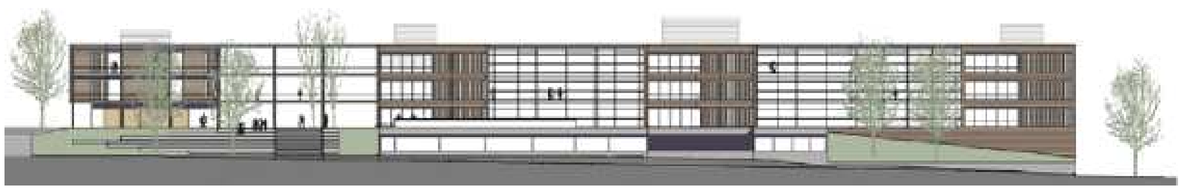
Color superconductivity in dense quark matter

Igor A. Shovkovy



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Outline

I. Introduction into color superconductivity

- General properties of dense matter
- Cooper instability and the ground state
- $N_f = 2$ color superconductivity
- Color-flavor locked phase ($N_f = 3$)
- Spin-1 color superconductivity ($N_f = 1$)

II. Color superconductivity in neutral matter

- Neutrality vs. color superconductivity
- Gapless phases of color superconductivity
- Current status
- Outlook

QCD at large baryon density

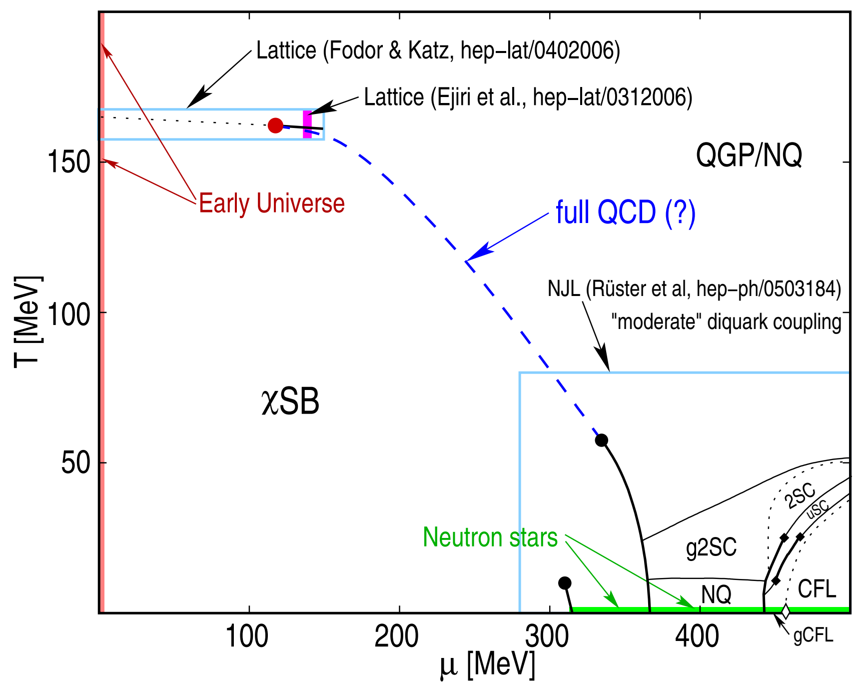
One would like to know the fundamental properties of QCD at

$$\mu \gtrsim \Lambda_{QCD} \gtrsim T$$

- So far, there are no reliable lattice results at $\mu \gtrsim \Lambda_{QCD}$
- Effective models have a limitted predictive power

⊕ Effects of charge neutrality and β equilibrium are not under control

⊕ Difficulties in determining stable ground states



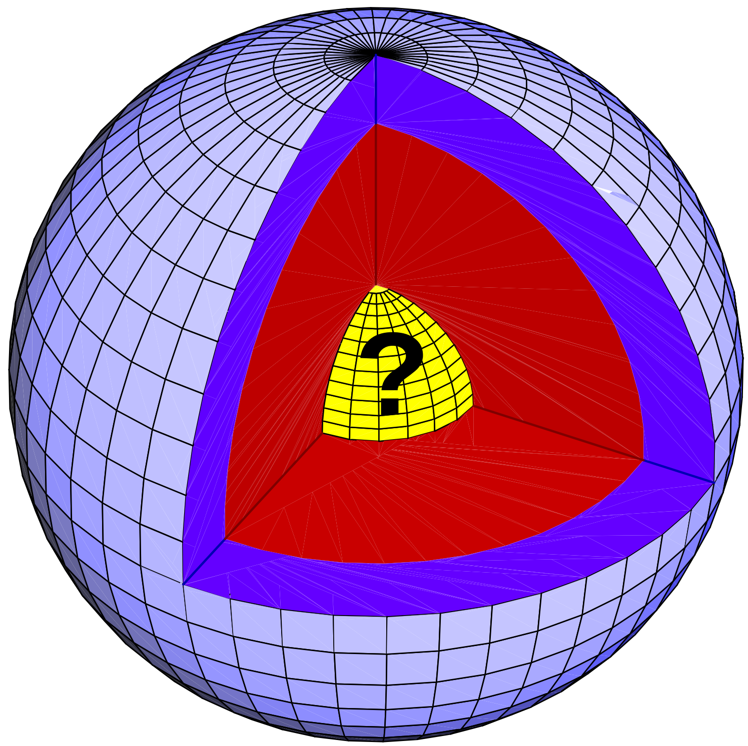
Is it of phenomenological interest?

Dense baryonic matter in Nature

Dense baryonic matter exists in the Universe

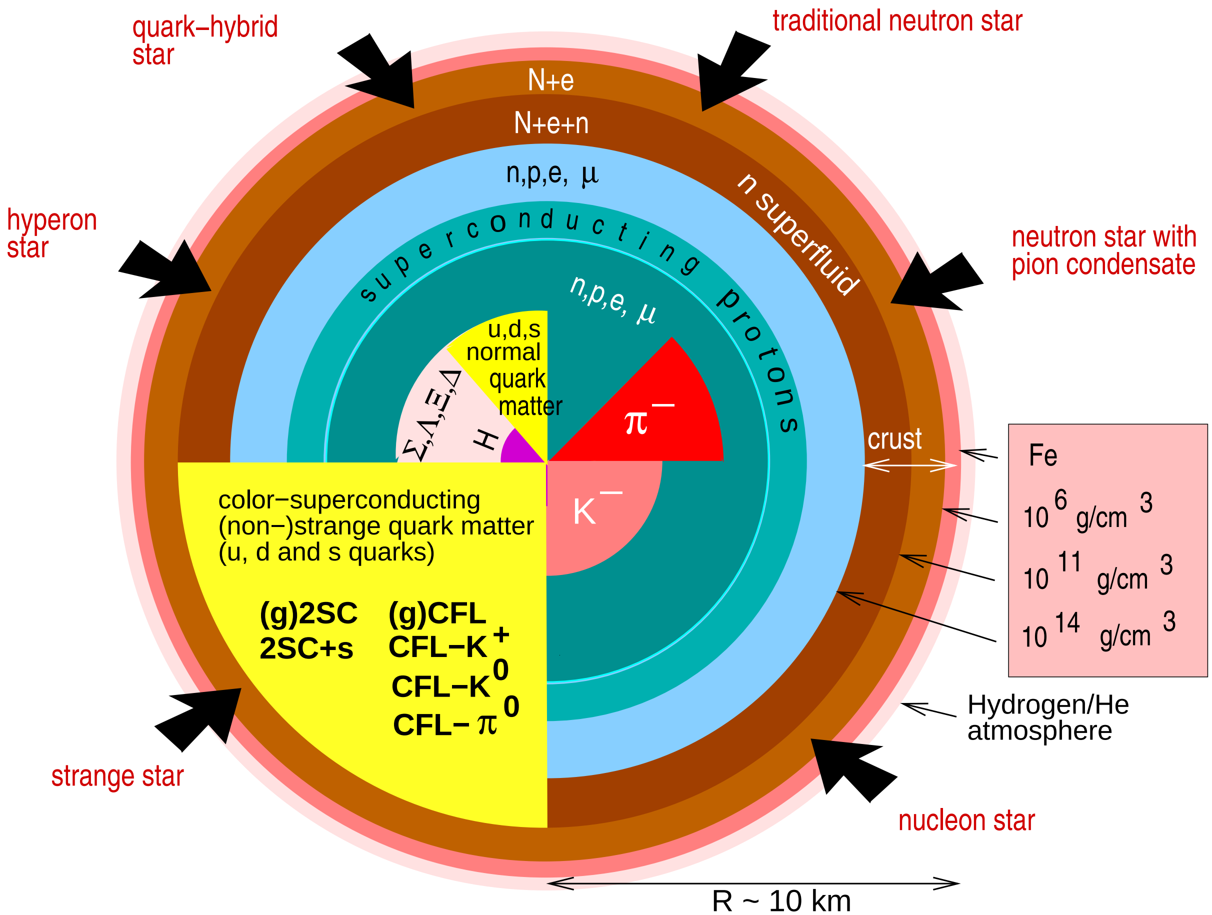
Compact (neutron) stars

- Radius:
  $R \simeq 10$ km
- Mass:
  $1.25 M_\odot \lesssim M \lesssim 2 M_\odot$
- Core temperature:
  10 keV $\lesssim T \lesssim 10$ MeV
- Surface magnetic field:
  10^8 G $\lesssim B \lesssim 10^{14}$ G



What is the state of matter at the highest stellar densities, $\rho_c \gtrsim 5\rho_0$?

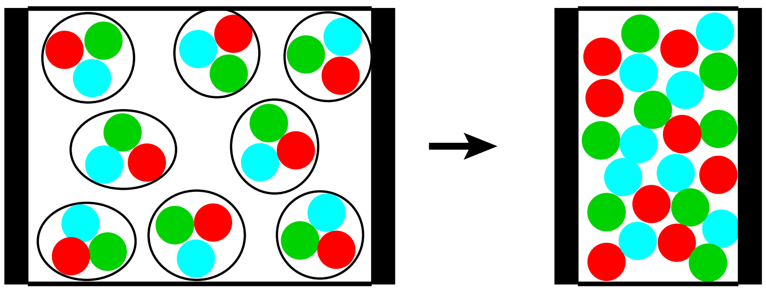
Phases of matter inside compact stars



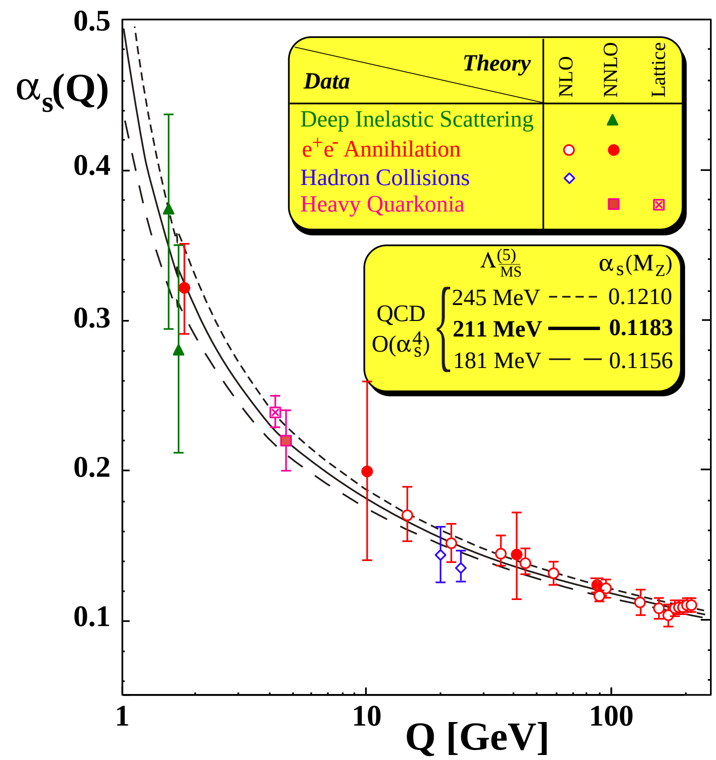
[figure from F. Weber, astro-ph/0407155 (modified)]

Very dense baryonic matter

- “Squeezing” baryonic matter hard should produce quark matter:



- Asymptotic freedom: $\alpha_s(\mu) \ll 1$
  $\mu \gg \Lambda_{QCD}$ [Gross&Wilczek; Politzer,’73]
- Very dense quark matter is **weakly** interacting [Collins&Perry,’75]



Note: realistic densities in stars are not sufficiently large:

$$\rho \lesssim 10\rho_0, \text{ where } \rho_0 \approx 0.15 \text{ fm}^{-3} \quad \Rightarrow \quad \mu \lesssim 500 \text{ MeV}$$

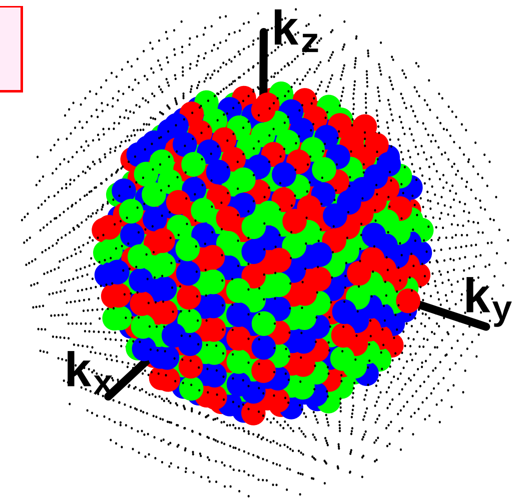
Ground state of dense quark matter

Noninteracting quarks:

(i) Deconfined quarks ($\mu \gg \Lambda_{QCD}$)

(ii) Pauli principle ($s = \frac{1}{2}$)

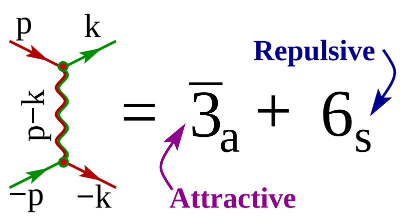
$\Rightarrow$



Interacting quarks:

(i) Effective models ($\mu \gtrsim \Lambda_{QCD}$)

(ii) One-gluon exchange ($\mu \gg \Lambda_{QCD}$)



$\Rightarrow$ Cooper instability

$\Downarrow$

Color superconductivity

$$\langle \left(\bar{\Psi}^C\right)_i^\alpha \gamma_5 \Psi_j^\beta \rangle \neq 0$$

Models

Densities of interest: $5\rho_0 \lesssim \rho \lesssim 10\rho_0$

(i) QCD (from first principles):

$$\mathcal{L}_{\rm QCD} = \bar{\psi}_f^{\alpha} \left(i\gamma^{\mu}\partial_{\mu} + \gamma^0 \mu_f + g T^a_{\alpha\beta} \gamma^{\mu} A^a_{\mu} - m_f \right) \psi_f^{\beta} - \frac{1}{4} F^a_{\mu\nu} F^{a,\mu\nu}$$

– results are reliable only when $\mu \gg \Lambda_{QCD}$

(ii) Phenomenological (e.g., NJL-type) models fitted to reproduce basic properties of vacuum QCD and/or nuclear matter, e.g.,

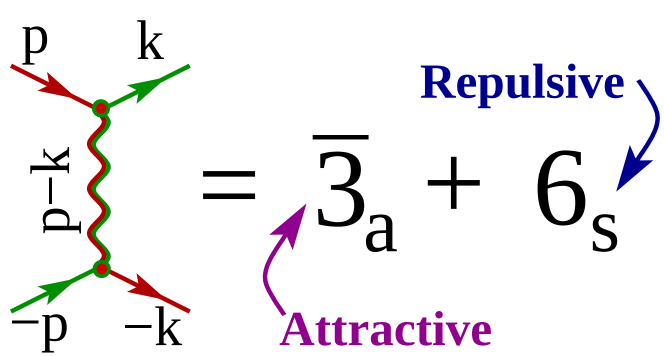
$$\mathcal{L}_{\rm NJL} = \bar{\psi}_f^{\alpha} \left(i\gamma^{\mu}\partial_{\mu} + \gamma^0 \mu_f - m_f \right) \psi_f^{\beta} + \frac{g^2}{2} (\bar{\psi}\gamma^{\mu} T^a \psi)(\bar{\psi}\gamma_{\mu} T^a \psi)$$

– may work only when $\rho \lesssim \rho_0$

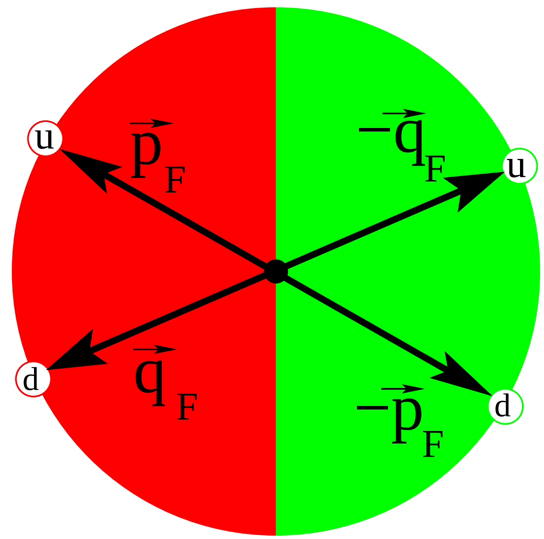
$N_f = 2$ color superconductivity

Simplest case, 2SC phase [Barrois,'78; Bailin&Love,'84]

- Assumption: $p_F^{\rm up} \approx p_F^{\rm down} \approx \mu$
- $N_c = 3$: "red", "green" and "blue"
- Quark-quark interaction in QCD:



$\langle u_p\, d_{-p} \rangle = -\langle u_q\, d_{-q} \rangle \neq 0$



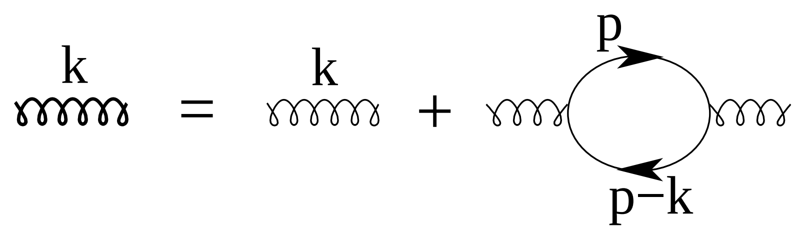
Cooper instability → color superconductivity

$(|\bullet\bullet\rangle - |\bullet\bullet\rangle)_{\bar{3}} \otimes (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)_0 \otimes (|u,d\rangle - |d,u\rangle)$ (⇐ Pauli principle)

QCD: screening of one-gluon interaction

Hard-dense-loop approximation:

$\mathcal{D}^{-1}_{\mu\nu}(k) = D^{-1}_{0,\mu\nu}(k) - \Pi_{\mu\nu}(k)$, i.e.,



Electrical Debye screening and magnetic dynamical screening
[Son,hep-ph/9812287]:

$$i\mathcal{D}_{\mu\nu}(k_4,|\vec{k}|) \simeq -\frac{O^{(el)}_{\mu\nu}}{k_4^2+|\vec{k}|^2+2M_D^2} - \frac{|\vec{k}|O^{(mag)}_{\mu\nu}}{|\vec{k}|^3+\pi M_D^2|k_4|/2},$$

where $M_D^2 = \alpha_s N_f \mu/\pi$ is the Debye mass

Region of dominant interaction: $\Delta \ll k_4 \lesssim |\vec{k}| \ll \mu$

Note: QCD analysis is reliable when $\Delta \gg \Lambda_{QCD}$

Some details of the analysis

Nambu-Gorkov quark field:

$$\Psi = \frac{1}{\sqrt{2}} \begin{pmatrix} \psi \\ C\bar{\psi}^T \end{pmatrix}, \quad \text{where} \quad C \equiv i\gamma^0\gamma^2$$

Quark propagator:

$$\mathcal{G}^{-1} = i \begin{pmatrix} p^\mu\gamma_\mu + \mu\gamma_0 & \boxed{\hat{\Delta}} \\ \boxed{\gamma_0\hat{\Delta}^\dagger\gamma_0} & p^\mu\gamma_\mu - \mu\gamma_0 \end{pmatrix}$$

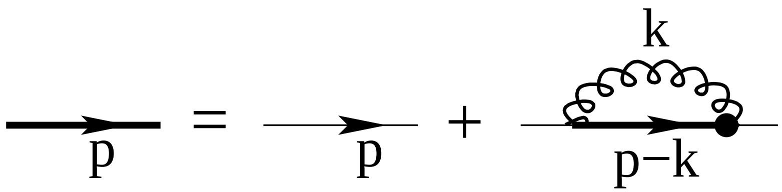
with $\hat{\Delta}^{\alpha\beta}_{ij} = \varepsilon^{3\alpha\beta}\epsilon_{ij}\gamma_5\Delta$

Quark-gluon vertex:

$$\Gamma^a_\mu = \gamma_\mu \begin{pmatrix} T^a & 0 \\ 0 & -\left(T^a\right)^T \end{pmatrix}$$

Schwinger-Dyson (gap) equation

[hep-ph/9906478;hep-ph/9906512;nucl-th/9907041;hep-ph/9909574;hep-ph/9910225]



This reduces to

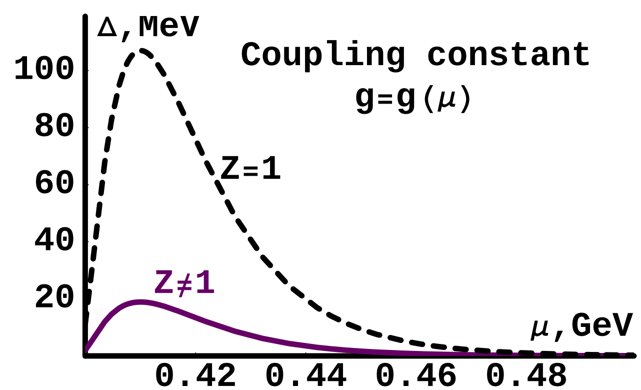
$$\Delta(p_4) \simeq \frac{g^2}{18\pi^2} \int \frac{dq_4 \Delta(q_4)}{\sqrt{q_4^2 + \Delta^2}} \ln \frac{\Lambda}{|q_4 - p_4|}$$

where

$\Lambda = 2(4\pi)^4 \mu g^{-5} \exp\left(-\frac{4+\pi^2}{8}\right)$

Solution for the gap:

$$\Delta(0) \simeq \Lambda \exp\left(-\frac{3\pi^2}{\sqrt{2}g}\right)$$



Instanton induced interaction

Local interaction [Alford et al., '98; Rapp et al., '98]

$$\mathcal{L}_{\rm int}^{(\rm eff)} \simeq -\frac{G}{8N_c^2(N_c-1)}\left[\left(\psi^T C\tau_2\lambda_A^a\gamma_5\psi\right)\left(\bar\psi\tau_2\lambda_A^a\gamma_5 C\bar\psi^T\right)+\ldots\right]$$

where τ_2 and λ_A^a are antisymmetric Pauli and Gell-Mann matrices

Diquark condensate:

$$\Delta^3 \sim \varepsilon^{3\alpha\beta}\langle\psi_\alpha^i C(\tau_2)_{ij}\gamma_5\psi_\beta^j\rangle$$

Choice of parameters:

e.g., $m_q^{(\rm vacuum)} = 400$ MeV $\Rightarrow$ $G \simeq 490$ GeV^{-2}

(or $N_{\rm inst}/V \simeq 1.44$ fm^{-4}) Cut-off: $\Lambda \lesssim 1$ GeV

Result for the gap: $\Delta \simeq 100$ MeV

Symmetries of 2SC ground state

- Diquark condensate:

$$\langle \left(\bar{\Psi}^C\right)_i^{\alpha} \gamma_5 \Psi_j^{\beta} \rangle \sim \varepsilon^{3\alpha\beta} \epsilon_{ij} \Delta$$

When $\Delta \neq 0$,

– chiral $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ — intact

– baryon number $\mathrm{U}(1)_B \to \tilde{\mathrm{U}}(1)_B$ with $\tilde{B} = B - \frac{2}{\sqrt{3}} T_8$

– gauge symmetry $\mathrm{U}(1)_{\mathrm{em}} \to \tilde{\mathrm{U}}(1)_{\mathrm{em}}$ with $\tilde{Q} = Q - \frac{1}{\sqrt{3}} T_8$

– approximate axial $\mathrm{U}(1)_A$ is broken $\to$ 1 pseudo-NG boson

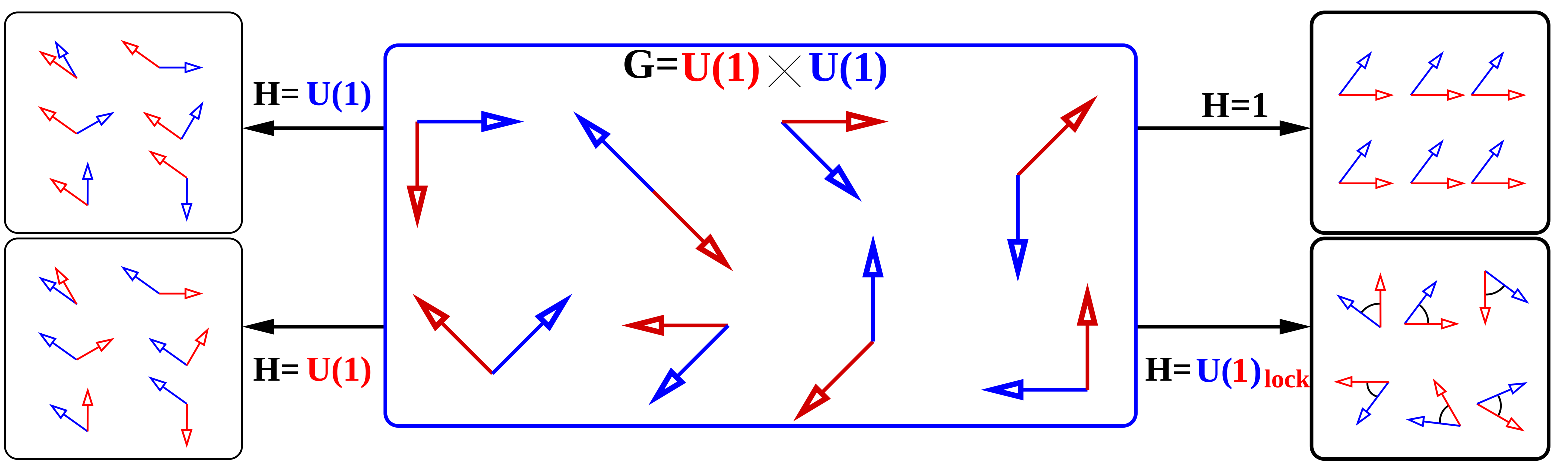
– color gauge symmetry $\mathrm{SU}(3)_c \to \mathrm{SU}(2)_c$ by Anderson-Higgs mechanism $\to$ 5 massive gluons

$N_f = 3$ color superconductivity

- Cooper pair: $(|\bullet\bullet\rangle - |\bullet\bullet\rangle)_{\bar{3}} \otimes (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)_{J=0} \otimes (|\mathrm{u,d}\rangle - |\mathrm{d,u}\rangle)_{\bar{3}}$
- Diquark condensate:

$$\langle \left(\bar{\Psi}_L^C\right)_i^\alpha \left(\Psi_L\right)_j^\beta \rangle = \langle \left(\bar{\Psi}_R^C\right)_i^\alpha \left(\Psi_R\right)_j^\beta \rangle \simeq \sum_I \varepsilon^{\alpha\beta I} \epsilon_{ijI} \Delta$$

- $\mathrm{SU}(3)_L \times \mathrm{SU}(3)_R \times \mathrm{SU}(3)_c \rightarrow \mathrm{SU}(3)_{L+R+c}$ via "color-flavor locking", without $\langle \bar{q}_L q_R \rangle$ condensates! [Alford et al. hep-ph/9804403]



Symmetries of CFL ground state

- chiral $SU(3)_L \times SU(3)_R$ is broken down to $SU(3)_{L+R+c}$
  $\rightarrow$ 8 (pseudo-)NG bosons, i.e., π^0, $\pi^\pm$, $K^\pm$, K^0, $\bar{K}^0$, η
  (like in vacuum QCD)

- baryon number $U(1)_B$ is broken $\rightarrow$ 1 NG boson (ϕ)
  (quark matter is superfluid)

- approximate axial $U(1)_A$ is broken $\rightarrow$ 1 pseudo-NG boson (η')

- color gauge symmetry $SU(3)_c$ is broken by Anderson-Higgs mechanism $\rightarrow$ 8 massive gluons

- gauge symmetry $U(1)_{\text{em}} \rightarrow \tilde{U}(1)_{\text{em}}$ with $\tilde{Q} = Q + \frac{2}{\sqrt{3}} T_8$
  (there is no Meissner effect)

$N_f = 1$ color superconductivity

- Cooper pair: $\left(|\bullet\bullet\rangle - |\bullet\bullet\rangle\right)_{\bar{3}} \otimes |\uparrow\uparrow\rangle_{J=1}$
- Diquark condensate:

$$\langle \left(\bar{\Psi}^C\right)^{\alpha} \gamma_5 \Psi^{\beta} \rangle \simeq \varepsilon^{\alpha\beta c} \Delta_{c\delta} \left(\hat{\mathbf{k}}^{\delta} \sin\theta + \gamma_{\perp}^{\delta}(\vec{\mathbf{k}}) \cos\theta\right)$$

- Many possibilities, e.g., see [A.Schmitt, nucl-th/0412033]:
  - Color-spin-locked phase: $\Delta_{c\delta} = \delta_{c\delta}$ → largest pressure (?)
  - Planar phase: $\Delta_{c\delta} = \delta_{c\delta} - \delta_{c3}\delta_{\delta 3}$
  - Polar phase: $\Delta_{c\delta} = \delta_{c3}\delta_{\delta 3}$
  - A-phase: $\Delta_{c\delta} = \delta_{c3}\left(\delta_{\delta 1} + i\delta_{\delta 2}\right)$

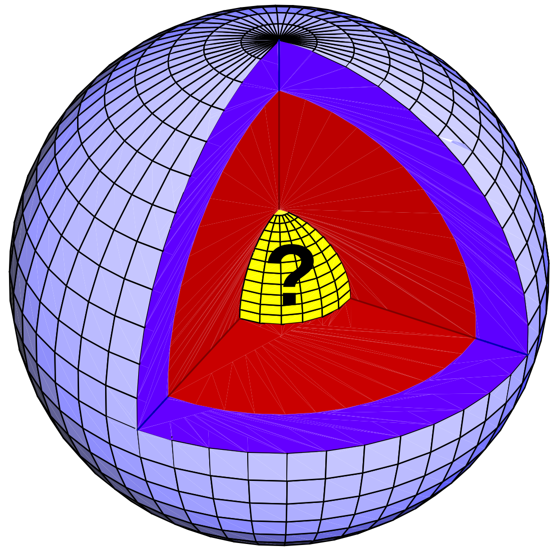
Color superconductivity in neutral matter

Matter in the bulk of a star is

(i) β-equilibrated: $\mu_d = \mu_u + \mu_e$

(ii) electrically and color neutral:

$n_Q^{\rm el} = 0, \qquad n_Q^{\rm color} = 0$



If $n_Q \neq 0$, the Coulomb energy is

$$E_{\rm Coulomb} \sim n_Q^2 R^5 \sim M_\odot c^2 \left(\frac{n_Q}{10^{-15} e/{\rm fm}^3}\right)^2 \left(\frac{R}{1~{\rm km}}\right)^5$$

e.g., if $10^{-2} \lesssim n_Q \lesssim 10^{-1} e/{\rm fm}^3 \quad \Rightarrow \quad E_{\rm Coulomb}^{\rm 2SC} \gg M_\odot c^2$

Unconventional Cooper pairing, $N_f = 2$

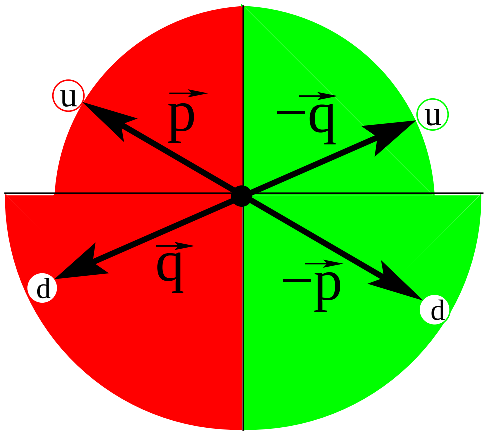
- The "best" 2SC phase appears when $n_d \approx n_u$
- Neutral matter appears when $n_d \approx 2n_u$
- Electrons, required by β equilibrium, **cannot** help:

$$n_d \approx 2n_u \quad \Rightarrow \quad \mu_d \approx 2^{1/3}\mu_u \;\Rightarrow\; \mu_e = \mu_d - \mu_u \approx \frac{1}{4}\mu_u$$

$$\text{i.e.,} \qquad n_e \approx \frac{1}{4^3}\frac{n_u}{3} \ll n_u$$

Cooper pairing is distorted by the following "mismatch" parameter:

$$\delta\mu \equiv \frac{p_F^{\text{down}} - p_F^{\text{up}}}{2} = \frac{\mu_e}{2} \neq 0$$



Gapless 2SC phase

Competition: $\delta\mu$ vs. Δ_0 (where Δ_0 is the gap at $\delta\mu = 0$)

The "winner" is determined by the diquark coupling strength [Shovkovy&Huang, hep-ph/0302142]

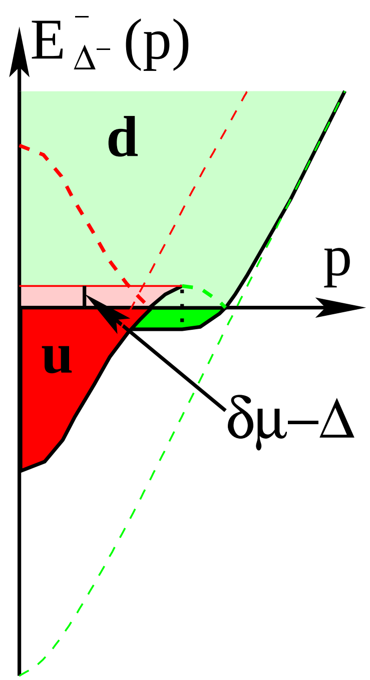
1. $\delta\mu \gtrsim \Delta_0$ — the mismatch does not allow Cooper pairing: normal phase is the ground state

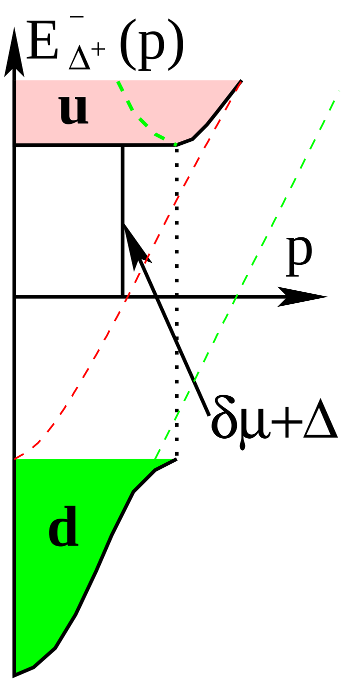
2. $\delta\mu \lesssim \frac{1}{2}\Delta_0$ — coupling is strong enough to win over the mismatch: 2SC is the ground state

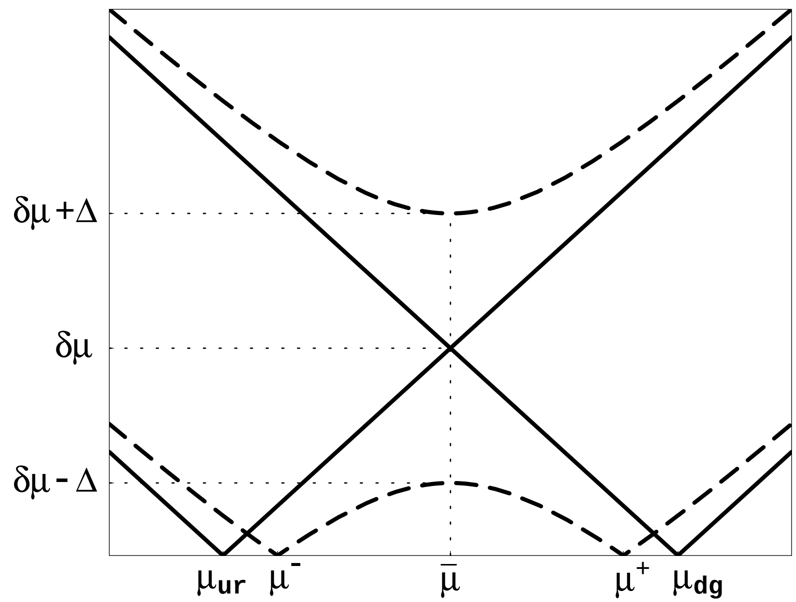
3. $\frac{1}{2}\Delta_0 \lesssim \delta\mu \lesssim \Delta_0$ — regime of intermediate coupling strength: the ground state is the gapless 2SC phase

Quasiparticle spectrum in g2SC phase

"Intermediate" coupling



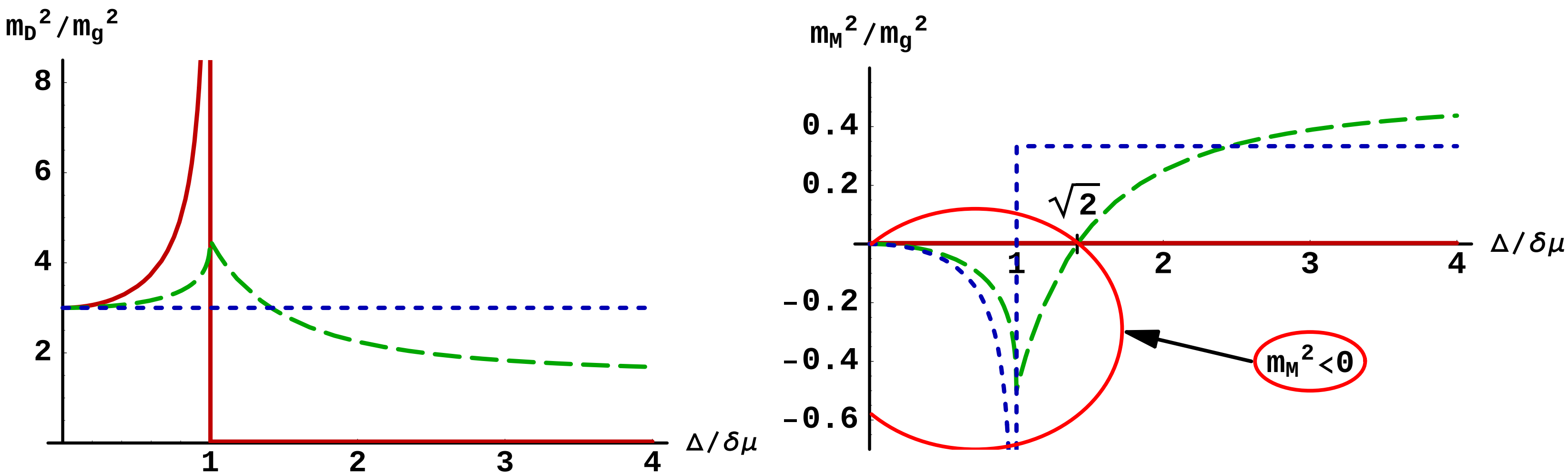




The energy gaps in the quasiparticle spectra are $\boxed{0}$ & $\boxed{\Delta + \delta\mu}$

Chromomagnetic instability

Recent results for gluon screening masses
[Huang & Shovkovy, hep-ph/0407049]:



$A = 1, 2, 3$ — red solid line

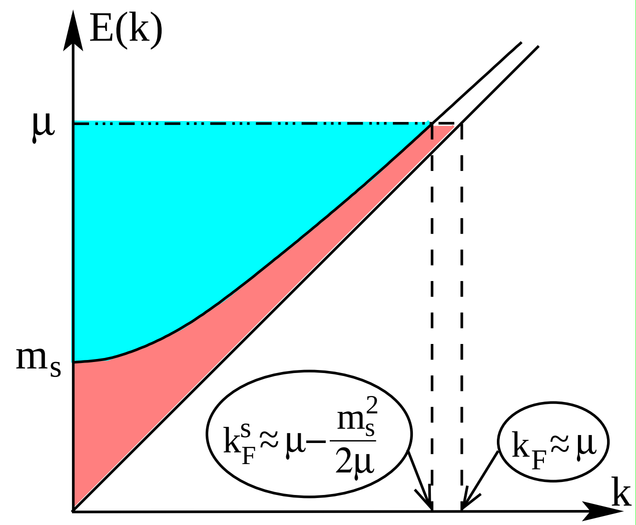
$A = 4, 5, 6, 7$ — green long-dash line

$A = \tilde{8}$ — blue short-dash line

$N_f = 2+1$ color superconductivity, $0 < m_s < \infty$

Fermi momentum of strange quarks is lowered:

$$k_F^s \simeq \mu - \frac{m_s^2}{2\mu}$$



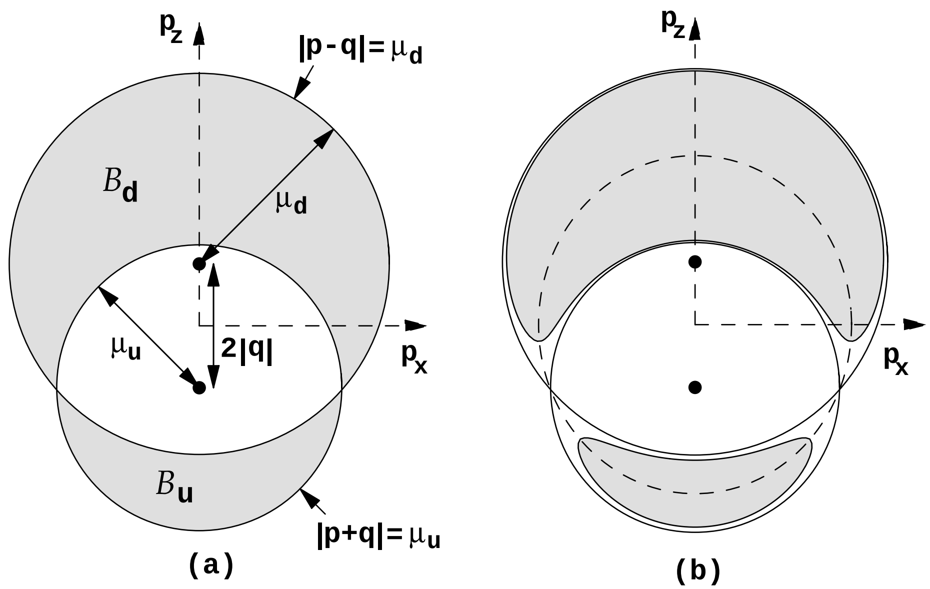
The ground state of strange quark matter may have:

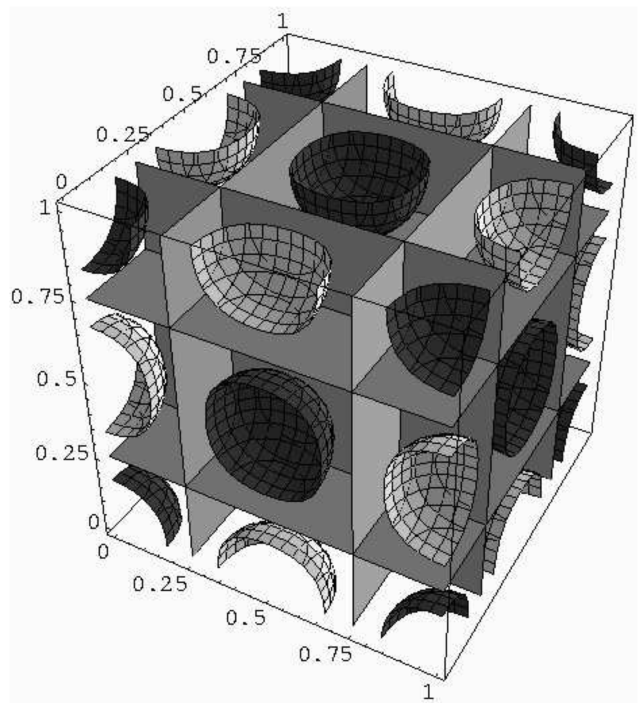
- only spin-1 condensates of same flavor
- only superconductivity of up and down quarks (2SC or g2SC)
- crystaline pairing (nonzero momentum pairing, LOFF)
- ...

LOFF phase

Crystalline color superconductivity
[Alford, Bowers & Rajagopal, hep-ph/0008208]

Cooper pairs with nonzero momenta:





[Bowers, hep-ph/0305301]

Gapless CFL phase

[Alford, Kouvaris & Rajagopal, hep-ph/0311286]

- Distorted color-flavor pairing:

$$\Delta_{ij}^{\alpha\beta} \simeq \Delta_1\ \epsilon_{1ij}\varepsilon^{1\alpha\beta} + \Delta_2\ \epsilon_{2ij}\varepsilon^{2\alpha\beta} + \Delta_3\ \epsilon_{3ij}\varepsilon^{3\alpha\beta} + \ldots$$

- Control (mismatch) parameter:

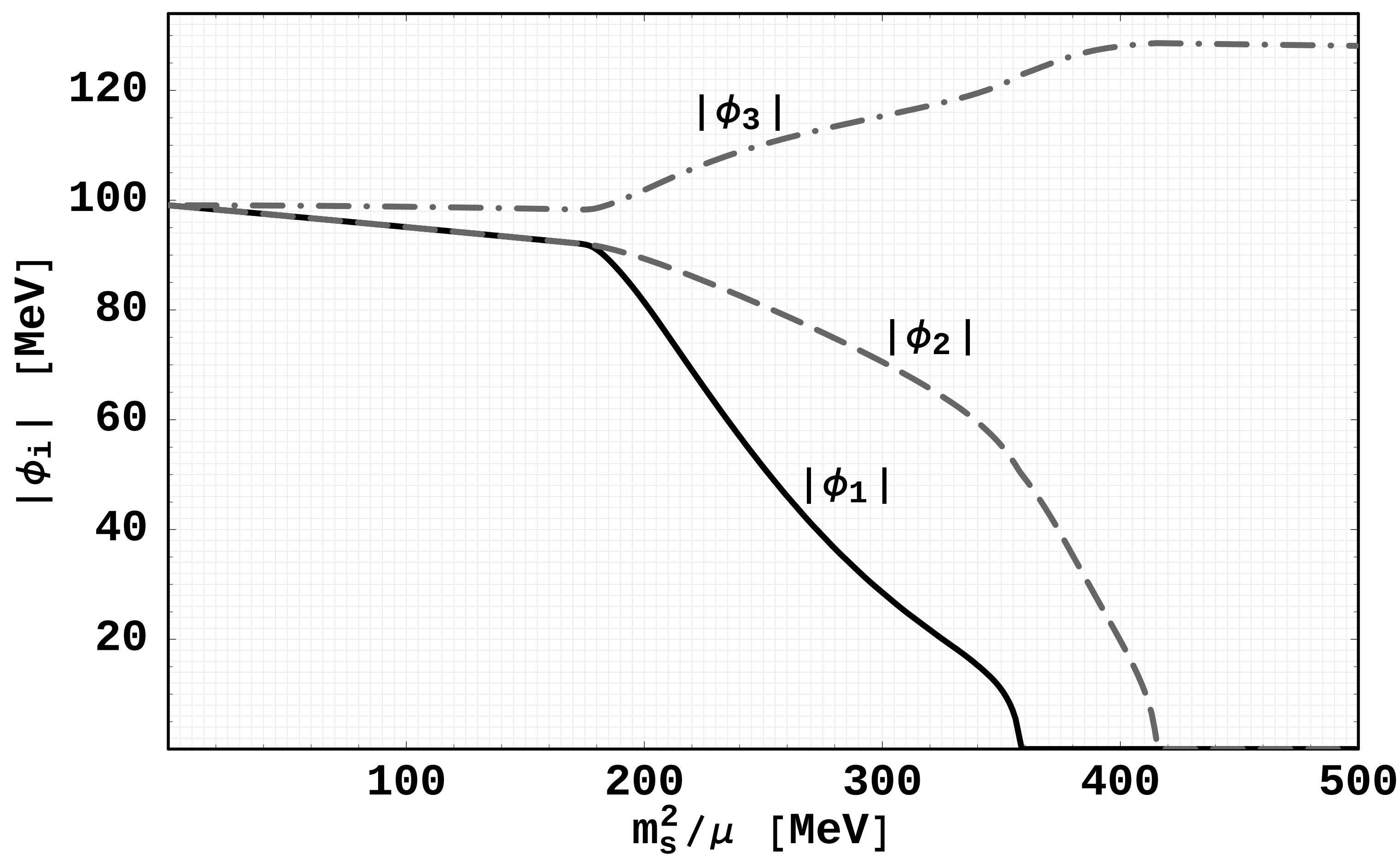
$$\delta\mu \equiv \frac{\mu_{bd} - \mu_{gs}^{\text{eff}}}{2} \approx -\frac{\mu_8}{2} + \frac{m_s^2}{4\mu} \approx \boxed{\frac{m_s^2}{2\mu}}$$

where $\mu_{gs}^{\text{eff}} \simeq \mu_{gs} - \dfrac{m_s^2}{2\mu}$ and $\mu_8 \simeq -\dfrac{m_s^2}{2\mu}$ (blue color is special)

- At $T = 0$, the gapless CFL phase occurs when

$$\delta\mu \equiv \frac{m_s^2}{2\mu} > \Delta_0$$

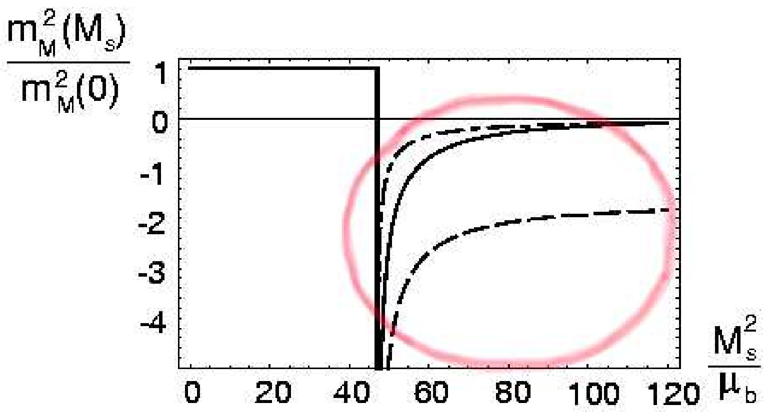
Gap parameters

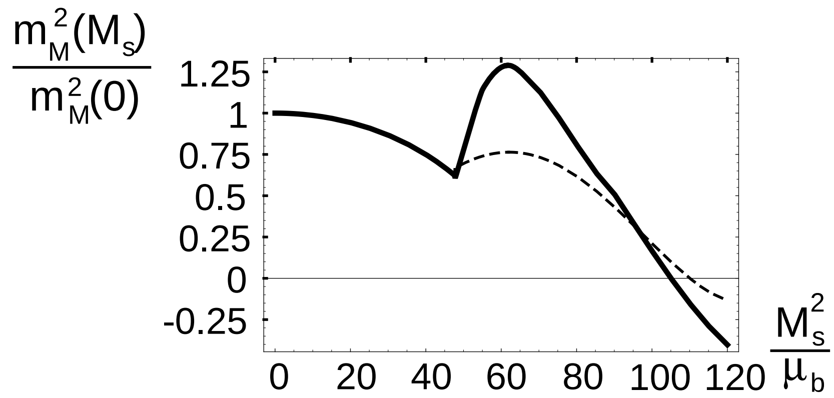


NJL model [nucl-th/0411040]

Chromomagnetic instability in gCFL phase

Recent results for Meissner screening masses
[Casalbuoni, Gatto, Mannarelli, Nardulli, Ruggieri, hep-ph/0410401]:





$A = 1, 2$ — solid line

$A = 3$ — short-dashed line

$A = 8$ — long-dashed line

$A = 4, 5$ — dashed line

$A = 6, 7$ — solid line

State of the art

At least one thing is clear,

- Sometimes, ground state of neutral dense quark matter is

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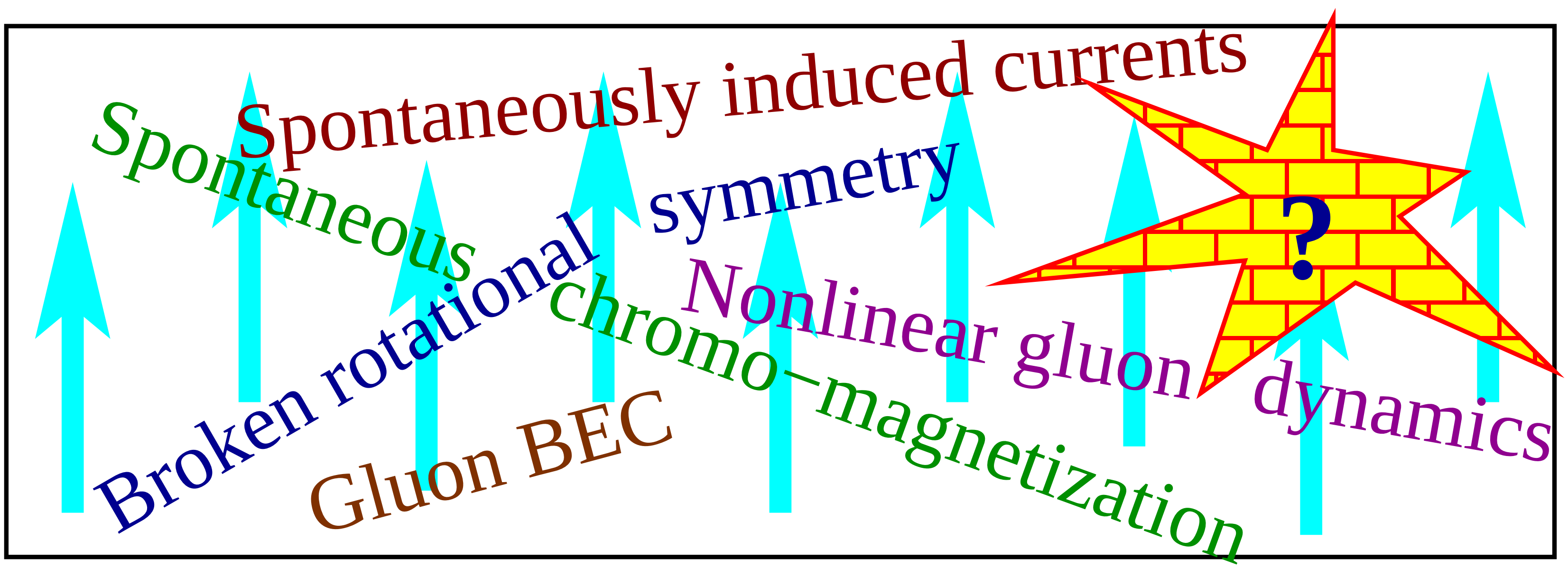
At least one thing is clear,

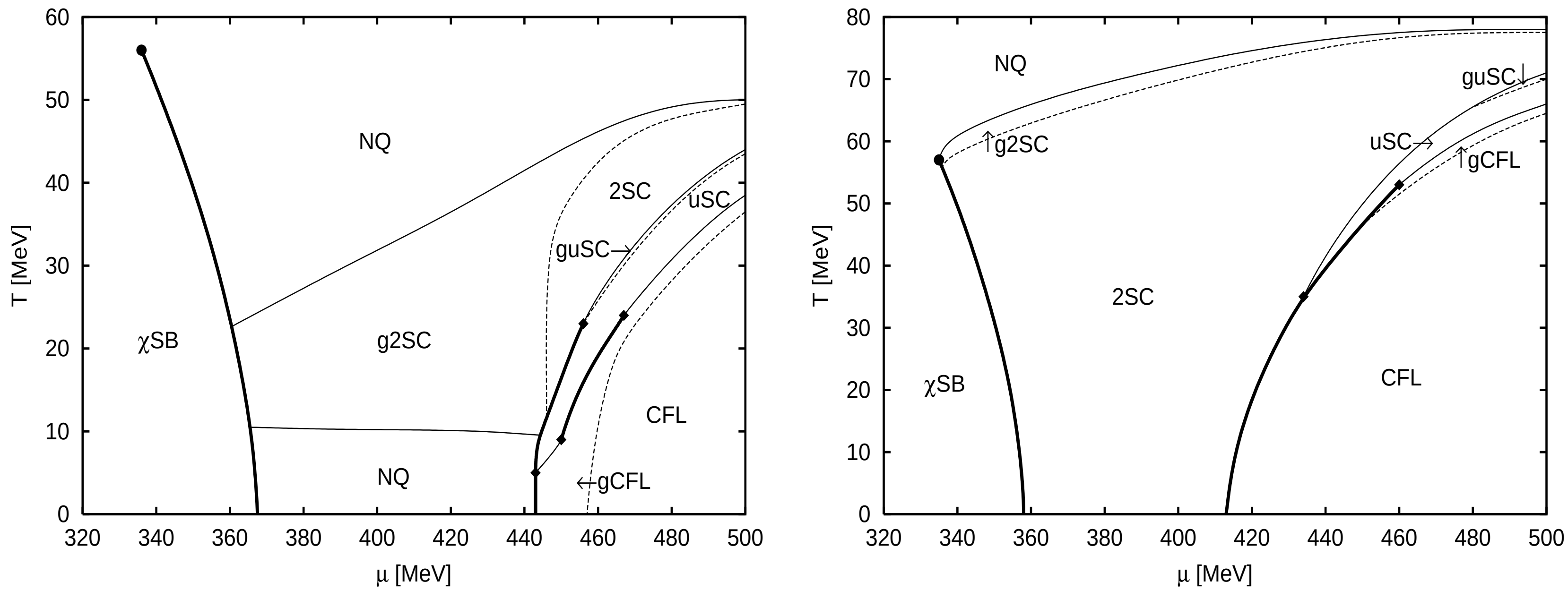
- Sometimes, ground state of neutral dense quark matter is

something like this



Phase diagrams of neutral dense QCD

[Rüster, Werth, Buballa, Shovkovy & Rischke, hep-ph/0503184]



$G_D = \frac{3}{4} G_S$ (intermediate coupling) $G_D = G_S$ (strong coupling)

Note: Gapless phases play little role at strong coupling, $G_D = G_S$

See also [Blaschke, Fredriksson, Grigorian, Sandin & Öztaš, hep-ph/0503194]

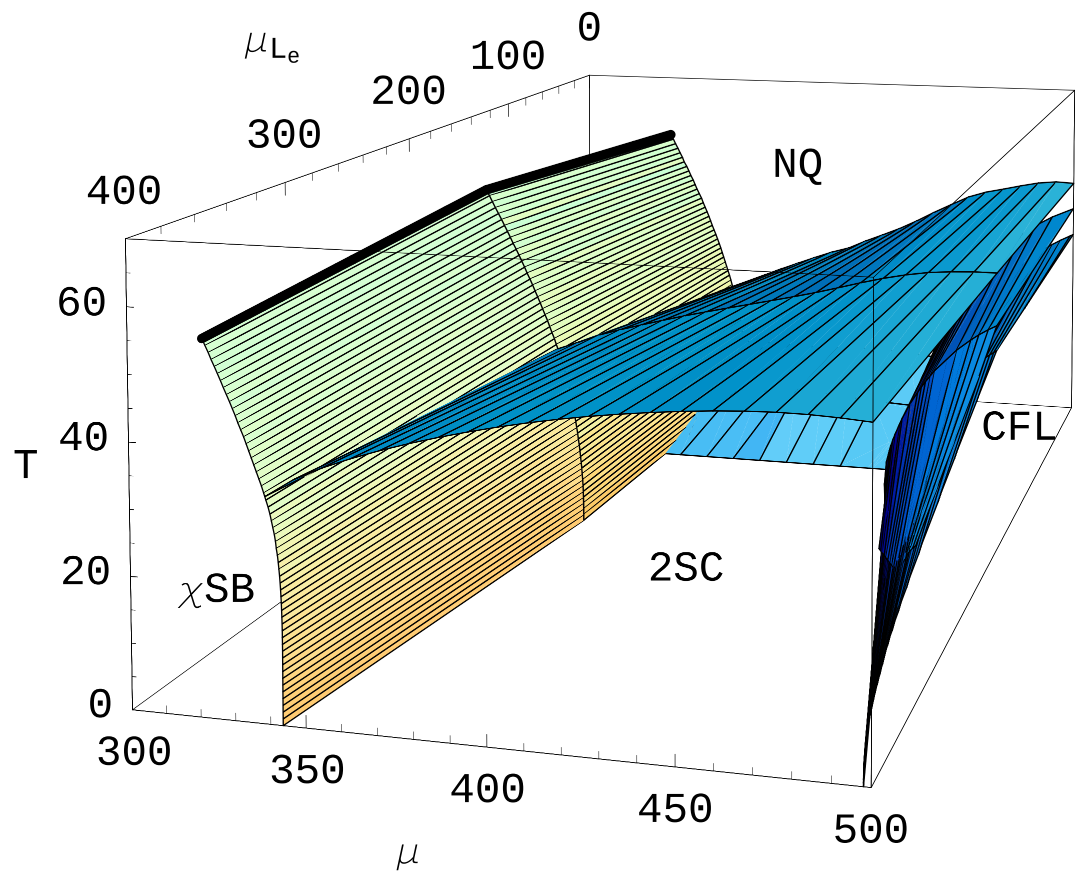
3D phase diagram

Neutrino trapping

$\mu_{L_e} > 0$

inside hot matter

$T \lesssim 40$ MeV



[Rüster, Werth, Buballa, Shovkovy & Rischke, in preparation]

Current status

- At $\mu \gg \Lambda_{QCD}$, QCD dynamics is weakly coupled, but non-perturbative

– In this limit, QCD can be studied from first principles

- Some features of $T - \mu$ phase diagram start to develop

– In particular, sufficiently dense matter is a color superconductor

- Neutrality and β-equilibrium strongly affect the properties of CSC matter

– There can exist many different phases (e.g., 1SC, 2SC, g2SC, CFL, gCFL, mCFL, uSC, dSC, LOFF, CFL+K^0, CFL+η)

- Current problems: (i) instabilities of gapless phases, (ii) inhomogeneous ground states, (iii) search for observables, etc.

Outlook

- Systematic studies of different phases in models of dense QCD
- Detailed physical properties (transport properties, in particular) of various quark phases
- Phases with unconventional Cooper pairing and their role in different branches of physics
- The search for promising observable(s), (dis-)proving the presence of quark matter inside stars
- Developing rigorous approaches to treat QCD at nonzero densities

Some reviews on color superconductivity

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