Overview of Heavy Quark Spectroscopy from a Lattice Perspective

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Purpose of this talk

- Provide an overview of $T = 0$ heavy-quark spectroscopy
- Review current lattice methods: possibilities and limitations
- Not a review of results
- See talks by Prelovsek, Thomas
- First, the challenge
Charmonium spectroscopy before the B-factories
Charmonium spectroscopy after the B-factories

Mariana Nielsen (CHARM 2010)
Theoretical challenges

- Test of our ability/methodology for solving QCD.
- Precision spectroscopy of the low levels.
- Characterize and predict states.
  - Spin exotics: \( J^{PC} \) inaccessible through \( \bar{Q}Q \).
  - Hybrids: \( \bar{Q}Q + \text{glue} \)
  - Tetraquarks: \( \bar{Q}Q\bar{q}q \) as in the \( Z_c^+ \) (?)
  - Molecules: deuteron-like \( D - \bar{D} \)
Our only *ab initio* method

Limitations
- Including multihadronic states is difficult.
- Real-time behavior is only indirectly accessible (see next talk)

To extend its reach: supplement with models, effective field theory.
Lattice Spectroscopy Methods

- Hadronic correlator. Hermitian interpolating operators $\mathcal{O}_i$.

$$C_{ij}(t) = \langle 0 | \mathcal{O}_i(t) \mathcal{O}_j^\dagger(0) | 0 \rangle$$

- Spectral decomposition

$$C_{ij}(t) = \sum_n \langle 0 | \mathcal{O}_i(0) | n \rangle e^{-E_n t} \langle n | \mathcal{O}_j^\dagger(0) | 0 \rangle$$
Low lying charmonium spectrum

- HPQCD Galloway et al. [1411.1318]
- Two lattice spacings, three sea-quark mass sets, two smearings
- Bayesian multiexponential fit (up to 9 exponentials)
Variational Method

- Spectral decomposition

\[ C_{ij}(t) = \sum_n \langle 0 | \mathcal{O}_i(0) | n \rangle e^{-E_n t} \langle n | \mathcal{O}_j^\dagger(0) | 0 \rangle \]

- Matrix form

\[ C(t) = Z T^t Z^\dagger, \]

- Effective transfer matrix and overlap matrix

\[ T = \text{diag} \exp(-E_n) \quad Z_{i,n} = \langle 0 | \mathcal{O}_i(0) | n \rangle \]

- Choose a large basis set of interpolating operators.
- Solve the generalized eigenvalue problem (reference time \( t_0 \))

\[ C(t) u_n = \lambda_n(t, t_0) C(t_0) u_n \]

- If we have \( N \) linearly independent interpolating operators, and exactly \( N \) energy levels, we get, exactly,

\[ \lambda_n(t, t_0) = \exp[-E_n(t - t_0)]. \]

- Otherwise, there are corrections that die exponentially as \( t_0 > t - t_0 \to \infty \).
- Low-lying levels are more reliable, higher levels are difficult.
Example for low-lying charmonium

Bilinear $\bar{c}O_{\text{nc}}$. Here operators for the $T^{PC}_1$ irrep are shown. In the notation below, $\nabla_i$ generates a discrete covariant difference in direction $i$, $D_k = |\varepsilon_{ijk}| \nabla_i \nabla_j$, and $B_i = \varepsilon_{ijk} \nabla_i \nabla_j$.

<table>
<thead>
<tr>
<th>$T^{--}_1$</th>
<th>$T^{+-}_1$</th>
<th>$T^{-+}_1$</th>
<th>$T^{++}_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma_i$</td>
<td>$\gamma_4 \gamma_5 \gamma_i$</td>
<td>$\gamma_4 \nabla_i$</td>
<td>$\gamma_5 \gamma_i$</td>
</tr>
<tr>
<td>$\gamma_4 \gamma_i$</td>
<td>$\gamma_5 \nabla_i$</td>
<td>$\varepsilon_{ijk} \gamma_4 \gamma_5 \gamma_j \nabla_k$</td>
<td>$\varepsilon_{ijk} \gamma_j \gamma_k \nabla_k$</td>
</tr>
<tr>
<td>$\nabla_i$</td>
<td>$\gamma_4 \gamma_5 \nabla_i$</td>
<td>$\varepsilon_{ijk} \gamma_4 \gamma_5 \gamma_j D_k$</td>
<td>$\varepsilon_{ijk} \gamma_j B_k$</td>
</tr>
<tr>
<td>$\varepsilon_{ijk} \gamma_5 \gamma_j \nabla_k$</td>
<td></td>
<td>$\varepsilon_{ijk} \gamma_4 \gamma_5 \gamma_j D_k$</td>
<td>$\varepsilon_{ijk} \gamma_4 \gamma_5 \gamma_j B_k$</td>
</tr>
<tr>
<td>$</td>
<td>\varepsilon_{ijk}</td>
<td>\gamma_4 \gamma_5 \gamma_j D_k$</td>
<td></td>
</tr>
<tr>
<td>$\gamma_5 B_i$</td>
<td></td>
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<tr>
<td>$\gamma_4 \gamma_5 B_i$</td>
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</tbody>
</table>
Low-lying charmonium: FNAL/MILC result

- 5 lattice spacings, 2 sea quark masses. Can take physical limit.
- Spin-averaged 1P and 1S masses:

\[
M_{1P} = \frac{(M_{\chi c_0} + 3M_{\chi c_1} + 5M_{\chi c_2})}{9}
\]
\[
M_{1S} = \frac{(M_{\eta c} + 3M_{J/\psi})}{4}
\]

- Splittings

1S hyperfine = \(M_{J/\psi} - M_{\eta c}\)
1P - 1S splitting = \(M_{1P} - M_{1S}\)
1P spin - orbit = \(\frac{(5M_{\chi c_2} - 3M_{\chi c_1} - 2M_{\chi c_0})}{9}\)
1P tensor = \(\frac{(3M_{\chi c_1} - M_{\chi c_2} - 2M_{\chi c_0})}{9}\)
1P hyperfine = \(\overline{1P} - M(h_c)\)
Mohler, Lattice 2014 [1412.1057]

- Fit to shapes derived from NRQCD power counting.
- Experiment $\Delta M_{HF_1} = 0.178$.
- Note width of $\eta_c$ is 32 MeV!
## Low-lying charmonium: FNAL/MILC result

<table>
<thead>
<tr>
<th>Mass difference</th>
<th>This analysis [MeV]</th>
<th>Experiment [MeV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1P-1S splitting</td>
<td>457.3 ± 3.6</td>
<td>457.5 ± 0.3</td>
</tr>
<tr>
<td>1S hyperfine</td>
<td>118.1 ± 2.1</td>
<td>113.2 ± 0.7</td>
</tr>
<tr>
<td>1P spin-orbit</td>
<td>49.5 ± 2.5</td>
<td>46.6 ± 0.1</td>
</tr>
<tr>
<td>1P tensor</td>
<td>17.3 ± 2.9</td>
<td>16.25 ± 0.07</td>
</tr>
<tr>
<td>1P hyperfine</td>
<td>−6.2 ± 4.1</td>
<td>−0.10 ± 0.22</td>
</tr>
</tbody>
</table>

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Higher-lying lattice cc spectroscopy

Hadron Spectrum Collaboration 1204.5425 (2012)
Problems

- Not safe to omit open charm (multihadronic states).
- Only one lattice spacing ($a_s = 0.12$ fm, $a_t \approx 0.032$ fm)
- Heavy up/down quarks: 400 MeV pion
- Got charmonium HFS = 80 MeV vs 113.2(7) MeV (expt).
Optimized interpolating operators

Variationally optimized state vector

\[ Z_{i,n} = \langle 0 | \mathcal{O}_i(0) | n \rangle \]
\[ |\psi_n\rangle = \sum_i Z_{n,i}^{-1} \mathcal{O}_i |0\rangle \]

\(|\psi_n\rangle\) is an approximate eigenstate with eigenvalue \(E_n\).

Could be helpful.
Optimized interpolating operators

- Customized smearing.
- von Hippel et al [1306.1440]: “Free form” interpolating operator
- Gauge invariant form giving complete control over relative wave function
- Replace Gaussian smearing with a more realistic wave function.
- e.g. Mark Wurtz et al [1409.7103] for first excited D-wave bottomonium.

\[ \psi_{ij}(x) = \sin\left(\frac{2\pi x_i}{L}\right) \sin\left(\frac{2\pi x_j}{L}\right)(r - b) \exp\left(\frac{r}{a}\right) \]
Crafting interpolating operators

![Graph](image)

- **S, P, D, F, G**
- **0-0, 1+, 1+, 0++, 1++, 2++, 2++, 3++, 3++, 4++, 4++**

- **mass (GeV)**
- **this work**
- **experiment**
- **B̅B threshold (experiment)**

C. DeTar (University of Utah) Royal Society January 28, 2015 18/24
Including two-hadron states

- Discrete scattering states at finite volume, center of mass.

\[ E_n = 2\sqrt{m^2 + p^2} \quad p = (2\pi/L)n \]  
(Noninteracting, equal mass, two-body)

- Lüscher method. With interaction, \( E_n \) is shifted.

- As long as the box is larger than the size of the interaction region \( L/2 > R \), and scattering is only elastic, the shift in energy carries information about the elastic scattering phase shift.

- Replace \( p = (2\pi/L)q \) from (noninteracting \( q = n \)).

\[ E = 2\sqrt{m^2 + (2\pi/L)^2 q^2} \]

- Then we get the phase shift at momentum \( p = (2\pi/L)q \) from

\[ p \cot \delta(p) = \frac{2Z_{00}(1; q^2)}{\sqrt{\pi L}} \]

- This is for S-wave. Generalizations apply to higher angular momentum and moving frames.
Near-threshold bound states

- Partial wave scattering amplitude.
  \[ T = \frac{1}{p \cot \delta(p) - ip} \]

- Effective range formula
  \[ p \cot \delta(p) = \frac{1}{a_0} + \frac{1}{2} r_0^2 p^2 \]

- So use lattice result at discrete \( p \) to determine \( a_0 \) and \( r_0^2 \). Then do analytic continuation.

- Poles in \( T \) represent bound states \( (a < 0) \) or resonances.

- For example, Lang et al [1301.7670] use this method to conclude that the \( D_{s1} \) is a \( D^*K \) bound state and the \( D_{s0}^* \) is a \( DK \) bound state.

- Also applied to charmonium excitations (Prelovsek talk.)

- For a recent example, Lang et al. [1501.01646] [1403.8103]
Perturbative mixing model

- Treat light quark annihilation/production perturbatively
  \[ H = H_0 + V \]
- Unperturbed states \( H_0 \): \( DD^* \) and \( cc \)
- Mixing term \( V \) connects these two sectors.
- Use Euclidean lattice correlators to compute matrix elements of \( V \).
- Compute level shifts in conventional real-time perturbation theory.
Application to X(3872)

- Model: The X(3872) is the result of mixing the $\chi_{c1}(2P)$ with $DD^*$ scattering states.

- Second order energy shift (continuum) of the unperturbed $\chi_{c1}(2P)$:

$$E = E^{(0)}(\chi_{c1}(2P)) + \int d^3p \frac{x(p)^2}{E - E(p)}$$

where $x(p)$ is the transition hamiltonian matrix element taking the $\chi_{c1}(2P)$ to the $DD^*$ scattering state with momentum $p$, and $E(p)$ is the energy of the $DD^*$ scattering state.

- Measure $x(p)$ and $E^{(0)}(\chi_{c1}(2P))$ on the lattice. (Work in progress)
Conclusions

▶ Lattice QCD is needed to characterize, predict states
▶ State of the art
  ▶ Multiple interpolating operators, including multihadronic states where appropriate
  ▶ Multiple lattice spacings permitting a continuum extrapolation
  ▶ Ability to reach physical quark masses
  ▶ Sufficiently large volume
  ▶ Careful tuning of the heavy quark masses.
▶ Variational method
▶ Elastic scattering phase shift determination
▶ Effective range approximation for states near threshold
▶ Lattice-inspired phenomenology
Thank you

University of Glasgow above the River Kelvin, photo by Laurel Casjens