

Particle creation and warm inflation

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Abstract

During cosmological inflation, it has been suggested that fields coupled to the inflaton can be excited by the slow-rolling inflaton into a quasi-stable non-vacuum state. Within this scenario of “warm inflation”, this could allow for a smooth transition to a radiation dominated Universe without a separate reheating stage and a modification of the slow roll evolution, as the heat-bath backreacts on the inflaton through friction. In order to study this from first principles, we investigate the dynamics of a scalar field coupled to the inflaton and N light scalar boson fields, using the 2PI-1/ N expansion for nonequilibrium quantum fields. As a first step we restrict ourselves to Minkowski spacetime, interpret the inflaton as a time-dependent background, and use vacuum initial conditions. We find that the dominant effect is particle creation at late stages of the evolution due to the effective time-dependent mass. The further transfer of energy to the light degrees of freedom and subsequent equilibration only occurs after the end of inflation. As a consequence, the adiabatic constraint, which is assumed in most studies of warm inflation, is not satisfied when starting from an initial vacuum state.

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1. Introduction

In recent years quantitative cosmological observations have become available, resulting in a model for our Universe that is consistent with an early period of inflation and predictions of simple inflationary theories [1]. In the most elementary setup, the dynamics of the inflaton mean field $\phi(t) = \langle \varphi(t, \mathbf{x}) \rangle$ is determined by

$$\ddot{\phi}(t) + 3H(t)\dot{\phi}(t) + V'[\phi(t)] = 0, \quad (1.1)$$

subject to the Friedmann equation $H^2 = (\frac{1}{2}\dot{\phi}^2 + V[\phi])/3M_{\text{Pl}}^2$, where M_{Pl} is the Planck mass.

In warm inflation [2–7] the key idea is that interactions between the inflaton and other quantum fields are important during inflation and that they result in continuous energy transfer from the inflaton to these other fields. If this transfer is sufficiently fast and equilibration is rapid, a quasi-stable state could

be achieved, different from the inflationary vacuum. An additional effective friction term would then be expected in the inflaton equation of motion. A simple phenomenological modification of Eq. (1.1) incorporating this reads [2,3]

$$\ddot{\phi}(t) + [3H(t) + \Upsilon_\phi(t)]\dot{\phi}(t) + V'[\phi(t)] = 0, \quad (1.2)$$

where $\Upsilon_\phi(t)$ is a time- and field-dependent friction coefficient.^{1,2}

Since warm inflation necessarily involves multiple interacting quantum fields, which are dynamically evolving in real time, a full, quantitative understanding is difficult. In particular, a first-principle investigation requires all tools available to study quantum field dynamics far from equilibrium beyond the mean-field approximation. This may be contrasted with the theory of preheating due to parametric resonance after inflation, which can be understood from a combination of mean-field dynamics and the classical random field approximation [10–12].

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¹ For a critical analysis of warm inflation, see Ref. [8].

² For a study of particle creation and friction during power law inflation, see Ref. [9].

For the purpose of this Letter, the scenario of warm inflation separates naturally into three parts [5]:

1. The inflaton field ϕ is coupled to a second scalar field χ . While the inflaton rolls down the effective potential, it interacts with the χ field, resulting in the excitation of χ degrees of freedom.
2. The field χ is coupled to other degrees of freedom, which can be light fermion (ψ) or scalar (σ) fields. These fields are excited as well and may thermalize. Various interaction terms are possible.
3. The backreaction of the χ field on the inflaton leads to effective friction in the inflaton equation of motion, resulting in overdamped inflaton dynamics.

A commonly used interaction term between the inflaton field and the second field χ is given by $\frac{1}{2}g^2\phi^2\chi^2$. Due to the time dependence of the inflaton $\phi(t)$, an effective time-dependent mass term appears in the quantum dynamics of the χ field, $m_\chi^2 + g^2\phi^2(t)$, where m_χ is the mass in absence of the inflaton. In this Letter, we consider a large field model, such that the inflaton is rolling down its effective potential and $\phi(t)$ is decreasing. As a first step, we treat the inflaton as a time-dependent background and assume the most extreme case, in which the slow-rolling of the inflaton is caused by the interactions with the heat-bath rather than the expansion of the Universe. Taking this overdamped limit with $\Upsilon \gg H$, and assuming a quadratic potential $V[\phi] = \frac{1}{2}m_\phi^2\phi^2$ for simplicity, the dominant solution of Eq. (1.2) in the overdamped regime reads $\phi(t) = \phi_0 \exp[-(m_\phi^2/\Upsilon)t]$. We assumed that Υ is time- and field-independent. The inflaton is treated as a dynamical field in a companion paper [13]. Here, our goal is to study the dynamics of parts (1) and (2) described above, by combining a mode function analysis [14] and the techniques of the two-particle irreducible (2PI) effective action [15] for nonequilibrium quantum fields [16]. Since the inflaton acts as a time-dependent background, we find that there is particle production in the χ sector, akin to particle production in curved spacetime [14]. In Section 2 we study this process using a mode function analysis. We find that details of χ particle creation depend crucially on the size of the zero-temperature mass m_χ and the momentum k . Most importantly, we find that particle production only takes place towards the end of the evolution and that the amount of particles is, to a large extent, independent from the initial conditions and the duration of the inflationary stage.

In order to determine the range of validity of the mode function analysis and study the effect of interactions, we continue in Section 3 with a full far-from-equilibrium numerical study in quantum field theory using the 2PI effective action, and include interactions between the scalar field χ and N light quantum fields. Specifically, we include N scalar fields σ_a ($a = 1, \dots, N$) and use a truncation of the 2PI effective action determined by the 2PI-1/ N expansion to next-to-leading order (NLO) [17,18]. This approximation has been well studied in recent years and is known to give a quantitative description of both the early evolution far from equilibrium as well as the later stages of equilibration and thermalization (see, e.g.,

Refs. [17,19–21]). We consider both a trilinear coupling $\chi\sigma_a^2$ and a quartic coupling $\chi^2\sigma_a^2$. Irrespective of the interaction term, we find that as long as the inflaton is evolving, the mode function analysis gives an accurate description of the dynamics in the χ sector. In particular, processes leading to equilibration and thermalization are not yet relevant.

In this first study, we ignore the expansion of the Universe and work in Minkowski spacetime. If anything, neglecting the dilution caused by expansion should increase the chances of realizing a “warm” state.

2. Mode function analysis

As a first step, we perform a mode function analysis for a free χ field, subject to a time-dependent mass

$$M_\chi^2(t) = m_\chi^2 + \delta m^2 e^{-\gamma t}. \quad (2.1)$$

Here δm contains the details on the coupling to the inflaton. Specifically for the $\frac{1}{2}g^2\phi^2\chi^2$ interaction, we find that $\delta m^2 e^{-\gamma t} = g^2\phi^2(t)$, but these details are not required here. Using the standard decomposition, we write

$$\chi(t, \mathbf{x}) = \int \frac{d^3k}{(2\pi)^3} [a_{\mathbf{k}} f_{\mathbf{k}}(t) e^{i\mathbf{k}\cdot\mathbf{x}} + a_{\mathbf{k}}^\dagger f_{\mathbf{k}}^*(t) e^{-i\mathbf{k}\cdot\mathbf{x}}], \quad (2.2)$$

and find that $f_{\mathbf{k}}(t)$ is a solution of

$$\ddot{f}_{\mathbf{k}}(t) + [\mathbf{k}^2 + m_\chi^2 + \delta m^2 e^{-\gamma t}] f_{\mathbf{k}}(t) = 0. \quad (2.3)$$

A change of variables to $x = (2\delta m/\gamma) e^{-\gamma t/2}$ shows that this is a Bessel equation of the form

$$x^2 f_{\mathbf{k}}''(x) + x f_{\mathbf{k}}'(x) + \left[x^2 + \frac{4\omega_{\mathbf{k}}^2}{\gamma^2} \right] f_{\mathbf{k}}(x) = 0, \quad (2.4)$$

where $\omega_{\mathbf{k}} = \sqrt{\mathbf{k}^2 + m_\chi^2}$. We note that only the combination $\omega_{\mathbf{k}}/\gamma$ appears in this equation, while the dependence on δm will enter via the initial conditions. The general solution is

$$f_{\mathbf{k}}(t) = A_{\mathbf{k}}^+ J_{2i\omega_{\mathbf{k}}/\gamma} \left(\frac{2\delta m}{\gamma} e^{-\gamma t/2} \right) + A_{\mathbf{k}}^- J_{-2i\omega_{\mathbf{k}}/\gamma} \left(\frac{2\delta m}{\gamma} e^{-\gamma t/2} \right), \quad (2.5)$$

where $J_\nu(z)$ is the Bessel function of the first kind. The constants $A_{\mathbf{k}}^\pm$ are determined by the initial conditions, which are fixed by demanding that $f_{\mathbf{k}}(0) = 1/\sqrt{2\Omega_{\mathbf{k}}}$ and $\dot{f}_{\mathbf{k}}(0) = -i\Omega_{\mathbf{k}} f_{\mathbf{k}}(0)$, where $\Omega_{\mathbf{k}} = \sqrt{\mathbf{k}^2 + M_\chi^2(0)}$ and $M_\chi(0)$ is the initial mass. This determines $A_{\mathbf{k}}^\pm$ as

$$A_{\mathbf{k}}^\pm = \frac{\mp i}{\sqrt{2\Omega_{\mathbf{k}}}} \frac{\pi}{\gamma \sinh(2\pi\omega_{\mathbf{k}}/\gamma)} \left[J_{\mp 2i\omega_{\mathbf{k}}/\gamma} \left(\frac{2\delta m}{\gamma} \right) + i\Omega_{\mathbf{k}} J_{\mp 2i\omega_{\mathbf{k}}/\gamma} \left(\frac{2\delta m}{\gamma} \right) \right], \quad (2.6)$$

where

$$J_{\mp 2i\omega_{\mathbf{k}}/\gamma} \left(\frac{2\delta m}{\gamma} \right) = \left. \frac{d}{dt} J_{\mp 2i\omega_{\mathbf{k}}/\gamma} \left(\frac{2\delta m}{\gamma} e^{-\gamma t/2} \right) \right|_{t=0}. \quad (2.7)$$

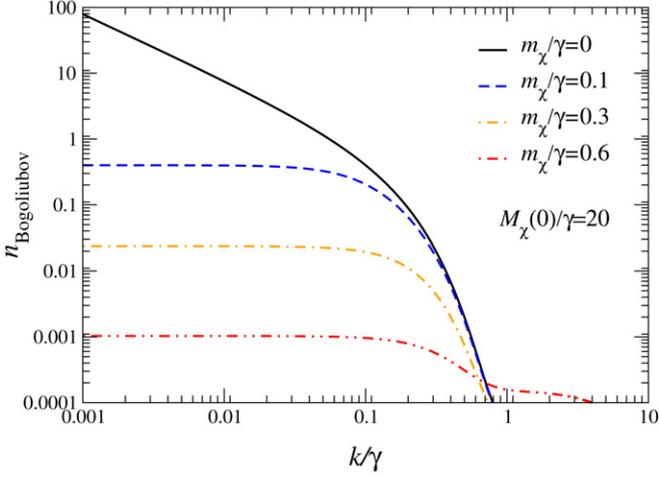


Fig. 1. Particle number $n_{\mathbf{k}}$ as a function of k/γ according to the Bogoliubov relation for four values of the final mass m_{χ} . The initial mass is $M_{\chi}(0)/\gamma = 20$.

One may verify that the Wronskian $f_{\mathbf{k}}(t)\dot{f}_{\mathbf{k}}^*(t) - \dot{f}_{\mathbf{k}}(t)f_{\mathbf{k}}^*(t) = i$ is preserved during the time evolution.

In order to identify the produced particle number at asymptotically late times, we use some elementary properties of Bessel functions and find that

$$\lim_{t \rightarrow \infty} J_{\pm 2i\omega_{\mathbf{k}}/\gamma} \left(\frac{2\delta m}{\gamma} e^{-\gamma t/2} \right) = C_{\mathbf{k}}^{\pm} e^{\mp i\omega_{\mathbf{k}} t}, \quad (2.8)$$

with

$$C_{\mathbf{k}}^{\pm} = \left(\frac{\delta m}{\gamma} \right)^{\pm 2i\omega_{\mathbf{k}}/\gamma} \frac{1}{\Gamma(1 \pm 2i\omega_{\mathbf{k}}/\gamma)}. \quad (2.9)$$

At late times, we find therefore that the mode functions oscillate with the expected frequency $\omega_{\mathbf{k}}$, and

$$\lim_{t \rightarrow \infty} f_{\mathbf{k}}(t) = A_{\mathbf{k}}^+ C_{\mathbf{k}}^+ e^{-i\omega_{\mathbf{k}} t} + A_{\mathbf{k}}^- C_{\mathbf{k}}^- e^{i\omega_{\mathbf{k}} t}. \quad (2.10)$$

A comparison with the standard form of the mode functions at $t \rightarrow \infty$, $\tilde{f}_{\mathbf{k}}(t) = e^{-i\omega_{\mathbf{k}} t}/\sqrt{2\omega_{\mathbf{k}}}$ yields the Bogoliubov coefficient [14]

$$\begin{aligned} \beta_{\mathbf{k}} &= i \left[\tilde{f}_{\mathbf{k}}(t) \partial_t f_{\mathbf{k}}(t) - f_{\mathbf{k}}(t) \partial_t \tilde{f}_{\mathbf{k}}(t) \right] \\ &= -\sqrt{2\omega_{\mathbf{k}}} A_{\mathbf{k}}^- C_{\mathbf{k}}^-. \end{aligned} \quad (2.11)$$

Therefore, the final particle number (starting in vacuum initially) is $n_{\mathbf{k}} = |\beta_{\mathbf{k}}|^2 = 2\omega_{\mathbf{k}} |A_{\mathbf{k}}^- C_{\mathbf{k}}^-|^2$, which, after some algebra, can be written as³

$$\begin{aligned} n_{\mathbf{k}} &= \frac{1}{2\Omega_{\mathbf{k}}} \frac{\pi}{\gamma \sinh(2\pi\omega_{\mathbf{k}}/\gamma)} \left\{ \left| J_{2i\pi\omega_{\mathbf{k}}/\gamma} \left(\frac{2\delta m}{\gamma} \right) \right|^2 \right. \\ &\quad \left. + \Omega_{\mathbf{k}}^2 \left| J_{2i\pi\omega_{\mathbf{k}}/\gamma} \left(\frac{2\delta m}{\gamma} \right) \right|^2 \right\} - \frac{1}{2}. \end{aligned} \quad (2.12)$$

The resulting particle numbers are shown in Fig. 1 for four different values of the asymptotic mass m_{χ} and an initial mass

³ If the initial particle number is nonzero and equals $\langle a_{\mathbf{k}}^{\dagger} a_{\mathbf{k}} \rangle = n_{\mathbf{k}}^{(0)}$, we find that $n_{\mathbf{k}} = (1 + 2n_{\mathbf{k}}^{(0)})\omega_{\mathbf{k}} |C_{\mathbf{k}}^-|^2 (|A_{\mathbf{k}}^+|^2 + |A_{\mathbf{k}}^-|^2) - \frac{1}{2}$.

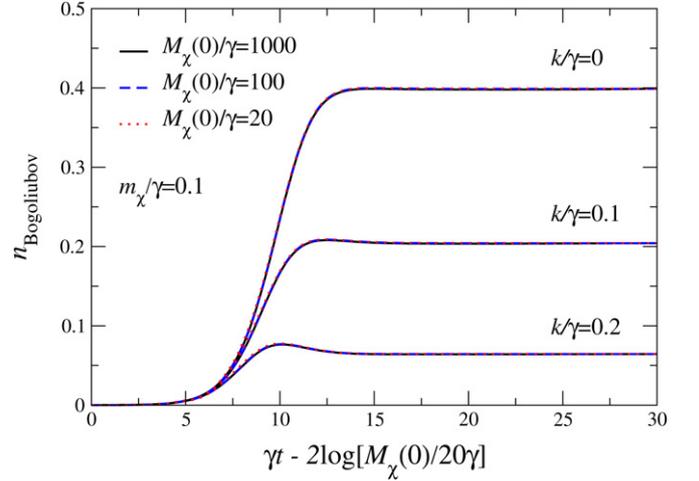


Fig. 2. Particle number $n_{\mathbf{k}}$ as a function of time, according to the Bogoliubov relation, for three values of the initial mass $M_{\chi}(0)$ and three momentum modes. After shifting time by an amount $2\log[M_{\chi}(0)/M_{\chi}^{\text{ref}}]$, where $M_{\chi}^{\text{ref}}/\gamma = 20$ is a reference mass, there is no dependence on the initial mass remaining. The final mass is $m_{\chi}/\gamma = 0.1$.

$M_{\chi}(0)/\gamma = 20$. We observe that a significant amount of particles are only produced when $m_{\chi}/\gamma \ll 1$, and only with momentum $k/\gamma \lesssim 1$.

In order to determine when particles are created during the inflationary stage, we show the time-dependent particle number,

$$n_{\mathbf{k}}(t) = \frac{1}{2\omega_{\mathbf{k}}(t)} \left[|\dot{f}_{\mathbf{k}}(t)|^2 + \omega_{\mathbf{k}}^2(t) |f_{\mathbf{k}}(t)|^2 \right] - \frac{1}{2}, \quad (2.13)$$

where $\omega_{\mathbf{k}}(t) = \sqrt{\mathbf{k}^2 + M_{\chi}^2(t)}$, in Fig. 2. We comment on possible other definitions of particle number elsewhere [13]. The time-dependent particle number is shown for three different initial masses $M_{\chi}(0)$, the largest mass corresponding to the longest period of inflation (recall that $M_{\chi}(0) \sim \phi_0$). However, we find that a trivial shift of the time variable is sufficient to take the initial mass dependence into account. In other words, the amount of particles produced is independent of the initial conditions and therefore the duration of the inflationary stage. Generically, particles can only be produced when $|\dot{\omega}_{\mathbf{k}}(t)| \gtrsim \omega_{\mathbf{k}}^2(t)$, which in our model translates to $\gamma \gtrsim M_{\chi}(t)$. If we denote the time when most particles are created with t_* , we find from Fig. 2 that $\gamma t_* - 2\log[M_{\chi}(0)/M_{\chi}^{\text{ref}}] \approx 8$, where $M_{\chi}^{\text{ref}}/\gamma = 20$ is a reference mass. The size of the effective mass is then $M_{\chi}(t_*)/\gamma \approx 0.37 < 1$, confirming that χ particles are only produced when $M_{\chi}(t)/\gamma$ is sufficiently small.

3. Interactions

In the mode function analysis of the previous section, interactions besides the time-dependent mass are ignored. In order to assess the validity of that analysis, we now include interactions and consider the coupling of the scalar field χ to N light degrees of freedom, either fermionic or bosonic. We start with some parametric estimates and then present the numeri-

cal results obtained with the help of the 2PI-1/ N expansion for nonequilibrium quantum fields.

3.1. Parametric estimates

In order to get an estimate for the magnitude of various parameters, we use here an inflaton potential $V[\varphi] = \frac{1}{2}m_\varphi^2\varphi^2$. During slow-roll inflation, the Hubble parameter is approximately given by $H \sim m_\varphi\phi/M_{\text{Pl}}$. In order to have inflation, the initial amplitude should be $\phi_0 \sim$ a few M_{Pl} , and to satisfy CMB constraints [1], we require that $m_\varphi \sim 10^{-6}M_{\text{Pl}}$. We then find that $H \sim 10^{-6}\phi_0 \gtrsim 10^{-6}M_{\text{Pl}}$. In warm inflation, the effective damping term Υ in the inflaton equation wins over the expansion rate. Taking $\Upsilon \sim 100H$ yields $\Upsilon \sim 10^{-4}M_{\text{Pl}}$. Using exponential time dependence, $\phi(t) = \phi_0 \exp[-(m_\varphi^2/\Upsilon)t]$, then yields a value of $\gamma \sim 10^{-8}M_{\text{Pl}}$ for the rate in the time-dependent mass (2.1). In order to have any particle production, we found from the mode function analysis that $m_\chi/\gamma \lesssim 0.1$, or $m_\chi \lesssim 10^{-9}M_{\text{Pl}}$, which means that the χ particle can be some degree of freedom beyond the Standard Model.

We now consider the couplings to the light degrees of freedom. First we consider the coupling to N fermion fields, with the interaction term

$$\sum_{a=1}^N \frac{h}{\sqrt{N}} \chi \bar{\psi}_a \psi_a. \quad (3.1)$$

The factor \sqrt{N} is introduced to allow for a proper $1/N$ expansion. Following Ref. [5], we impose $h^2 \lesssim 1$ such that perturbation theory is reasonable.⁴ From a standard one-loop calculation one can compute the (zero-temperature) onshell decay width for $\chi \rightarrow \psi\psi$, in the case that $M_\chi > 2M_\psi$. Here the masses $M_{\chi,\psi}$ include possible background field dependence. The width is given by

$$\Gamma_{\mathbf{p}}^\chi = \frac{h^2 M_\chi^2}{8\pi \sqrt{\mathbf{p}^2 + M_\chi^2}} \left(1 - \frac{4M_\psi^2}{M_\chi^2}\right)^{3/2}. \quad (3.2)$$

The important assumption made in most studies of warm inflation is that the dynamics takes place in the so-called adiabatic approximation, $|\dot{\phi}/\phi| \ll \Gamma_{\mathbf{p}}^\chi$, leading to quick decay (and possibly thermalization) during inflation.⁵ In our model we have to compare $\Gamma_{\mathbf{p}}^\chi$ with γ . Taking for simplicity $M_\psi \ll M_\chi$ and $\mathbf{p} = \mathbf{0}$, we find

$$\frac{\Gamma_{\mathbf{0}}^\chi}{\gamma} = \frac{h^2}{8\pi} \frac{M_\chi}{\gamma}. \quad (3.3)$$

When it is assumed that $M_\chi \sim g\phi_0$ and $\phi_0 \gg \gamma$ (as discussed above), this ratio can be much larger than one. However, in the mode function analysis we found that χ particles are only produced when the (time-dependent) mass M_χ is much smaller, specifically $M_\chi/\gamma \lesssim 1$. In that case the important conclusion is

that the dynamics is not taking place in the adiabatic regime, but rather in the opposite limit $|\dot{\phi}/\phi| \gg \Gamma_{\mathbf{p}}^\chi$.

In the numerical study below, we couple the χ field to N light bosonic fields, with the interaction term

$$\sum_{i=1}^N \frac{h}{\sqrt{N}} \chi \sigma_a^2. \quad (3.4)$$

In this case, the coupling constant h is dimensionful and it is natural to write $h = m_\chi \tilde{h}$, where \tilde{h} is dimensionless. We consider again the decay process $\chi \rightarrow \sigma\sigma$ and find for the onshell width, assuming that $M_\chi > 2M_\sigma$,

$$\Gamma_{\mathbf{p}}^\chi = \frac{\tilde{h}^2 m_\chi^2}{8\pi \sqrt{\mathbf{p}^2 + M_\chi^2}} \sqrt{1 - \frac{4M_\sigma^2}{M_\chi^2}}. \quad (3.5)$$

To test the adiabatic approximation, we compare again $\Gamma_{\mathbf{p}}^\chi$ with γ . Taking $M_\sigma \ll M_\chi$ and $\mathbf{p} = \mathbf{0}$, we find

$$\frac{\Gamma_{\mathbf{0}}^\chi}{\gamma} = \frac{\tilde{h}^2}{8\pi} \frac{m_\chi}{\gamma} \frac{m_\chi}{M_\chi} \ll 1. \quad (3.6)$$

Since all factors are strictly less than one, we are not in the adiabatic limit. This is in agreement with the results from the numerical analysis carried out below.

3.2. Nonequilibrium dynamics

To determine the range of validity of the mode function analysis and estimates carried out above, we continue with a numerical study using the 2PI effective action for quantum field dynamics in real time. We refrain from providing details on the 2PI effective action and the Schwinger–Keldysh formalism for nonequilibrium field theory, instead we refer to Ref. [18], whose notation we follow closely.

We consider the following action,

$$\begin{aligned} S[\chi, \sigma] = & - \int d^4x \left\{ \frac{1}{2} (\partial_\mu \chi)^2 + \frac{1}{2} [m_\chi^2 + \delta m^2 e^{-\gamma t}] \chi^2 \right. \\ & + \frac{\lambda_\chi}{4!N} \chi^4 + \frac{1}{2} (\partial_\mu \sigma_a)^2 + \frac{1}{2} m_\sigma^2 \sigma_a^2 \\ & \left. + \frac{\lambda_\sigma}{4!N} (\sigma_a \sigma_a)^2 + V_{\text{int}}[\chi, \sigma] \right\}. \end{aligned} \quad (3.7)$$

We consider two different interaction terms $V_{\text{int}}[\chi, \sigma]$ between the χ field and the scalar fields σ_a , a trilinear and a quartic coupling. Both interaction terms preserve the $O(N)$ symmetry in the σ sector, such that the σ two-point function can be written as $G_{\sigma ab}(x, y) = \delta_{ab} G_\sigma(x, y)$. We have scaled all couplings in such a way that a proper $1/N$ expansion is possible. We use $N = 4$ throughout.

Trilinear coupling. Motivated by Ref. [5], we start with the trilinear coupling,

$$V_{\text{int}}[\chi, \sigma] = \frac{h}{\sqrt{N}} \chi \sigma_a^2 + c_\chi \chi. \quad (3.8)$$

⁴ This corresponds to $Nh^2 \lesssim 1$ in the conventions of Ref. [5].

⁵ The origin of the adiabatic approximation can be traced back to Refs. [22, 23].

Because this term breaks the symmetry $\chi \rightarrow -\chi$, we have to allow for a nonzero expectation value $\bar{\chi}(t) = \sqrt{N}\langle\chi(t, \mathbf{x})\rangle$.⁶ The term linear in χ is used to shift the minimum of the (effective) potential at the initial time to $\langle\chi\rangle = 0$. The 2PI part of the effective action is written as $\Gamma_2[G_\chi, G_\sigma] = \Gamma_{\chi\sigma}[G_\chi, G_\sigma] + \Gamma_\sigma[G_\sigma]$, where the first term is given by the two-loop diagram

$$\Gamma_{\chi\sigma}[G_\chi, G_\sigma] = ih^2 \int d^4x d^4y G_\chi(x, y) G_\sigma^2(x, y). \quad (3.9)$$

The second part, $\Gamma_2[G_\sigma]$, is the standard NLO contribution for N scalar fields. This contribution has been discussed in detail in Refs. [17,18] and will not be shown here explicitly. Other 2PI diagrams are suppressed by $1/N$ and are not included. The resulting equations of motion follow from a variation of the 2PI effective action with respect to $\bar{\chi}$, G_χ and G_σ , and read

$$\begin{aligned} \frac{d^2}{dt^2} \bar{\chi} + \left[m_\chi^2 + \delta m^2 e^{-\gamma t} + \frac{\lambda_\chi}{6} \bar{\chi}^2 + \frac{\lambda_\chi}{2N} G_\chi(x, x) \right] \bar{\chi} \\ = -h G_\sigma(x, x) - c_\chi, \end{aligned} \quad (3.10)$$

and for the two-point functions ($j = \chi, \sigma$)

$$\begin{aligned} -[\square_x + M_j^2(t)] G_j(x, y) \\ = i \int d^4z \Sigma_j(x, z) G_j(z, y) + i \delta^4(x - y), \end{aligned} \quad (3.11)$$

with the effective masses

$$\begin{aligned} M_\chi^2(t) &= m_\chi^2 + \delta m^2 e^{-\gamma t} + \frac{\lambda_\chi}{2} \bar{\chi}^2, \\ M_\sigma^2(t) &= m_\sigma^2 + 2h \bar{\chi} + \lambda_\sigma \frac{N+2}{6N} G_\sigma(x, x). \end{aligned} \quad (3.12)$$

The nonlocal contributions to the self-energies $\Sigma_{\chi, \sigma}$ follow from variation of $\Gamma_2[G_\chi, G_\sigma]$ in the usual manner [13,18].

Quartic coupling. In order to assess the importance of the possibility of onshell decay $\chi \rightarrow \sigma\sigma$ in the trilinear case, we also consider the following quartic potential,

$$V_{\text{int}}[\chi, \sigma] = \frac{h}{2N} \chi^2 \sigma_a^2. \quad (3.13)$$

In the 2PI effective action we include the two- and three-loop diagrams

$$\begin{aligned} \Gamma_{\chi\sigma}[G_\chi, G_\sigma] &= -\frac{h}{2} \int d^4x G_\chi(x, x) G_\sigma(x, x) \\ &\quad + \frac{ih^2}{2N} \int d^4x d^4y G_\chi^2(x, y) G_\sigma^2(x, y), \end{aligned} \quad (3.14)$$

as well as $\Gamma_\sigma[G_\sigma]$ to NLO as above. Strictly speaking, we deviate here from the $1/N$ expansion, since the three-loop diagram only appears at next-to-next-to leading order. We include it nevertheless, since it plays the same role as the two-loop diagram in the trilinear case and results in $\chi\sigma$ interaction beyond the mean-field approximation. In this case the symmetry $\chi \rightarrow -\chi$ is preserved, so that we can take $\langle\chi\rangle = 0$ consistently for the entire evolution. Hence Eq. (3.10) is absent, while the effective

masses appearing in Eq. (3.11) are

$$M_\chi^2(t) = m_\chi^2 + \delta m^2 e^{-\gamma t} + h G_\sigma(x, x), \quad (3.15)$$

$$M_\sigma^2(t) = m_\sigma^2 + \lambda_\sigma \frac{N+2}{6N} G_\sigma(x, x) + \frac{h}{N} G_\chi(x, x). \quad (3.16)$$

The nonlocal self-energies follow again from the 2PI effective action.

We solve the resulting equations numerically, following the approach in [17,19–21]. Space is discretized on a three-dimensional lattice with 32^3 sites and a physical size of $\gamma L = 32$. The length of the memory kernel is $\gamma t = 20$, containing 800 time steps. The masses are $m_\chi/\gamma = 0.1$ and $m_\sigma/m_\chi = 1/3$. The coupling constants are $\lambda_\chi = 6$, $\lambda_\sigma = 6$. In the trilinear case the coupling $h/m_\chi = 5/3$, while in the quartic case we use $h = 1$. We initialize the two-point functions in vacuum. Renormalization is carried out in such a way that the set of equations is initialized at the fixed point of the renormalized mean field equations [13,24].

Particle number is a derived concept and not always well defined in an interacting field theory out of equilibrium. Instead of comparing time-dependent particle numbers, we prefer to study basic quantities appearing in the dynamical equations: the equal-time two-point functions $G_\chi(t, t; \mathbf{k})$ and $G_\sigma(t, t; \mathbf{k})$. In Figs. 3 and 4 we show the evolution of the nine lowest momentum modes in time (the evolution of the zero momentum mode is not shown). In the top left corners, the dynamics without interactions is presented, in which case $G_\chi(t, t; \mathbf{k}) = |f_{\mathbf{k}}(t)|^2$ and $G_\sigma(t, t; \mathbf{k}) = \frac{1}{2}(\mathbf{k}^2 + m_\sigma^2)^{-1}$. The latter is exactly time-independent, because of our choice of initialization. In the other three frames, we show the dynamics with trilinear interaction (bottom left), and with quartic interaction using the Hartree approximation, i.e. including the self-consistently determined masses (3.15) and (3.16) but not the nonlocal terms (top right), and the full evolution (bottom right).

As can be seen in Fig. 3, the evolution of G_χ is in all cases nearly identical to the free case, except for a small additional growth in the case that the nonlocal self energies are included. From this we conclude that the mode function analysis describes the important aspects of this part of the dynamics extremely well, even in the presence of interactions. In particular, the process of particle production can be studied in a satisfactory manner using the techniques of Section 2.

The response of the σ propagator is shown in Fig. 4. Recall that without interactions, the equal-time propagator is constant in time. In the trilinear case interactions with the χ field have a small effect, in the low-momentum modes only and with a characteristic time scale much longer than $1/\gamma$. In particular, there is no notion of equilibration and thermalization. This is consistent with the estimates given above and also with previous studies of nonequilibrium scalar field dynamics using the 2PI effective action [17,20,21], in which it was shown that thermalization occurs only after a time of the order of at least several hundred elementary oscillations. In the quartic case, the situation is slightly different because of the presence of the χ tadpole in the effective σ mass (3.16). During the evolution, the mass M_σ grows as $G_\chi(x, x)$ increases due the time-dependent

⁶ This was overlooked in Ref. [5].

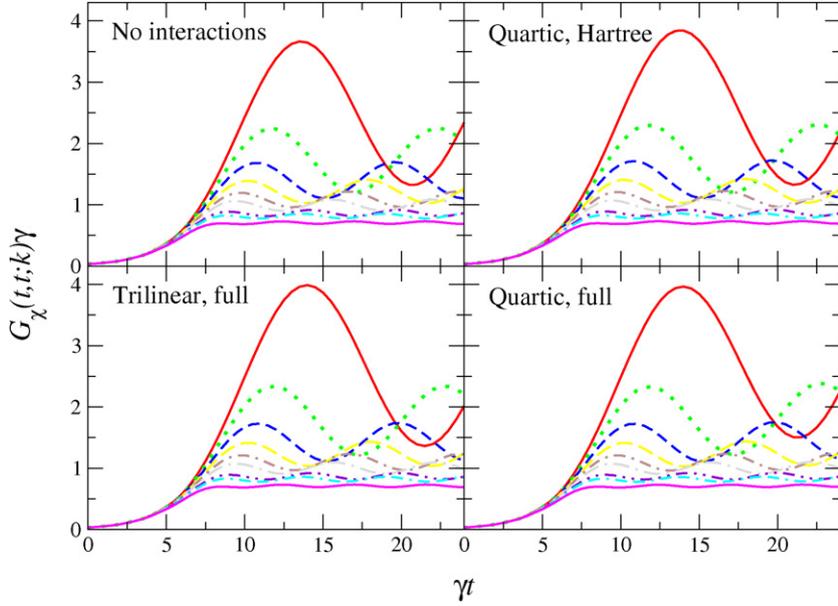


Fig. 3. Equal-time two-point function $G_\chi(t, t; \mathbf{k})$ as a function of time t in units of γ , for nine different momenta k (the ordering is such that the smallest momentum corresponds to the largest value at $\gamma t \approx 10$), using the analytical expression (top left), the trilinear interaction (bottom left), the quartic interaction term in the Hartree approximation (top right), and the quartic interaction term (bottom right). In all cases the initial mass is $M_\chi(0)/\gamma = 20$ and the final mass is $m_\chi/\gamma = 0.1$.

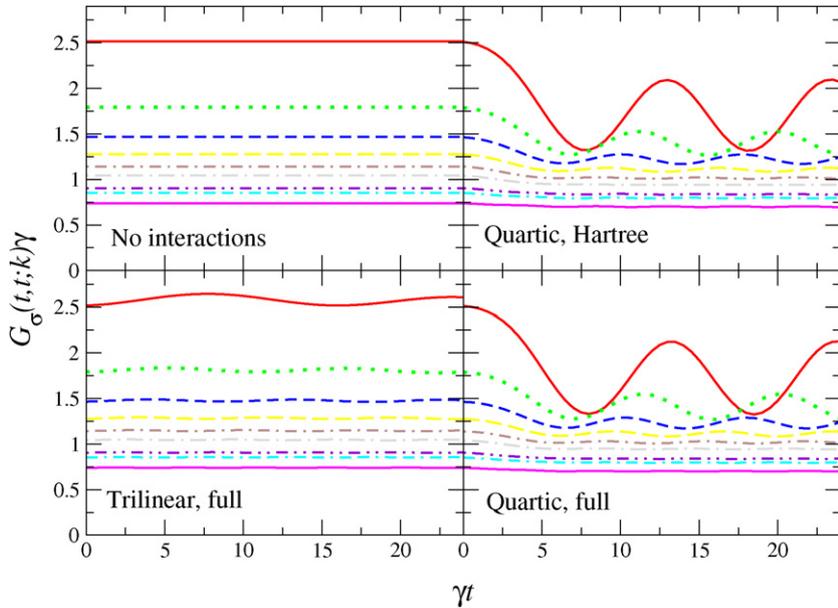


Fig. 4. Same as in Fig. 3, for the σ two-point function $G_\sigma(t, t; \mathbf{k})$. The largest value at $t = 0$ corresponds to the smallest momentum, since $G_\sigma(0, 0; \mathbf{k}) = \frac{1}{2}(\mathbf{k}^2 + m_\sigma^2)^{-1}$.

inflaton background. The response of the σ propagator is captured very well in the Hartree approximation. Additional non-local diagrams (Fig. 4, bottom right) play only a minor role at this stage, since in the theory with a quartic coupling onshell decay $\chi \rightarrow \chi\sigma\sigma$ is kinematically not allowed, while scattering processes $\chi\chi \rightarrow \sigma\sigma$ are suppressed due to small amount of χ particles present.

4. Outlook

In order to investigate the dynamics of warm inflation, we studied the quantum evolution of a scalar field χ coupled to the

inflaton as well as N light scalar fields. In this first analysis we treated the inflaton as a background field and ignored the expansion of the Universe. We took vacuum initial conditions. From a comparison with nonequilibrium quantum field dynamics using the 2PI-1/ N expansion to next-to-leading order, we found that the response in the χ sector can be accurately understood from a mode function analysis and that further interactions with light scalar fields are subdominant. The important conclusion is that χ particle production only takes place towards the end of the evolution, independent of the duration of the inflationary stage. An immediate consequence is that under these conditions the so-called adiabatic approximation, $|\dot{\phi}/\phi| \ll \Gamma^\chi$, assumed

in most studies of warm inflation, is violated. Therefore, the dynamics during inflation takes place far from equilibrium and processes important for thermalization become important only later. These findings are complementary to those obtained in Ref. [8].

Although our investigation used a specific $\varphi^2\chi^2$ interaction and assumed an exponential time dependence for the inflaton background field, we believe that most of the results obtained here are generic for systems initially in vacuum. In particular, χ particles can only be produced when the rate of time variation of the effective mass is comparable with the effective mass itself. This observation is a potential hurdle in all models where the effective χ mass receives a contribution from interactions with the inflaton and the inflaton initially has a large expectation value (large field models). Concerning the time dependence, we have also studied a linear evolution, $\phi(t) \sim t$, and found that the specific time dependence is not crucial [13].

As a next step, we plan to treat the inflaton as a dynamical quantum field, and use 2PI effective action techniques to analyse the effective inflaton equation of motion, including the backreaction from χ particles. Since we found in this Letter that the role of the additional light degrees of freedom is subdominant, we can focus entirely on the inflaton- χ sector.

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