Deconfinement dynamics and thermalization in the QGP

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Heavy ion collision data is well described by the hydrodynamics of a near-perfect fluid provided very rapid thermalization, i.e in $\sim 0.5fm/c$ is assumed as initial condition.

What is the mechanism of this rapid thermalization?

At asymptotically high temperatures above the deconfinement T_c, where the running coupling $g(T)$ is small, QCD is well-described as a gas of weakly coupled quasi-particles.

At the energy densities achieved in the high energy heavy ion collisions, however, perturbative treatment of the equilibration process appears not to be applicable.

Various estimates of thermalization times based on perturbative scattering processes have been obtained:

Parton-cascade approach to the time evolution of hard partons; bottom-up scenario incorporating saturation picture initial conditions, ...
D. Molnar and M. Gyulassy, Bjoraker and R. Venugopalan
R. Baier, A.H. Mueller, D. Schiff, D.T. Son, ...

They all result into thermalization times much longer than those needed by the hydrodynamical simulations.

It has been argued that this is a generic feature of any dynamical evolution based only on perturbative scattering processes (Kovchegov).

Even within a weak coupling analysis, however, non-perturbative effects may contribute to the dynamics.

It has been pointed out that, in a plasma with an anisotropic hard parton distribution, instabilities may develop in soft gauge field modes generated within the linear response approximation: P. Arnold, J. Lenaghan, G.D. Moore, Strickland, S. Mrowczynski, Heinz, ...

It has been argued that these small-amplitude unstable modes are not stabilized by (non-Abelian) non-linearities. Thus they can grow to amplitudes large enough to contribute $O(1)$ fraction to the total energy density, and drive isotropization of the hard modes faster than any hard collision equilibration time scale: P. Arnold, J. Lenaghan, G.D. Moore and L.G. Yaffe.

Whether the evolution of these particular semiclassical instabilities can be reliably followed beyond the linear response level at which they were identified is certainly far from clear. (A. Dumitru and Y. Nara; C. Manuel and S. Mrowczynski; Rebhan and Strickland; P. Arnold, G.D. Moore and L.G. Yaffe; ...)

All such attempts are basically perturbative (hard loop effective actions).

Other non-perturbative dynamics at strong effective couplings defined at scales appropriate to the nonlinear interaction of such growing long range modes must enter.

Still, consideration of such potential instabilities properly point to the basic underlying question.

What is the general type of response to a rapid transition across the confinement-deconfinement boundary in QCD?

How can we address this question?

Starting with the system in thermalized equilibrium, perform a temperature quench and follow the system over successive simulation sweeps on its path toward regaining equilibrium.

This *cannot* be directly identified with the exact real-time evolution,

It is known, however, from many studies, mostly of condensed matter systems, to accurately reflect it. At the very least, the method provides a consistent picture of possible features of the system's real-time response.

It has been extensively and successfully used for many systems exhibiting, in particular, first-order transitions. Indeed, in studies of binary alloys and binary fluids it has been found to reproduce the experimentally observed behavior in quantitative detail.

Very few studies for gauge theories (Miller, Ogilvie; Bazavov, Berg, Heller, Meyer-Ortmanns, Velytsky)

Procedure (hep-ph/0507197)

- Thermalize in the confining phase;
  perform quench into deconfinement phase.

- Follow the subsequent evolution of system under varying conditions of
  - spatial expansion
  - temperature fall-off

- Measure Polyakov loops and structure functions.

Polyakov loop

$$l(\vec{r},t) = l_0 + u(\vec{r},t),$$

Structure function

$$S(\vec{k},t) = \left\langle u(\vec{k},t)\, v(-\vec{k},t) \right\rangle$$

Simulations only in pure $SU(3)$ LGT at the moment.
Note: Issue of gauge invariant (non-local) observalbes in gauge theories.

Anisotropic action:

$$S = -\beta_\xi/3 \sum_x \mathrm{Re}\left[\xi^{-1} \sum_{i>j} \mathrm{Tr} U_{x,ij} + \xi \sum_i \mathrm{Tr} U_{x,0i}\right],$$

where i, j runs over space-like directions, $\xi = a/a_\tau$ is the space-time anisotropy, $\beta_\xi = 6/g_\xi^2$, and $g_\xi^2 = g_\sigma \cdot g_\tau$ is the anisotropic coupling.

Then, by varying two parameters it is possible to carry out expansion of the system while maintaining constant temperature, or also let the temperature drop thus allowing for cooling of the expanding plasma.

Spatial expansion

Hubble-like uniform expansion of the metric:

Vary the space-like lattice spacing a as

$$a = a_0(1 + \frac{\tau}{\tau_0}),$$

where τ is the proper time variable, and τ_0 is the parameter which controls the rate of expansion.

Pure expansion: after the initial quench, temperature $T = 1/(N_\tau a_\tau)$ is kept constant throughout the expansion by fixing the time-like spacing a_τ.

The anisotropy $\xi = a/a_\tau$ evolves as:

$$\xi(\tau) = 1 + \frac{\tau}{\tau_0}.$$

assuming zero time anisotropy $\xi(\tau = 0) = 1$.

Dependence of the space-like lattice spacing on the coupling and anisotropy

One-loop order RG relationship:

$$a\Lambda(\xi) = \exp\{-1/(2b_0 g_\xi^2)\},$$

where $b_0 = 11 \cdot 3/(48\pi^2)$, and $\Lambda(\xi)$ is dependent on ξ through

$$\Lambda(\xi)/\Lambda_E = \exp\{-(c_\sigma(\xi) + c_\tau(\xi))/4b_0,$$

where $c_\sigma(\xi)$ and $c_\tau(\xi)$ are known functions.

Inclusion of the next order terms adds minor corrections for the range of the couplings and anisotropies used and is straightforward. Following a known procedure (Karsch) one can compute $\Lambda(\xi)$ for a range of anisotropy values. This allows us to estimate the necessary time evolution of β_ξ

$$\beta_\xi(\tau) = \beta(0) - 6 \cdot 2b_0 \log\left[(1 + \frac{\tau}{\tau_0})\frac{\Lambda(\xi)}{\Lambda_E}\right].$$

A non-perturbative further correction need to be applied, when appropriate (Boyd, Engels, Karsch, Laermann, Legeland, Lutgemeier, Petersson).

Note: In the standard application of anisotropic lattices, the anisotropy is varied by decreasing the *time-like* lattice spacing. This procedure keeps the coupling within the scaling window provided the initial coupling is close to the continuum limit.

Here we keep the time-like spacing constant (or, later, slowly increase it) as we vary the *space-like* coupling.This induces changes in the coupling that may drive its value out of the scaling regime ($\beta \sim 5$). Therefore, our expansion has to be truncated whenever the value of β falls below this cut-off value. This condition implies that, in order to follow the system evolution for longer time, one needs lattices with larger N_τ, thus rendering the problem more computationally intensive.

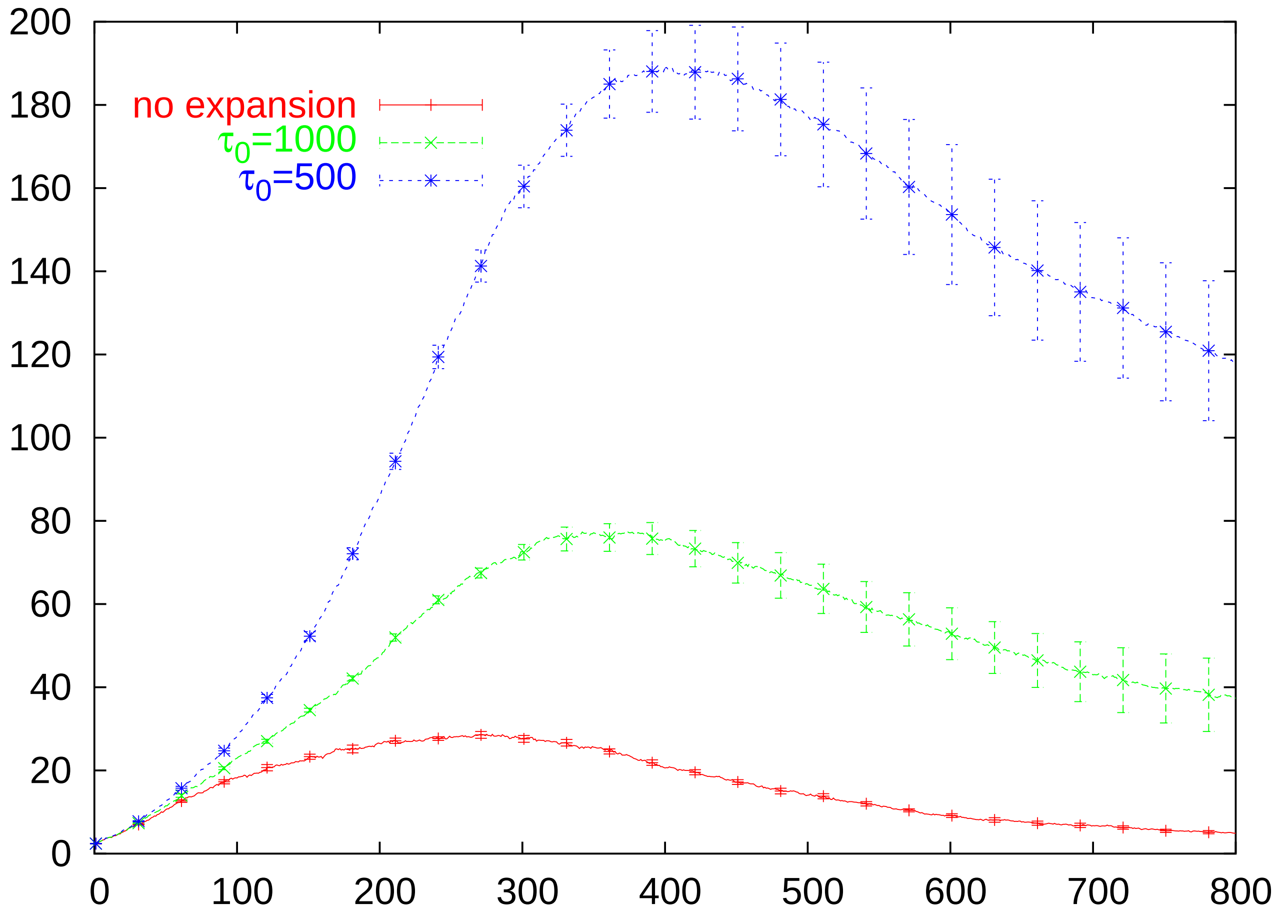

Figure 1: First mode of the structure function at different inverse expansion rates $\tau_0 = 500, 1000, \infty$. Quench $\beta_\xi = 5.5 \to 5.92$ on 16^3x4 lattice.This corresponds to a temperature after the quench $T_{\text{final}} = 1.57 T_c$. The phase transition on this lattice at $\xi = 1$ is at $\beta = 5.6902(2)$.

We present here only averages of on-axis modes, such as permutations of $(n, 0, 0)$ - this is the n-th mode in our notation.

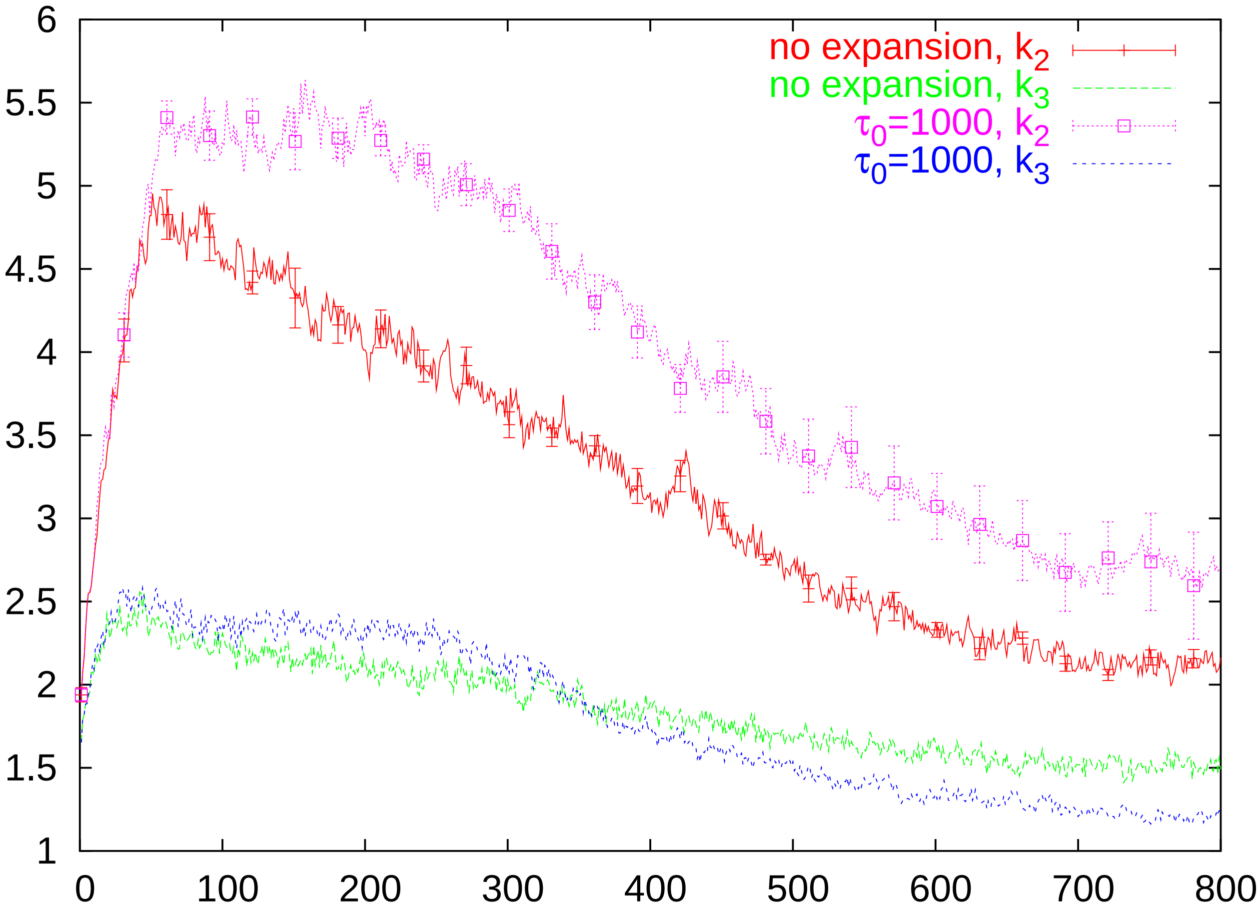

Figure 2: Higher (k_2 and k_3) structure function modes for non-expanding and $\tau_0 = 1000$ expanding system.

Fits to the exponent for the structure function data

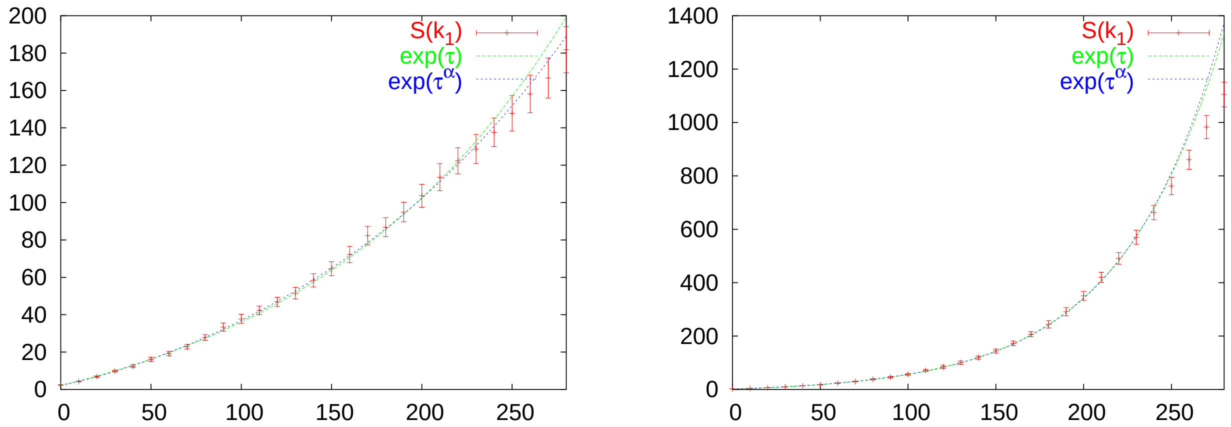

Figure 3: Fits to the exponent. Quenches on the 32^3x4 lattice in non-expanding (left) and $\tau_0 = 500$ expanding (right) systems.

There is no substantial change in the behavior of the exponent between the expanding and non-expanding systems.

There is no substantial deviation from exponential growth with linear τ dependence in the exponent.

A fit to

$$S(\tau) \sim \exp(C \cdot \tau^{\alpha})$$

gives $\alpha = 1.22$ for the expanding system, whereas it gives $\alpha = 1.18$ for the non-expanding system. In Fig. 3 we also show fits to $S(\tau) \sim \exp(C \cdot \tau)$, which in fact provides the best fit per parameter degree of freedom. Both fits are over the τ range from 0 to 250.

There is no substantial deviation from exponential growth with linear τ dependence in the exponent. Furthermore, from the figures we observe divergence from exponential behavior at later times $\tau > 200$.

All this provides a manifestation of the limitations of the linear response effective models. (There are 10 times as many points for the fitting as on the plot.)

Expansion accompanied by temperature fall-off

Temperature drop:

$$T = \frac{T_0}{(1 + \tau/\tau_0')^\alpha}.$$

In the fire-tunnel (in the proper frame) the temperature is expected to decrease with the proper time τ as $T \sim \tau^{-1/3}$ (Bjorken). So we take $\alpha = 1/3$.

Effect of temperature drop on the structure function

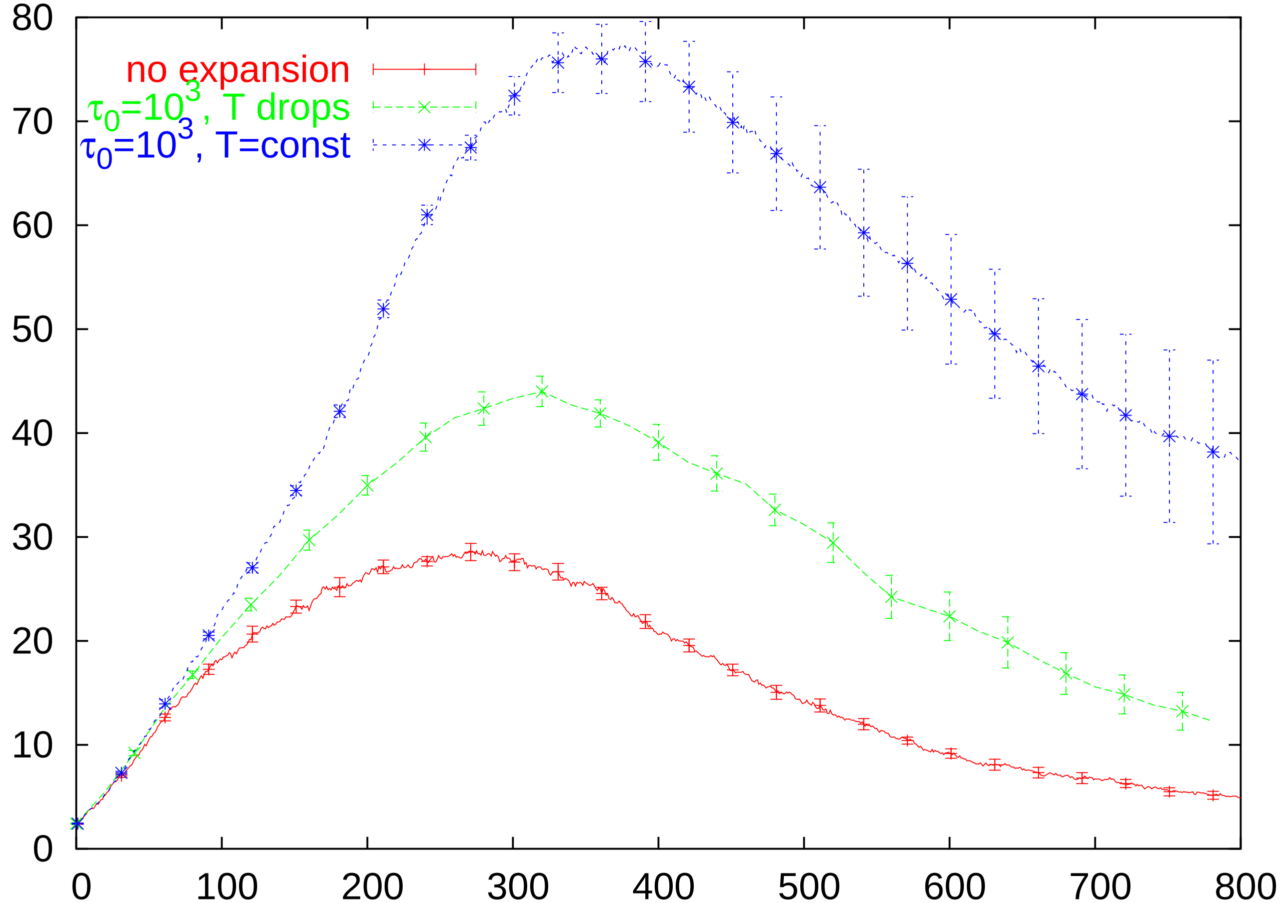

Figure 4: Comparison of first mode of structure functions including drop in temperature on a $16^3 \times 4$ lattice (same quench as in Fig. 1).

The important feature here is that *the location of the peaks is little affected by the presence of expansion and/or temperature evolution*. This implies that the system's response to the sudden quench is set by an internal dynamics scale that is faster than that of the expansion and accompanying temperature fall-off rates considered here.

After the structure function is past its peak, the system is 'isotropized', and, after a relatively short time, any memory of the initial fast, spinodal-like, long range response to the violent quench across the deconfinemnt transition boundary disappears.

The subsequent evolution is that of (quasi)equilibrium evolution of the system as it expands and cools towards its return to the confinement phase.

Time profile of the Polyakov loop average

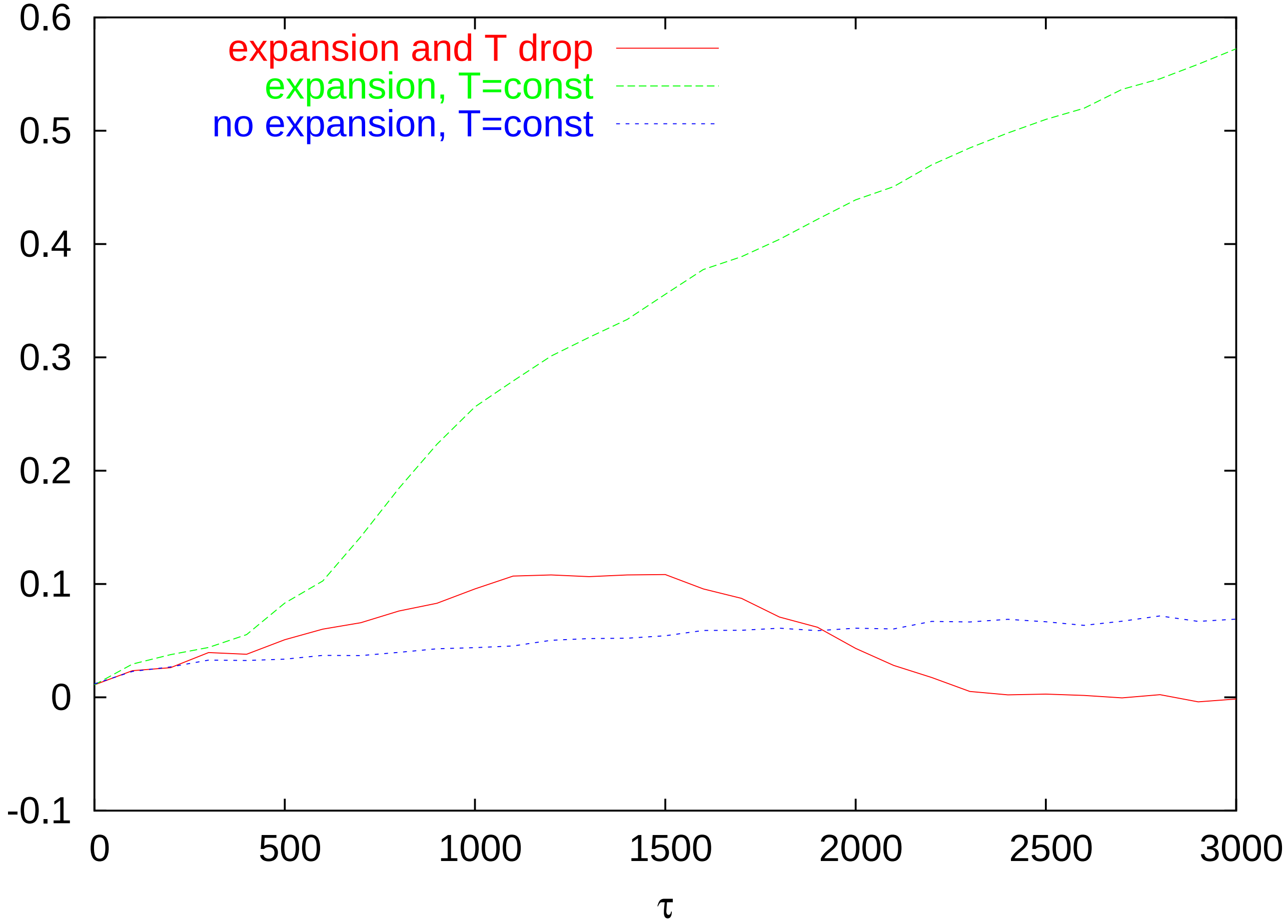

Figure 5: The Polyakov loop evolution after a quench in decreasing temperature on $32^3 \times 8$ lattice (same initial and final temperature quench as in Fig. 1).

Basic features

- Pure expansion drives the Polyakov loop towards larger values, i.e. more pronounced deconfined behavior. To get some insight into this behavior observe that spatial expansion enhances the timelike part of the action (2nd term in the square brackets on the r.h.s.) while suppressing the spacelike parts. This tends to increase the expectation of timelike Polyakov lines.

- Decreasing temperature counteracts the expansion effect. Eventually, under the combined effects of expansion and temperature fall-off, the Polyakov loop expectations drops to zero signaling the return of the system to the confinement phase.

  These qualitative features of are rather generic, being stable under changes in the expansion and temperature fall-off rates.

- The crucial feature characterizing the system's overall evolution following the rapid quench into the deconfinement region is the fact that *there appear to be two scales involved in this evolution*. The first is the scale set by the location of the peak of the structure function; the second is the scale set by the interval to return to the confinement phase. Furthermore, *there is wide separation between these two scales.*

A closer look

Quench to $T_{\mathrm{final}} \sim 3T_c$. (Note that T_c here means critical temperature of pure gluodynamics).

$$\begin{aligned} \beta_{\mathrm{initial}} &= 5.90 \qquad 0.76T_c \\ \beta_c &= 6.0625 \qquad T_c \\ \beta_{\mathrm{final}} &= 6.85 \qquad 2.95T_c\ , \end{aligned}$$

where 'initial' and 'final' refer to before and after the quench.

We model the spatial expansion and temperature drop on the lattice in terms of the two parameters τ_0 and τ_o' as:

$$\begin{aligned} a_s &= a_0(1+\frac{\tau}{\tau_0}), \\ a_\tau &= a_0(1+\frac{\tau}{\tau_0'})^{1/3} \\ &= a_0(1+y\frac{\tau}{\tau_0})^{1/3}, \end{aligned}$$

where $y = \tau_0/\tau_0'$ is the ratio of the 'speeds'.

The evolution of the anisotropy then is

$$\xi(\tau) = \frac{1 + \frac{\tau}{\tau_0}}{(1 + y\,\frac{\tau}{\tau_0})^{1/3}}$$

Before chemical freeze-out the plasma undergoes an x-fold expansion

$$x = \frac{a_s}{a_0} = 1 + \frac{\tau_{fo}}{\tau_0}$$

On the other hand

$$\frac{T_{fo}}{T_{final}} = \frac{1}{(1 + y\frac{\tau_{fo}}{\tau_0})^{1/3}}$$

From these two we get

$$y = \frac{(T_{final}/T_{fo})^3 - 1}{x - 1}$$

Hydrodynamical model phenomenology 'yields' all parameters Take $x \sim 9$ for the conversion to be completed. As the chemical freeze-out temperature take $T_{fo} = T_c$. This corresponds to $y \sim 3.25$.

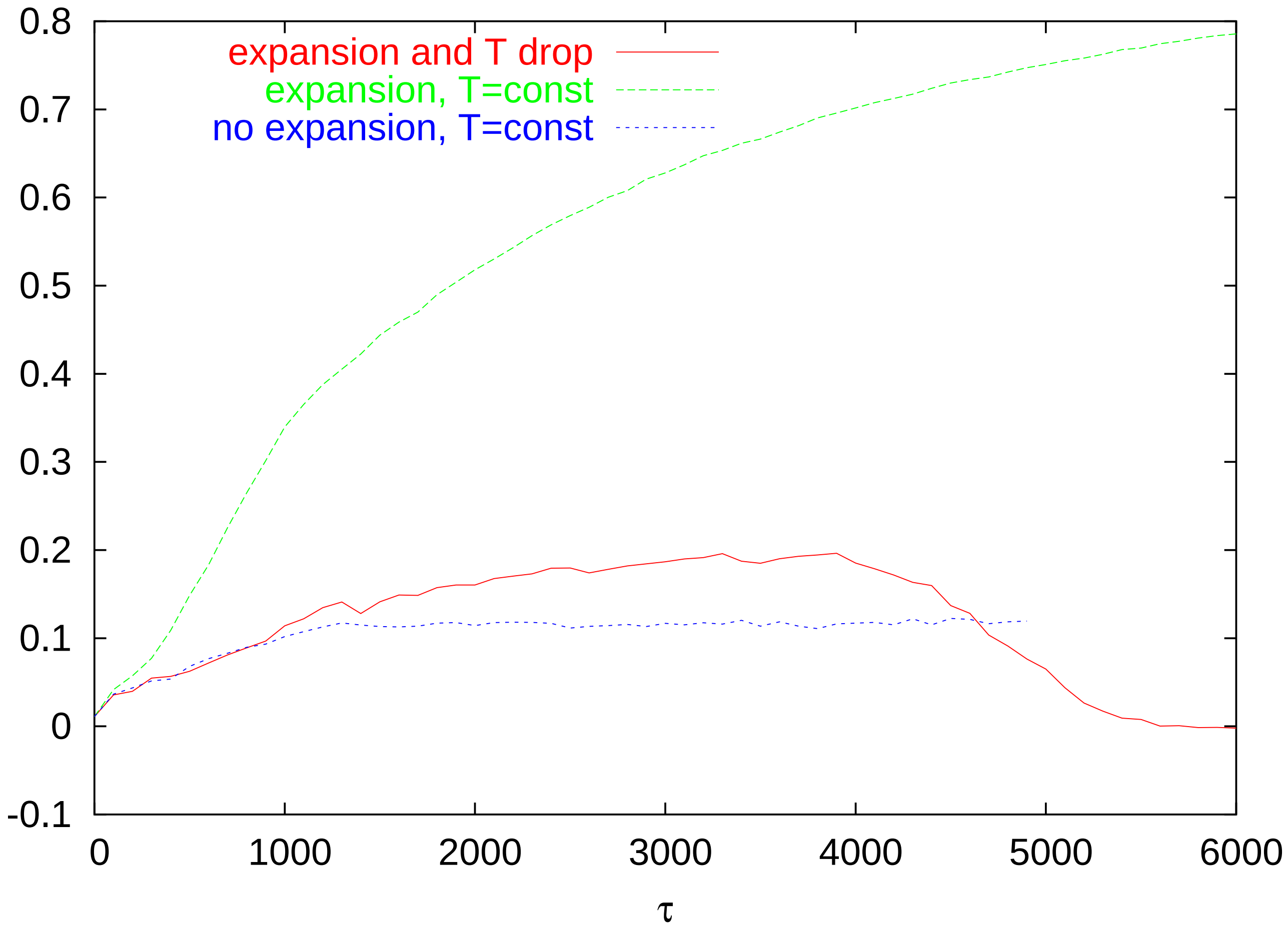

Figure 6: The Polyakov loop evolution after a quench to $2.95T_c$, $\tau_0 = 1000$

Structure function

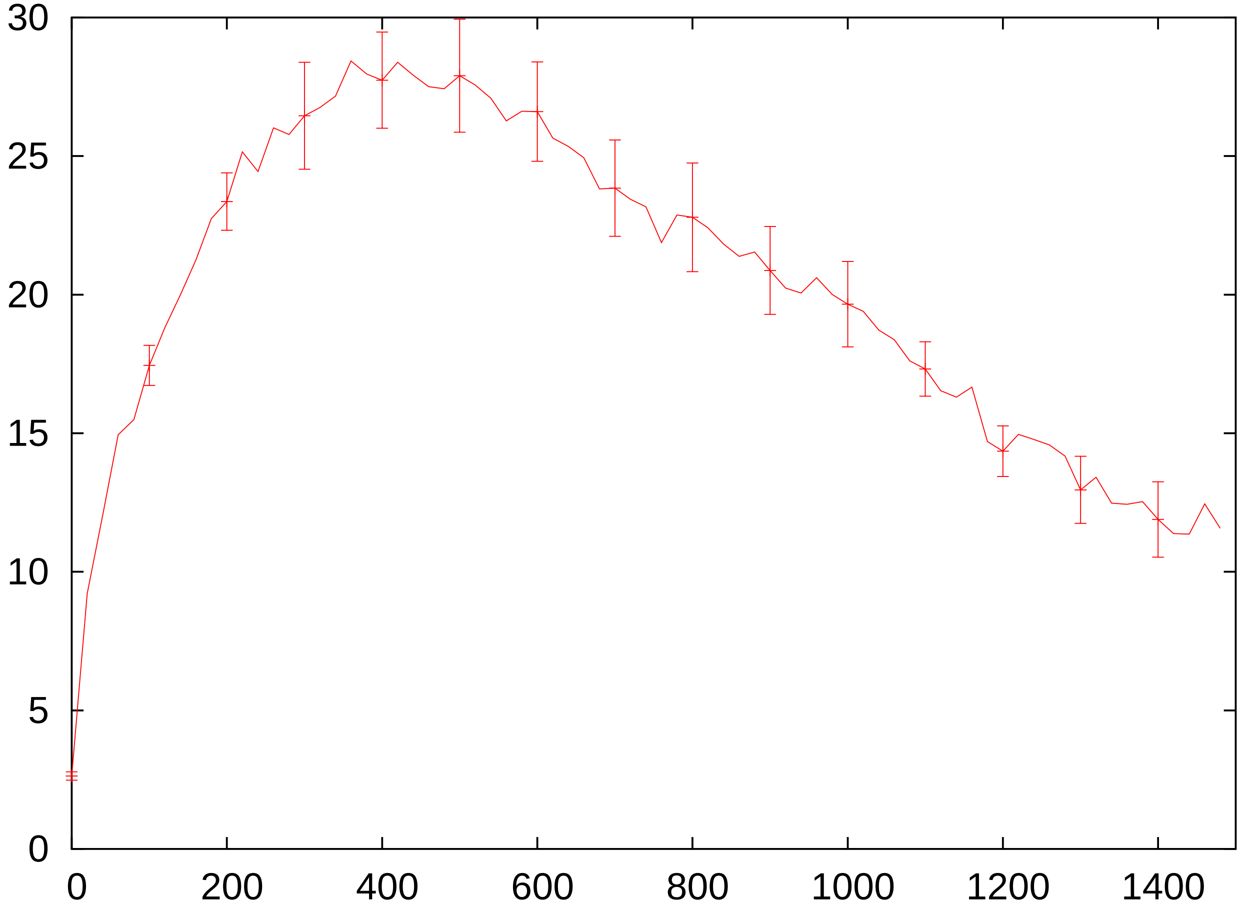

Figure 7: The first mode of structure function after a quench.

Peak reached at $\tau = 400$.

Polyakov loop evolution

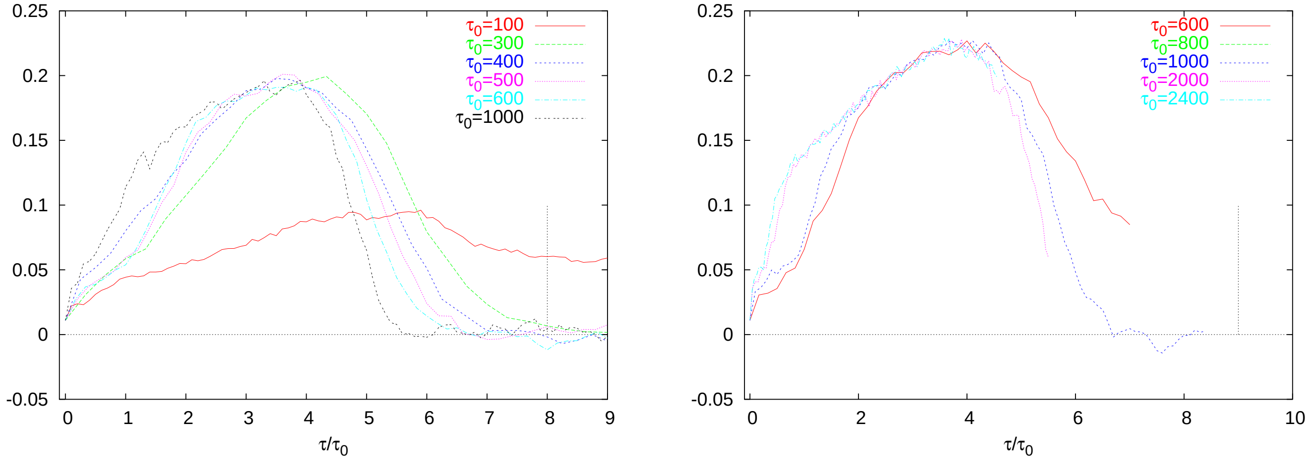

Figure 8: The Polyakov loop evolution after a quench to $2.95T_c$ for various values of τ_0 and $x = 9$ (left panel), $x = 10$ (right panel).

left panel: $x = 9$, $\frac{\tau_{fo}}{\tau_0} = 8$. Drops to zero before this point in the range of $\tau_0 \sim 400 - 600$.
Upper bound time estimate for the peak of the structure function in physical units: $\lesssim 0.75 - 1.1\ fm/c$.

right panel: $x = 10$, $\frac{\tau_{fo}}{\tau_0} = 9$. Drops to zero before this point in the range of $\tau_0 \gtrsim 700 - 800$.
Upper bound time estimate for the peak of the structure function in physical units: $\lesssim 0.56 - 0.63\ fm/c$.

Conclusions-Outlook

- We studied the response of the pure $SU(3)$ gauge theory to a rapid quench from its confined to its deconfined phase. We followed the subsequent evolution of the system under varying conditions of temperature fall-off and/or spatial expansion.

- There are two distinct scales characterizing this evolution.
  - There is one scale set by the development of very long range modes continuously from zero to their maximum amplitude over a short time interval. Thermalization, manifested by the decay of the structure function, follows shortly afterwards.
  - The second scale is set by the time interval to return to the confinement phase under quasi-equilibrium evolution, and is affected by the details of expansion and temperature fall-off.

- There is a robust wide separation between the two scales.

- Translated to physical units under reasonable assumptions about the lifetime of the plasma, this gives estimates of the ‘fast’ dynamics in the range of $0.5 - 1\ fm/c$.

- Future directions
  - Include anisotropy among the different space directions, thus allowing a more ‘realistic’ treatment of spatial expansion.
  - Explore the effect of having a really finite physical system. In our simulations we, as usual, employed periodic boundary conditions. But one may explore different ones that would mimic a finite rather than an infinite system.
  - Measure other observables.
  - Consider fermionic operators (in quenched approximation).