Strong coupling lattice QCD at finite temperature and density

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with Michael Fromm (ETH)

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and in progress

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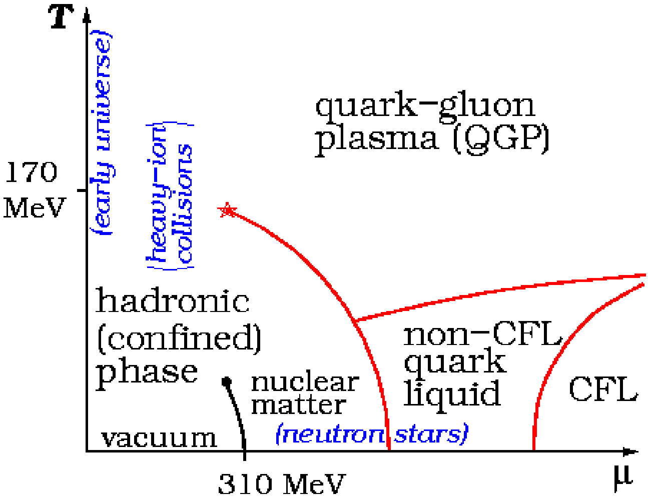
QCD phase diagram according to Wikipedia



This talk is about the **hadron** $\leftrightarrow$ **nuclear matter** transition

Motivation (1)

Strong coupling LQCD: why bother ?

Asymptotic freedom: $a(\beta_{\rm gauge}) \propto \exp(-\frac{\beta_{\rm gauge}}{4N_c b_0})$

ie. $a \to 0$ when $\beta_{\rm gauge} \equiv \frac{2N_c}{g^2} \to +\infty$. Here $\boxed{\beta_{\rm gauge} = 0}$:

- Lattice “infinitely coarse”
- Physics not universal

Nevertheless:

- Properties similar to QCD: confinement and χSB
- Include (perhaps) next term in strong coupling expansion, ie. $\beta_{\rm gauge} > 0$
- When you can't find the solution to the sign problem,

live with it

When $\beta_{\rm gauge} = 0$, sign problem is **manageable** $\to$ **full phase diagram**

Motivation (2)

- 25^{+} years of analytic predictions:

  80's: Kluberg-Stern et al., Kawamoto-Smit, Damgaard-Kawamoto

  $$T_c(\mu = 0) = 5/3, \;\; \mu_c(T = 0) = 0.66$$

  90's: Petersson et al., $1/g^2$ corrections

  00's: detailed (μ, T) phase diagram: Nishida, Kawamoto,...

  08: Ohnishi, Münster & Philipsen,...

  How accurate is mean-field ($1/d$) approximation?

- Almost no Monte Carlo crosschecks:

  89: Karsch-Mütter $\longrightarrow$ MDP formalism $\longrightarrow$ $\mu_c(T = 0) \sim 0.63$

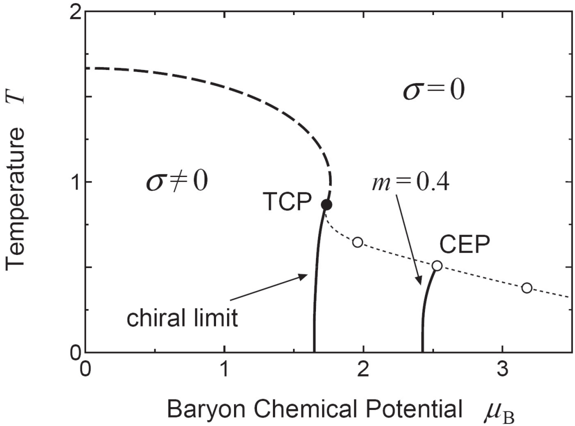
  92: Karsch et al. $T_c(\mu = 0) \approx 1.40$

  99: Azcoiti et al., MDP ergodicity ??

  06: PdF-Kim, HMC $\longrightarrow$ hadron spectrum $\sim 2\%$ of mean-field

Can one trust the details of analytic phase-diagram predictions?

Phase diagram according to Nishida (2004)



- Very similar to conjectured phase diagram of $N_f = 2$ **QCD**
- But no deconfinement here: high density phase is **nuclear matter**
- Baryon mass $= M_{\rm proton}$ $\Rightarrow$ lattice spacing $a^{-1} \sim 300$ MeV
- What happens to hadron $\leftrightarrow$ nuclear matter transition as m_π is varied?

Strong coupling $SU(3)$ with staggered quarks

$Z = \int \mathcal{D}U \mathcal{D}\bar\psi \mathcal{D}\psi \exp(-\bar\psi(\not{D}(U)+m)\psi)$, no plaquette term ($\beta_{\rm gauge} = 0$)

- One colored fermion field per site (6 d.o.f. – no Dirac indices)
- $\not{D}(U) = \frac{1}{2}\sum_{x,\nu}\eta_\nu(x)(U_\nu(x) - U_\nu^\dagger(x-\hat\nu))$, $\eta_\nu(x) = (-)^{x_1+..+x_{\nu-1}}$

$U(1)_V \times U(1)_A$ symmetry:

$$\left.\begin{array}{l} \psi(x) \to e^{i\theta}\psi(x) \\ \bar\psi(x) \to e^{-i\theta}\bar\psi(x) \end{array}\right\} \text{unbroken} \Rightarrow \text{quark number} \Rightarrow \text{chem. pot.}$$

$$\left.\begin{array}{l} \psi(x) \to e^{i\varepsilon(x)\theta}\psi(x) \\ \bar\psi(x) \to e^{i\varepsilon(x)\theta}\bar\psi(x) \\ \varepsilon(x) = (-)^{x_0+x_1+x_2+x_3} \end{array}\right\} \text{spont. broken } (m=0) \Rightarrow \text{quark condensate}$$

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- Chemical potential $\mu \quad \rightarrow \exp(\pm a\mu)U_{\pm 4}$

- Alternative 1: integrate over fermions

$$Z = \int \mathcal{D}U \; \det(\not{D}(U)+m) \quad \rightarrow \quad \text{HMC, etc...}$$

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- Alternative 2: integrate over links — Rossi & Wolff

$\rightarrow$ **Color singlet** degrees of freedom:

- **Monomer** (meson $\bar\psi\psi$) $M(x) \in \{0,1,2,3\}$
- **Dimer** (meson hopping), non-oriented $n_\nu(x) \in \{0,1,2,3\}$
- **Baryon** hopping, oriented $\bar{B}B_\nu(x) \in \{0,1\} \quad \rightarrow$ *self-avoiding loops C*

Point-like, hard-core baryons in pion bath

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$$Z(m,\mu) = \sum_{\{M,n_\nu,C\}} \prod_x \frac{m^{M(x)}}{M(x)!} \prod_{x,\nu} \frac{(3-n_\nu(x))!}{n_\nu(x)!} \prod_{\text{loops}\, C} \rho(C)$$

with constraint $(M+\sum_{\pm\nu} n_\nu)(x) = 3 \ \forall x \notin \{C\}$

MDP Monte Carlo

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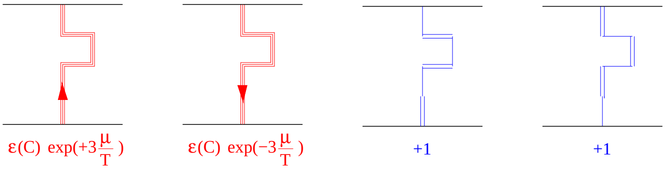
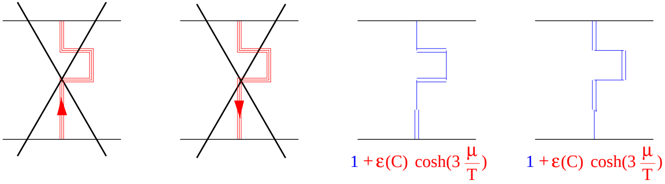


Karsch & Mütter: Regroup into "MDP ensemble" $\longrightarrow$ sign pb. eliminated at $\mu = 0$



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Further difficulties:

- changing monomer number difficult: weight $\sim m^{\sum_x M(x)}$
  monomer-changing update (Karsch & Mütter) restricted to $m \sim \mathcal{O}(1)$

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Solved with **worm algorithm** (Prokof’ev & Svistunov)

Worm algorithm for MDP

Here for chiral limit $m = 0$ (no monomers: $M(x) = 0 \; \forall x$)

- Break a dimer bond and introduce a pair of adjacent monomers $M(x), M(y)$
- Choose among neighbours of y by local heatbath and move $M(y)$ there
  heatbath: sampling of 2-point function $\frac{1}{Z_{||}} M(x) M(y) \exp(-S_{||})$
- Keep moving "head" y until $y \longrightarrow x$, ie. "worm closes" $\longrightarrow$ new configuration in $Z_{||}$

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Global change obtained from sequence of local updates

Each local step gives information on 2-point function

cf. Adams & Chandrasekharan for $U(N)$

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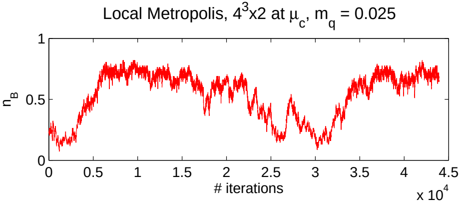
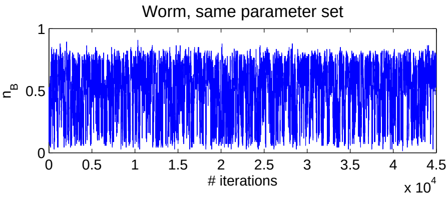
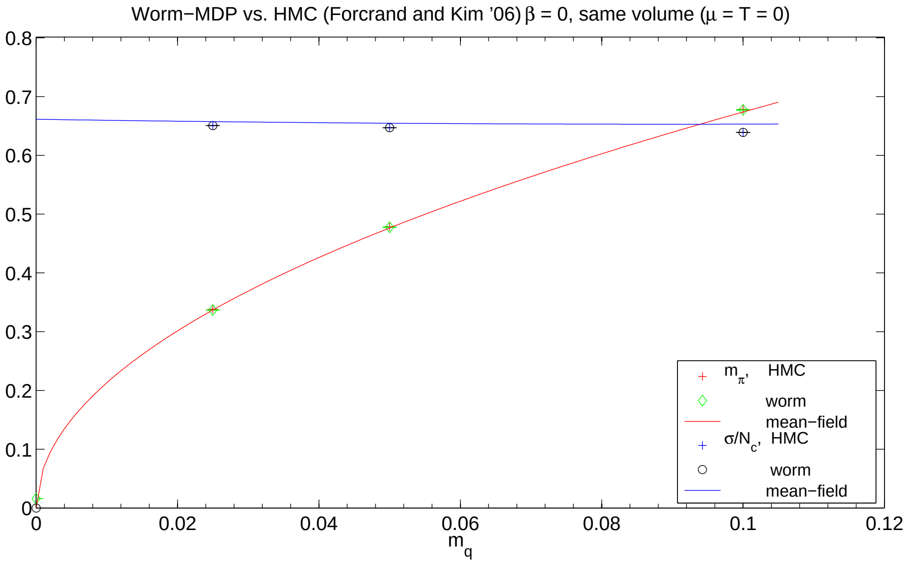
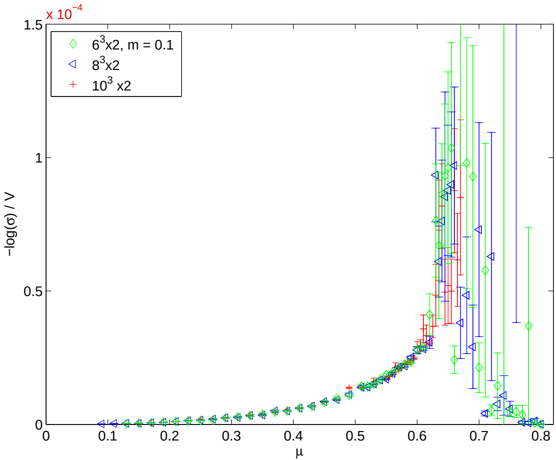




[Non-trivial] consistency check with HMC



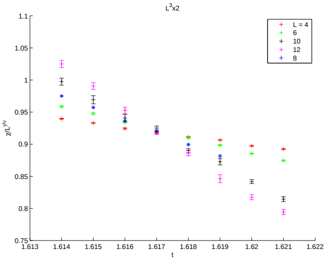
Sign problem? Monitor $-\frac{1}{V}\log\langle\text{sign}\rangle$

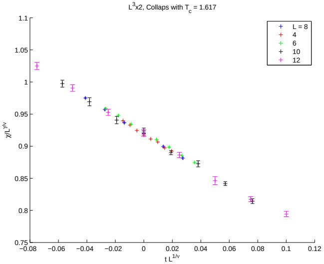


- $\langle\text{sign}\rangle = \frac{Z_{||}}{Z} \sim \exp(-{\color{red}V} f(\mu^2))$ as expected
- Can reach $\sim 16^3 \times 4\ \forall\mu$, ie. adequate

$\mu = 0$ finite-T chiral transition

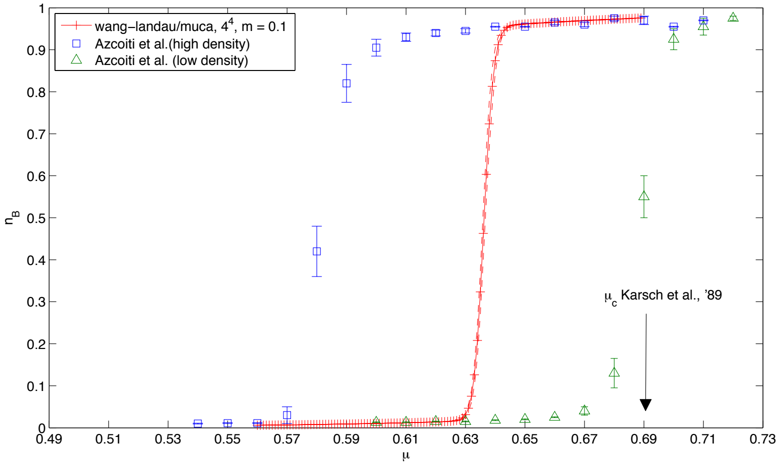
- Mean-field: $T_c = 5/3$ — Anisotropy γ: $T = \gamma^2/N_t$
- Previously: extrapolation $m \to 0$ with $N_t = 4 \to \gamma_c = 2.37(2)$
  ie. $T_c(N_t = 4) = 1.40(3)$ Karsch et al.
- Now, $m = 0$ exactly, $N_t = 2$ and $N_t = 4$: chiral susc. versus γ





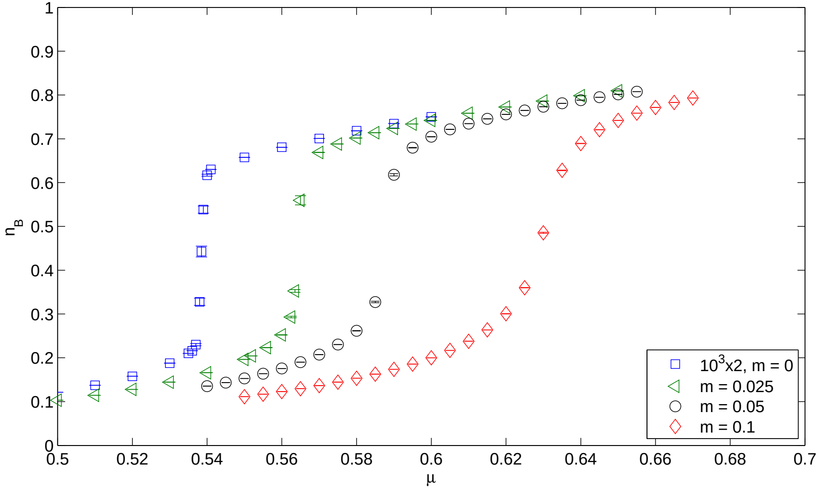
- $N_t = 2$, $\gamma_c = 1.617(1)$, ie. $T_c(N_t = 2) = 1.307(2)$ with $3d$ $O(2)$ exponents
  mean-field off by 20%
- $N_t = 4$, $\gamma_c = 2.350(5)$, ie. $T_c(N_t = 4) = 1.381(6)$ "scaling" with N_t ?

Consistency check with Karsch & Mütter: $T = 1/4, m = 0.1$



- High-density phase: saturation at 1 baryon per site (cf. nuclear matter)
- Azcoiti was right: ergodicity restored only with multicanonical (Wang-Landau)
- Karsch & Mütter value too large; obtained from metastable branch ?

Varying the quark mass at $T = 1/2$ fixed

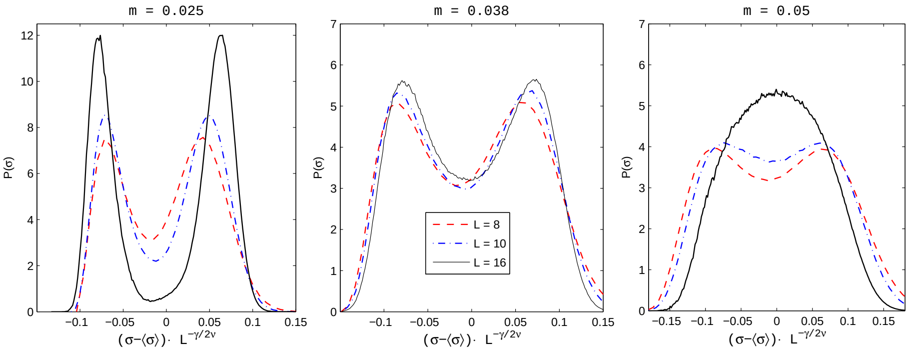


As $m \longrightarrow 0$, μ_c decreases and transition becomes stronger

critical mass m_c ?

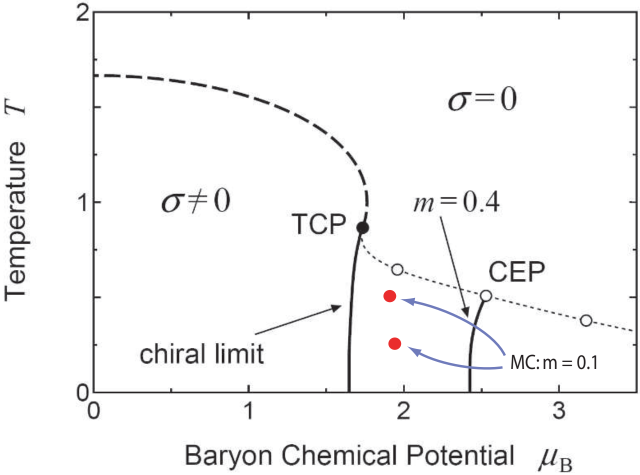
Critical mass $m_c(T = 1/2)$?

Distribution of quark condensate: finite-size scaling ?



- Critical mass: $m_c(T = 1/2) \sim 0.038$
- Universality class: γ/ν consistent with $3d$ Ising

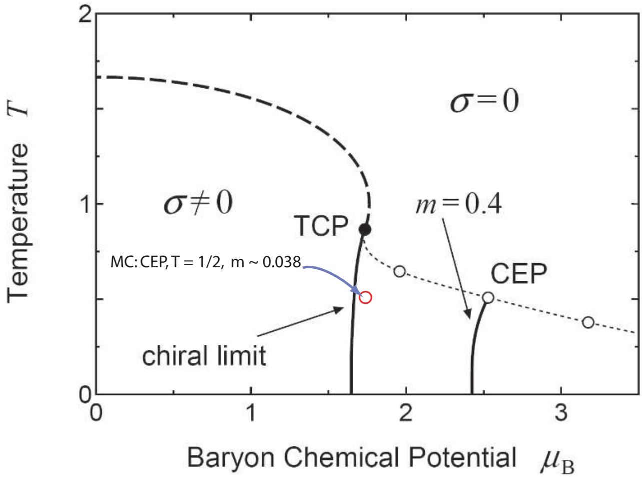
Compare with Nishida (2004): transition line for $m = 0.1$



Clausius-Clapeyron: $\frac{dT}{d\mu} = -\frac{\Delta n}{\Delta S} \longrightarrow$ vertical at $T = 0$ (non-deg. ground-state)

- Nishida: at low T, $S|_{\text{high}} < S|_{\text{low}}$, because $n_B|_{\text{high}} \approx$ saturation
- sensitive to $n_B|_{\text{low}}$ vs $(1 - n_B|_{\text{high}})$: not borne out by MC

Compare with Nishida (2004): CEP at $T = 1/2$



$m_{CEP} \approx 0.038$ vs prediction ≈ 0.4: wrong by $\mathcal{O}(10)$!

Beware of quantitative mean-field predictions for phase diagram

Transition to nuclear matter: $T = 0, \mu = \mu_c$

Puzzle:

- Mean-field baryon mass is ≈ 3 $\Rightarrow$ expect $\mu_c = \frac{1}{3} F_B(T=0) \approx 1$
- Mean-field estimate $\mu_c \sim 0.55 - 0.66$ much smaller, ie. $\mu_c^B \sim 600$ MeV !

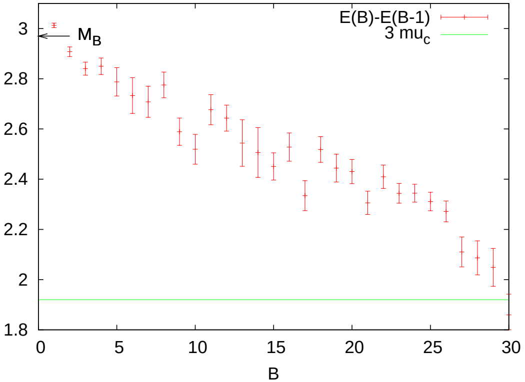
Wrong M_B? wrong μ_c?

- Baryon mass ≈ 3 checked by HMC PdF & Kim
- $\mu_c \approx 0.64$ (see earlier) for $m = 0.1, T = 1/4$

Explanation: nuclear attraction $\sim$ **1/3** baryon mass ??

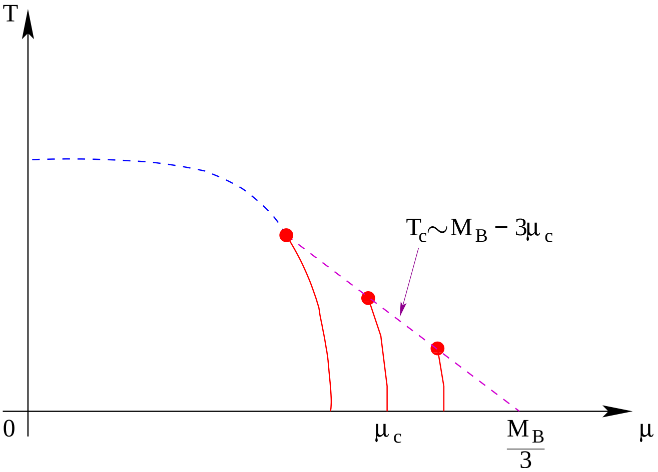
Monitor $T = 0$ energy as a function of baryon number, ie. $(E(B) - E(B-1))$ vs B

Internal energy vs baryon number at $T = 1/4, m = 0.1$



- $E(B=1) \sim M_B$: temperature is low ($T \sim 75$ MeV, $m_\pi \sim 200$ MeV)
- $E(B=2) - 2E(B=1) \sim -0.1$, ie. "deuteron" binding energy ca. 30 MeV
- Further binding with ca. 20 "nearest-neighbours" to give μ_c ca. 600 MeV
- $E(B) \sim (3\mu_c B + a_S B^{2/3})$, ie. (bulk + surface tension) (Weizsäcker)
- "Magic numbers" with increased stability?
- Attraction due to bath of neutral pions: cf. Casimir effect

Hadron $\leftrightarrow$ nuclear matter transition vs pion mass



- Transition becomes weaker for heavier pions
- In strong coupling LQCD, for physical m_π/M_B,
  interactions enhanced by $\mathcal{O}(10)$ $\longrightarrow$ good laboratory

Conclusions

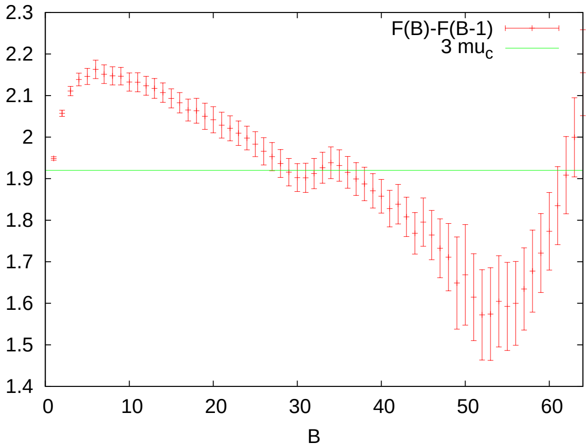
Summary

- Take mean-field results with a grain of salt
- "Clean-up" of phase diagram justified
- Nuclear matter from QCD

Outlook

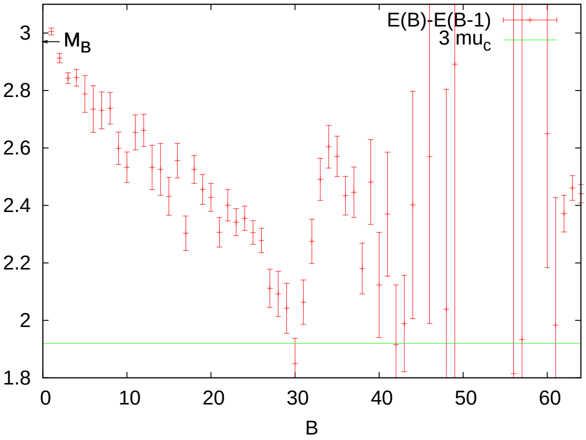
- Improve systematics:
  - Check mean-field "scaling" $T = \gamma^2 / N_t$
  - Compare real and imaginary μ
- Determine phase diagram:
  - Tricritical point for $m = 0$
  - Critical end-point as a function of m
- Include second quark species $\longrightarrow$ isospin
- Include $\mathcal{O}(\beta)$ effects ?

Backup slide: Maxwell construction



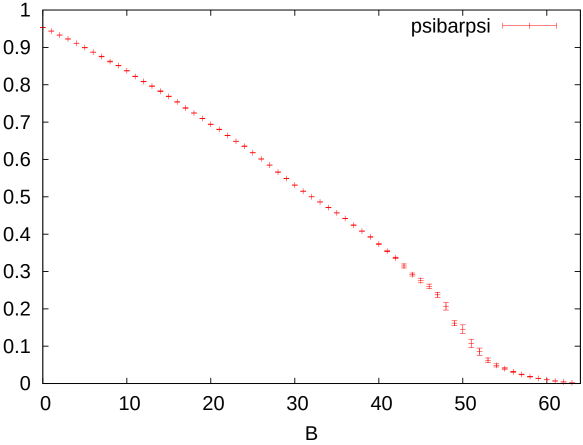
- μ_c consistent with Maxwell construction
- Area gives interface tension of nuclear matter

Backup slide: energy for each additional baryon



The energy of an additional baryon in the dense phase is $> \ 3\mu_c$

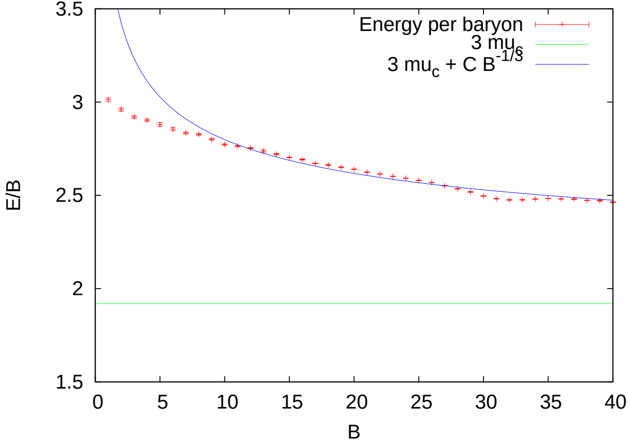
Backup slide: chiral symmetry restoration



$\langle \bar{\psi}\psi \rangle$ decreases $\sim$ linearly with B

$\longrightarrow$ chiral symm. restored in nuclear matter

Backup slide: energy per baryon



$E(B)$ well described by $(a_V B + a_S B^{2/3})$, ie. (bulk + surface tension) (Weizsäcker)