Heavy quark diffusion and lattice correlators

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- Numerical results on 1S quarkonia correlators
A. Jakovác, P.P., K. Petrov and A. Velytsky

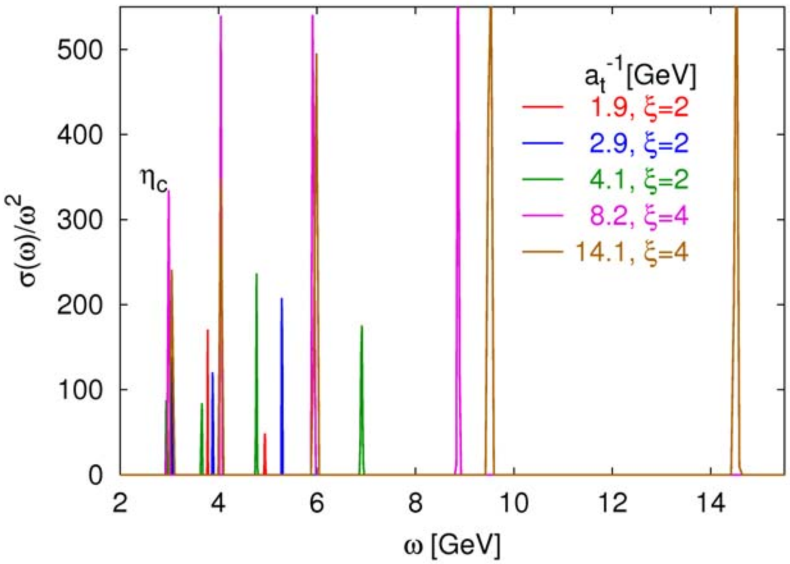
- Low energy part of quarkonia spectral functions from Langevin effective theory
P.P. and D. Teaney, hep-ph/0507318

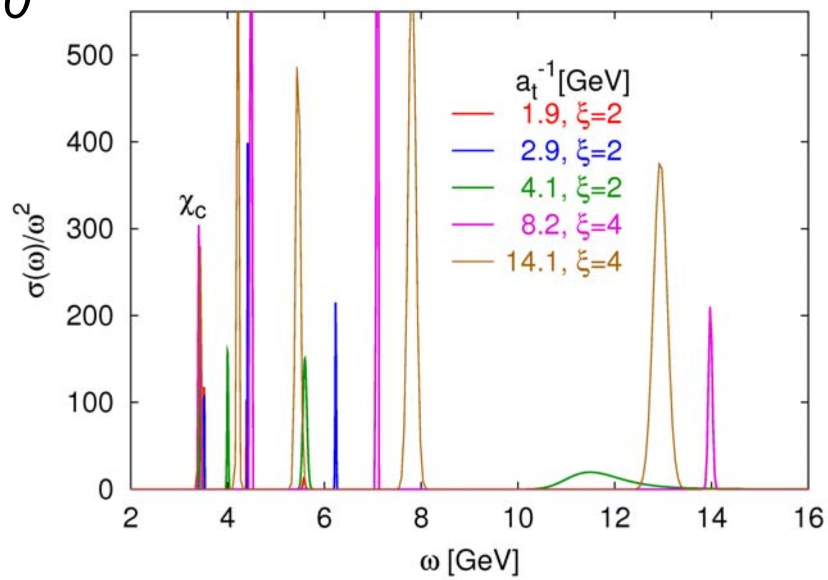
- Heavy quark diffusion constant from lattice correlators ?
Existing lattice data on Euclidean correlators cannot give any information about transport coefficients ! <1% accuracy is needed for the correlators
see also G. Aarts and J.M. Martinez Resco, JHEP 0204, 053 (2002)

- Why heavy quarks ? $t_{tran} \sim M/T^2 \gg$ any timescale
Light quarks : $t_{tran} \sim 1/(g^4 T)$ large only in the weak coupling

Charmonia spectral functions at zero and finite T

T=0





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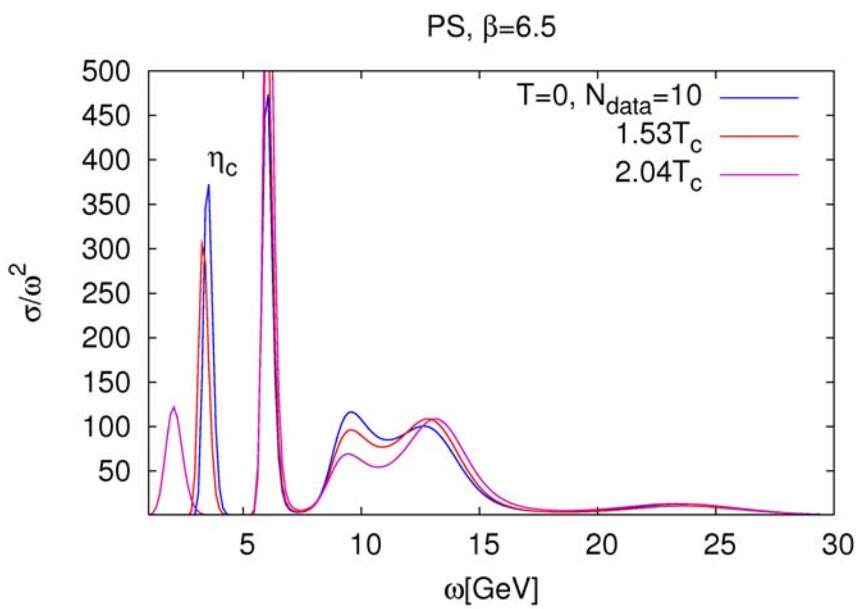
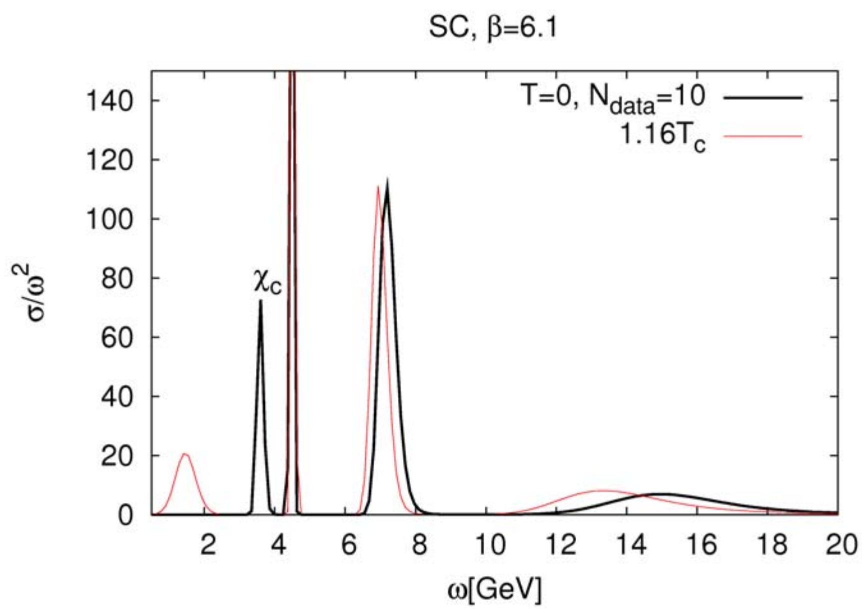
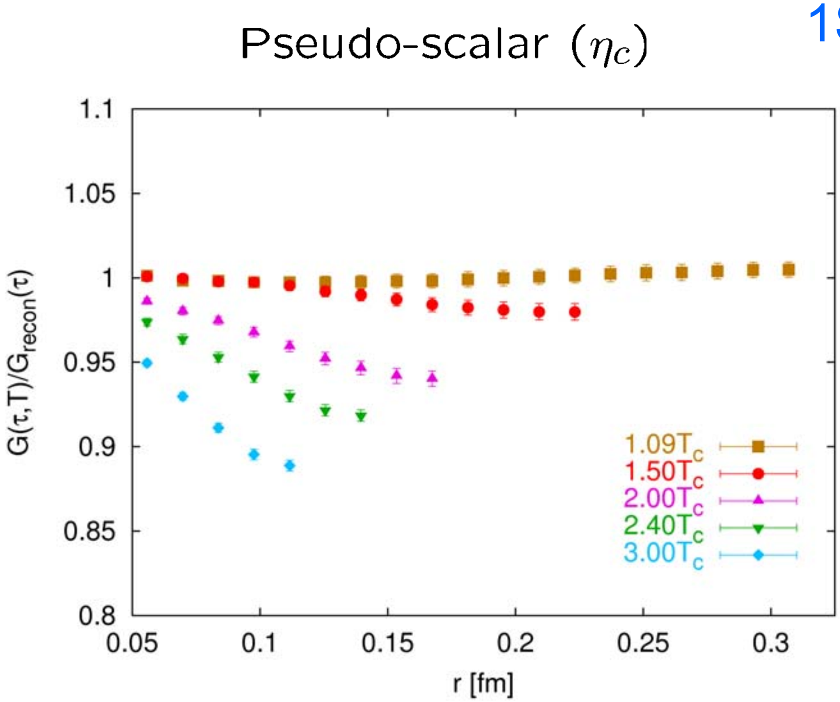


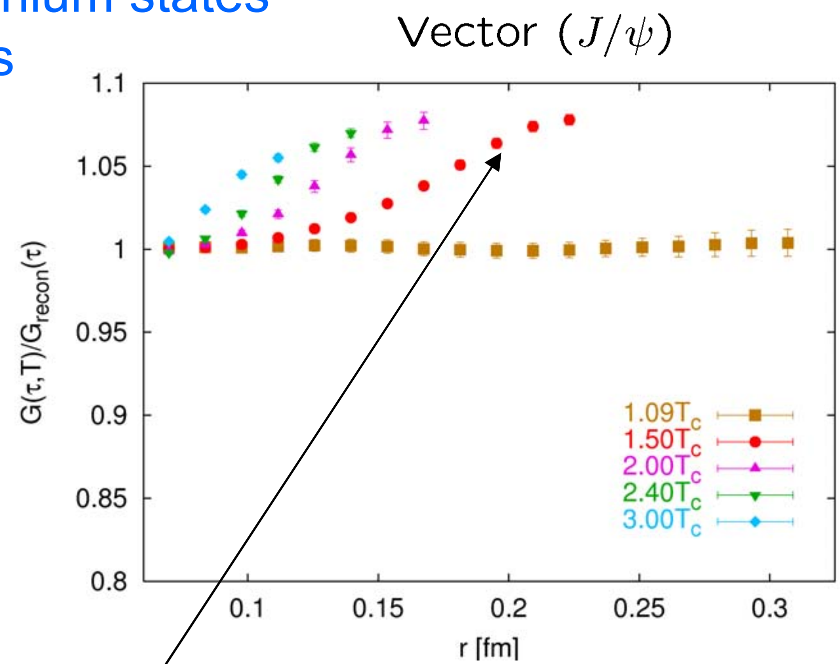


in agreement with Datta, Karsch, P.P., Wetzorke, PRD 69 (04) 094507

Vector correlator and heavy quark diffusion



1S charmonium states survies



Vector current is conserved $\longrightarrow$ fluctuations of charm number

$$\sigma_V^{ii}(\omega) = F_{J/\psi}^2(T)\delta(\omega^2 - m_{J/\psi}^2(T)) + \frac{1}{4\pi^2}\omega^2\sqrt{1 - \frac{4m_D^2(T)}{\omega^2}} + \chi_s(T)\left(\frac{T}{M}\right)\omega\delta(\omega)$$

$$\frac{1}{3}\chi_s(T)\frac{T}{M}\omega \cdot \frac{1}{\pi} \cdot \frac{\eta}{\omega^2 + \eta^2}$$

Interactions

Free streaming :
Collision less Botzmann equation

Effective Langevin theory

$$\eta = \frac{T}{M}\frac{1}{D} \qquad \partial_t N_c + D\nabla^2 N_c = 0$$

Heavy quark diffusion, linear response and Euclidean correlatots

Linear response :

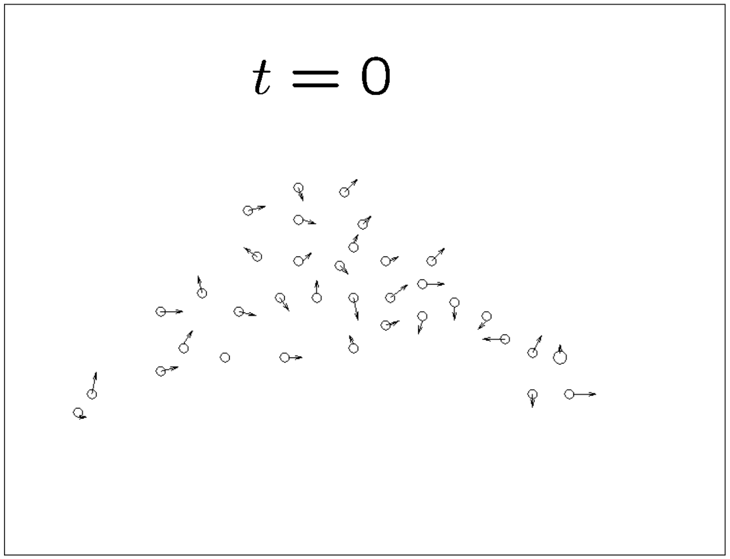
$$H = H_0 - \int d^3x \mu(x,t),\ N(x,t)\ N(x,t) = \bar{q}(x,t)\gamma_0 q(x,t),\ \mu(x,t) = e^{\epsilon t}\theta(-t)\mu(x)$$

$$\langle \delta N(x,t) \rangle = \int_{-\infty}^{\infty} dt' \chi_{NN}(x, t'-t)\mu(x,t')$$

$$\sigma_{NN}(k,\omega) = \frac{1}{\pi}\mathrm{Im}\chi_{NN}(k,\omega)$$

$$\chi_{JJ}^{ij}(k,\omega) = (\delta_{ij} - \frac{k_i k_j}{k^2})\chi_{JJ}^{T}(k,\omega) + \frac{k_i k_j}{k^2}\chi_{JJ}^{L}(k,\omega)$$

$$\frac{\omega^2}{k^2}\chi_{NN}(k,\omega) = \chi_{JJ}^{L}(k,\omega)$$



Euclidean correlators:

$$G^{00}(k,\tau) = \int d^3x e^{i\mathbf{kx}} \langle J_E^0(x,\tau) J_E^0(0,0)\rangle = -D_{NN}(k,-i\tau) = -\int_0^{\infty} d\omega \sigma_{NN}(k,\omega) K(\tau,\omega,T)$$

$$G^{ij}(k,\tau) = \int d^3x e^{i\mathbf{kx}} \langle J_E^i(x,\tau) J_E^j(0,0)\rangle = D_{JJ}^{ij}(k,-i\tau) = \int_0^{\infty} d\omega \sigma_{JJ}^{ij}(k,\omega) K(\tau,\omega,T)$$

$$K(\tau,\omega,T) = \frac{\cosh(\omega(\tau - 1/(2T)))}{\sinh(\omega/(2T))}$$

Correlators and diffusion

$$\rho_{NN,JJ}(k,\omega) = \rho^{\text{high}}_{NN,JJ}(k,\omega) + \rho^{\text{low}}_{NN,JJ}(k,\omega)$$

$$G^{L,\text{low}}_{JJ}(k,\tau) \simeq 2T\int_0^\infty \frac{d\omega}{\omega}\,\rho^{L,\text{low}}_{JJ}(k,\omega)\left[1-\frac{1}{6}\left(\frac{\omega}{2T}\right)^2 + \omega^2\frac{1}{2}(\tau-\beta/2)^2+\ldots\right]$$

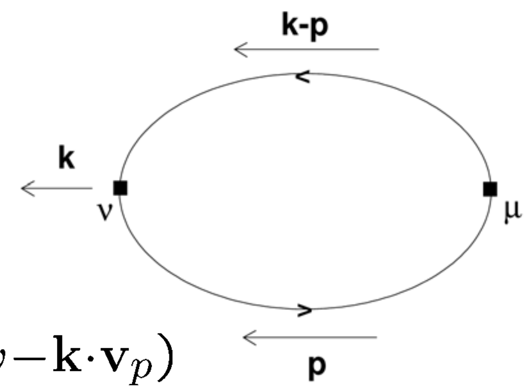
$$G^{L,\text{low}}_{JJ}(k,\tau) = \frac{T}{k^2}\left[\partial_t^{(1)}\chi_{NN}(k,t) + \frac{1}{24\,T^2}\partial_t^{(3)}\chi_{NN}(k,t)\; - \;\partial_t^{(3)}\chi_{NN}(k,t)\,\frac{1}{2}(\tau-\beta/2)^2+\ldots\right]_{t=0}$$

$$\beta = 1/T$$

Collisionless Boltzmann equation :

$$\left(\frac{\partial}{\partial t} + v_p^i\frac{\partial}{\partial x^i}\right) f(x,p,t) = 0$$

Bare 1-loop :



$$\rho^{\text{low}}_{NN}(k,\omega) = \frac{1}{T}\int\frac{d^3p}{(2\pi)^3} f_p(1\pm f_p)\,\mathbf{k}\cdot\mathbf{v}_p\,\delta(\omega-\mathbf{k}\cdot\mathbf{v}_p)$$

$$\rho^{\text{low}}_{JJ}(k,\omega) = \chi_s\frac{\omega^3}{k^2}\frac{1}{\sqrt{2\pi k^2\frac{T}{M}}}\exp\left(-\frac{\omega^2}{2k^2\frac{T}{M}}\right)$$

$\mathbf{k}\to 0$:

$$\rho^{L,\text{low}}_{JJ}(k,\omega) = \chi_s\frac{T}{M}\,\omega\delta(\omega)$$

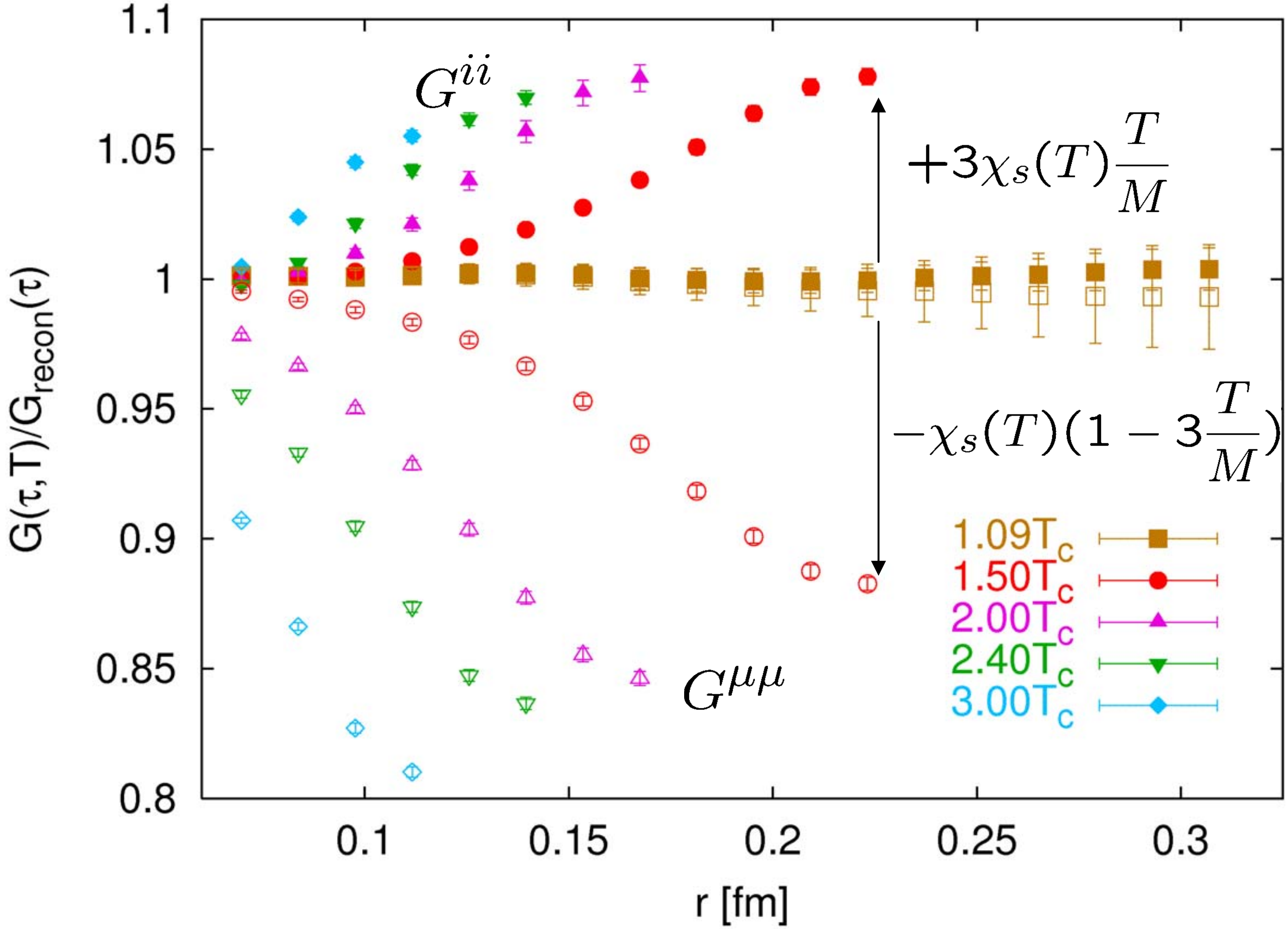
$$\rho^{L,\text{low}}_{NN}(k,\omega) = \chi_s\,\omega\delta(\omega)$$

$$G^{\text{low}}_{JJ}(k,\omega) = T\chi_s(k)\frac{T}{M}$$

$$G^{\text{low}}_{NN}(k,\omega) = T\chi_s(k)$$

Lattice data on the vector correlator

$$\beta = 6.5,\ \xi = a_s/a_t = 4,\ a_t = 14.12\text{GeV},\ 24^3 \times N_t$$



$$\frac{M}{T} \simeq 4.6$$

Transport contribution can be clearly seen !

Correlators and diffusion

$$t_{tran} \sim M/T^2 \gg 1$$

Moore, Teaney, PRC 71 (05) 064904

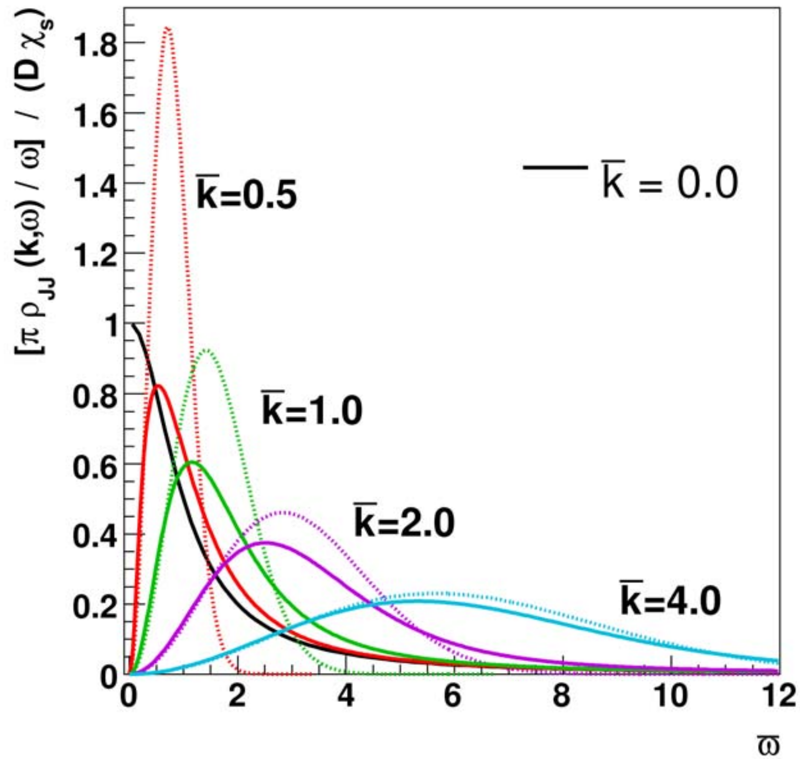
$$\frac{dx^i}{dt} = \frac{p^i}{M}, \frac{dp^i}{dt} = \xi^i(t) - \eta p^i,$$

$$\langle \xi^i(t) \xi^j(t') \rangle = \kappa \delta^{ij} \delta(t - t')$$

$$\eta = \frac{\kappa}{2MT}, \; D = \frac{T}{M\eta}$$

$$t \gg 1/\eta : \partial_t N(x,t) + D\nabla^2 N(x,t) = 0$$

$$\bar{k} = kD\sqrt{M/T}, \; \bar{\omega} = \omega D(M/T)$$



$k \ll \eta\sqrt{M/T}$:

$$\chi_{NN}(k,\omega) = \frac{\chi_s D k^2}{-i\omega + k^2 D} - \frac{\chi_s D k^2}{-i\omega + \eta}$$

$$\rho_{NN}(k=0,\omega) = \chi_s \omega \delta(\omega)$$

$$\rho_{JJ}(k=0,\omega) = \chi_s \omega \frac{1}{\pi} \frac{T}{M} \frac{\eta}{\omega^2 + \eta^2}$$

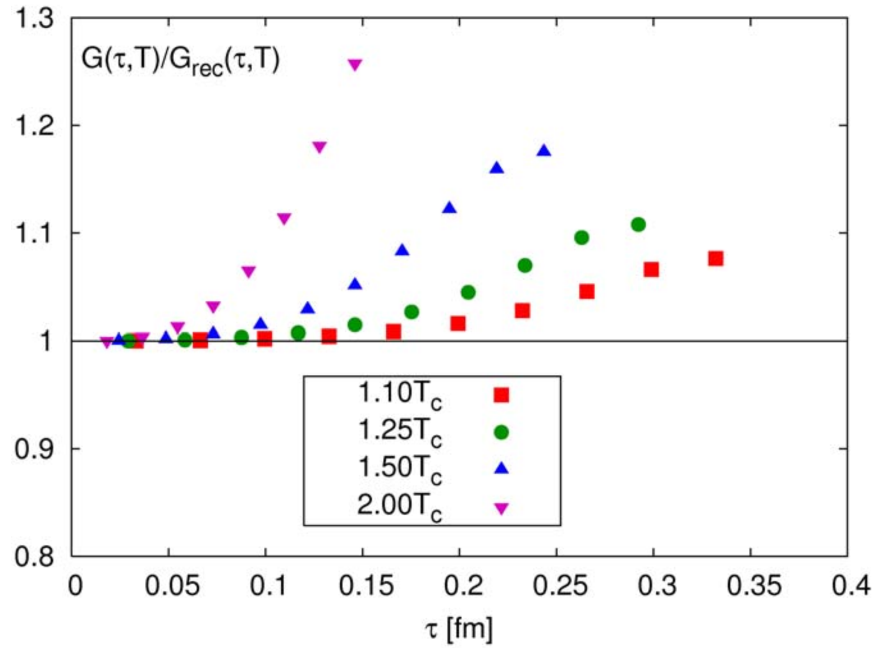
Transport contribution to the Euclidean correlators

J/psi survives in the plasma with almost no modifications of its properties

Umeda et al, '02, Asakawa ad Hatsuda, '04 Datta et al, '04,
Petrov, Lat '05

$$\rho^{\text{high}}_{JJ}(\omega) = m^3_{J/\psi} f^2_{J/\psi} \delta(\omega^2 - m^2_{J/\psi}) + \frac{1}{3}\frac{N_c}{8\pi^2}\theta(\omega^2 - 4m_D^2)\omega^2\sqrt{1 - \frac{4m_D^2}{\omega^2}}(2 + \frac{4m_D^2}{\omega^2})$$

$M_{J/\psi},\ f_{J/\psi}, m_D$ at $T=0$ from PDG



Examine *1/D=0* case :

$$\chi_s = \int \frac{d^3p}{(2\pi)^3} \exp(-\sqrt{p^2 + M^2}/T)$$

$$G^{rec}_{JJ}(\tau, T) = \int_0^\infty d\omega \rho_{JJ}(\omega, T=0) K(\omega, \tau, T)$$

$$\delta G_{JJ} \equiv G_{JJ}(\tau, T, \mu) - G_{JJ}(\tau, T, \mu = 0)$$

$$\simeq (\cosh(\mu/T) - 1) \int_0^\infty d\omega \rho^{\text{low}}_{JJ}(\omega) K(\tau, \omega, T) = G^{\text{low}}_{JJ}(\tau)$$

Transport contribution to the Euclidean correlators

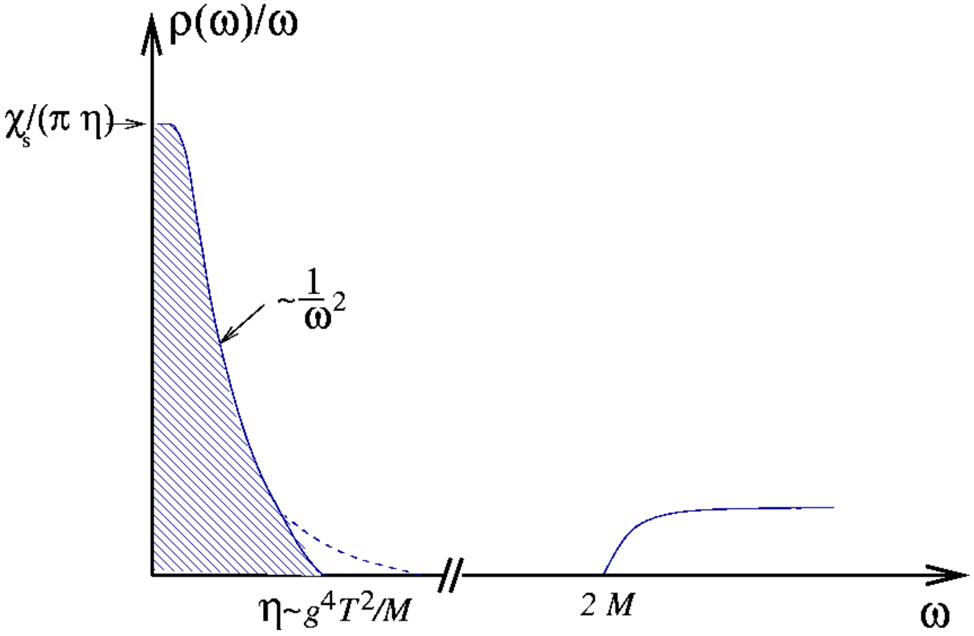
Interactions smear out the
$\chi_s \omega \delta(\omega)$ term
$width \sim \eta$

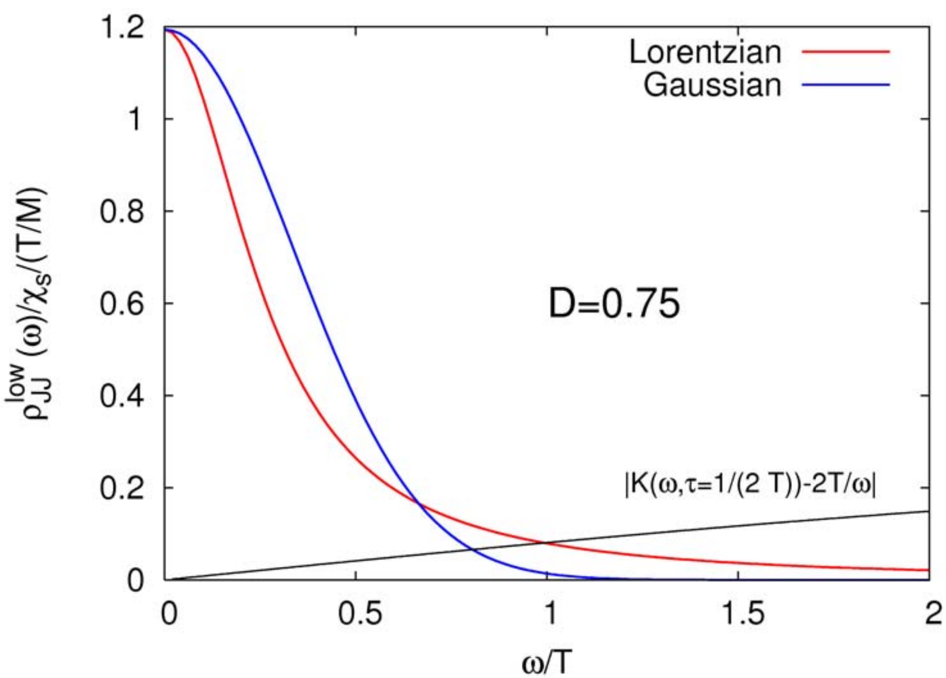
valid only for $\omega < \eta \ll T$ but
$K(\omega,\tau,T) - \frac{2T}{\omega}$ has support for $\omega \sim T$

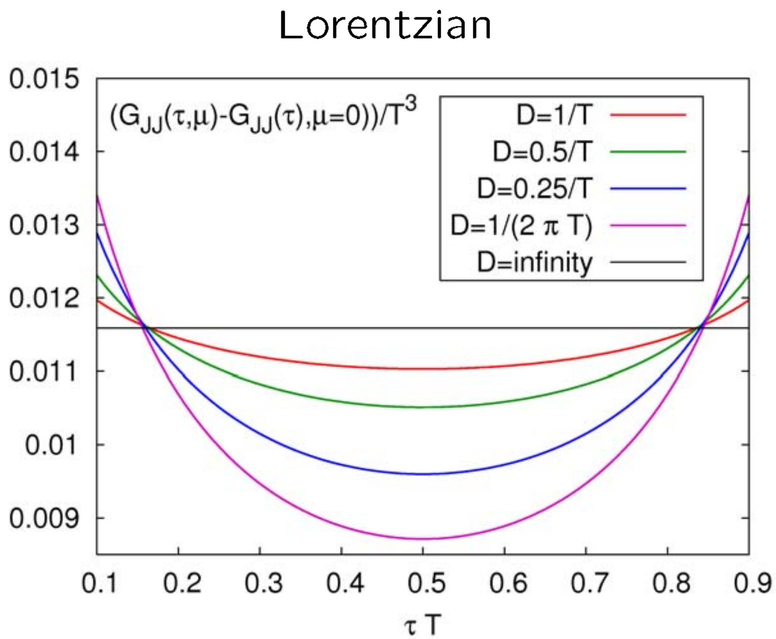
$G^{\rm low}_{JJ}(\tau) \neq const$ but has a small curvature around $\tau = 1/(2T)$

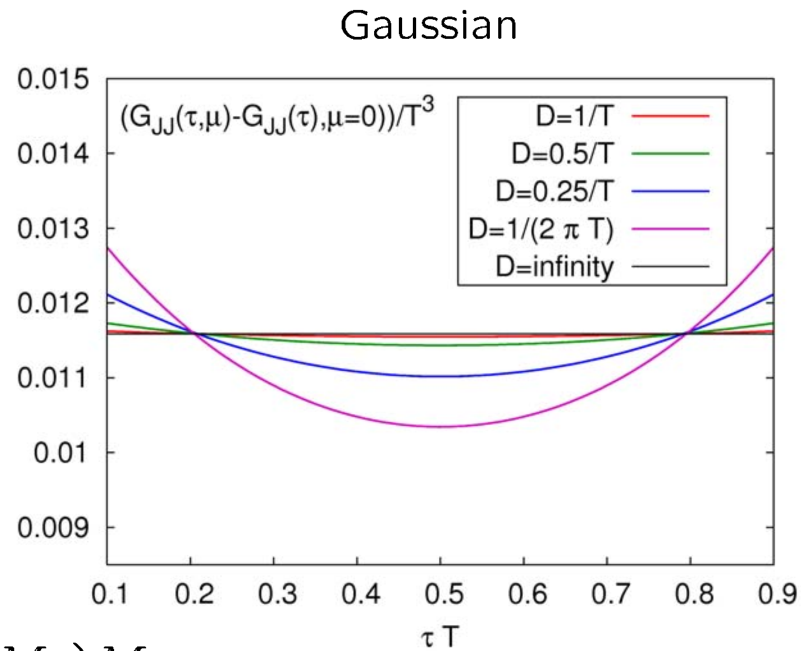
$$\rho^{\rm low}_{JJ}(\omega) = \chi_s \omega \frac{1}{\pi}\frac{T}{M}\frac{\eta}{\omega^2+\eta^2}$$

$$\rho^{\rm low}_{JJ}(\omega) = \chi_s \omega \frac{T}{M}\frac{1}{\sqrt{2\pi\eta_g^2}} e^{-\frac{\omega^2}{2\eta_g^2}},\ \ \eta_g = \sqrt{\frac{\pi}{2}}\frac{T}{MD}$$

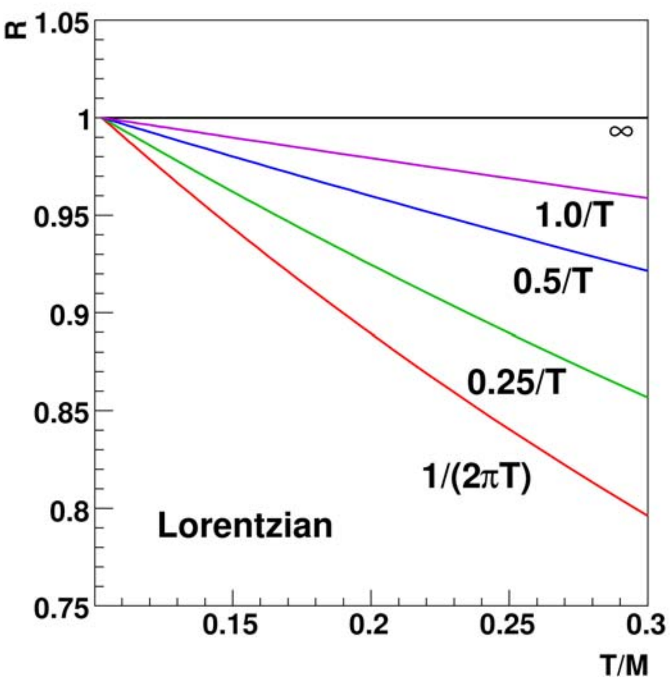


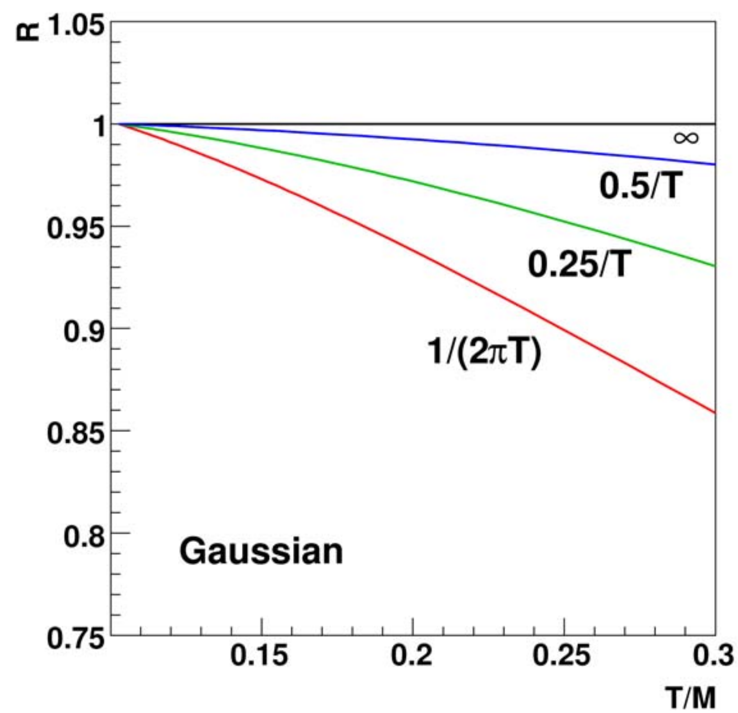






$$R \equiv \frac{\delta G_{JJ}(M)\chi_s(M_0)M_0}{\delta G_{JJ}(M_0)\chi_s(M)M}|_{\tau=1/(2T)}$$





Summary and Outlook

- It has been shown how heavy quark transport of heavy quarks contributes to the Euclidean meson correlators

- Transport contribution to the Euclidean correlator is related to time derivatives of the real time correlators at $t=0$ and therefore is insensitive to transport coefficients

- Recently large azimuthal anisotropies in the charm meson distribution were observed at RHIC and interpreted as evidence for heavy quark thermalization;
  this requires $DT<1$
  If $DT<1$ the Euclidean correlators may become sensitive to the value of D and can confirm or rule out the scenarios of heavy quark thermalization

- Future :
  analyze correlators at finite spatial momentum ➡ further constraints on D
  study the vector correlator in the light quark sector ➡ electric conductivity