Continuum Results of the Heavy Quark Momentum Diffusion Coefficient κ

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in collaboration with
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[arXiv:1409.3724, 1311.3759 and arXiv:1109.3941]

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Transport Coefficients are important ingredients into **hydro/transport models for the evolution of the system.**

Usually determined by matching to experiment (see right plot)

Need to be determined from QCD using first principle lattice calculations!
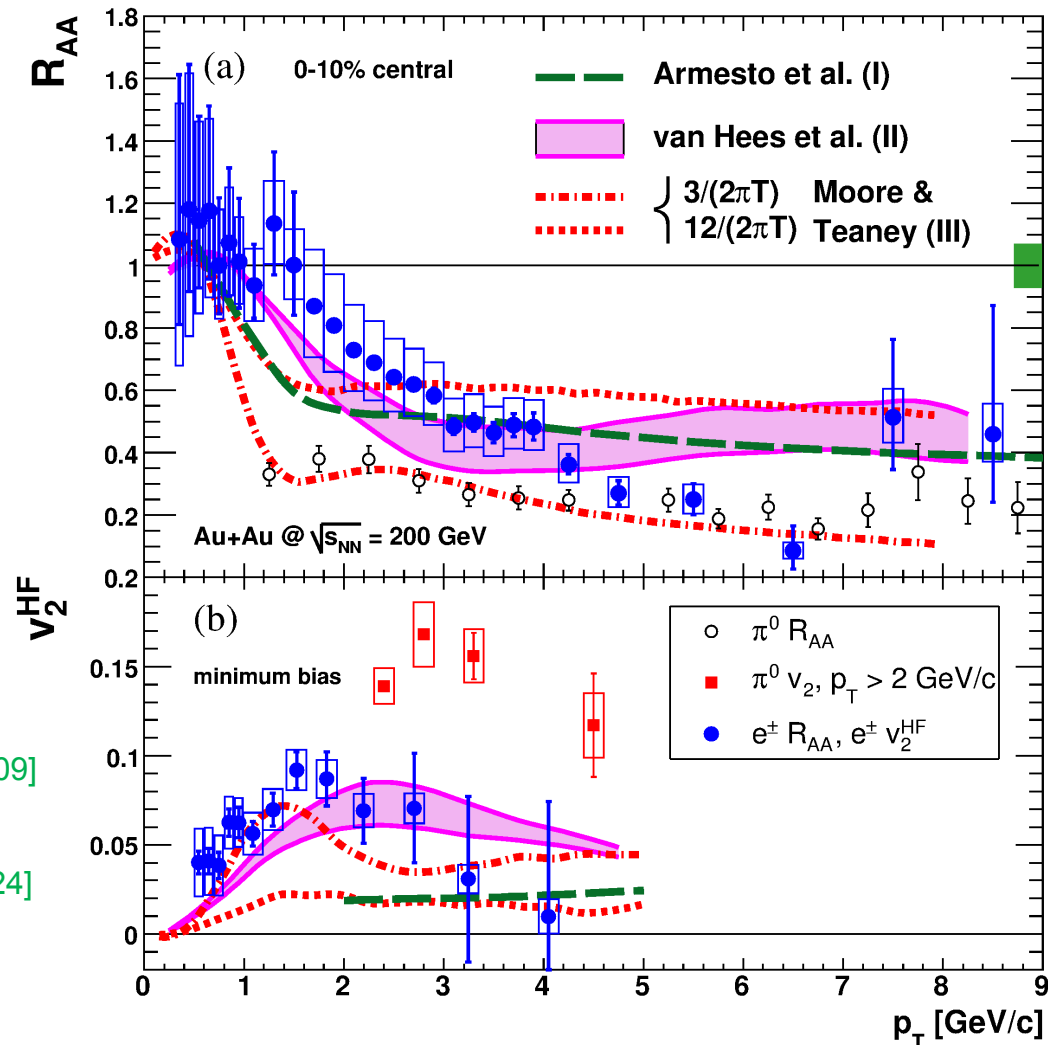
here heavy flavour:

Heavy Quark Diffusion Constant D
[H.T.Ding, OK et al., PRD86(2012)014509]

Heavy Quark Momentum Diffusion κ
[OK, arXiv:1409.3724]

or for light quarks:

Light quark flavour diffusion

Electrical conductivity
[A.Francis, OK et al., PRD83(2011)034504]

[PHENIX Collaboration, Adare et al.,PRC84(2011)044905 & PRL98(2007)172301]

Transport coefficients usually calculated using correlation function of conserved currents

$$G(\tau, \mathbf{p}, T) = \int_0^\infty \frac{d\omega}{2\pi} \rho(\omega, \mathbf{p}, T) K(\tau, \omega, T)$$
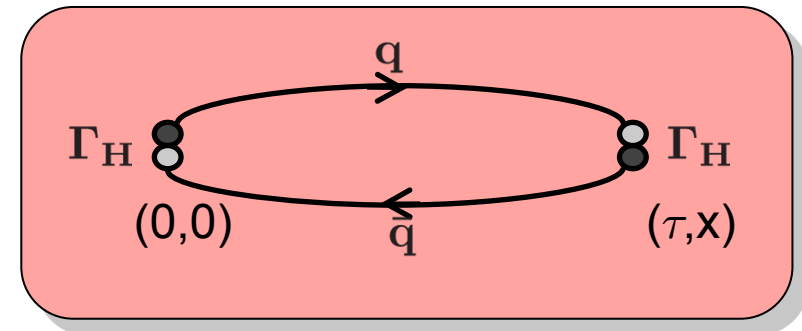
$$K(\tau, \omega, T) = \frac{\cosh\left(\omega(\tau - \frac{1}{2T})\right)}{\sinh\left(\frac{\omega}{2T}\right)}$$

Lattice observables:

$$G_{\mu\nu}(\tau, \vec{x}) = \langle J_\mu(\tau, \vec{x}) J_\nu^\dagger(0, \vec{0}) \rangle$$

$$J_\mu(\tau, \vec{x}) = 2\kappa Z_V \bar{\psi}(\tau, \vec{x}) \Gamma_\mu \psi(\tau, \vec{x})$$

$$G_{\mu\nu}(\tau, \vec{p}) = \sum_{\vec{x}} G_{\mu\nu}(\tau, \vec{x}) e^{i\vec{p}\vec{x}}$$

related to a conserved current

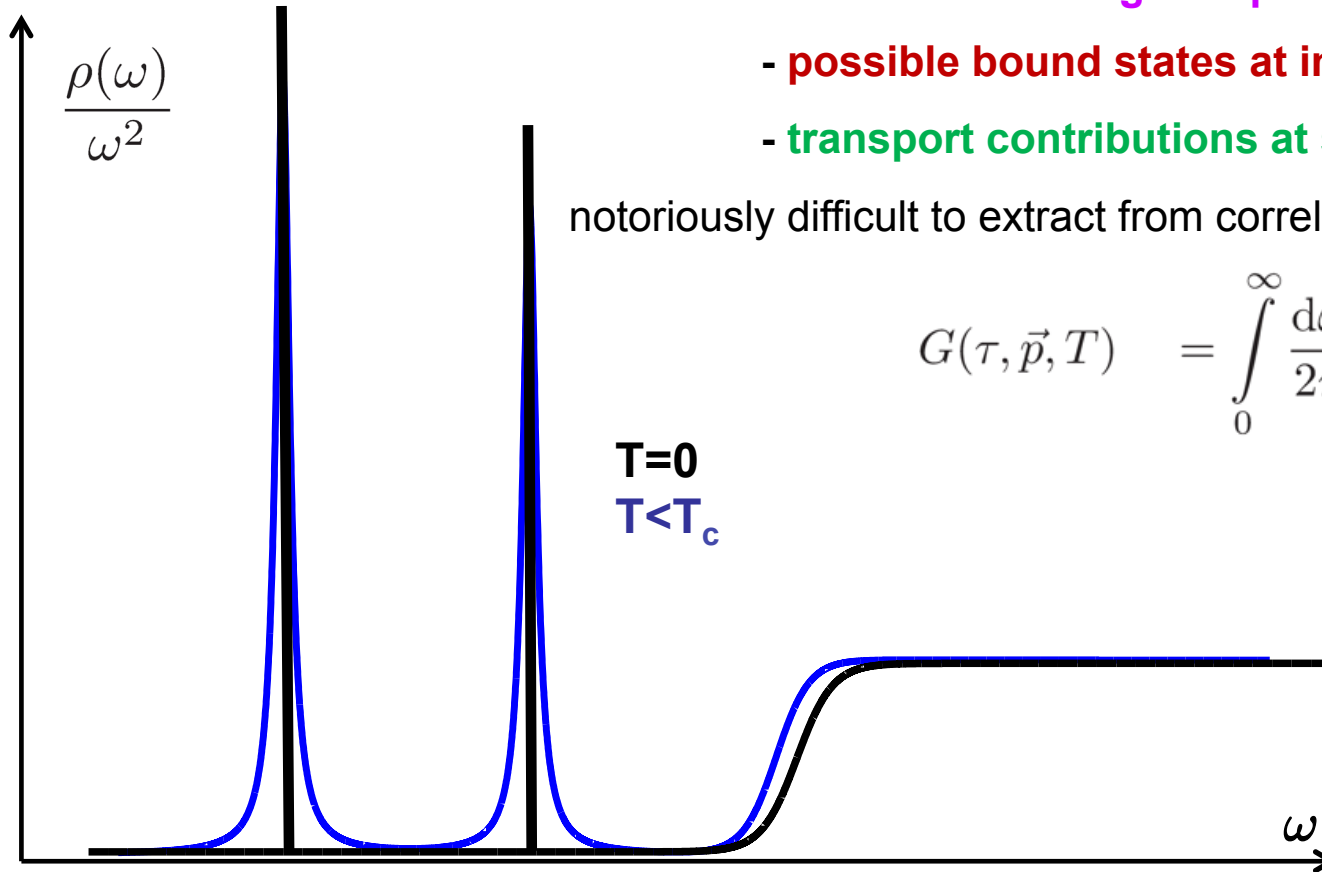only correlation functions calculable on lattice but

Transport coefficient determined by slope of spectral function at $\omega = 0$ (Kubo formula)

$$D = \frac{\pi}{3\chi_{00}} \lim_{\omega \to 0} \frac{\rho_{ii}(\omega, \vec{p} = 0, T)}{\omega T}$$

Different contributions and scales enter in the spectral function

- **continuum at large frequencies**

- **possible bound states at intermediate frequencies**

- **transport contributions at small frequencies**

notoriously difficult to extract from correlation functions

$$G(\tau, \vec{p}, T) = \int_0^\infty \frac{\mathrm{d}\omega}{2\pi} \rho(\omega, \vec{p}, T) K(\tau, \omega, T)$$

$\dfrac{\rho(\omega)}{\omega^2}$

T=0
T<T$_c$

ω

+ zero-mode contribution at ω=0: $\quad \rho(\omega) = 2\pi\chi_{00} \ \omega\delta(\omega)$

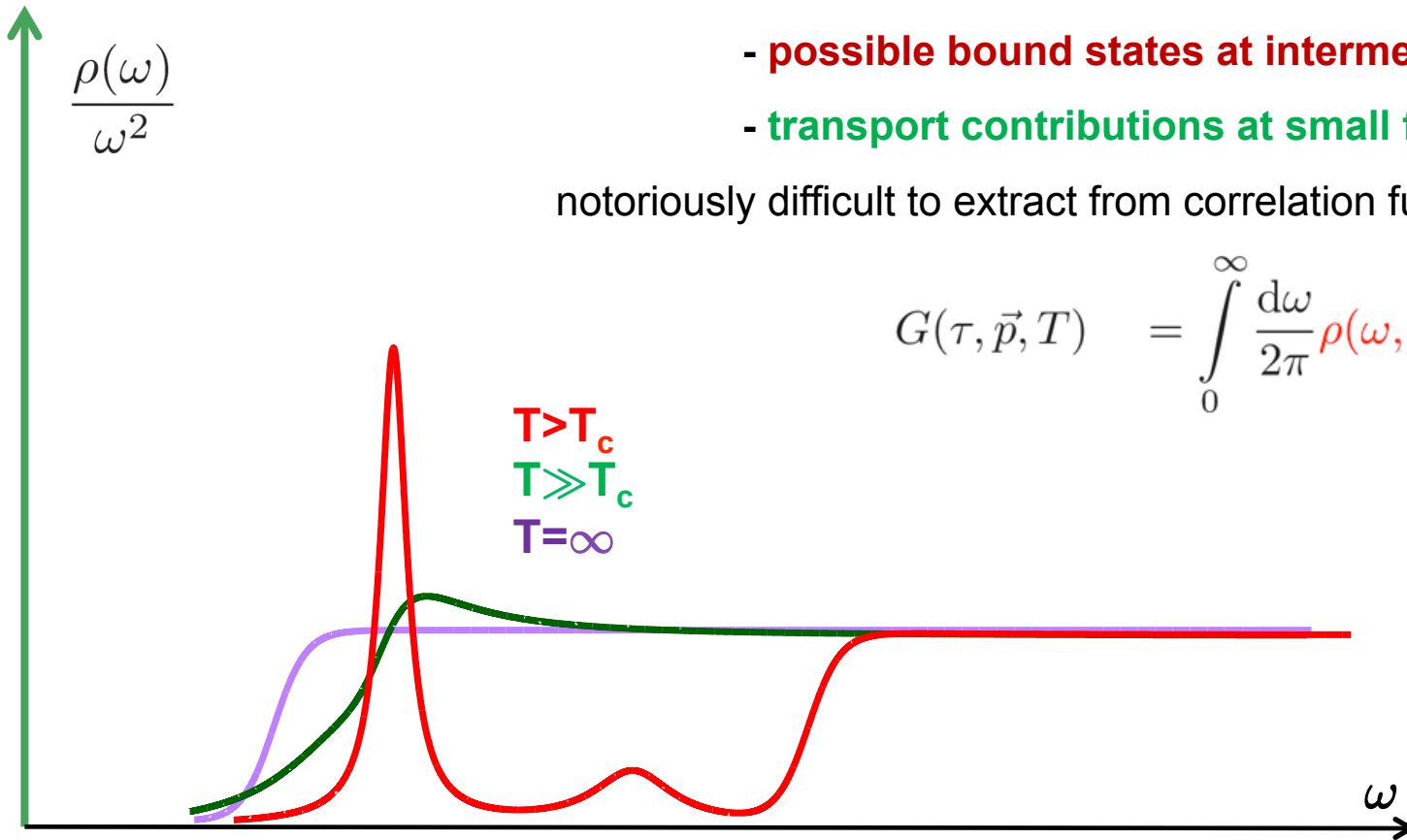+ transport peak at small ω: $\qquad \rho(\omega \ll T) = 2\chi_{00} \dfrac{T}{M} \dfrac{\omega\eta}{\omega^2 + \eta^2} \, , \qquad \eta = \dfrac{T}{MD}$

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$$G(\tau, \vec{p}, T) = \int\limits_{0}^{\infty} \frac{d\omega}{2\pi} \rho(\omega, \vec{p}, T) K(\tau, \omega, T)$$

$$\frac{\rho(\omega)}{\omega^2}$$

T>T$_c$
T≫T$_c$
T=∞

ω

+ zero-mode contribution at ω=0: $\quad \rho(\omega) = 2\pi\chi_{00}\ \omega\delta(\omega)$

+ transport peak at small ω: $\quad \rho(\omega \ll T) = 2\chi_{00}\dfrac{T}{M}\dfrac{\omega\eta}{\omega^2+\eta^2}\,, \qquad \eta = \dfrac{T}{MD}$
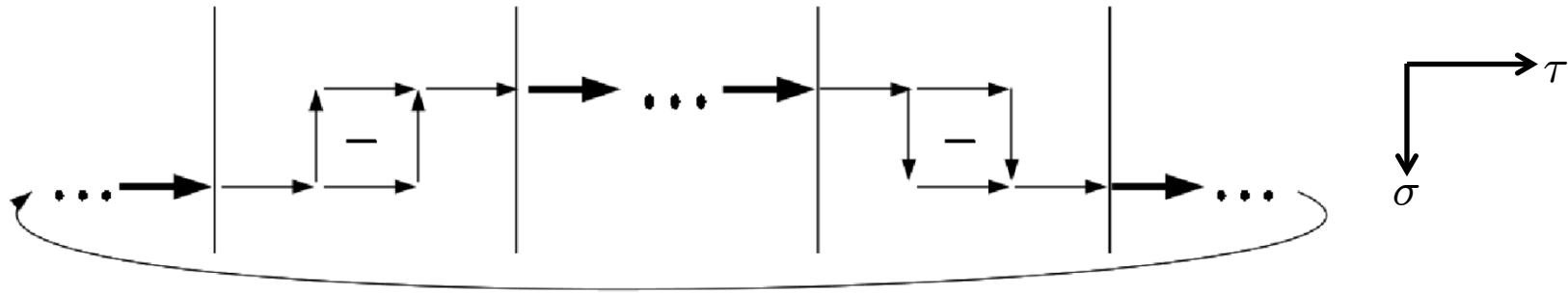
Heavy Quark Effective Theory (HQET) in the large quark mass limit

for a single quark in medium

leads to a (pure gluonic) "color-electric correlator"

[J.Casalderrey-Solana, D.Teaney, PRD74(2006)085012,
S.Caron-Huot,M.Laine,G.D. Moore,JHEP04(2009)053]

$$G_{\mathrm{E}}(\tau) \equiv -\frac{1}{3} \sum_{i=1}^{3} \frac{\left\langle \mathrm{Re\,Tr}\left[U(\frac{1}{T};\tau)\,gE_i(\tau,\mathbf{0})\,U(\tau;0)\,gE_i(0,\mathbf{0})\right]\right\rangle}{\left\langle \mathrm{Re\,Tr}[U(\frac{1}{T};0)]\right\rangle}$$
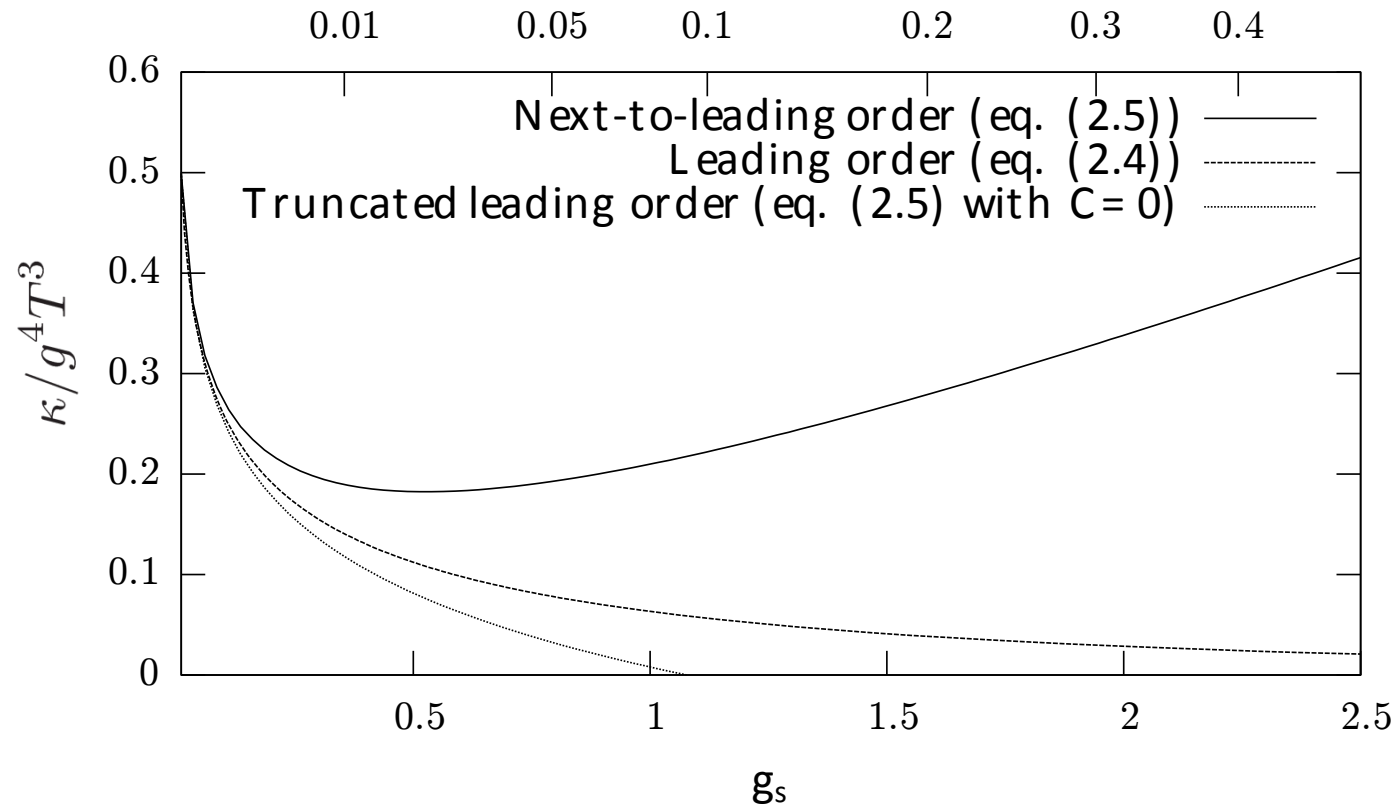
Heavy quark (momentum) diffusion:

$$\kappa = \lim_{\omega\to 0} \frac{2T\rho_{\mathrm{E}}(\omega)}{\omega} \qquad D = \frac{2T^2}{\kappa}$$

can be related to the thermalization rate:
$$\eta_D = \frac{\kappa}{2M_{kin}T}\left(1 + O\left(\frac{\alpha_s^{3/2}T}{M_{kin}}\right)\right)$$

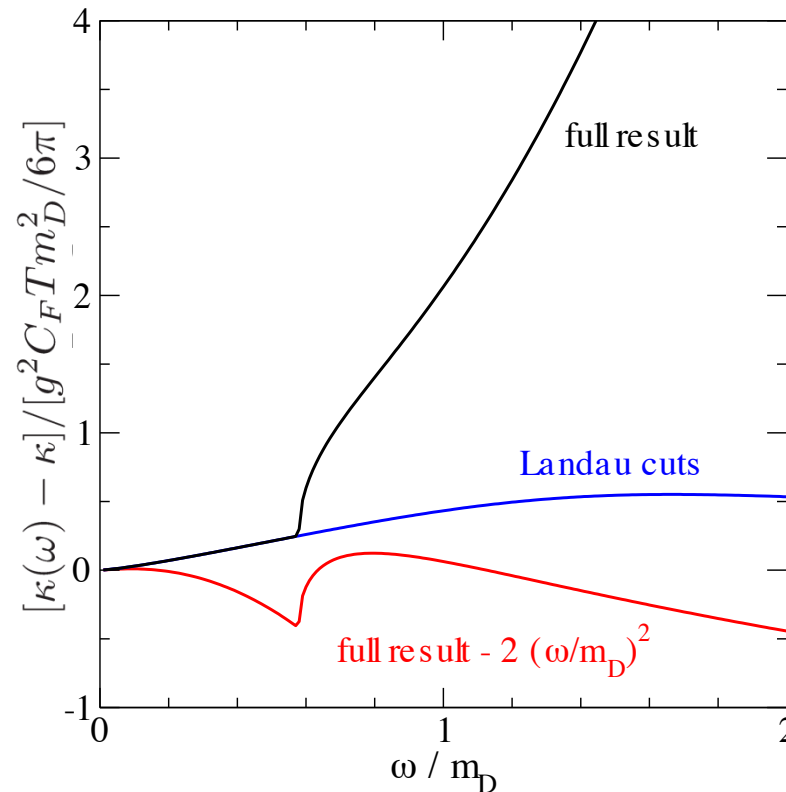NLO in perturbation theory: [Caron-Huot, G.Moore, JHEP 0802 (2008) 081]

very poor convergence

→ **Lattice QCD study required in the relevant temperature region**

NLO spectral function in perturbation theory: [Caron-Huot, M.Laine, G.Moore, JHEP 0904 (2009) 053]

in contrast to a narrow transport peak, from this a smooth limit

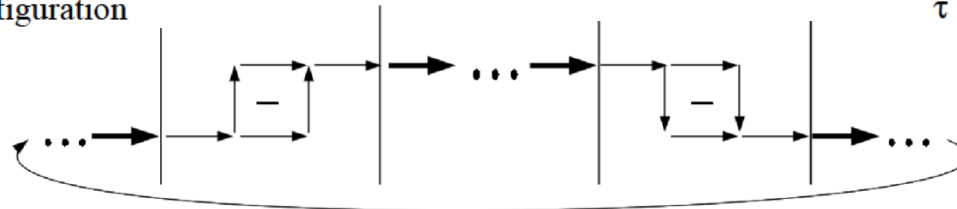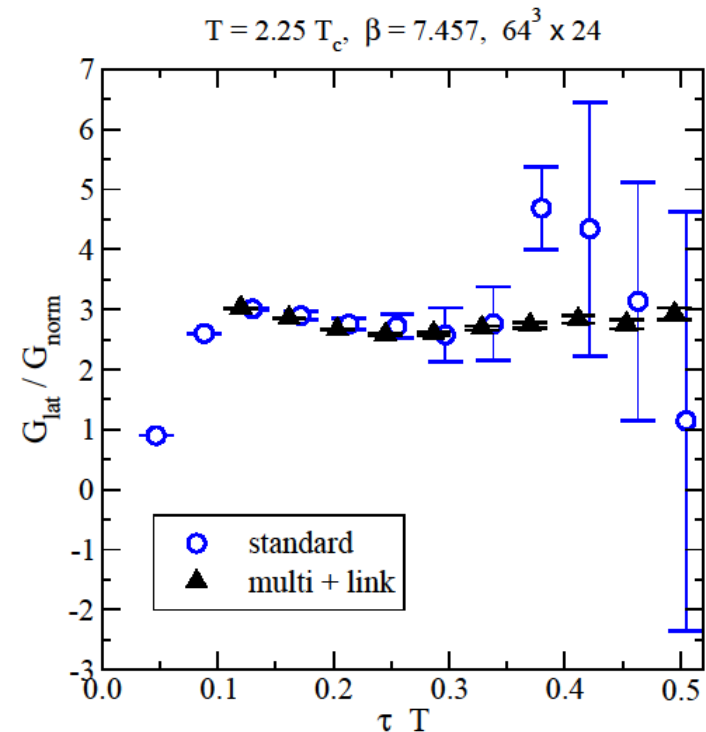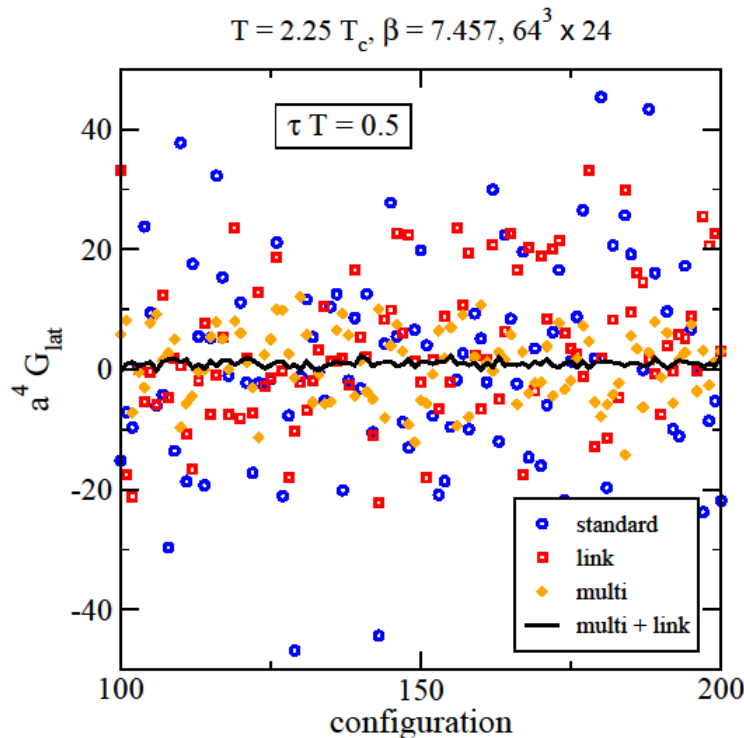$$\kappa/T^3 = \lim_{\omega \to 0} \frac{2T\rho_{\rm E}(\omega)}{\omega}$$

is expected

Qualitatively similar behaviour also found in AdS/CFT [S.Gubser, Nucl.Phys.B790 (2008)175]

Heavy Quark Momentum Diffusion Constant – Lattice algorithms

[A.Francis,OK,M.Laine,J.Langelage, arXiv:1109.3941 and arXiv:1311.3759]

due to the gluonic nature of the operator, signal is extremely noisy
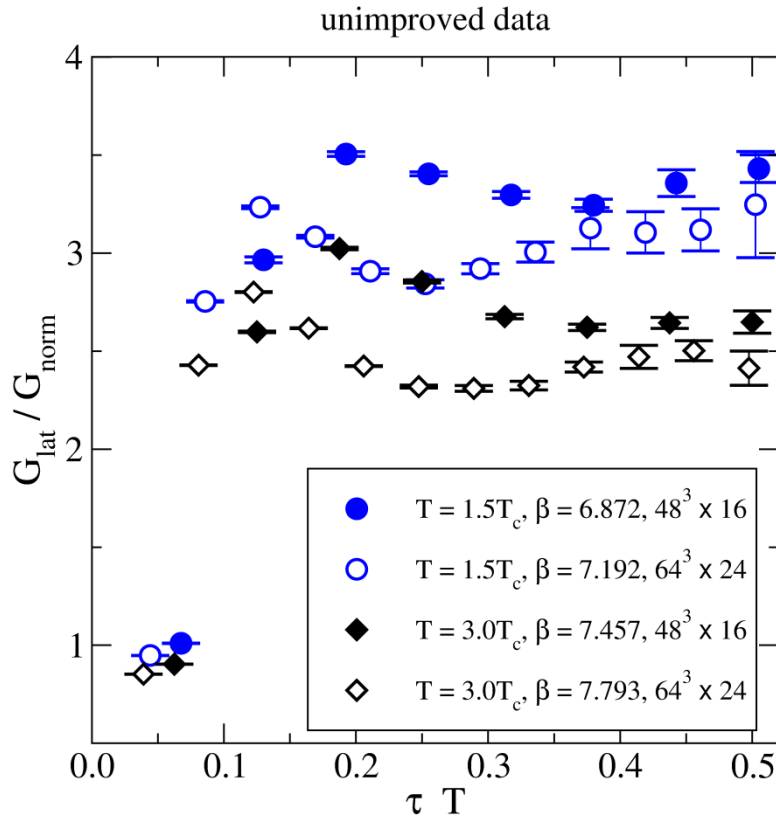
→ **multilevel combined with link-integration techniques to improve the signal**

[Lüscher,Weisz JHEP 0109 (2001)010 and H.B.Meyer PRD (2007) 101701]

[Parisi,Petronzio,Rapuano PLB 128 (1983) 418, and de Forcrand PLB 151 (1985) 77]

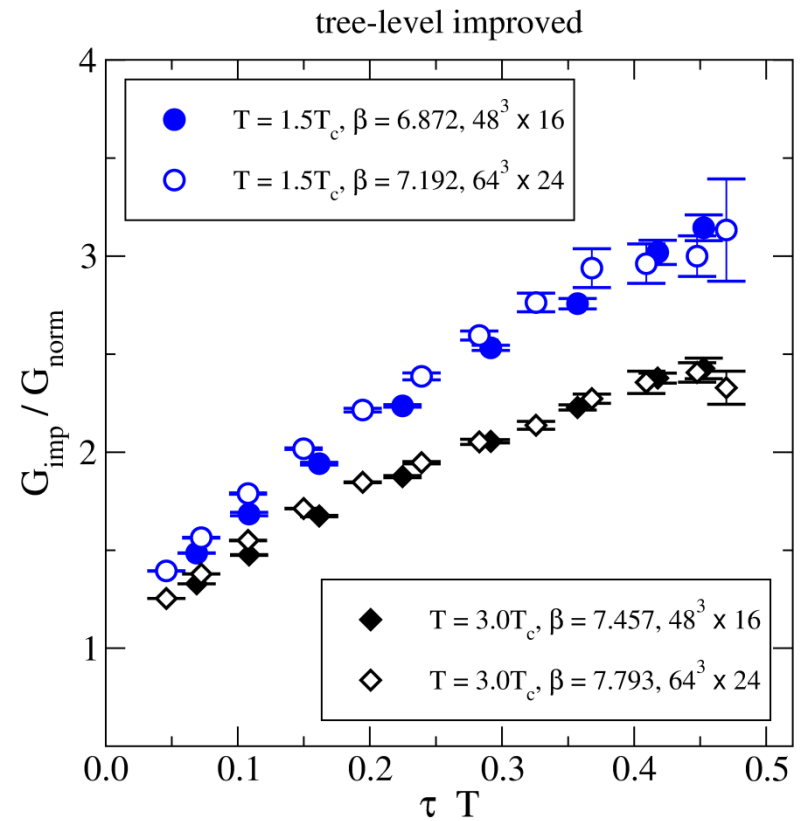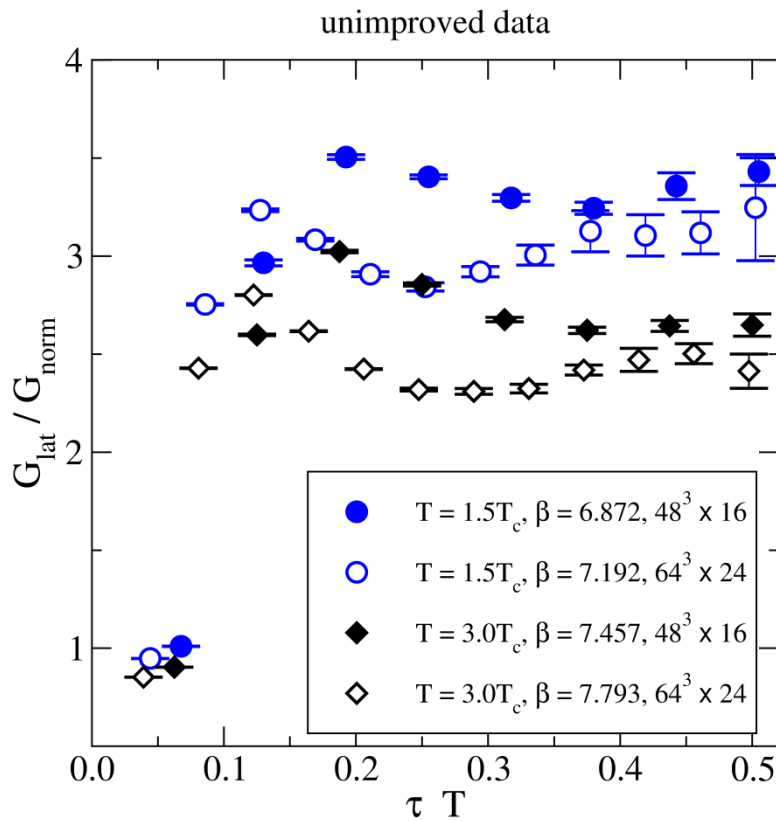[A.Francis,OK,M.Laine,J.Langelage, arXiv:1109.3941 and arXiv:1311.3759]

normalized by the LO-perturbative correlation function:

$$G_{\mathrm{norm}}(\tau T) \equiv \frac{G_{\mathrm{cont}}^{\mathrm{LO}}(\tau T)}{g^2 C_F} = \pi^2 T^4 \left[\frac{\cos^2(\pi\tau T)}{\sin^4(\pi\tau T)} + \frac{1}{3\sin^2(\pi\tau T)} \right] \qquad C_F \equiv \frac{N_c^2 - 1}{2N_c}$$

and renormalized using NLO renormalization constants $Z(g^2)$

[A.Francis,OK,M.Laine,J.Langelage, arXiv:1109.3941 and arXiv:1311.3759]

unimproved data

tree-level improved

lattice cut-off effects visible at small separations (left figure)

→ **tree-level improvement** (right figure) to reduce discretization effects

$$G_{\mathrm{cont}}^{\mathrm{LO}}(\overline{\tau T}) = G_{\mathrm{lat}}^{\mathrm{LO}}(\tau T)$$
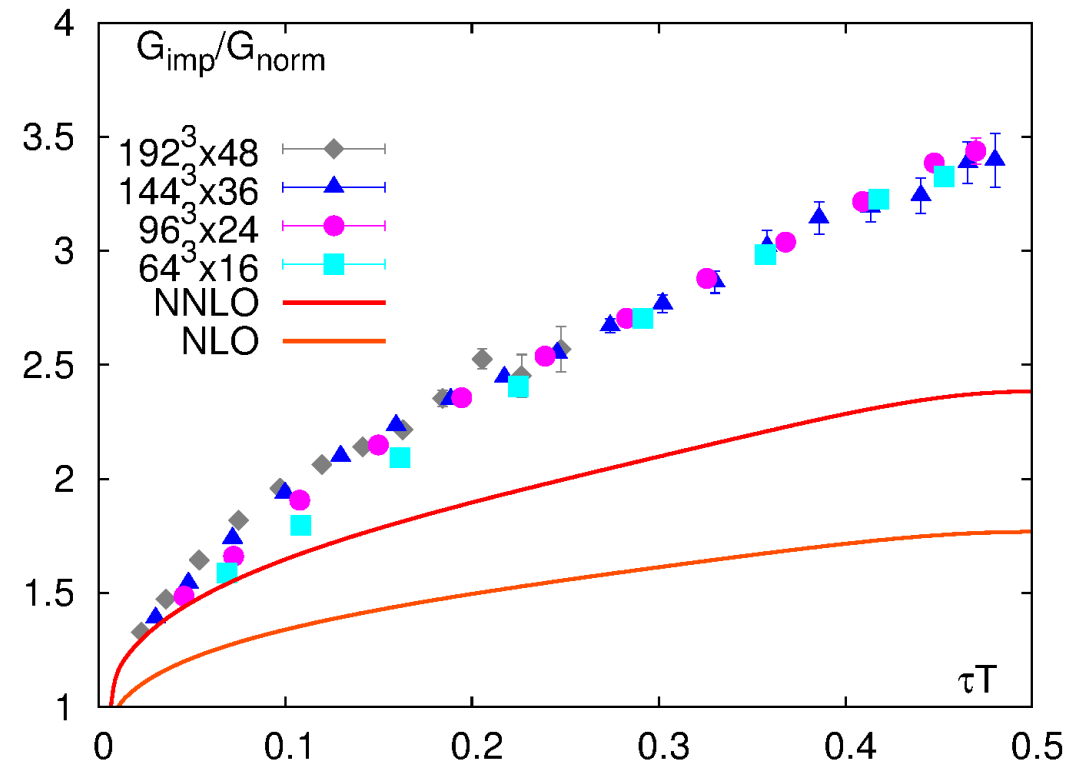
leads to an effective reduction of cut-off effect for all τT

Quenched Lattice QCD on large and fine isotropic lattices at $T \simeq 1.4\, T_c$

- standard Wilson gauge action

- algorithmic improvements to enhance signal/noise ratio

- fixed aspect ration $N_s/N_t = 4$, i.e. fixed physical volume $(2\text{fm})^3$

- perform the continuum limit, $a \to 0 \ \leftrightarrow \ N_t \to \infty$

- determine κ in the continuum using an Ansatz for the spectral fct. $\rho(\omega)$

N_σ	N_τ	β	$1/a[\text{GeV}]$	$a[\text{fm}]$	#Confs
64	16	6.872	7.16	0.03	100
96	24	7.192	10.4	0.019	160
144	36	7.544	15.5	0.013	584
192	48	7.793	20.4	0.010	223

Heavy Quark Momentum Diffusion Constant – Lattice results

finest lattices still quite noisy at large τT

but only

small cut-off effects at intermediate τT
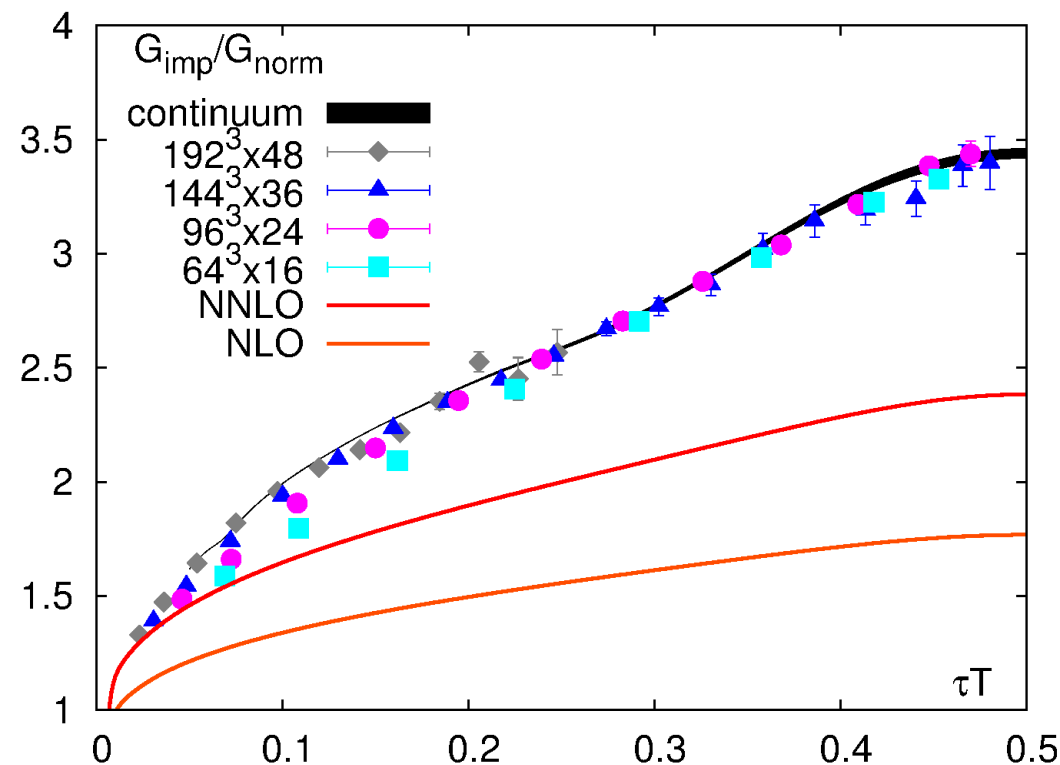
cut-off effects become visible at small τT

need to extrapolate to the continuum

perturbative behavior in the limit τT $\to$ 0

N_σ	N_τ	β	$1/a[\text{GeV}]$	$a[\text{fm}]$	#Confs
64	16	6.872	7.16	0.03	100
96	24	7.192	10.4	0.019	160
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192	48	7.793	20.4	0.010	223

allows to perform continuum extrapolation, $a \to 0 \leftrightarrow N_t \to \infty$ **, at fixed** T=1/a N_t

finest lattices still quite noisy at large τT

but only

small cut-off effects at intermediate τT

cut-off effects become visible at small τT

need to extrapolate to the continuum

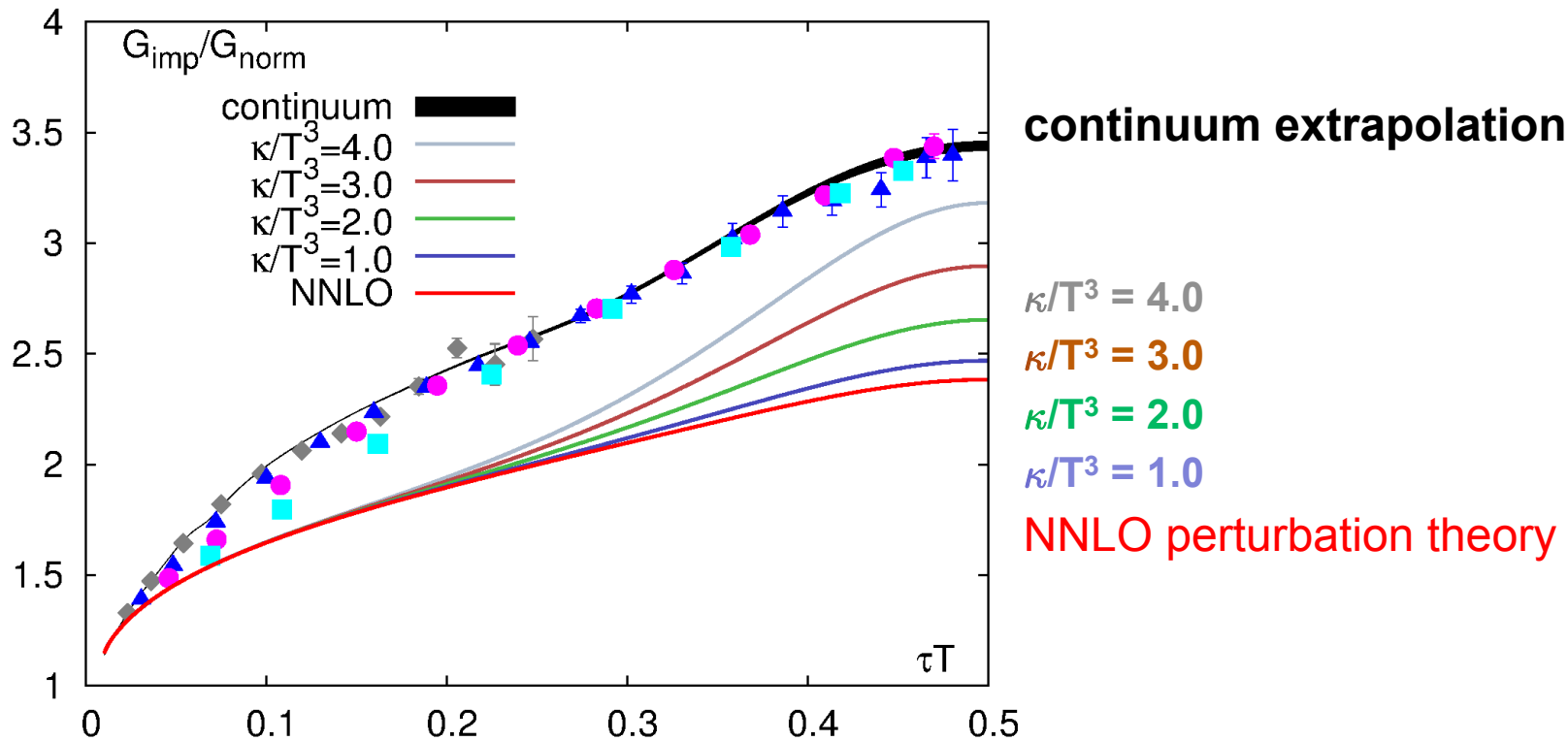**perturbative behavior in the limit τT → 0**

well behaved continuum extrapolation for $0.05 \leq \tau$T$ \leq 0.5$

finest lattice already close to the continuum

coarser lattices at larger τT close to the continuum

how to extract the spectral function from the correlator?

continuum extrapolation

$\kappa/T^3 = 4.0$

$\kappa/T^3 = 3.0$

$\kappa/T^3 = 2.0$
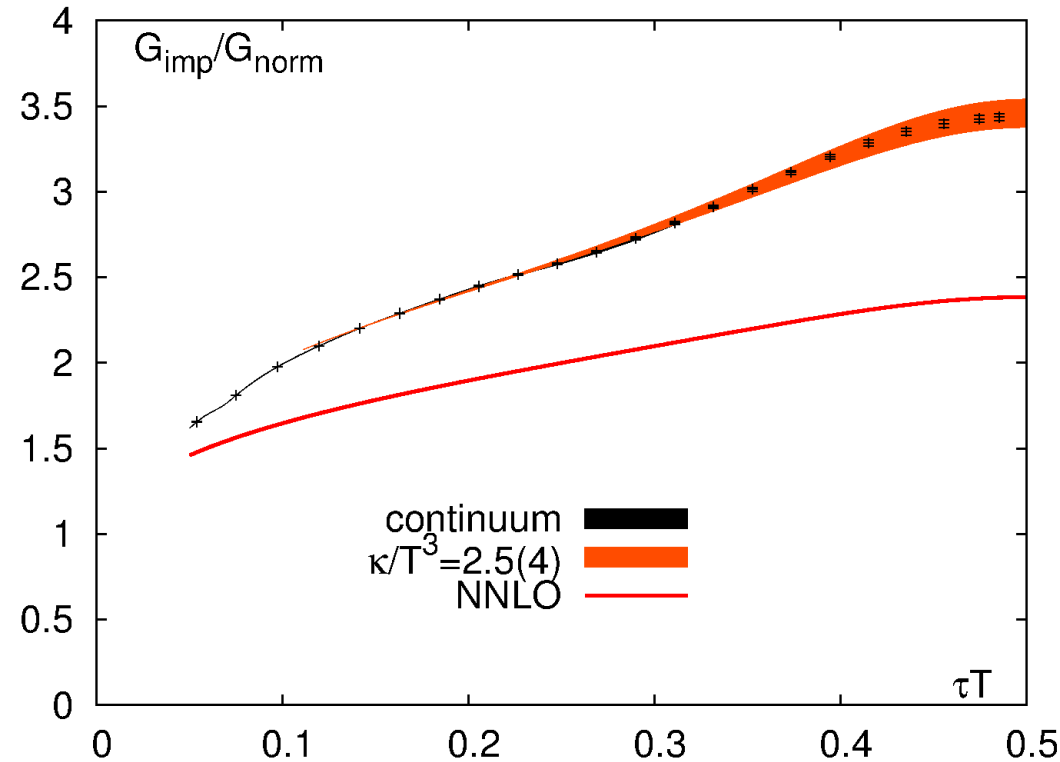
$\kappa/T^3 = 1.0$

NNLO perturbation theory

Model spectral function: transport contribution + NLO [Y.Burnier et al. JHEP 1008 (2010) 094)]

$$\rho_{\mathrm{model}}(\omega) \equiv \max\left\{ \rho_{\mathrm{NNLO}}(\omega), \frac{\omega\kappa}{2T} \right\}$$

$$G_{\mathrm{model}}(\tau) \equiv \int_0^\infty \frac{\mathrm{d}\omega}{\pi} \rho_{\mathrm{model}}(\omega) \frac{\cosh\left(\frac{1}{2} - \tau T\right)\frac{\omega}{T}}{\sinh\frac{\omega}{2T}}$$

some contribution at intermediate distance/frequency seems to be missing

result of the fit to $\rho_{model}(\omega)$

with three parameters: κ, A, B

Model spectral function: transport contribution + NNLO + correction

$$\rho_{\mathrm{model}}(\omega) \equiv \max\left\{ A\rho_{\mathrm{NNLO}}(\omega) + B\omega^3, \frac{\omega\kappa}{2T} \right\} \qquad G_{\mathrm{model}}(\tau) \equiv \int_0^\infty \frac{\mathrm{d}\omega}{\pi} \rho_{\mathrm{model}}(\omega) \frac{\cosh\left(\frac{1}{2} - \tau T\right)\frac{\omega}{T}}{\sinh\frac{\omega}{2T}}$$

used to fit the continuum extrapolated data

→ first continuum estimate of κ :

$$\kappa/T^3 = \lim_{\omega\to 0} \frac{2T\rho_{\mathrm{E}}(\omega)}{\omega} \simeq 2.5(4)$$

result of the fit to $\rho_{model}(\omega)$

$A\,\rho_{NLO}(\omega) + B\,w^3$

NNLO perturbation theory

$\dfrac{\omega\kappa}{2T}$ small but relevant contribution at $\tau T > 0.2$!

Model spectral function: transport contribution + NLO + correction

$$\rho_{\mathrm{model}}(\omega) \equiv \max\left\{ A\rho_{\mathrm{NNLO}}(\omega) + B\omega^3, \frac{\omega\kappa}{2T} \right\}$$

$$G_{\mathrm{model}}(\tau) \equiv \int_0^\infty \frac{d\omega}{\pi}\,\rho_{\mathrm{model}}(\omega)\,\frac{\cosh\left(\frac{1}{2} - \tau T\right)\frac{\omega}{T}}{\sinh\frac{\omega}{2T}}$$
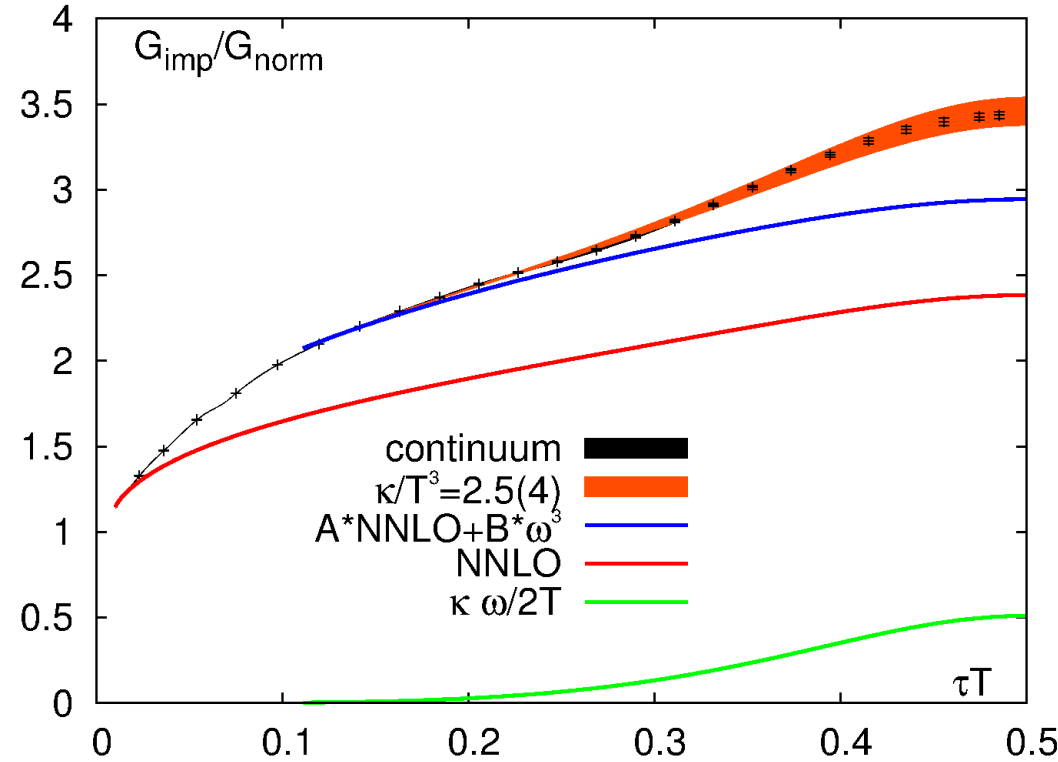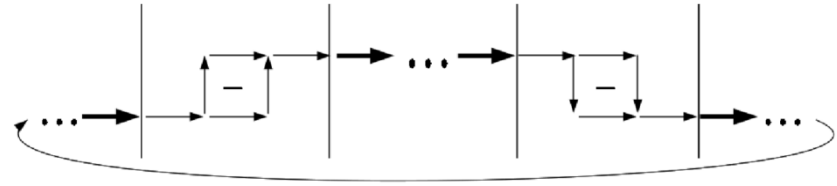
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→ **first continuum estimate of** κ :

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$$G_{\mathrm{E}}(\tau) \equiv -\frac{1}{3} \sum_{i=1}^{3} \frac{\left\langle \operatorname{Re} \operatorname{Tr}\left[U(\tfrac{1}{T}; \tau)\, g E_i(\tau, \mathbf{0})\, U(\tau; 0)\, g E_i(0, \mathbf{0}) \right] \right\rangle}{\left\langle \operatorname{Re} \operatorname{Tr}[U(\tfrac{1}{T}; 0)] \right\rangle}$$

Continuum extrapolation for the color electric correlation function

$\qquad\qquad\qquad$ **extracted from Quenched Lattice QCD**

$\qquad$ **- using noise reduction techniques to improve signal**

$\qquad$ **- and an Ansatz for the spectral function**

→ **first continuum estimate for the Heavy Quark Momentum Diffusion Coefficient κ**

More detailed analysis of the systematic uncertainties needed

$\qquad$ **- Different Ansätze for the spectral function**

$\qquad$ **- Other techniques to extract the spectral function**

Other Transport coefficients from Effective Field Theories?