The curvature of the QCD phase transition line

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Sign Problem and Complex Actions
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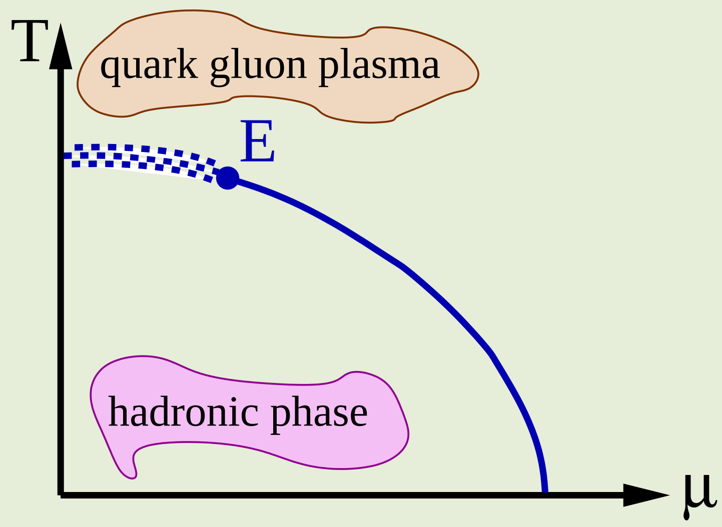
Introduction, motivation



- $\mu = 0$ area is relevant for - the early Universe
  - high energy collisions
- QCD transition at $\mu = 0$ is found to be a crossover

  [Y. Aoki, GE, Z. Fodor, S.D. Katz, K.K. Szabó]
- Different observables give different values for T_c namely, $T_c(\chi_{\bar{\psi}\psi}) \approx 151$ MeV, $T_c(\chi_s) \approx 175$ MeV

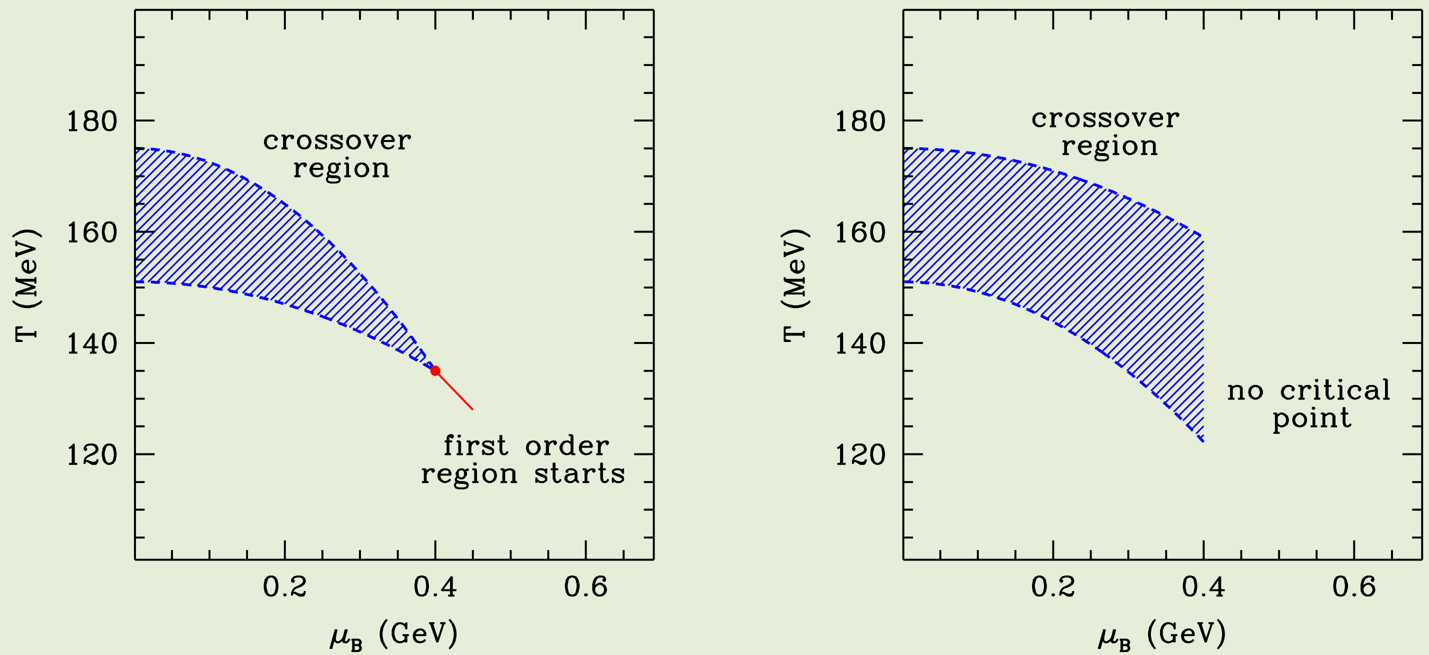
  [Y. Aoki, Z. Fodor, S.D. Katz, K.K. Szabó]

The role of the curvature

- Explore the $\mu \neq 0$ region of the phase diagram
- At $\mu \neq 0$ the fermion determinant is complex
  $\rightarrow$ importance sampling not possible
- Use Taylor-expansion in μ, around $\mu = 0$
  first term vanishes
  second term given by the curvature (κ)
- Aims:
  determine the curvature for different observables
  $\chi_{\bar{\psi}\psi}$, $\bar{\psi}\psi$; and for L, χ_s
- Comparison: $N_T = 4$ and 6 results; the curvature is in the range of $\kappa = 0.003 \ldots 0.01$

[Bielefeld-Swansea; Philipsen, de Forcrand; D'Elia, Lombardo; Fodor, Katz]

Scenarios



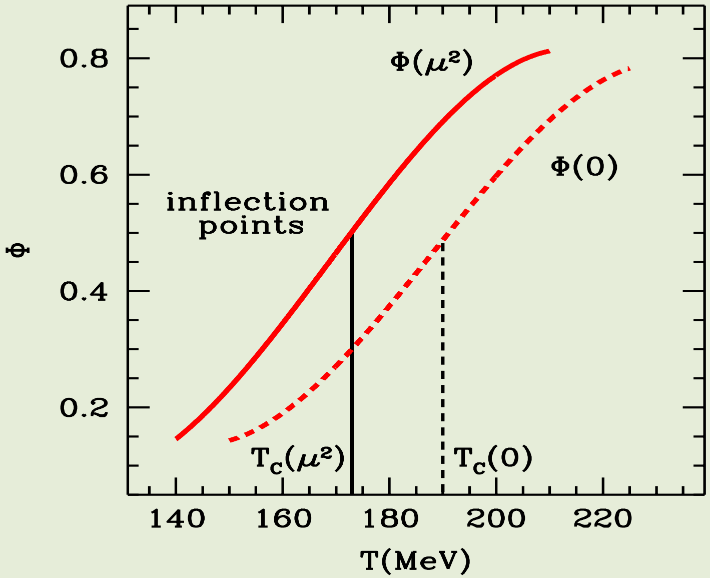
- Does the crossover region shrink or expand?
- The curvature can affect the existence of the critical endpoint
- Estimation: if $\mu_{crit} = 360$ MeV $\rightarrow \Delta\kappa \approx 0.02$
- $\mu \equiv \mu_B$

Curvature determination I.

- Equation of transition line is $T_c(\mu) = T_c\left(1 - \kappa\frac{\mu^2}{T_c^2}\right)$

$$\rightarrow \kappa = -T_c \left.\frac{\mathrm{d}T_c(\mu)}{\mathrm{d}\mu^2}\right|_{\mu=0}$$

- Determination of κ demonstrated on $\chi_s/T^2 \equiv \Phi$



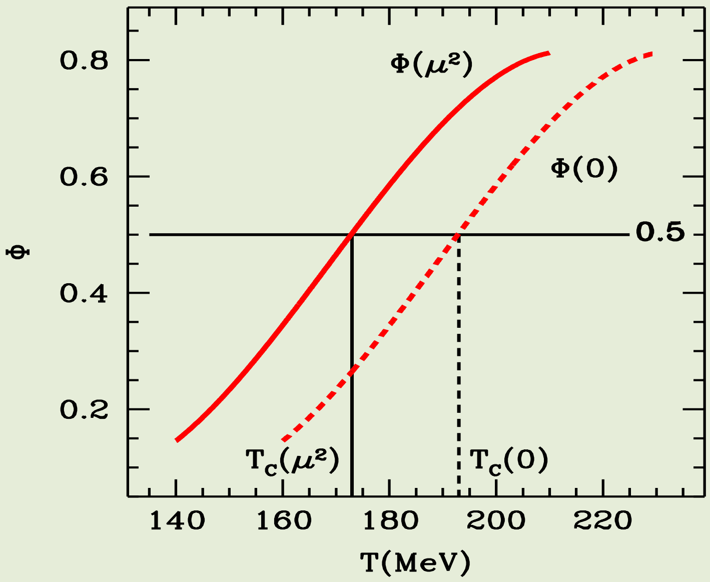
- Procedure #1: shape of Φ may get distorted at finite μ
- T_c defined as inflection point of Φ

$$\rightarrow \frac{\mathrm{d}T_c}{\mathrm{d}\mu^2} = \frac{T_c(\Delta\mu^2) - T_c(0)}{\Delta\mu^2}$$

- Computation very expensive

Curvature determination II.

- Procedure #2: suppose shape of Φ unchanged
- Let us define T_c as $\Phi(T_c) = 0.5$
- For $\Phi(T, \mu^2)$: $\mathrm{d}\Phi = \frac{\partial \Phi}{\partial T} \cdot \mathrm{d}T + \frac{\partial \Phi}{\partial \mu^2} \cdot \mathrm{d}\mu^2$



- along the $T_c(\mu)$ line
  $\mathrm{d}\Phi = 0$ by definition

  $\rightarrow \frac{\mathrm{d}T_c}{\mathrm{d}\mu^2} = -\left(\frac{\partial \Phi}{\partial \mu^2}\right) \Big/ \left(\frac{\partial \Phi}{\partial T}\right)$

- This method is much cheaper in CPU-time
- For $\frac{\partial \Phi}{\partial \mu^2}$ we need to measure new operators

Required operators

- Consider $\mathcal{Z} = \int \mathcal{D}U e^{-S_g(U)} \det M^{N_f/4}$
- $\frac{\partial \log \mathcal{Z}}{\partial \mu_{u,d}} = \langle n_{u,d} \rangle$; $\frac{\partial^2 \log \mathcal{Z}}{\partial \mu_{u,d}^2} = \langle \chi_{u,d} \rangle$

  $n_{u,d} = \frac{N_f}{4} \mathrm{Tr} \left(M^{-1} M' \right)$ and
  $\chi_{u,d} = n_{u,d}^2 + \frac{N_f}{4} \mathrm{Tr} \left(M^{-1} M'' - M^{-1} M' M^{-1} M' \right)$
  $(' \equiv \frac{\partial}{\partial \mu_{u,d}})$
- Observables $\mathcal{O}$ that don't depend on $\mu_{u,d}$ (L, χ_s):

$$\frac{\partial \langle \mathcal{O} \rangle}{\partial (\mu_{u,d}^2)} = \frac{1}{2} \frac{\partial^2 \langle \mathcal{O} \rangle}{\partial \mu_{u,d}^2} = \langle \mathcal{O} \chi_{u,d} \rangle - \langle \mathcal{O} \rangle \langle \chi_{u,d} \rangle$$

- Observables $\mathcal{O}$ that depend on $\mu_{u,d}$ ($\bar{\psi}\psi$, $\chi_{\bar{\psi}\psi}$):

$$\frac{\partial \langle \mathcal{O} \rangle}{\partial (\mu_{u,d}^2)} = \frac{1}{2} \frac{\partial^2 \langle \mathcal{O} \rangle}{\partial \mu_{u,d}^2} = \langle \mathcal{O} \chi_{u,d} \rangle - \langle \mathcal{O} \rangle \langle \chi_{u,d} \rangle + \langle 2\mathcal{O}' n_{u,d} + \mathcal{O}'' \rangle$$

Simulation details

- Symanzik improved gauge and stout-link improved staggered fermionic lattice action
- Physical masses for $m_{u,d}$ and for m_s
- LCP determined by fixing m_K/f_K and m_K/m_π
- Scale set by f_K
- Lattice spacings used: $N_T = 4, 6, 8, 10$ ($a \approx 0.3 \ldots 0.12$fm)
- with aspect ratios $N_S/N_T = 4$ and 3
- Measurements carried out with 80 random vectors (measurements and config. production balanced)
- Derivatives $\mathcal{O}'$ and $\mathcal{O}''$ calculated numerically using a purely imaginary chemical potential

Observables I.

- Polyakov loop $L = \frac{1}{N_S^3} \sum_x \mathrm{Tr} \prod_{t=0}^{N_T-1} U_4(x,t)$

  renormalization: $L_r = L \exp(V(r_0)/2T)$

- Strange susceptibility $\chi_s = \frac{T}{V} \frac{\partial^2 \log \mathcal{Z}}{\partial \mu_s^2}$

  no renormalization necessary

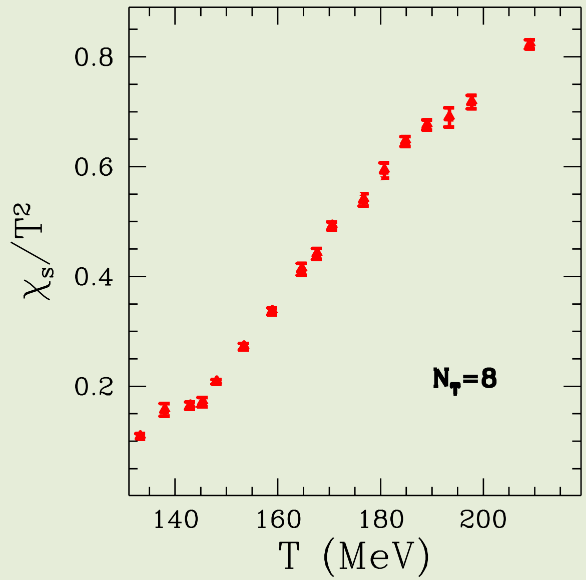
- Chiral condensate $\bar\psi\psi = \frac{T}{V} \frac{\partial \log \mathcal{Z}}{\partial m}$

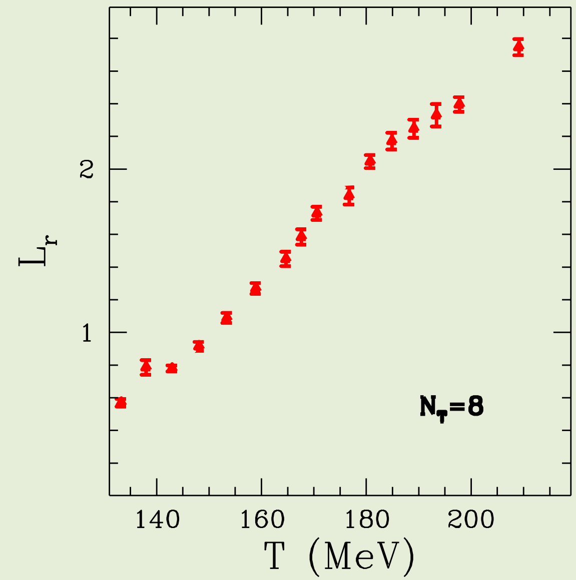
  renormalization: $\bar\psi\psi_r = (\bar\psi\psi - \bar\psi\psi(T=0)) \cdot m \cdot \frac{1}{m_\pi^4}$

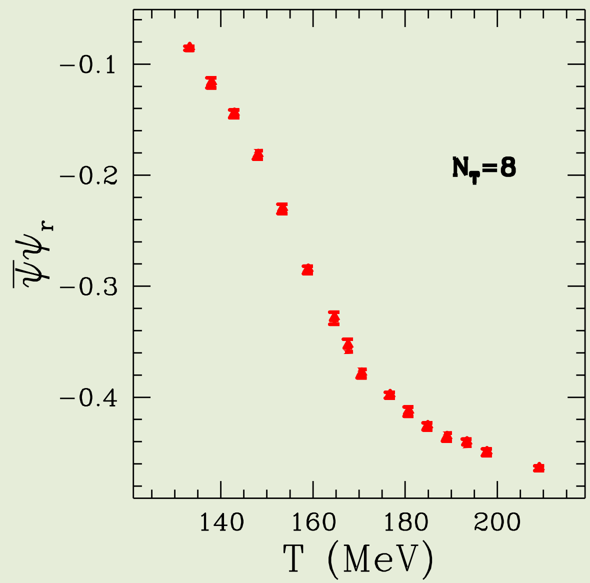
- Chiral susceptibility $\chi_{\bar\psi\psi} = \frac{T}{V} \frac{\partial^2 \log \mathcal{Z}}{\partial m^2}$

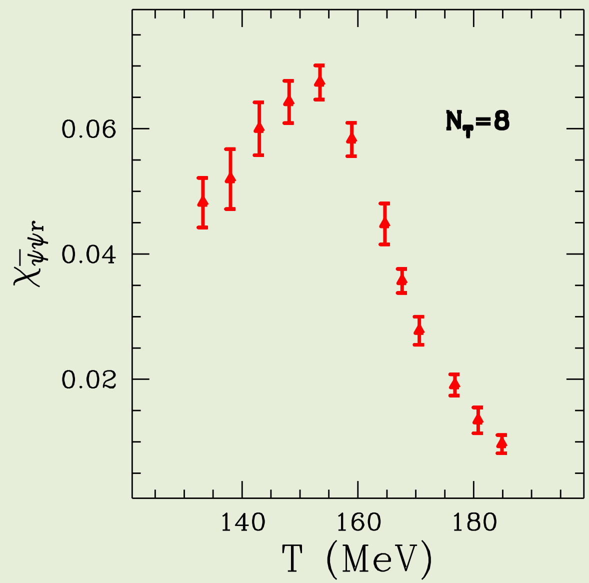
  renormalization: $\chi_{\bar\psi\psi r} = (\chi_{\bar\psi\psi} - \chi_{\bar\psi\psi}(T=0)) \cdot m^2 \cdot \frac{1}{T^4}$

Observables II.

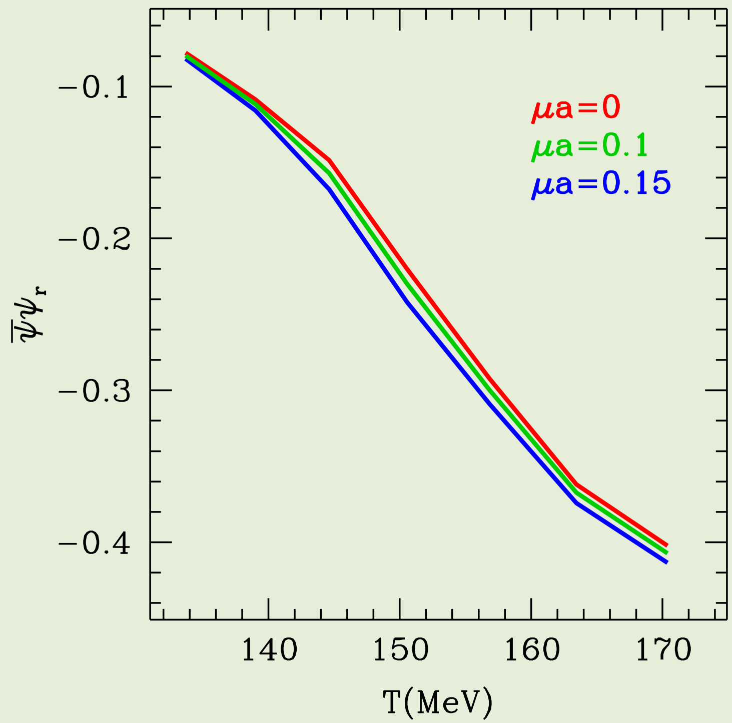


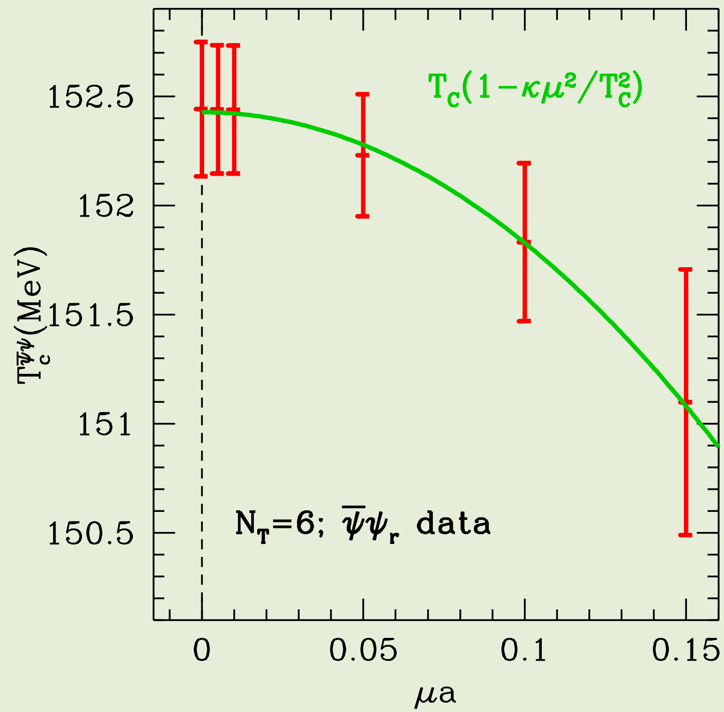






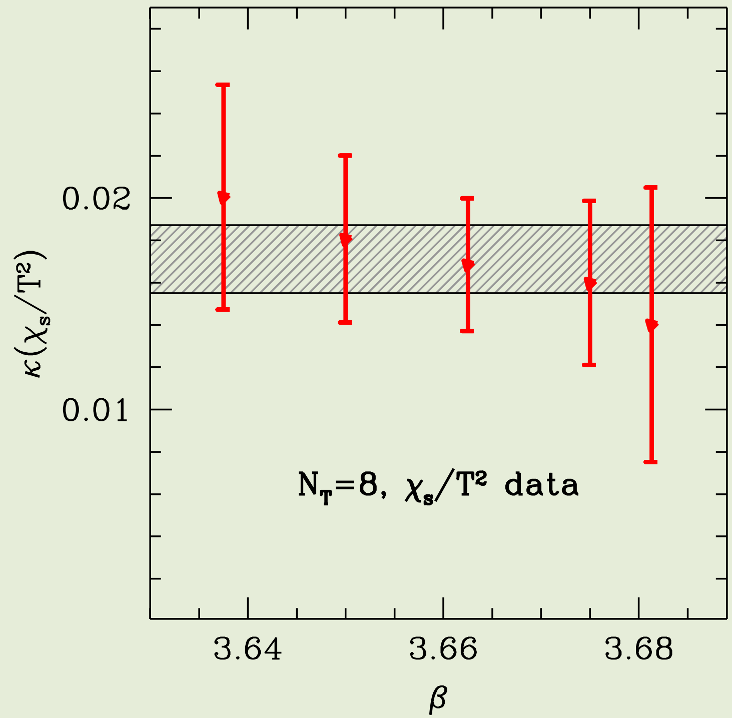
Results using procedure #1

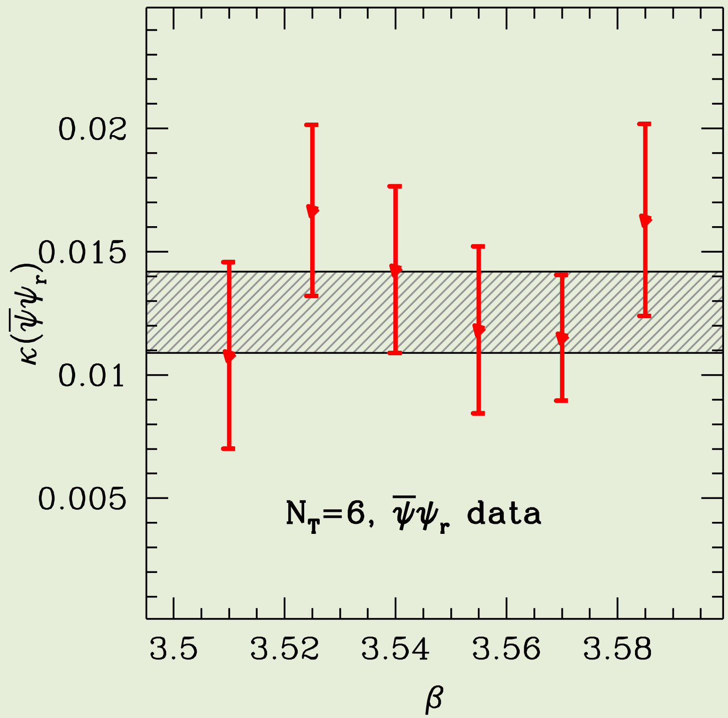




- $\Phi(\Delta\mu^2) = \Phi(0) + \frac{\partial\Phi}{\partial\mu^2} \cdot \Delta\mu^2$
- Inflection point of curves corresponding to various μ values

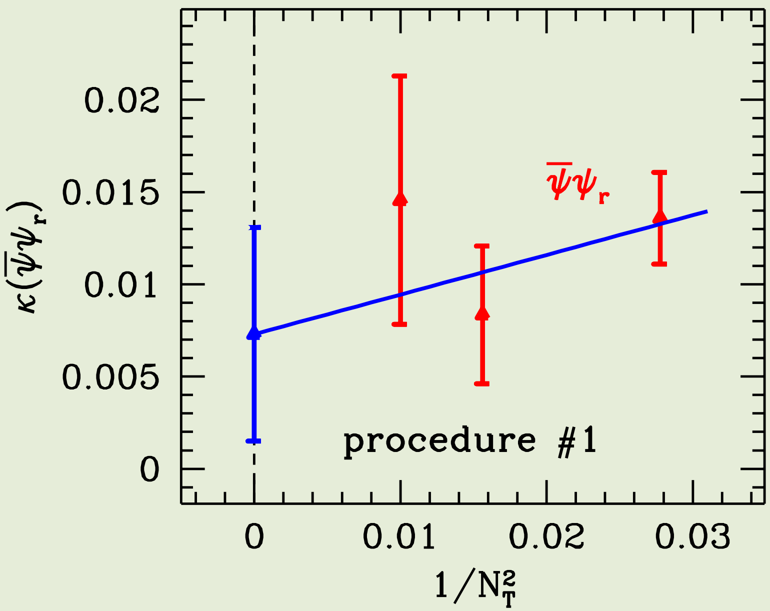
Results using procedure #2

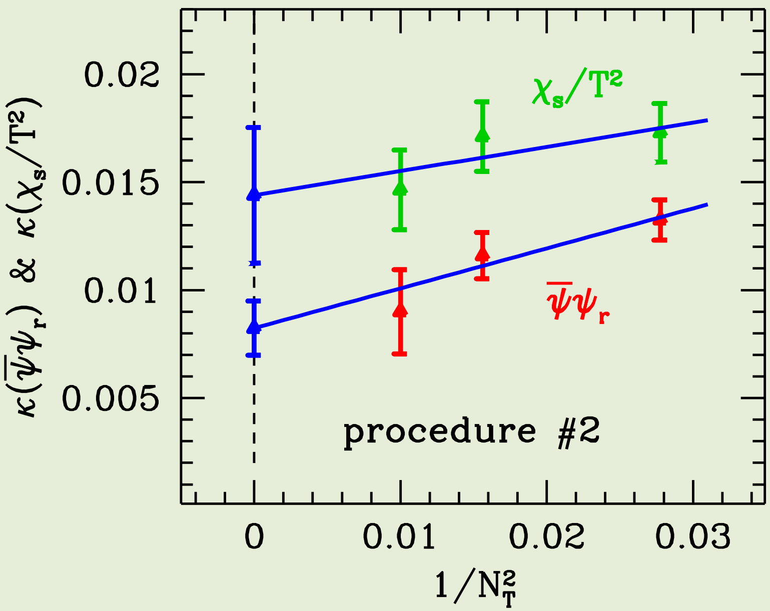




- Using procedure #2, κ can be calculated at arbitrary T
- Independent measurements at different β values
- Constant behaviour $\longrightarrow$ this procedure also reliable!

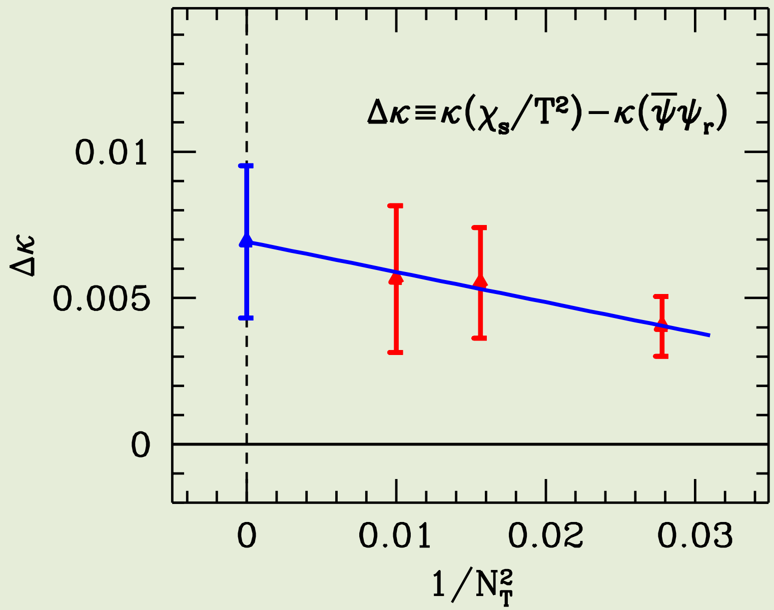
Continuum results

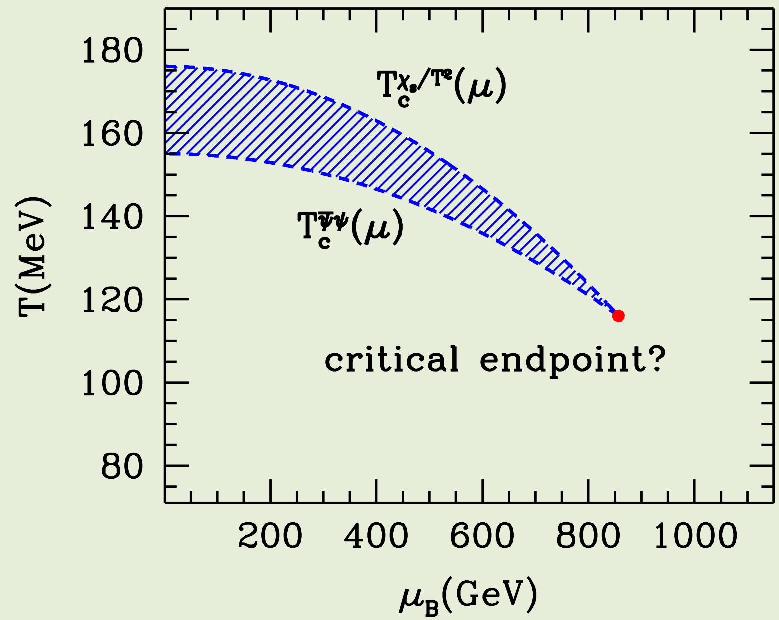




- Continuum extrapolated results from $N_T = 6, 8$ and 10; using both procedures
- Polyakov loop result consistent with χ_s/T^2
- Since procedure #2 can really be trusted, let's continue with those results

Preliminary results





- Difference $\Delta\kappa \equiv \kappa(\chi_s/T^2) - \kappa(\bar{\psi}\psi_r)$ not consistent with zero
- Indicates the strengthening of the transition
- Hint for existence / location of critical endpoint

Summary

- Two different procedures to determine the curvature
- In principle, procedure #1 is to be favoured to #2, but latter turns out to be also reliable
- In leading order in μ^2, transition curves of $\bar{\psi}\psi_r$ and χ_s/T^2 converge to each other
- Hint for critical endpoint
- Third observable? Higher orders?