x^2\rangle_{\mu=0}$$

$$d_4^x = \frac{\partial^4 \ln \mathcal{Z}}{\partial(\mu_x/T)^4} = \frac{1}{VT^3}\left(\langle(\delta N_x)^4\rangle - 3\langle(\delta N_x)^2\rangle\right)_{\mu=0} = \frac{1}{VT^3}\left(\langle N_x^4\rangle - 3\langle N_x^2\rangle\right)_{\mu=0}$$

$$\text{with} \quad , \quad x = q, I, Q \quad \text{and} \quad \partial_Q \equiv \frac{2}{3}\frac{\partial}{\partial\mu_u/T} - \frac{1}{3}\frac{\partial}{\partial\mu_d/T}$$

Quark number and charge fluctuations at $\mu_B = 0$; 2-flavor QCD ($m_\pi \simeq 770\ MeV$)

C. Allton et al. (Bielefeld-Swansea), PRD71 (2005) 054508



monotonic increase;
close to ideal gas value for $T \gtrsim 1.5 T_c$

develops cusp at T_c

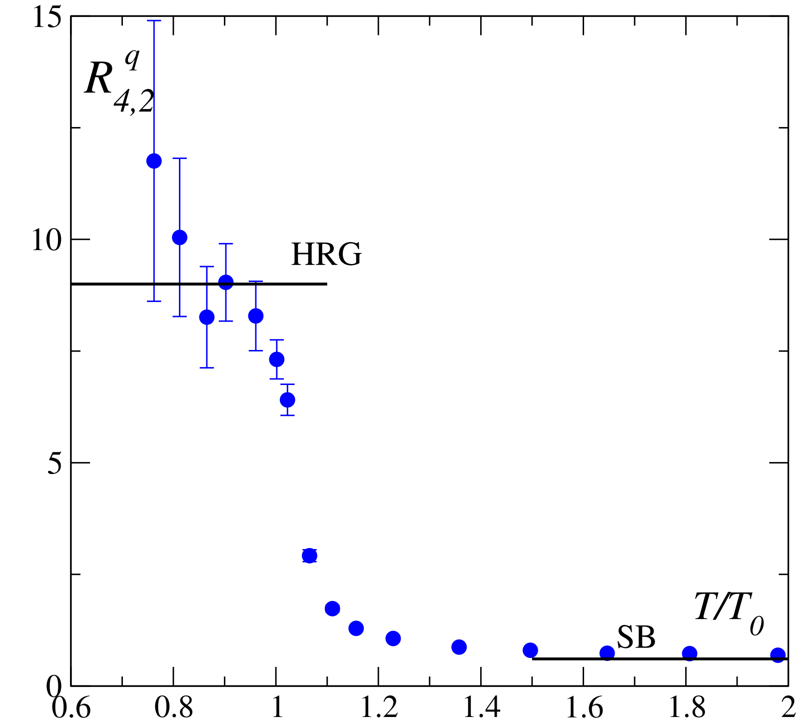
reaches ideal gas value for $T \gtrsim 1.5 T_c$

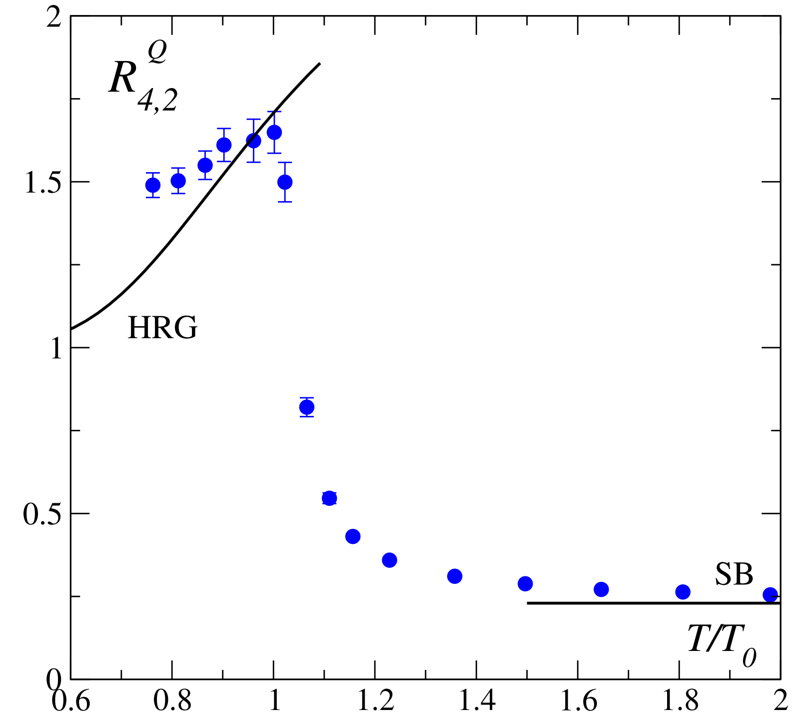
Cumulant ratios

- ratios of cumulants reflect carriers of baryon number and charge

$$R^x_{4,2} = d^x_4/d^x_2 \quad , \quad x = q,\ Q$$

$$R^q_{4,2} = \begin{cases} 9 & ,\mathrm{HRG} \\ \frac{6}{\pi^2} + \mathcal{O}(g^3) & ,\mathrm{high}-T \end{cases} \qquad R^Q_{4,2} = \begin{cases} 1 & ,\mathrm{HRG}, T \to 0 \\ \frac{34}{15\pi^2} + \mathcal{O}(g^3) & ,\mathrm{high}-T \end{cases}$$





Hadronic resonance gas $\Rightarrow$ Boltzmann approximation

heavy resonances, $T \;<<\; m_H \quad \Rightarrow \quad$ Boltzmann statistics

$\mu_B \equiv B\mu_q$

thermodynamics: $p(T,\mu_B) = \frac{T}{V} \ln Z(T,\mu_B,V) = \sum_m p_m(T,\mu_B)$

$$\ln Z(T,\mu_B,V) \;=\; \sum_{i\,\in\,\mathrm{mesons}} \ln Z^B_{m_i}(T,V) + \sum_{i\,\in\,\mathrm{baryons}} \ln Z^F_{m_i}(T,\mu_B,V)$$

contribution of baryons (fermions, -) or mesons (bosons, +) with mass m

$$\frac{p_m}{T^4} = \frac{d}{\pi^2}\left(\frac{m}{T}\right)^2 \sum_{\ell=1}^{\infty} (\pm 1)^{\ell+1}\, \ell^{-2}\, K_2(\ell m/T)\, \cosh(\ell \mu_B/T)$$

$$\Uparrow$$

$$K_2(x) \simeq \sqrt{\pi/2x}\;\exp(-x)\;, \quad x >> 1$$

- only $\ell = 1$ contributes for $(m_H - \mu_B) \gtrsim T$

$\Rightarrow$ Boltzmann approximation: $\frac{p_m}{T^4} = \frac{d}{\pi^2}\left(\frac{m}{T}\right)^2 K_2(m/T)\,\cosh(\mu_B/T)$

Hadronic resonance gas
$\Rightarrow$ Boltzmann approximation

heavy resonances, $T << m_H$ $\Rightarrow$ Boltzmann statistics

$\mu_B \equiv B\mu_q$

thermodynamics: $p(T,\mu_B) = \frac{T}{V} \ln Z(T,\mu_B,V) = \sum_m p_m(T,\mu_B)$

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baryons:

$\mu_3 = 3\mu_q$

diquarks:

$\mu_2 = 2\mu_q$

quasi-part.:

$\mu_1 = \mu_q$

The sQGP model: $\mu > 0$

E.V. Shuryak, I. Zahed, PR D 70 (2004) 054507

- heavy colored bound states for $T_c \leq T \lesssim 2T_c$:

  $$m_{\rm col} \approx 11.5\, T_c \left((T/3T_c)^{0.5} + 0.1 T_c/(T - T_c)\right)$$

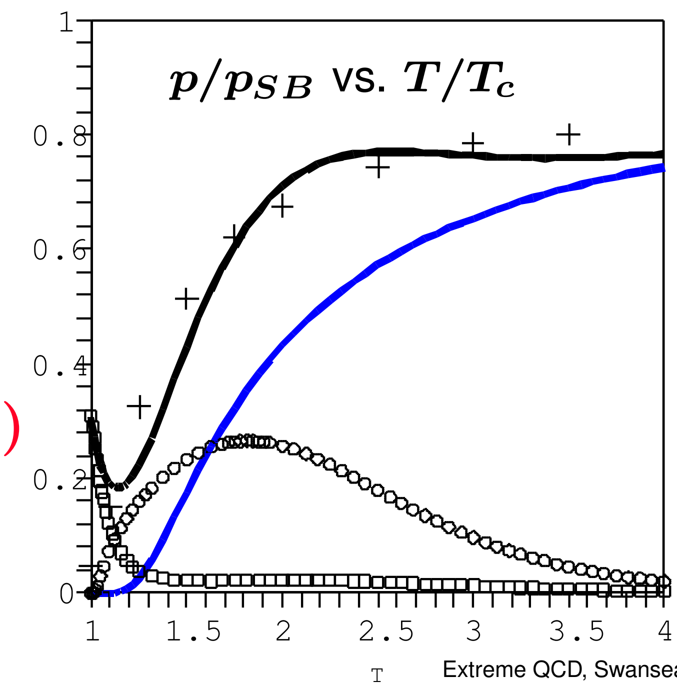
- $\sim$ 200 qg and $\bar{q}g$ states; $\sim$ 100 qq and $\bar{q}\bar{q}$ states ($n_f = 2$)
- contribute to the pressure like in a resonance gas (Boltzmann approximation) with reduced weight factor,

  $$R(T) = \frac{1}{1 + \exp[2(T - 2T_c)/T_c]}$$

  that reflects "melting" of the states

$\mu_q > 0$: $p/T^4 = F_q(T)\cosh(\mu_q/T)$

$$+R(T)\Big(F_{qg}(T)\cosh(\mu_q/T) + F_{qq}(T)\cosh(2\mu_q/T) \Big)$$



Fluctuations and colored bound states

- hadronic resonance gas:
  $p/T^4 = F_B(T)\cosh(3\mu_q/T)$: $d_n^q = 3^n F_B(T) \;\Rightarrow\; R_{4,2}^q = 9$
- if heavy colored diquarks contribute to fluctuations above T_c:

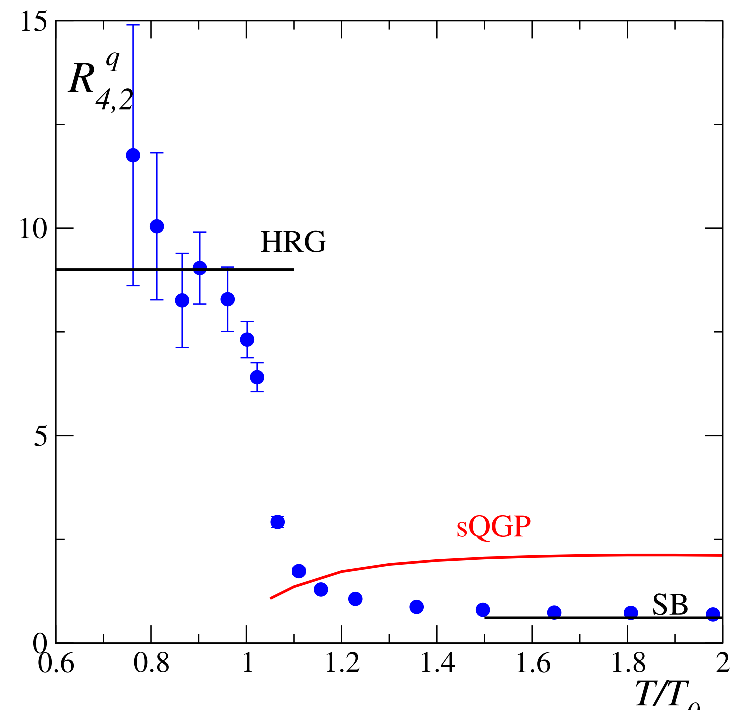
$$d_2^q = F_q(T) + R(T)\big(F_{qg}(T) + 4F_{qq}(T)\big)$$

$$d_4^q = F_q(T) + R(T)\big(F_{qg}(T) + 16F_{qq}(T)\big)$$

- increase in baryon number density fluctuations in the QGP due to "doubly charged" diquarks not confirmed by LGT:

sQGP: $\Rightarrow \; R_{4,2}^q = \dfrac{d_4^q}{d_2^q} \simeq 2.1$

at $T \simeq 2\,Tc$



Fluctuations and colored bound states

n_f-dependence:

$$d_2^q = F_q(T) + R(T)\big(F_{qg}(T) + 4F_{qq}(T)\big)$$

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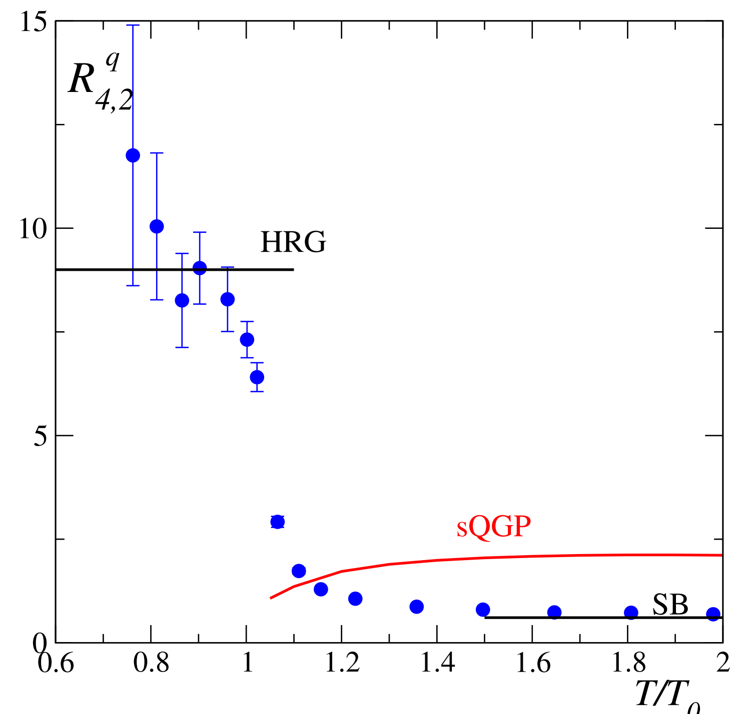

Fluctuations and colored bound states

n_f-dependence:

$\mathcal{O}(n_f^2)$ discrepancy will increase for $n_f = 3$

$$d_2^q = F_q(T) + R(T)\big(F_{qg}(T) + 4F_{qq}(T)\big)$$
$$d_4^q = F_q(T) + R(T)\big(F_{qg}(T) + 16F_{qq}(T)\big)$$

- increase in baryon number density fluctuations in the QGP due to "doubly charged" diquarks not confirmed by LGT:

sQGP: $\Rightarrow \; R_{4,2}^q = \dfrac{d_4^q}{d_2^q} \simeq 2.1$

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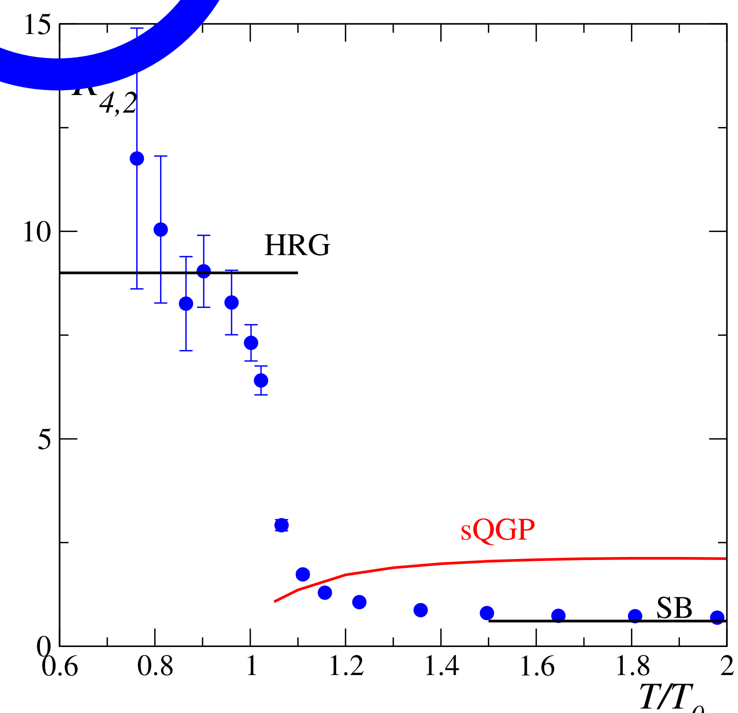
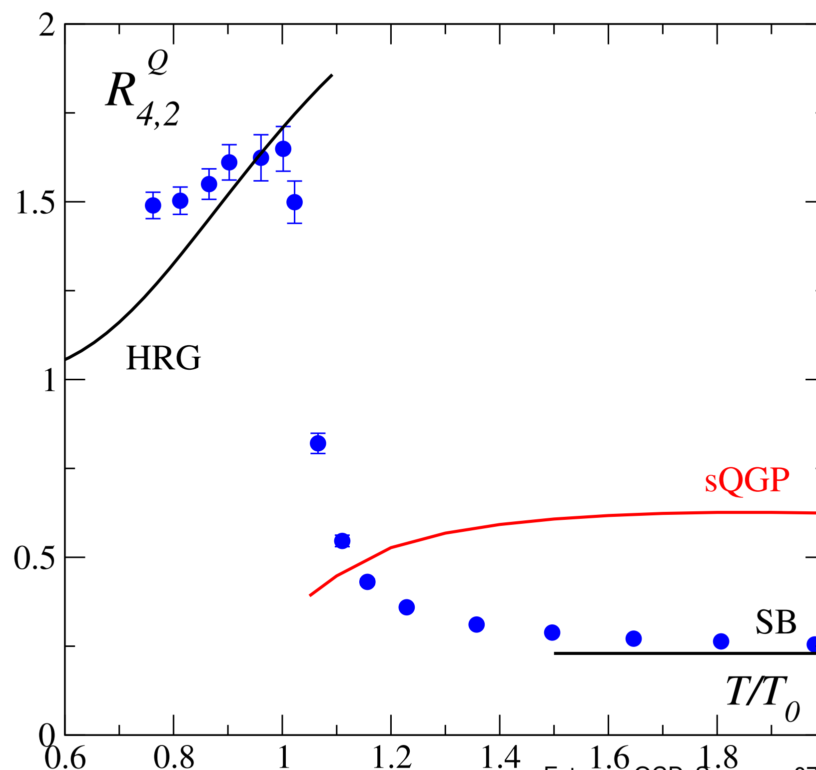


Charge fluctuations and colored bound states

- hadronic resonance gas: $R^Q_{4,2} \equiv \dfrac{d^Q_4}{d^Q_2} \longrightarrow 1$ for $T \longrightarrow 0$
- heavy colored diquarks dominate the fluctuations above T_c:

  sQGP: $\Rightarrow \ R^Q_{4,2} \simeq 0.62$ at $T \simeq 2\ T_c$
- large charge fluctuations in the QGP not confirmed by LGT



Similar conclusion drawn from baryon-strangeness correlations, V.Koch, A.Majumder, J.Randrup, nucl-th/0505052

Conclusions

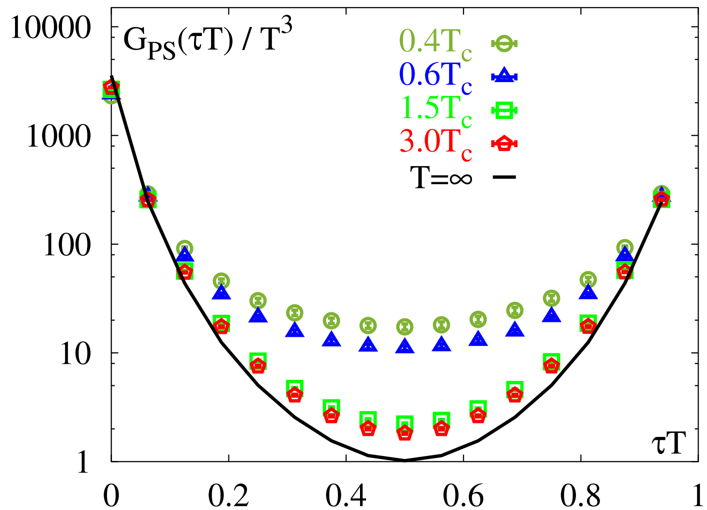
- running coupling:
  - T-independent at short distances;
  - $g(T) \simeq 1.5$ describes screening for $rT \gtrsim 1$;
  - free energies above T_c reflect "remnants of confinement"
- equation of state:
  - strong deviations from $\epsilon = 3p$ for $T \lesssim (2-3)T_c$
  - consistent with HTL-resummation for $T \gtrsim 3T_c$
- hadronic fluctuations above T_c
  - consistent with QCD perturbation theory for $T \gtrsim 2T_c$
  - no evidence for contributions from a large number of colored bound states in terms of a simple, non-interacting resonance gas

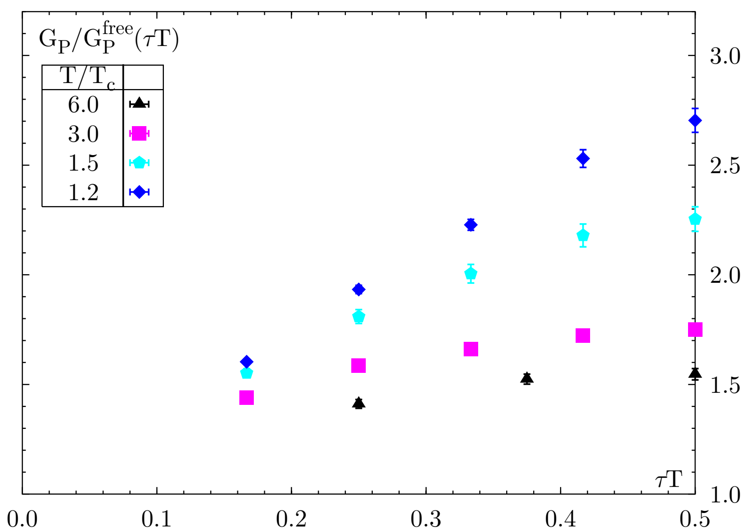
... another ingredient in the discussion of the sQCD model or its justification through lattice results

”resonance-like” structures in spectral functions of (light quark) pseudo-scalar and vector mesons

Light quark lattice correlators: pseudo-scalar and vector channel

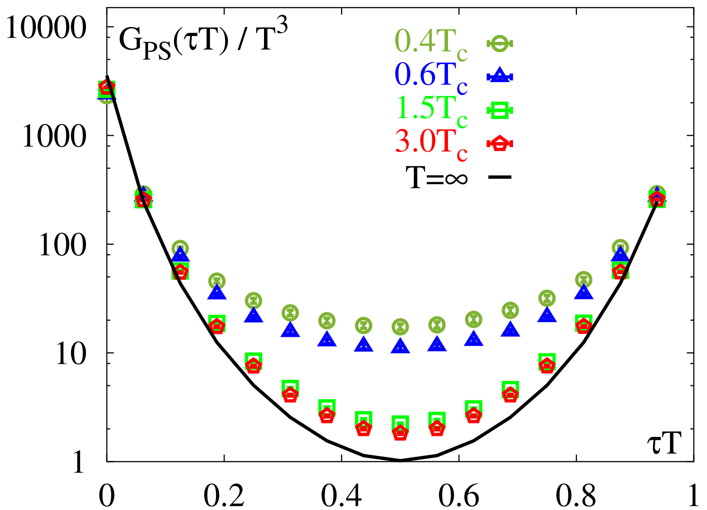
pseudo-scalar correlation functions

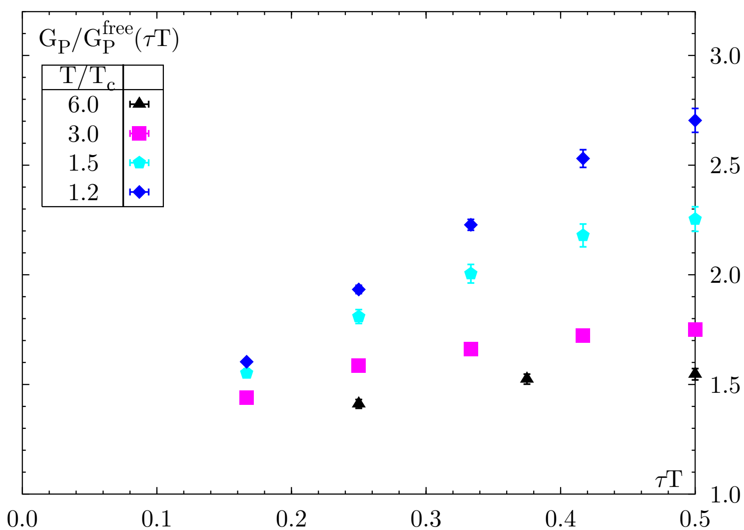




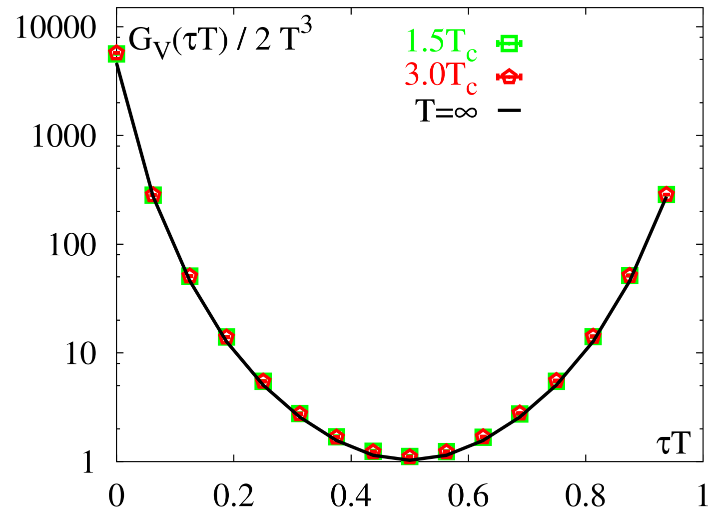
Light quark lattice correlators: pseudo-scalar and vector channel

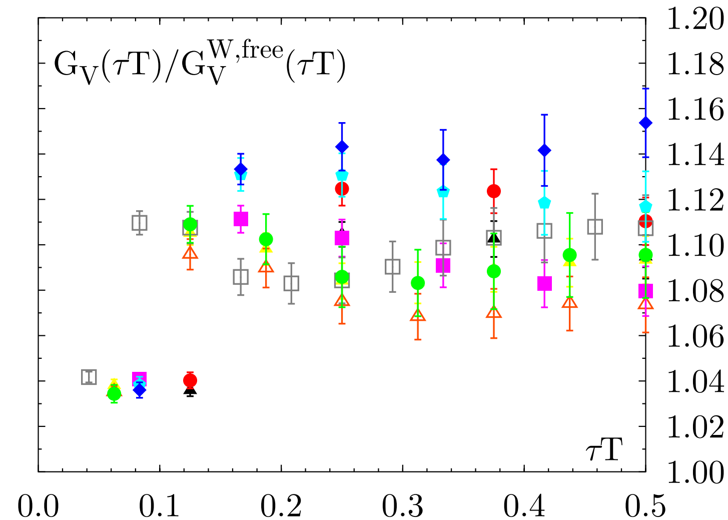
pseudo-scalar correlation functions



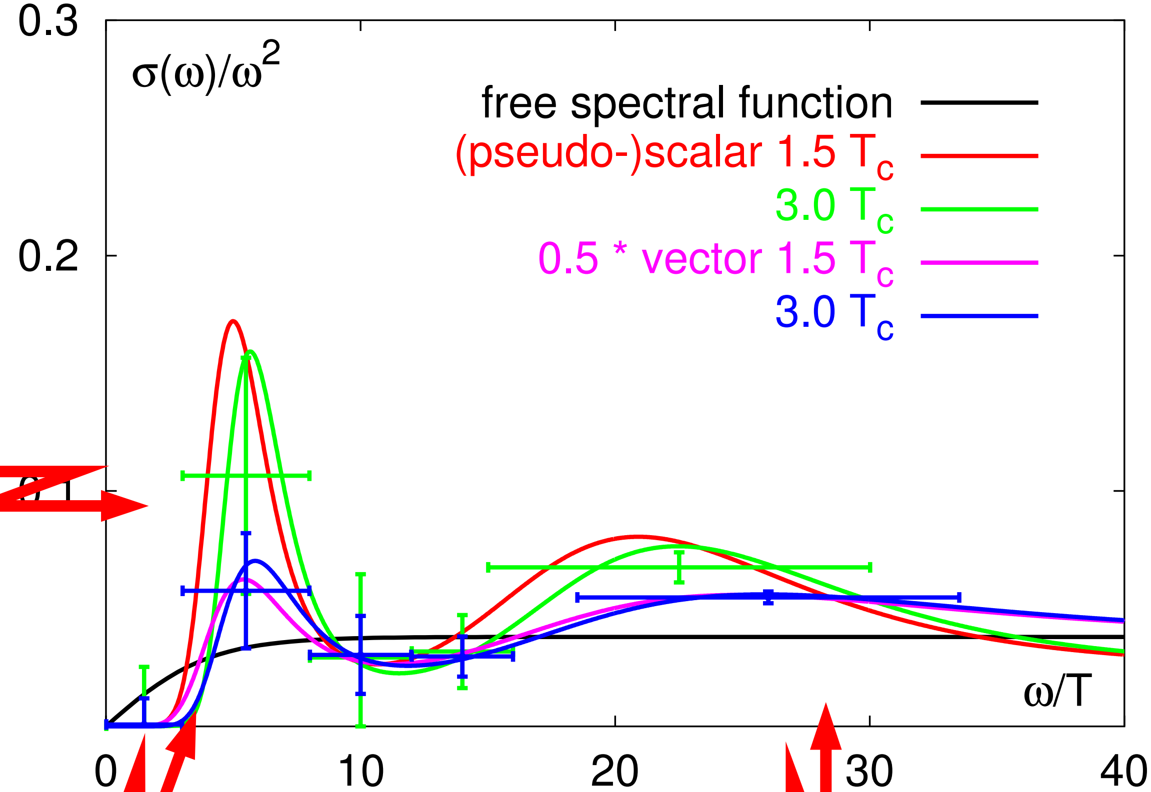


vector spectral functions





(Pseudo-)scalar and vector spectral functions above T_c



reconstructed

using MEM

enhancement

for $\omega \sim 5T$

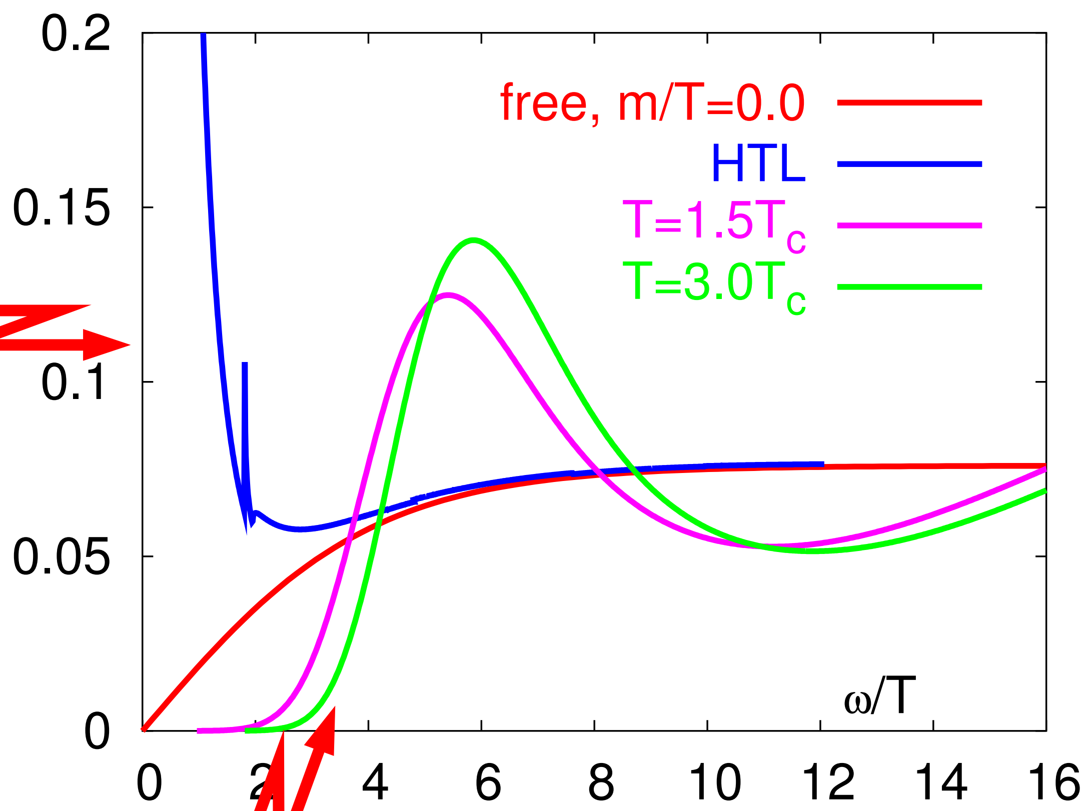
suppression relative
to free spec. fctn.
for $\omega \lesssim 3T$

lattice artefact:
enhancement due to
Wilson doublers

Lattice and HTL spectral functions

HTL-spf

$\sigma \sim 1/\omega$



HTL and LGT may agree for $\omega/T \gtrsim 4$

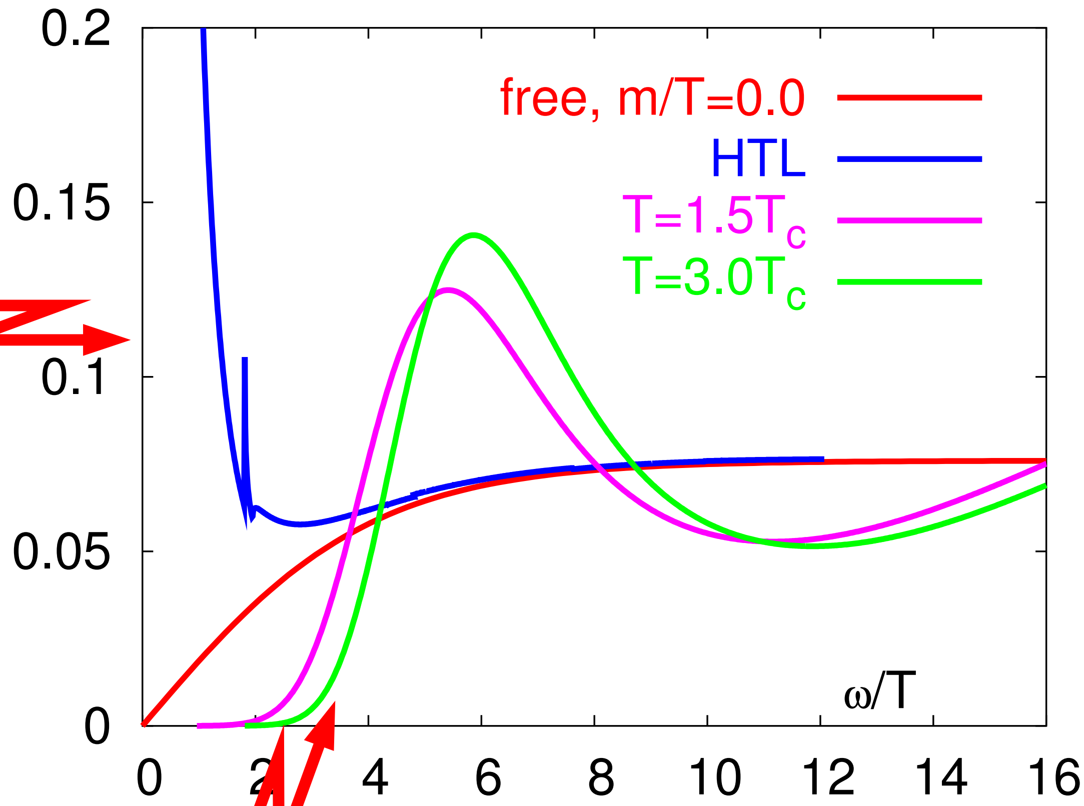
LGT-spf:

suppression relative to free spec. fctn. for $\omega \lesssim 3T$

Lattice and HTL spectral functions

HTL-spf

$\sigma \sim 1/\omega$



HTL and LGT may agree for $\omega/T \gtrsim 4$

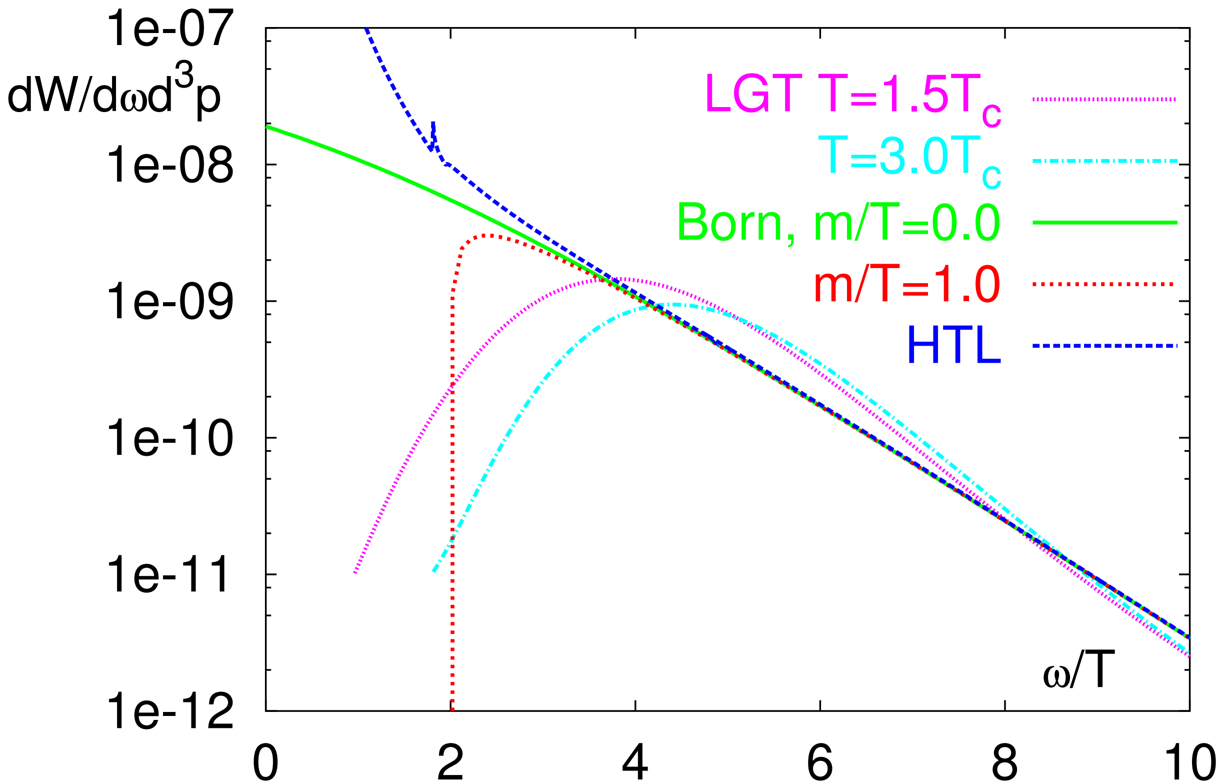
LGT-spf:

suppression relative to free spec. fctn. for $\omega \lesssim 3T$

correlators constructed from HTL-SPF with an energy cut-off at $\omega/T \simeq 2$ are similar to LGT result

Lattice and HTL dilepton rates

$$\frac{\mathrm{d}W}{\mathrm{d}\omega \mathrm{d}^3p} = \frac{5\alpha^2}{27\pi^2}\frac{\sigma_V(\omega,T)}{(\mathrm{e}^{\omega/T}-1)\omega^2} \quad , \quad \frac{\mathrm{d}W^{Born}}{\mathrm{d}\omega \mathrm{d}^3p} = \frac{5\alpha^2}{36\pi^4}\frac{1}{(\mathrm{e}^{\omega/2T}+1)^2}$$



only small enhancement over Born rate for $\omega \gtrsim 4T$

HTL and LGT may agree for $\omega/T \gtrsim 4$