# Effective field theories at finite temperature

Miguel A. Escobedo

Institut de Physique Théorique, CEA Saclay

28th of January, 2015


Work done in collaboration with N. Brambilla, J. Ghiglieri, J. Soto and A. Vairo

# Outline

# Review

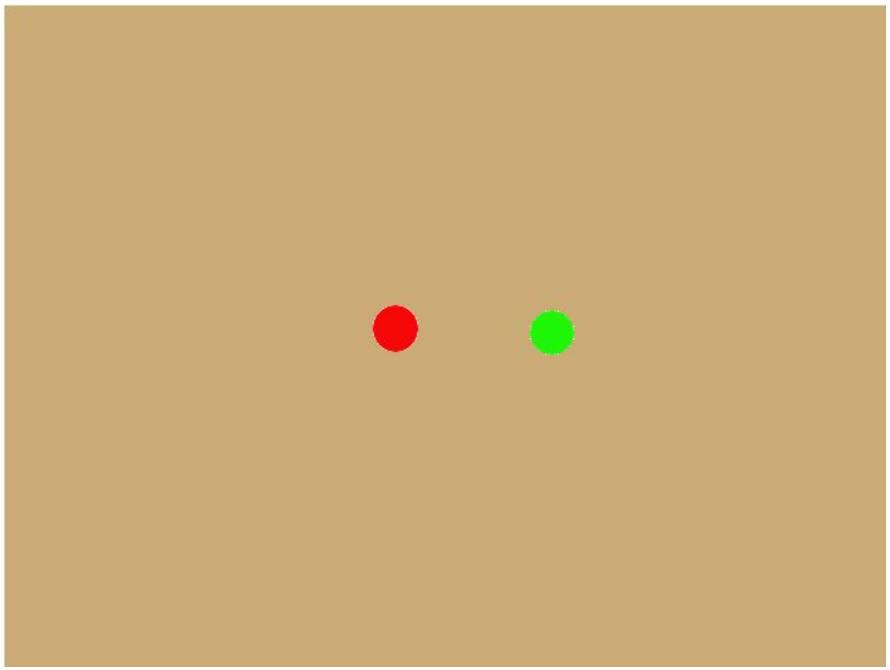
# The original idea of Matsui and Satz (1986)

- Quarkonia is quite stable in the vacuum.
- Deconfinement is due to colour screening, quantities measurable in Lattice QCD at finite temperature (static) support this. For example Polyakov loop.
- Dissociation of heavy quarkonium in heavy-ion collisions due to colour screening signals the creation of a quark-gluon plasma.

# Colour screening

$$V(r) = -\alpha_s \frac{e^{-m_D r}}{r}$$

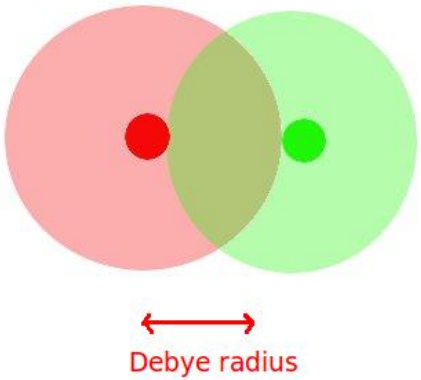
In the vacuum



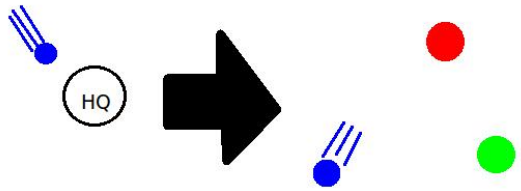
# Colour screening

$$V(r) = -\alpha_s \frac{e^{-m_D r}}{r}$$

At finite temperature



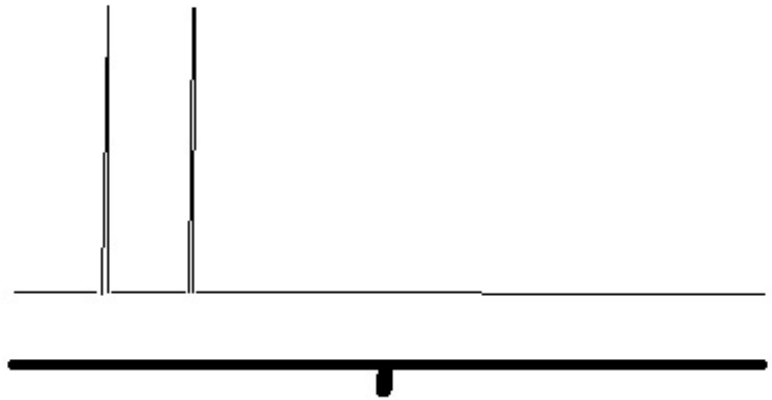
# Another mechanism, the decay width




- This effect makes the peak in the spectral function broader. It can arrive to a point where it is so broad that it does not make sense to speak of a bound state anymore.

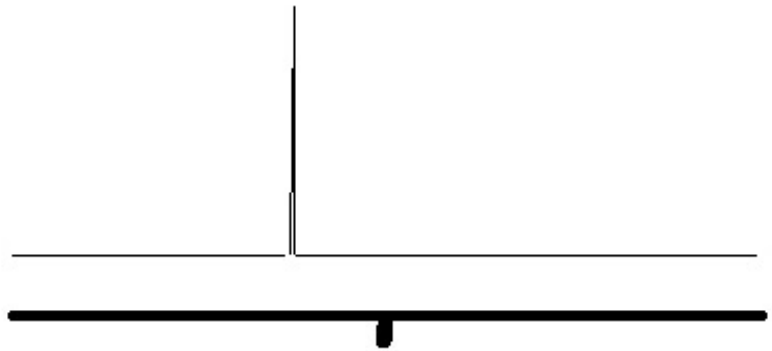
# (Very) Qualitative spectral function

$T = 0$



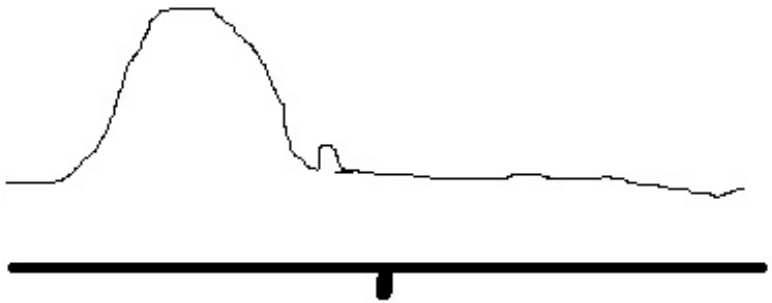
# (Very) Qualitative spectral function

Only screening



# (Very) Qualitative spectral function

Decay width



# Laine et al. perturbative potential (2007)

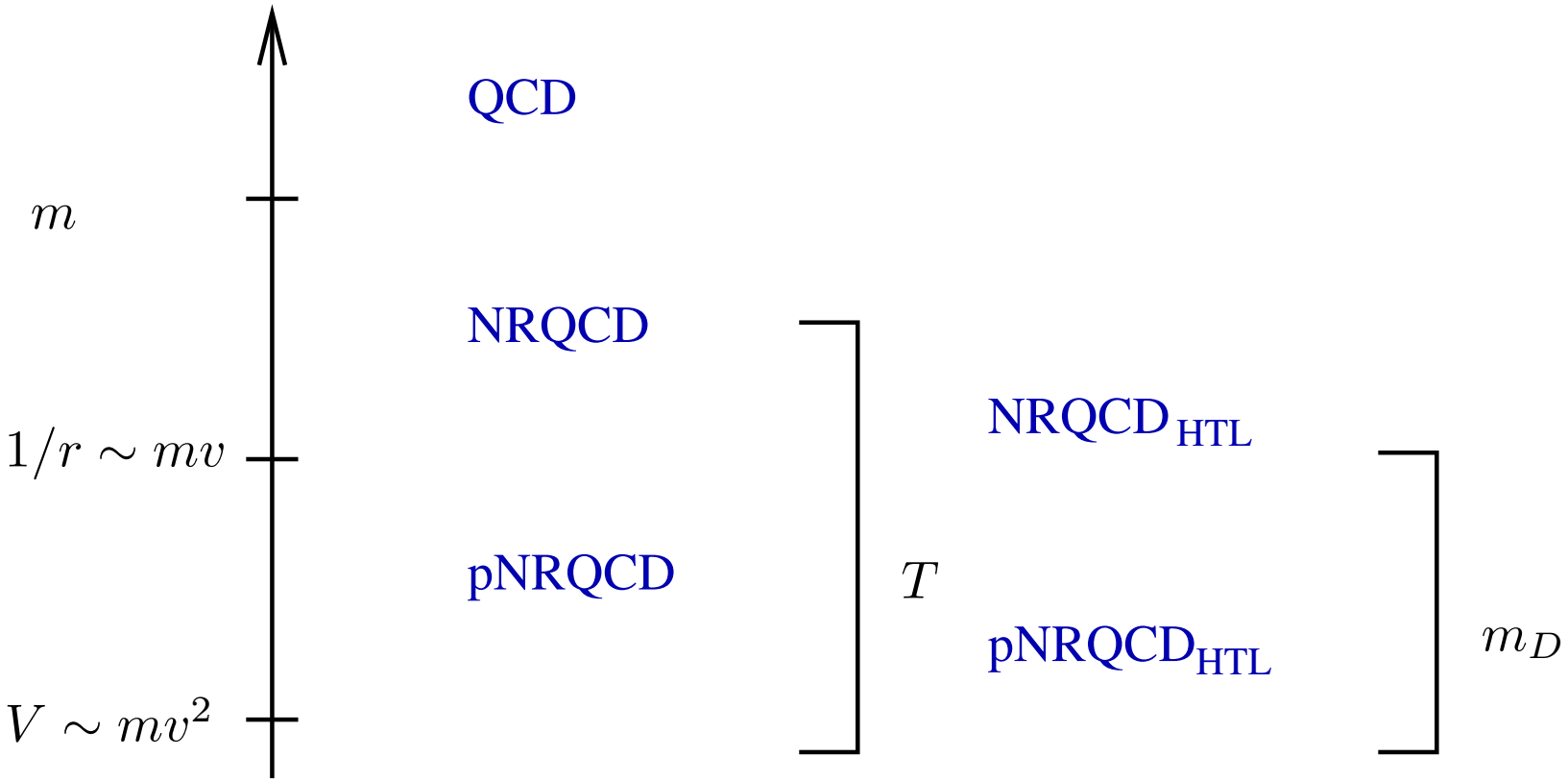
$$V(r) = -\alpha_s C_F \left[ m_D + \frac{e^{-m_D r}}{r} \right] - i\alpha_s T C_F \phi(m_D r)$$

with

$$\phi(x) = 2 \int_0^\infty \frac{dzz}{(z^2+1)^2} \left( 1 - \frac{\sin(zx)}{zx} \right)$$

- This potential was obtained through the Wilson loop in Minkownski space at finite temperature.
- It has an imaginary part that has to be related with a decay width.

# Effective field theories



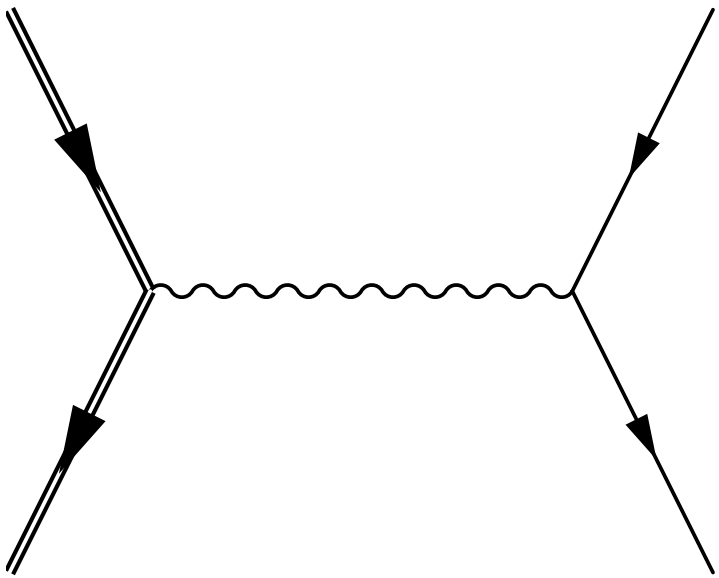

(Brambilla, Ghiglieri, Petreczky And Vairo (2008), M. A. E and Soto (2008))

# Energy scales for zero temperature heavy quarkonium

Heavy quarkonium at $T = 0$ is a system with a lot of different energy scales.

For example, for computing the decay of $J/\Psi$ to electrons...

- We need annihilation cross section of the quark and the anti-quark to electrons. The energies involved are of the order of $m_c$.



# Energy scales for zero temperature heavy quarkonium

Heavy quarkonium at $T = 0$ is a system with a lot of different energy scales.
For example, for computing the decay of $J/\Psi$ to electrons...

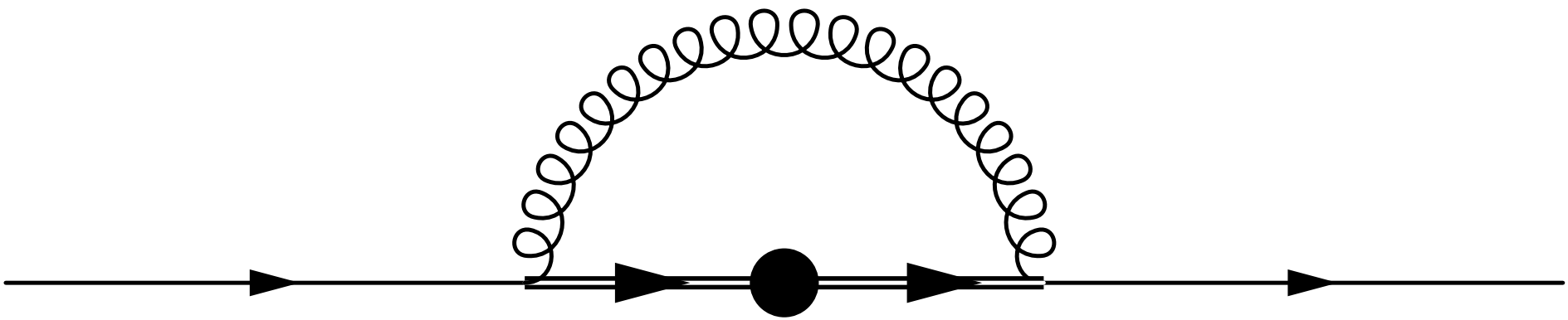
- We also need the probability that the quark and the anti-quark are at the same point, this is given by the wave-functions. The energies involved are of the order of $1/r$.

$$\Psi_{ab}(\mathbf{r})$$

# Energy scales for zero temperature heavy quarkonium

Heavy quarkonium at $T = 0$ is a system with a lot of different energy scales.
For example, for computing the decay of $J/\Psi$ to electrons...

- If we want to make a precision computation, we need to include the effects of the color octet component of $J/\Psi$. The energy involved here is of order of the binding energy.



# Energy scales for zero temperature heavy quarkonium

Heavy quarkonium at $T = 0$ is a system with a lot of different energy scales.
For example, for computing the decay of $J/\Psi$ to electrons...

- We need annihilation cross section of the quark and the anti-quark to electrons. The energies involved are of the order of $m_c$.
- We also need the probability that the quark and the anti-quark are at the same point, this is given by the wave-functions. The energies involved are of the order of $1/r$.
- If we want to make a precision computation, we need to include the effects of the color octet component of $J/\Psi$. The energy involved here is of order of the binding energy.

# Heavy quarkonium is non-relativistic

In a perturbative computation of the binding energy.

$$E = m_Q \alpha_s \sum_{n=0}^{\infty} \alpha_s^n A_n(v)$$

because $v$ is small we can not know the size of $A_n(v)$, for example, it could go like $1/v$.
If we use EFT the computation is an expansion in both $v$ and $\alpha_s$.

$$E = m_Q \alpha_s v^2 \sum_{n,m} \alpha_s^n v^m B_{n,m}$$

now $B_{n,m}$ is of order 1.
In perturbation theory $v \sim \alpha_s$.

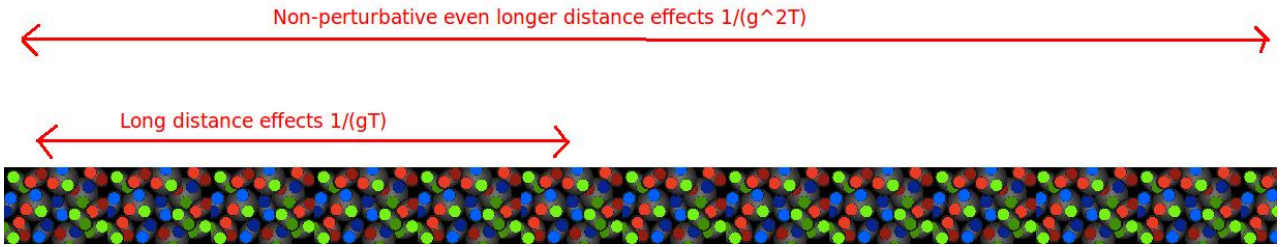
# Energy scales in a thermal bath

In the weak-coupling regime

- We have an almost free gas of quarks and gluons with typical energy $\pi T$.
- At long distances (order $\frac{1}{gT}$), non-trivial collective phenomena arise, as for example chromoelectric static fields screening.
- At even longer distances (order $\frac{1}{g^2 T}$), non-perturbative phenomena arise, as for example chromomagnetic static fields screening.

# Energy scales in a thermal bath

In the weak-coupling regime



# Heavy quarkonium in a thermal bath

In this case we have to combine the two types of energy scales.

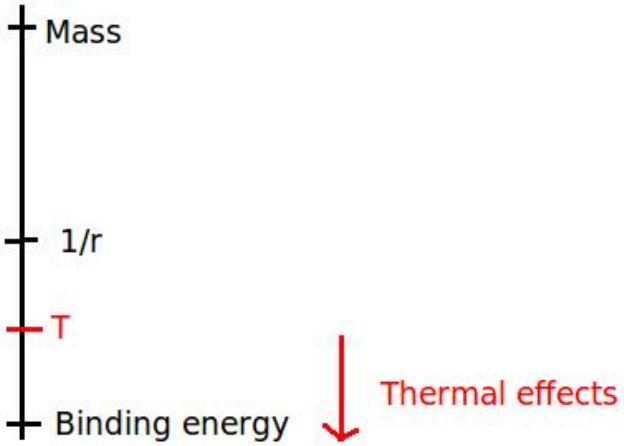
- The energy scales typical of a non-relativistic bound state. $m_Q$, $\frac{1}{r}$ and $\Delta E$.
- The energy scales of a weakly-coupled quark-gluon plasma. $\pi T$, $gT$...

Depending on the relation of $T$ with the energy scales of the bound state we are going to have very different physical situations.

# The case $1/r \gg T \gg E \gg m_D$

Brambilla, M.A.E, Soto and Vairo (2010)

# Thermal effects




- The physical results that come from energy scales higher than the temperature are not affected by the thermal bath.

$$\frac{1}{e^{q/T} \pm 1}$$

- Consequence: The EFT resulting from integrating out degrees of freedom higher than $T$ are still valid for this situation.
- These are NRQCD and pNRQCD.

# NRQCD

- Gives exactly the same results as QCD for all Green functions evaluated at distances bigger than $\frac{1}{m_Q}$.
- One can always compute the NRQCD from QCD (matching) using perturbation theory because $m_Q \gg \Lambda_{QCD}$.
- This EFT is also useful for Lattice computations as the UV cutoff of this theory is much smaller than $m_Q$.
- Suppressions in $\frac{1}{m_Q}$ that are not obvious from QCD are trivially seen with this Lagrangian.

# NRQCD

$$\mathcal{L}_{NRQCD} = \mathcal{L}_g + \mathcal{L}_q + \mathcal{L}_\psi + \mathcal{L}_\chi + \mathcal{L}_{\psi\chi}$$

$$\mathcal{L}_g = -\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu a} + \frac{d_2}{m_Q^2}F^a_{\mu\nu}D^2F^{\mu\nu a} + d_g^3\frac{1}{m_Q^2}gf_{abc}F^a_{\mu\nu}F^{\mu b}_{\alpha}F^{\nu\alpha c}$$

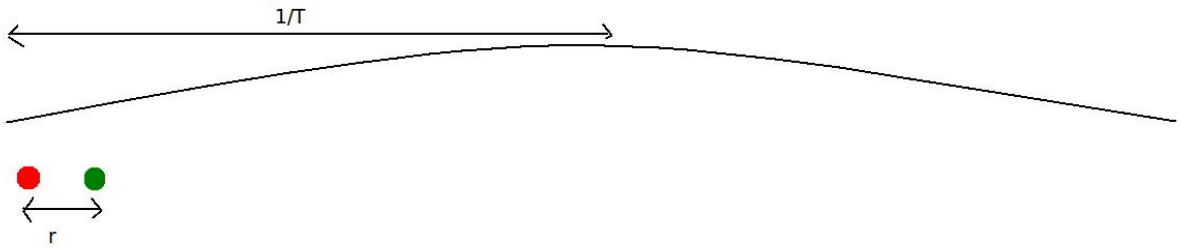
$$\mathcal{L}_\psi = \psi^\dagger\left(iD_0 + c_2\frac{\mathbf{D}^2}{2m_Q} + c_4\frac{\mathbf{D}^4}{8m_Q^3} + c_F g\frac{\sigma\mathbf{B}}{2m_Q} + c_D g\frac{\mathbf{DE}-\mathbf{ED}}{8m_Q^2}\right.$$
$$\left. + ic_S g\frac{\sigma(\mathbf{D}\times\mathbf{E}-\mathbf{E}\times\mathbf{D})}{8m_Q^2}\right)\psi$$

$$\mathcal{L}_\chi = c.c\ of\ \mathcal{L}_\psi$$

$$\mathcal{L}_{\psi\chi} = \frac{f_1(^1S_0)}{m_Q^2}\psi^\dagger\chi\chi^\dagger\psi + \frac{f_1(^3S_1)}{m_Q^2}\psi^\dagger\sigma\chi\chi^\dagger\sigma\psi + \frac{f_8(^1S_0)}{m_Q^2}\psi^\dagger T^a\chi\chi^\dagger T^a\psi$$
$$+\frac{f_8(^3S_1)}{m_Q^2}\psi^\dagger T^a\sigma\chi\chi^\dagger T^a\sigma\psi$$

# pNRQCD

There are still simplifications that are not obvious from NRQCD.




Thermal effects are going to see the bound state as a color dipole.

# pNRQCD

$$\mathcal{L}_{pNRQCD} = \int d^3\mathbf{r}\, Tr\left[S^\dagger\left(i\partial_0 - h_s\right)S + O^\dagger\left(iD_0 - h_o\right)O\right] + V_A(r)\, Tr(O^\dagger \mathbf{r} g\mathbf{E} S + S^\dagger \mathbf{r} g\mathbf{E} O) + \frac{V_B(r)}{2}\, Tr(O^\dagger \mathbf{r} g\mathbf{E} O + O^\dagger O \mathbf{r} g\mathbf{E}) + \mathcal{L}_g + \mathcal{L}_q$$

- Gives the same results as QCD and NRQCD for Green functions evaluated at distances much bigger than $r$.
- The matching between NRQCD and pNRQCD can be done perturbatively if $\frac{1}{r} \gg \Lambda_{QCD}$.
- The degrees of freedom for the heavy quarks are now a color singlet field and a color octet field.
- Using pNRQCD provides automatically with a Coulomb resummation.

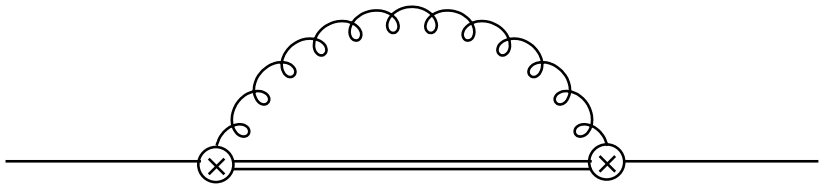
# From *pNRQCD* to $pNRQCD_{HTL}$

Now we take into account thermal effects. For this we integrate out degrees of freedom with virtuality of order $T^2$ and we go from *pNRQCD* to a new EFT $pNRQCD_{HTL}$.

- In the gluons and light quarks sector of $pNRQCD_{HTL}$ we will have the usual Hard Thermal Loop action.
- The potential of the singlet and the octet will have thermal corrections

# Integrating out the $T$ scale, Leading order in $\alpha_s$

Now we take into thermal effects into the singlet potential



$$-ig^2 C_F \frac{r^i}{D-1}\mu^{4-D}\int \frac{d^D k}{(2\pi)^D}\frac{i}{E-h_o-k_0+i\eta}[k_0^2 D_{ii}(k_0,k)+k^2 D_{00}(k_0,k)]r^i$$

All the thermal bath information is encoded in the gluon propagator $D$.

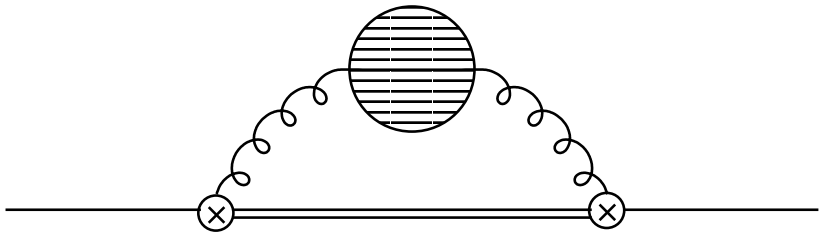
# Integrating out the $T$ scale, Leading order in $\alpha_{\rm s}$

$$\delta V_s = \frac{\pi}{9} N_c C_F \alpha_{\rm s}^2 T^2 r + \frac{2\pi}{3m} C_F \alpha_{\rm s} T^2$$
$$+ \frac{\alpha_{\rm s} C_F I_T}{3\pi} \left[ -\frac{N_c^3}{8} \frac{\alpha_{\rm s}^3}{r} - \frac{N_c(N_c+2C_F)\alpha_{\rm s}^2}{mr^2} + \frac{4(N_c-2C_F)\pi\alpha_{\rm s}}{m^2}\delta^3(\mathbf{r}) + N_c \frac{\alpha_{\rm s}}{m^2} \left\{ \nabla_{\mathbf{r}}^2, \frac{1}{r} \right\} \right]$$

There is an infrared divergence

$$I_T = \frac{2}{\epsilon} + \ln \frac{T^2}{\mu^2} - \gamma_E + \ln(4\pi) - \frac{5}{3}$$

# Integrating out the $T$ scale, next to leading order in $\alpha_{\rm s}$



$$\delta V_s^{(2\,\mathrm{loops})} = -\tfrac{3}{2}\zeta(3)\, C_F\, \tfrac{\alpha_{\rm s}}{\pi}\, r^2\, T\, m_D^2 + \tfrac{2}{3}\zeta(3)\, N_c\, C_F\, \alpha_{\rm s}^2\, r^2\, T^3$$
$$+i\left[\tfrac{C_F}{6}\alpha_{\rm s}\, r^2\, T\, m_D^2\, \left(-\tfrac{2}{\epsilon} + \gamma_E + \ln\pi - \ln\tfrac{T^2}{\mu^2} + \tfrac{2}{3} - 4\ln 2 - 2\tfrac{\zeta'(2)}{\zeta(2)}\right)\right.$$
$$\left. + \tfrac{4\pi}{9}\ln 2\; N_c\, C_F\, \alpha_{\rm s}^2\, r^2\, T^3\right]$$

This contribution was first found in the static limit by Brambilla, Ghiglieri, Petreczky and Vairo (2008).

# Computations in the $m\alpha_{\rm s}^2$ scale

- Because $T \gg m\alpha_{\rm s}^2$

$$\frac{1}{e^{\beta k}-1} \to \frac{T}{k} - \frac{1}{2} + \cdots$$

- The results coming from this energy scale can not be reproduced by a potential.

# Contribution from the $m\alpha_s^2$ scale

$$\delta E_{n,l} = -\frac{\pi \alpha_s C_F \ Tm_D^2}{3} \frac{a_0^2 n^2}{2} [5n^2 + 1 - 3l(l+1)]$$

$a_0$ is the Bohr radius of the fundamental state.

$$\delta\Gamma_{n,l} = \tfrac{1}{3} N_c^2 C_F \alpha_s^3 T - \tfrac{16}{3m} C_F \alpha_s T E_n + \tfrac{8}{3} N_c C_F \alpha_s^2 T \frac{1}{mn^2 a_0}$$
$$+ \frac{2E_n \alpha_s^3}{3} \left\{ \frac{4C_F^3 \delta_{l0}}{n} + N_c C_F^2 \left( \frac{8}{n(2l+1)} - \frac{1}{n^2} - \frac{2\delta_{l0}}{n} \right) + \frac{2N_c^2 C_F}{n(2l+1)} + \frac{N_c^3}{4} \right\}$$
$$- \frac{\alpha_s C_F T m_D^2}{6} \left( \frac{2}{\epsilon} + \ln \frac{E_1^2}{\mu^2} + \gamma_E - \frac{11}{3} - \ln \pi + \ln 4 \right) a_0^2 n^2 [5n^2 + 1 - 3l(l+1)]$$
$$+ \frac{2\alpha_s C_F T m_D^2}{3} \frac{C_F^2 \alpha_s^2}{E_n^2} I_{n,l}$$

$$I_{n,l} = \frac{E_n^2}{C_F^2 \alpha_s^2} \int \frac{d^3k}{(2\pi)^3} |\langle n, l|\mathbf{r}|\mathbf{k}\rangle|^2 \ln \frac{E_1}{E_n - k^2/m}$$

# Final result. $m\alpha_s \gg T \gg m\alpha_s^2 \gg m_D$

Sum of all thermal bath induced terms. (Non-thermally induced terms coming from the $m\alpha_s^2$ scale are subtracted).

$$
\begin{aligned}
\delta E_{n,l}^{(\text{thermal})} = & \frac{\pi}{9} N_c C_F\, \alpha_s^2\, T^2 \frac{a_0}{2} \left[3n^2 - l(l+1)\right] + \frac{\pi}{3} C_F^2\, \alpha_s^2\, T^2\, a_0 \\
& + \frac{E_n \alpha_s^3}{3\pi} \left[\log\left(\frac{2\pi T}{E_1}\right)^2 - 2\gamma_E\right] \left\{ \frac{4C_F^3 \delta_{l0}}{n} + N_c C_F^2 \left[\frac{8}{n(2l+1)} - \frac{1}{n^2} - \frac{2\delta_{l0}}{n}\right] \right. \\
& \qquad \left. + \frac{2N_c^2 C_F}{n(2l+1)} + \frac{N_c^3}{4} \right\} \\
& + \frac{2E_n C_F^3 \alpha_s^3}{3\pi} L_{n,l} \\
& + \frac{a_0^2 n^2}{2} \left[5n^2 + 1 - 3l(l+1)\right] \left\{ - \left[\frac{3}{2\pi}\zeta(3) + \frac{\pi}{3}\right] C_F\, \alpha_s\, T\, m_D^2 \right. \\
& \qquad \left. + \frac{2}{3}\zeta(3)\, N_c C_F\, \alpha_s^2\, T^3 \right\}
\end{aligned}
$$

where $E_n = -\dfrac{mC_F^2\alpha_s^2}{4n^2}$, $a_0 = \dfrac{2}{mC_F\alpha_s}$ and $L_{n,l}$ is the Bethe logarithm.

# Final result. $m\alpha_{\rm s} \gg T \gg m\alpha_{\rm s}^2 \gg m_D$

Sum of all thermal bath induced terms. (Non-thermally induced terms coming from the $m\alpha_{\rm s}^2$ scale are subtracted).

For a general number of colors $N_c$ ($C_F = (N_c^2-1)/(2N_c)$):

$$\delta E^{(\rm thermal)}_{n,l} = \frac{\pi}{9} N_c C_F\, \alpha_{\rm s}^2\, T^2 \frac{a_0}{2}\left[3n^2 - l(l+1)\right] + \frac{\pi}{3} C_F^2\, \alpha_{\rm s}^2\, T^2\, a_0 \qquad \sim m\alpha_{\rm s}^5 \frac{T^2}{E^2}$$

$$m\alpha_{\rm s}^5 \sim \begin{cases} +\dfrac{E_n \alpha_{\rm s}^3}{3\pi}\left[\log\left(\dfrac{2\pi T}{E_1}\right)^2 - 2\gamma_E\right]\left\{\dfrac{4C_F^3\delta_{l0}}{n} + N_c C_F^2\left[\dfrac{8}{n(2l+1)} - \dfrac{1}{n^2} - \dfrac{2\delta_{l0}}{n}\right]\right. \\ \qquad\qquad \left. + \dfrac{2N_c^2 C_F}{n(2l+1)} + \dfrac{N_c^3}{4}\right\} \\ +\dfrac{2E_n C_F^3 \alpha_{\rm s}^3}{3\pi} L_{n,l} \end{cases}$$

$$m\alpha_{\rm s}^6 \frac{T^3}{E^3} \sim +\frac{a_0^2 n^2}{2}\left[5n^2 + 1 - 3l(l+1)\right]\left\{-\left[\frac{3}{2\pi}\zeta(3) + \frac{\pi}{3}\right] C_F\, \alpha_{\rm s}\, T\, m_D^2 \right. \\ \left. + \frac{2}{3}\zeta(3)\, N_c C_F\, \alpha_{\rm s}^2\, T^3\right\}$$

where $E_n = -\dfrac{mC_F^2\alpha_{\rm s}^2}{4n^2}$, $a_0 = \dfrac{2}{mC_F\alpha_{\rm s}}$ and $L_{n,l}$ is the Bethe logarithm.

# Final result. $m\alpha_{\rm s} \gg T \gg m\alpha_{\rm s}^2 \gg m_D$

Sum of all thermal bath induced terms. (Non-thermally induced terms coming from the $m\alpha_{\rm s}^2$ scale are subtracted).

$$
\begin{aligned}
\Gamma_{n,l}^{(\text{thermal})} = \; & \frac{1}{3}N_c^2 C_F \alpha_{\rm s}^3 T + \frac{4}{3}\frac{C_F^2\alpha_{\rm s}^3 T}{n^2}(C_F+N_c) \\
& + \frac{2E_n\alpha_{\rm s}^3}{3}\left\{\frac{4C_F^3\delta_{l0}}{n} + N_c C_F^2\left[\frac{8}{n(2l+1)} - \frac{1}{n^2} - \frac{2\delta_{l0}}{n}\right] + \frac{2N_c^2 C_F}{n(2l+1)} + \frac{N_c^3}{4}\right\} \\
& - \left[\frac{C_F}{6}\alpha_{\rm s} T m_D^2\left(\ln\frac{E_1^2}{T^2} + 2\gamma_E - 3 - \log 4 - 2\frac{\zeta'(2)}{\zeta(2)}\right) + \frac{4\pi}{9}\ln 2\, N_c C_F\, \alpha_{\rm s}^2 T^3\right] \\
& \qquad \times a_0^2 n^2\left[5n^2 + 1 - 3l(l+1)\right] \\
& + \frac{8}{3}C_F\alpha_{\rm s}\, T m_D^2\, a_0^2 n^4\, I_{n,l}
\end{aligned}
$$

# Final result. $m\alpha_{\rm s} \gg T \gg m\alpha_{\rm s}^2 \gg m_D$

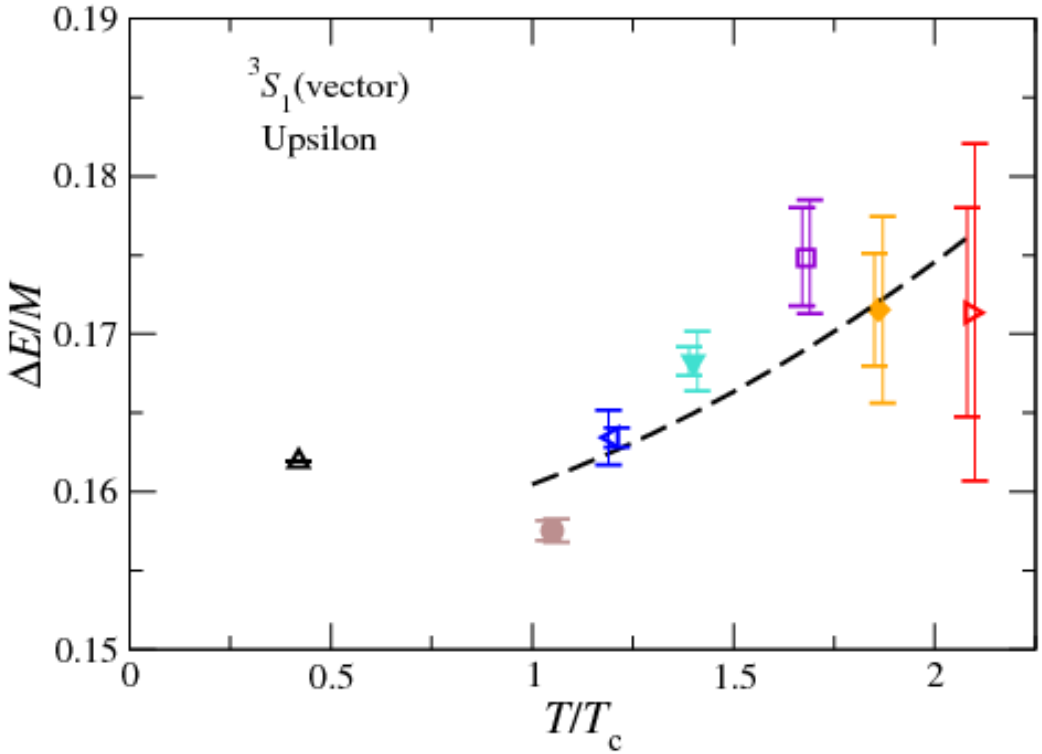
Sum of all thermal bath induced terms. (Non-thermally induced terms coming from the $m\alpha_{\rm s}^2$ scale are subtracted).

$$m\alpha_{\rm s}^5 \sim \Gamma^{(\rm thermal)}_{n,l} = \frac{1}{3}N_c^2 C_F \alpha_{\rm s}^3 T + \frac{4}{3}\frac{C_F^2\alpha_{\rm s}^3 T}{n^2}(C_F+N_c) \sim m\alpha_{\rm s}^5 \frac{T}{E}$$
$$+\frac{2E_n\alpha_{\rm s}^3}{3}\left\{\frac{4C_F^3\delta_{l0}}{n} + N_c C_F^2\left[\frac{8}{n(2l+1)} - \frac{1}{n^2} - \frac{2\delta_{l0}}{n}\right] + \frac{2N_c^2 C_F}{n(2l+1)} + \frac{N_c^3}{4}\right\}$$

$$m\alpha_{\rm s}^6\frac{T^3}{E^3} \sim \left\{ \begin{array}{l} -\left[\frac{C_F}{6}\alpha_{\rm s} T m_D^2\left(\ln\frac{E_1^2}{T^2} + 2\gamma_E - 3 - \log 4 - 2\frac{\zeta'(2)}{\zeta(2)}\right) + \frac{4\pi}{9}\ln 2\, N_c C_F\, \alpha_{\rm s}^2 T^3\right] \\ \qquad \times a_0^2 n^2\left[5n^2+1-3l(l+1)\right] \\ +\frac{8}{3}C_F\alpha_{\rm s}\, T m_D^2\, a_0^2 n^4\, I_{n,l} \end{array} \right.$$

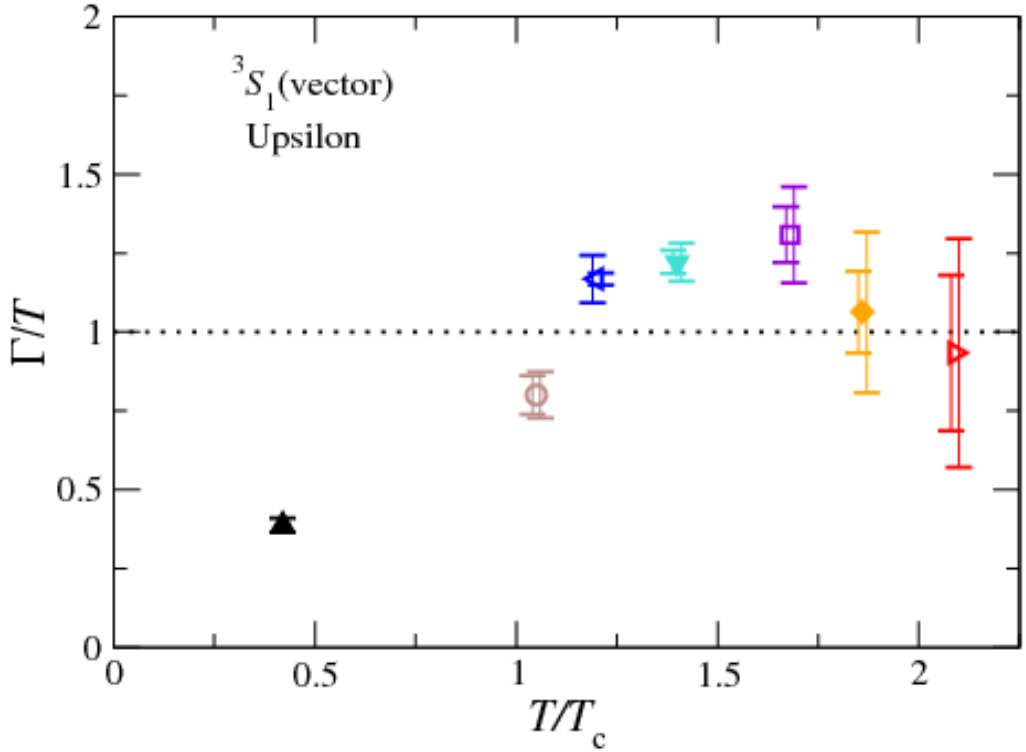
# Check results, compare with lattice

Comparison of MEM lattice computation (Aarts, Allton, Kim, Lombardo, Oktay, Ryan, Sinclair and Skullerud (2011)) with our computations in the $1/r \gg T \gg E \gg m_D$ regime fitting $\alpha_s$.



# Check results, compare with lattice

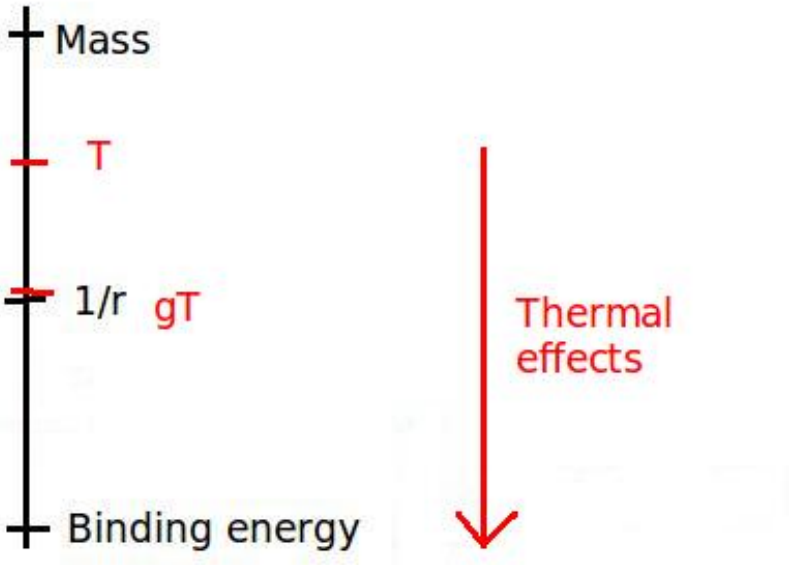
Comparison of MEM lattice computation (Aarts, Allton, Kim, Lombardo, Oktay, Ryan, Sinclair and Skullerud (2011)) with our computations in the $1/r \gg T \gg E \gg m_D$ regime fitting $\alpha_s$.



$m \gg T \gg \frac{1}{r} \sim gT$. Dissociation temperature.

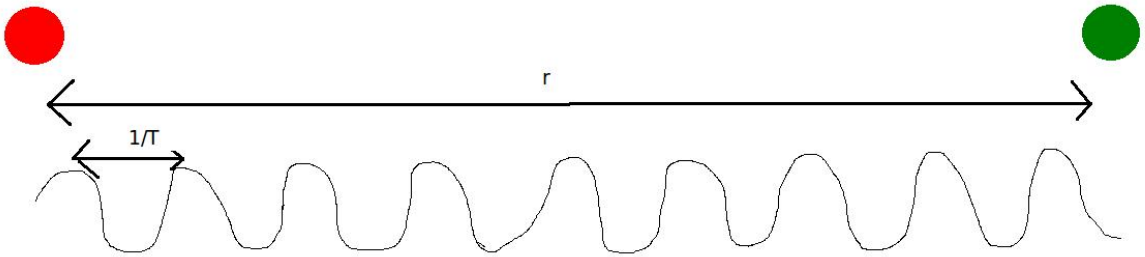
M.A.E and Soto (2010)

# Thermal effects




Now we can start with NRQCD Lagrangian at $T = 0$. Thermal effects can be included in a new EFT called $NRQCD_{HTL}$.

# Thermal effects




- Effects at the energy scale $T$ are going to see heavy quarks as elements that are very far away from each other.
- We will have to average this thermal fluctuations in the gluons that are interchanged by the heavy quarks.

# $NRQCD_{HTL}$

- We will have HTL corrections in the gluon and light quarks propagators. In fact, this is going to be the more relevant change for bound states phenomenology.
- NLO corrections to heavy quark sector.

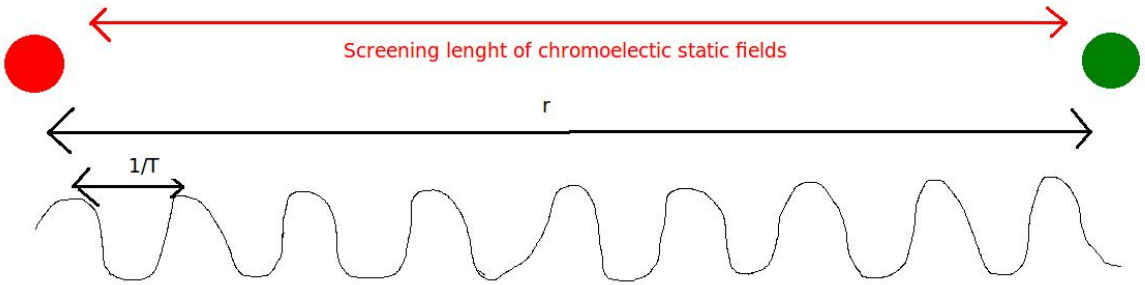
# $NRQCD_{HTL}$

- We will have HTL corrections in the gluon and light quarks propagators. In fact, this is going to be the more relevant change for bound states phenomenologically.
- NLO corrections to heavy quark sector.

Why is it NLO?

- In the Coulomb gauge at the scale $T$, only the spatial gluons are thermalized: $A_i$.
- In this gauge $A_0$ is not modified by the temperature at LO up to the scale $gT$.
- The coupling of heavy quarks with $A_i$ is always multiplied at least by one power of $\frac{1}{m_Q}$.

# $NRQCD_{HTL} \rightarrow pNRQCD_{HTL}$




- After averaging thermal fluctuations we find HTL propagator.
- This HTL modification is a leading order effect in the $A_0$ field at distances $r$ because $\frac{1}{r} \sim m_D$.
- The LO potential is going to be modified, and this can break the bound state.

# The potential at $m \gg T \gg m\alpha_s$

$$V(r) = -\frac{\alpha_s e^{-m_D r}}{r} - \alpha_s m_D$$
$$+\frac{i16\alpha_s^2 C_F T^3}{\pi m_D^2}\left(\frac{\pi^2 (m_D^{lq})^2}{4T^2 g^2} + g(m_c\beta) + \frac{m_c^2}{2T^2(e^{\beta m_c}+1)}\right)\phi(m_D r),$$

- We consider the effect of the charm mass that can be important for bottomonium. The $m_c \to 0$ result was first found by Laine, Philipsen, Romatschke and Tassler.
- $m_D^{lq}$ is the Debye mass that is found in the $m_c \to \infty$ limit.
- $g(0) = \frac{\pi^2}{12}$ and goes exponentially to zero at large values of $m_c\beta$.
- $$\phi(x) = 2\int_0^\infty \frac{dzz}{(z^2+1)^2}\left[\frac{sin(zx)}{zx} - 1\right]$$

# The dissociation temperature

- The temperature when the imaginary part of the potential is of the same order of magnitude as the real part is lower than the temperature where screening is important for bound states (in the limit of weak coupling).
- If both screening and dissipation are a perturbation the bound state survives.
- If the imaginary part of the potential is bigger than the real part then the the bound state does not exist anymore.
- It is natural then to propose as dissociation temperature the temperature where the real part of the potential is as big as the imaginary part.

$$\frac{1}{a_0^3} = 16\alpha_s(\pi T)C_F T^3 \left( \frac{\pi^2 (m_D^{lq})^2}{4T^2 g^2} + g(m_c\beta) + \frac{m_c^2}{2T^2(e^{\beta m_c}+1)} \right).$$

# Dissociation temperature for charmonium

Assuming $g$ small and $\frac{1}{a_0} \gg m_D$. For $J/\Psi$

| $\alpha_{\rm s}$ | $T_d$ (MeV) |
|---|---|
| $\alpha_{\rm s}(\pi T)$ | 230 |
| $\alpha_{\rm s}(2\pi T)$ | 280 |

Too close to $T_c$?

# Dissociation temperature for bottomonium

For $\Upsilon(1S)$

| $\alpha_s$ | $T_d$ (MeV) |
|---|---|
| $\alpha_s(\pi T)$ | 440 |
| $\alpha_s(2\pi T)$ | 500 |

# Dependence of bottomonium dissociation with charm mass

| $m_c$ (MeV) | $T_d$ (MeV) |
|---|---|
| $\infty$ | 480 |
| 5000 | 480 |
| 2500 | 460 |
| 1200 | 440 |
| 0 | 420 |

Table: Dissociation temperature for Upsilon (1S) for different values of the charm mass

# Cross-section

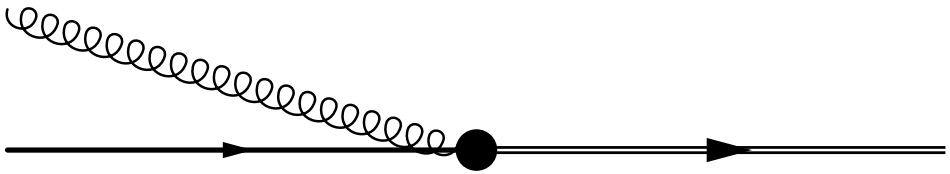
Brambilla, M.A.E, Ghiglieri and Vairo (2011)
Brambilla, M.A.E, Ghiglieri and Vairo (2013)

# Duality imaginary potential/cross-section

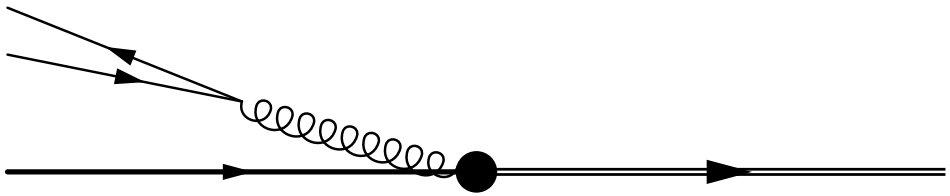
- Related by the cutting rules.
- No information is lost except for $m_D \sim 1/r$ or $m_D \gg 1/r$.
- More physical picture.
- Also true at finite chemical potential.

# Perturbative computations of cross-section for quarkonia in the literature

Gluo-dissociation



Bhanot and Peskin (1979)
Quasi-free dissociation

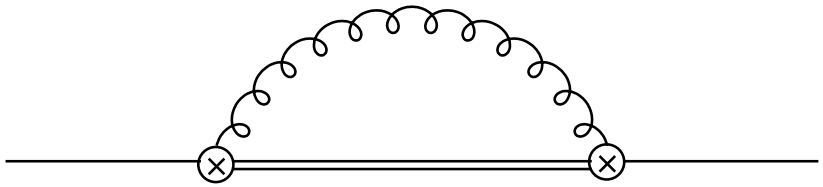


Combridge (1978), Park, Kim, Song, Lee and Wong (2007)

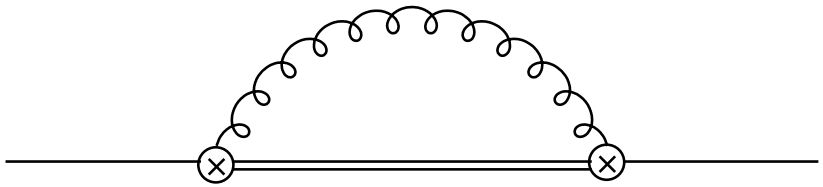
# Gluo-dissociation in EFT

$$HQ + g \to Q + \bar{Q}$$

Imaginary part of the diagram



# Power counting



- The gluon is on-shell.
- The energy difference between a heavy quarkonium state and two free heavy quarks is of order $m_Q v^2$.

This effect is found when taking into account the energy region $m_Q v^2$. It can be studied using pNRQCD

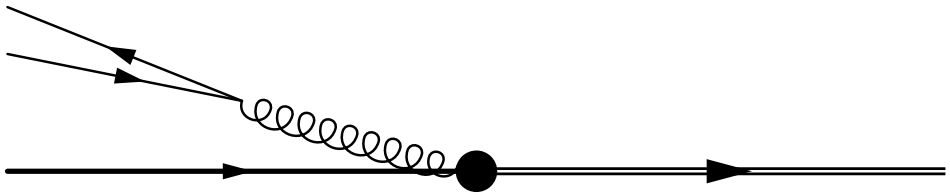
$$\delta\Gamma \propto \alpha_s r^2 T (\Delta E)^2$$

# Quasi-free dissociation

$$HQ + p \rightarrow Q + \bar{Q} + p$$

- p is a parton that can be a quark or a gluon.
- The parton and the heavy quark interact through a virtual gluon.
- Energy and momentum conservation does not impose any constraint to the parton energy, as opposite to what happened in gluo-dissociation. Dominates $m_D \gg E$.

# Quasi-free dissociation in the literature

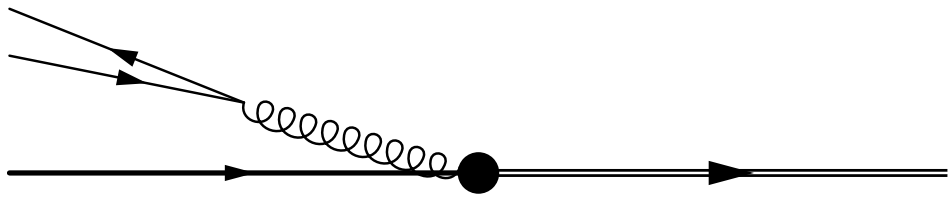


$$1/r \gg T \gg E$$

Park, Kim, Song, Lee and Wong (2007)

- Uses same techniques as Bhanot and Peskin computation of gluo-dissociation.
- Multipole expansion. This is also done in EFT.
- Large $N_c$ limit. This is the main difference with EFT computation.

# Quasi-free dissociation in the literature



$$T \gg 1/r$$

Combridge (1978)

$$\Gamma(HQ + p \to Q + \bar{Q} + p) = 2\Gamma(Q + p \to Q + p)$$

This is only true when $m_D a_0 \gg 1$. In this limit it coincides with the imaginary part of the potential computed by Laine, Philipsen, Romatschke and Tassler.

# Quasi-free dissociation in EFT

$$1/r \gg T$$

We can go beyond large-$N_c$ limit.
Power counting tells us that the contribution to the decay with goes like $\alpha_s^2 r^2 T^3$. This means quasi-free dissociation dominates if $m_D \gg \Delta E$.

$$1/r \sim T$$

Not computed before.

$$T \gg 1/r$$

The decay width information is encoded in the imaginary part of the potential, and must be ressumed.

# Some notation

$$\sigma(k, m_D) = \sigma_R f(x, y)$$

where

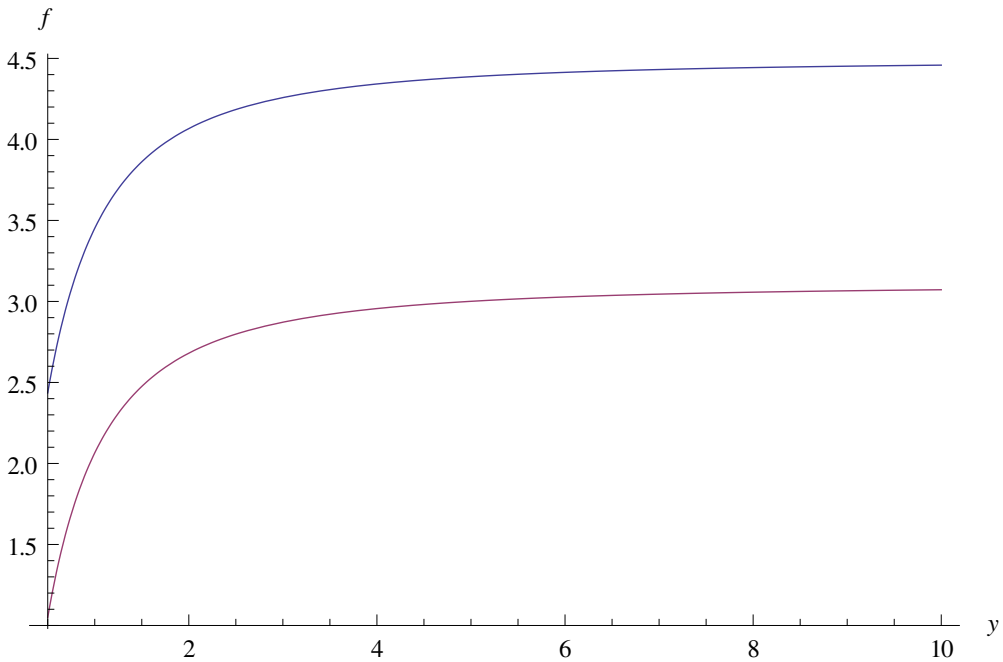
$$\sigma_R = 8\pi C_F \alpha_s^2 N_F a_0^2$$
$$x = m_D a_0$$
$$y = k a_0$$

I will only show the result for the fermion part, the boson part is quantitatively and qualitatively very similar.

# $T \sim \frac{1}{r} \gg m_D$ cross-section for $1S$

$$f(x,y) = -\frac{3}{2} + 2\log\left(\frac{2}{x}\right) + \log\left(\frac{y^2}{1+y^2}\right) - \frac{1}{y^2}\log(1+y^2)$$

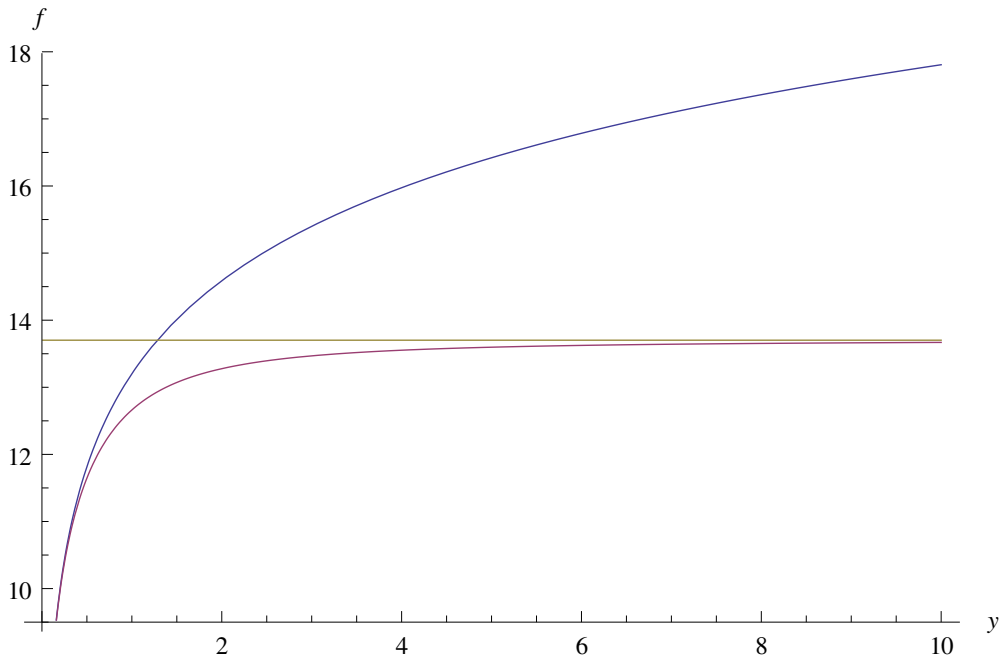
$x \ll 1$ and $y \sim 1$.




$m_D a_0 = 0.1$ and $m_D a_0 = 0.2$.

# Summary cross-section for $1S$

$m_D a_0 = 0.001$




$\frac{1}{r} \gg T \gg m_D$, $T \sim \frac{1}{r} \gg m_D$ and $T \gg \frac{1}{r} \sim m_D$. Discrepancy between blue and red lines signals a failure of color dipole approximation.

# Summary of results

In EFT we can compute the cross-section for a wide range of temperatures.

- $\Delta E \gg m_D$. Gluo-dissociation dominates. Multipole expansion is accurate.
- $1/r \gg T \gg m_D \gg \Delta E$. Quasi-free dissociation dominates. Multipole expansion is accurate.
- $1/r \sim T$. Quasi-free dissociation dominates. Multipole expansion is not accurate.
- $T \gg 1/r \sim m_D$. Quasi-free dissociation dominates. It can be encoded in the imaginary part of a potential, it needs to be resummed.

# Conclusions

# Conclusions

- Extract a Schrödinger equation from QCD and know at which precision we need something more. Quantum field theory definition of the potential.
- All relevant temperature regimes except $m_D \sim E$ are well understood in perturbation theory and thermal equilibrium
- There are two dissociation mechanism, screening and decay width, the later dominates in the perturbative regime.
- Dissociation temperature of $\Upsilon(1S)$ is around $450 MeV$. Similar to maximum temperature at LHC.