Heavy Quarkonium moving in a Quark-Gluon Plasma

Joan Soto

Dept. d'Estructura i Constituents de la Matèria

and

Institut de Ciències del Cosmos

Universitat de Barcelona

(with **Miguel Ángel Escobedo**, **Floriana Giannuzzi**, **Massimo Mannarelli**)

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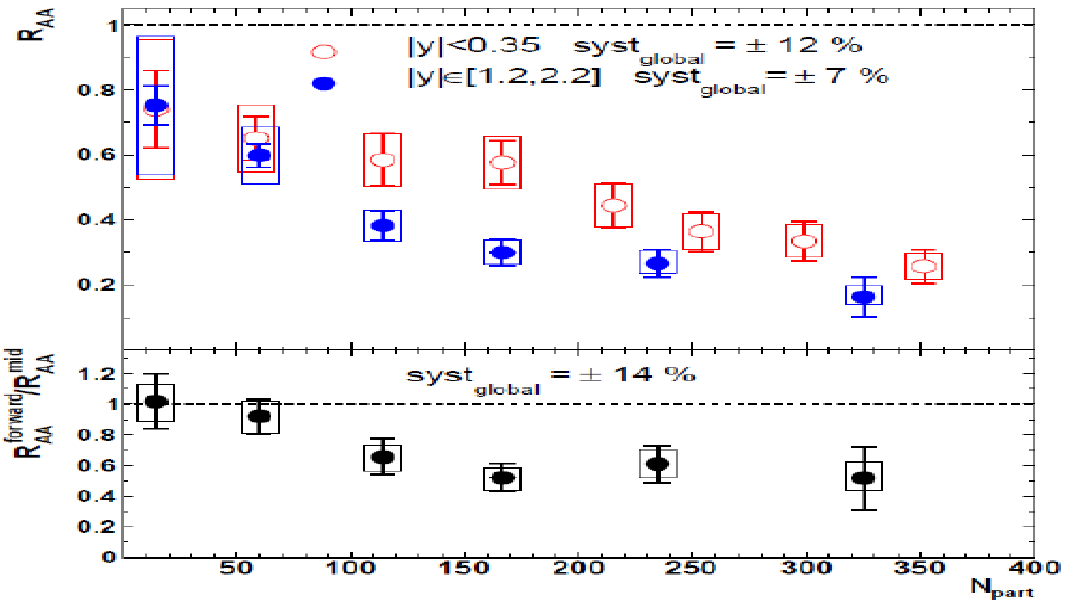
  - Fruitful strategy in the case at rest
    - Miguel Angel Escobedo, JS; Phys. Rev. A 78, 032520 (2008), Phys. Rev. A 82, 042506 (2010)
    - Nora Brambilla, Miguel Angel Escobedo, Jacopo Ghiglieri, JS, Antonio Vairo, JHEP 1009 (2010) 038

Motivation



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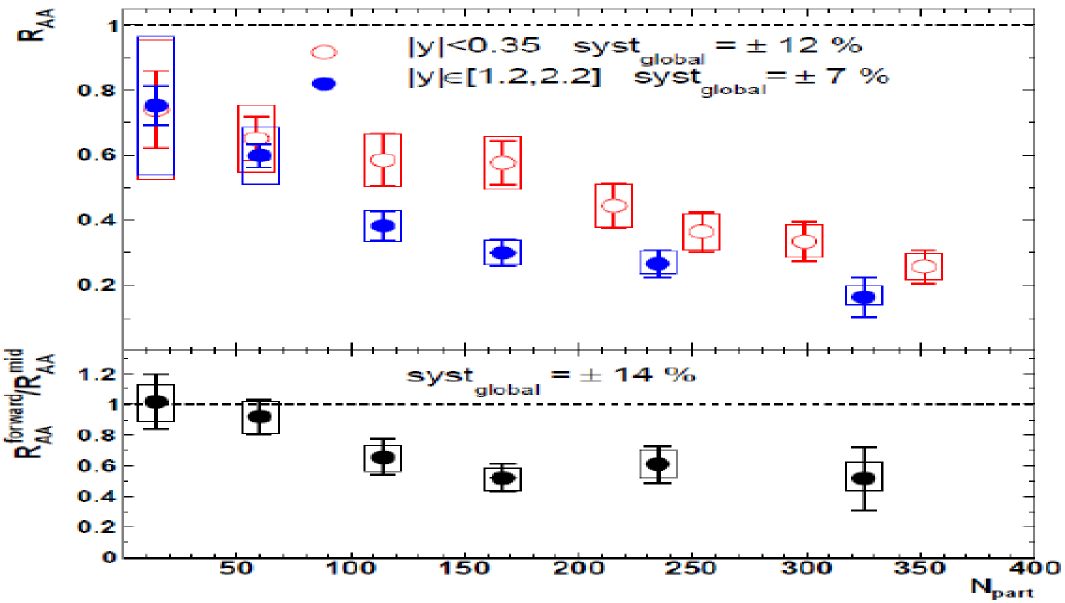

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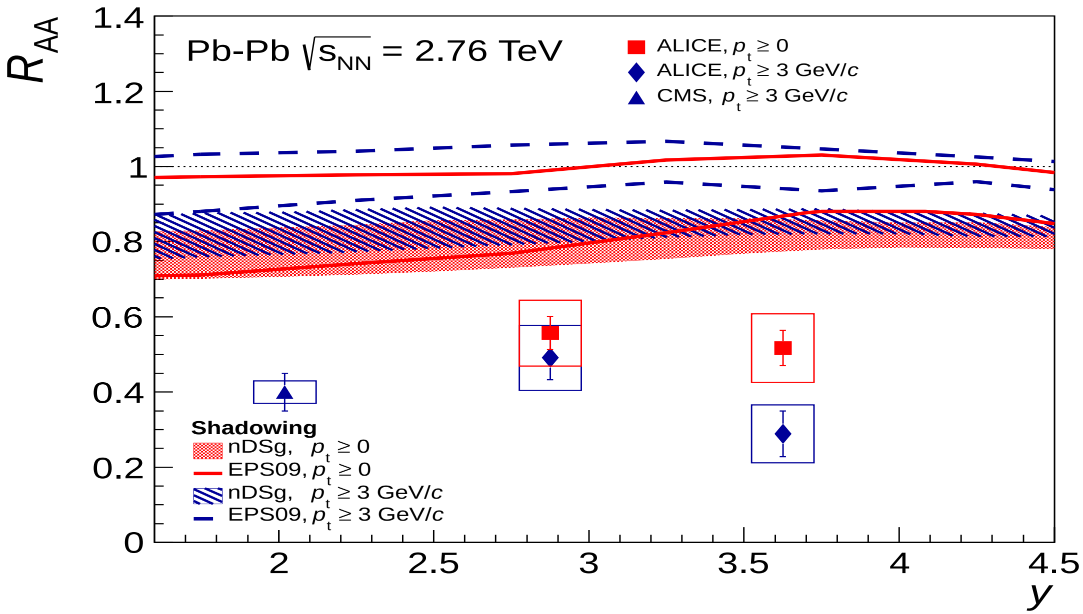

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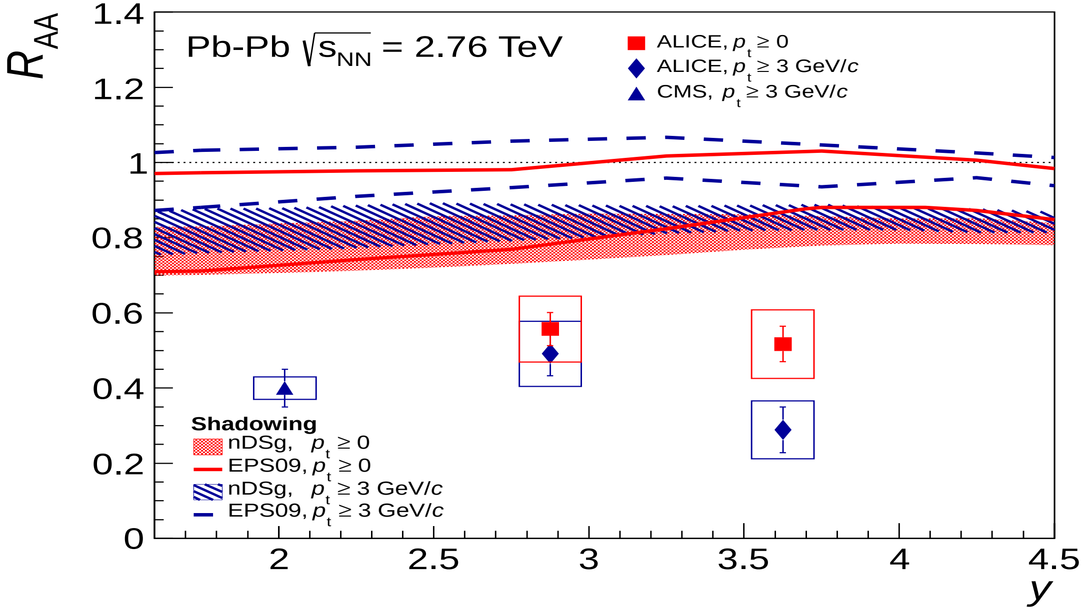

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- The J/ψ suppression may depend on the transverse momentum

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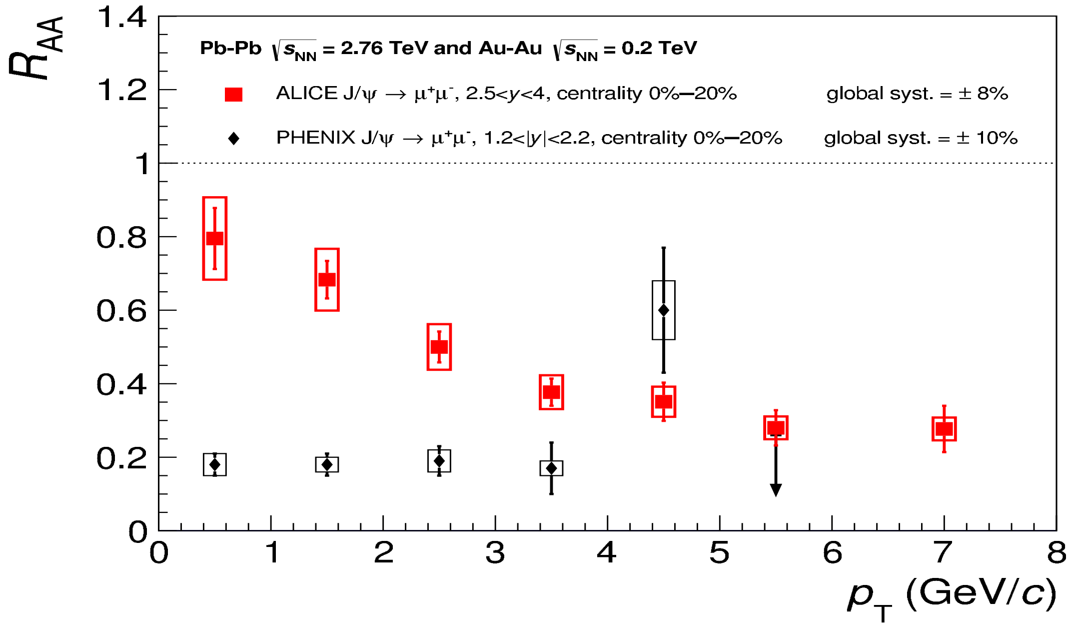

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- The J/ψ suppression does depend on the transverse momentum

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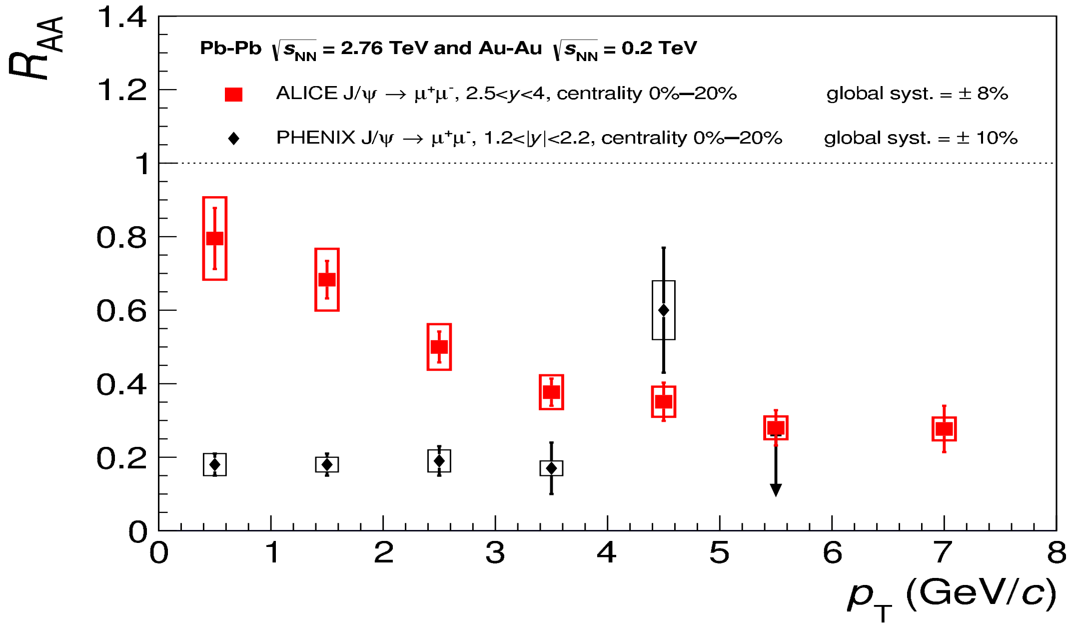

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- The J/ψ suppression does depend on the transverse momentum
- Is this due, at least in part, to the fact that the in-medium properties of J/ψ depend on the velocity ?

Relevant scales (hydrogen atom)

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- $m \neq 0$, $T = 0$ case:
  - m (hard), electron mass
  - $m\alpha/n$ (soft), inverse Bohr radius, $\alpha = e^2/4\pi$; e, electron charge
  - $m\alpha^2/n^2$ (ultrasoft), binding energy

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- $m = 0$, $T \neq 0$ case:
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- $m \neq 0$, $T \neq 0$ case: what is the interplay among the scales above?

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- $m \neq 0$, $T \neq 0$ case: contributions of energies above T are exponentially suppressed by Boltzmann factors

Non-Relativistic QED (T=0)

$$\mathcal{L}_{NRQED} = -\frac{1}{4}d_1 F_{\mu\nu}F^{\mu\nu} + \frac{d_2}{m^2}F_{\mu\nu}D^2F^{\mu\nu} + N^\dagger iD^0 N +$$
$$+\psi^\dagger(iD^0 + \frac{\mathbf{D}^2}{2m} + \frac{\mathbf{D}^4}{8m^3} + c_F e\frac{\boldsymbol{\sigma}\mathbf{B}}{2m} + c_D e\frac{\boldsymbol{\nabla}\mathbf{E}}{8m^2} +$$
$$+ic_S e\frac{\boldsymbol{\sigma}(\mathbf{D}\times\mathbf{E}-\mathbf{E}\times\mathbf{D})}{8m^2})\psi$$

(Caswell, Lepage, 1986)

Potential NRQED (T=0)

$$L_{pNRQED} = -\int d^3\mathbf{x}\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \int d^3\mathbf{x}S^\dagger(t,\mathbf{x})\Bigg(iD_0 + \frac{\boldsymbol{\nabla}^2}{2m} + \frac{Z\alpha}{|\mathbf{x}|} +$$

$$+\frac{\boldsymbol{\nabla}^4}{8m^3} + \frac{Ze^2}{m^2}\left(-\frac{c_D}{8} + 4d_2\right)\delta^3(\mathbf{x}) + ic_S\frac{Z\alpha}{4m^2}\boldsymbol{\sigma}\cdot\left(\frac{\mathbf{x}}{|\mathbf{x}|^3}\times\boldsymbol{\nabla}\right)\Bigg)S(t,\mathbf{x})$$

$$+\int d^3\mathbf{x}S^\dagger(t,\mathbf{x})e\mathbf{x}\cdot\mathbf{E}S(t,\mathbf{x})\,.$$

(Pineda, Soto, 1997)

Hard Thermal Loops EFT (m=0)

$$\delta\mathcal{L}_{HTL} = \frac{1}{2} m_D^2 F^{\mu\alpha} \int \frac{d\Omega}{4\pi} \frac{k_\alpha k_\beta}{-(k.\partial)^2} F^{\mu\beta} + m_e^2 \bar{\psi}\gamma^\mu \int \frac{d\Omega}{4\pi} \frac{k_\mu}{k.\partial} \psi$$

$$k = (1, \hat{\mathbf{k}}), \qquad m_D^2 = e^2 T^2/3, \qquad m_e^2 = e^2 T^2/8$$

(Braaten, Pisarsky, 1992)

The $v \neq 0$ case

- Bound state at rest, the medium moves at velocity v (Weldom, 82)

$$f(\beta k^0) \rightarrow f(\beta^\mu k_\mu) = \frac{1}{e^{|\beta^\mu k_\mu|} \pm 1}, \quad \beta^\mu = \frac{\gamma}{T}(1, \mathbf{v})$$

$v = |\mathbf{v}|$, $\gamma = 1/\sqrt{1 - v^2}$

- $O(3)$ rotational symmetry is reduced to $O(2)$
- In light cone coordinates $k_+ = k_0 + k_3$, $k_- = k_0 - k_3$

$$\beta^\mu k_\mu = \frac{1}{2}\left(\frac{k_+}{T_+} + \frac{k_-}{T_-}\right), \quad T_+ = T\sqrt{\frac{1+v}{1-v}}, \quad T_- = T\sqrt{\frac{1-v}{1+v}}$$

  - For $v \not\sim 1$ (moderate velocities), $T_+ \sim T \sim T_-$
  - For $v \sim 1$ (relativistic velocities), $T_+ \gg T \gg T_-$
    - Collinear region, $k_+ \sim T_+$, $k_- \sim T_-$
    - Soft (ultrasoft) region, $k_+ \sim k_- \sim T_-$

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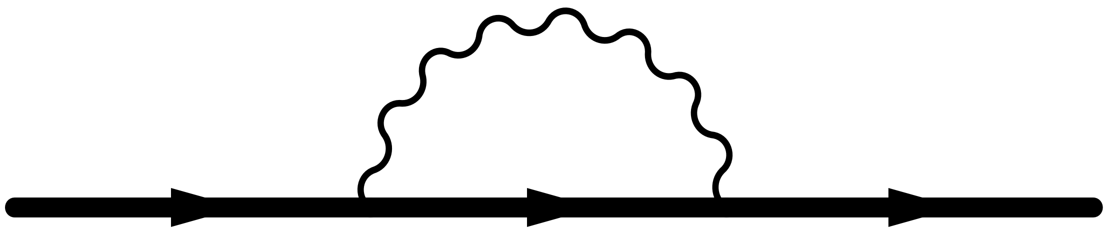
  - Thermal effects are encoded in the ultrasoft photons



$v \not\approx 1$: the $T \ll m\alpha/n$ case in QCD

- For $T = \beta^{-1} \gg m\alpha^2/n^2$,

$$\delta E_{nlm} = \frac{2\pi C_F T^2}{3}\left[\frac{\alpha_s}{m_Q} + \frac{N_c\alpha_s^2}{2}\langle r\rangle_{nlm} + \frac{N_c\alpha_s^2}{2}\langle r\rangle_{nlm}(1-3f(v))\langle 2l00|l0\rangle\langle 2l0m|lm\rangle\right] + \mathcal{O}(\alpha_s r^2 E^3, \alpha_s^2 r^2 T^3)$$

$$f(v) = \frac{1}{v^3}\Big(v(2-v^2) - 2(1-v^2)\tanh^{-1}(v)\Big)$$

- For $l \neq 0$ the energy shift already depends on the velocity at LO !

$v \not\approx 1$: the $T \ll m\alpha/n$ case in QCD

- For $T = \beta^{-1} \gg m\alpha^2/n^2$,

$$\delta\Gamma_{nlm} = \frac{\alpha_s C_F T\sqrt{1-v^2}}{3v}\left[4\left(-\frac{2E_n^c}{m_Q} + \frac{\alpha_s N_c}{m_Q a_0 n^2} + \frac{\alpha_s^2 N_c^2}{8}\right)\log\left(\frac{1+v}{1-v}\right) + \right.$$
$$\left. + \left(-\frac{4E_n^c}{m_Q} - \frac{\alpha_s N_c}{m_Q a_0 n^2} + \frac{\alpha_s^2 N_c^2}{4}\right) h_{lm}(v)\right] + \mathcal{O}(\alpha_s r^2 E^3, \alpha_s^2 r^2 T^3)$$

$$h_{lm}(v) = \left[\left(1 - \frac{3}{v^2}\right)\log\left(\frac{1+v}{1-v}\right) + \frac{6}{v}\right]\langle 2l00|l0\rangle\langle 2l0m|lm\rangle$$

- The decay width decreases as v increases !

Moderate velocity ($v \not\propto 1$)

- The $\frac{m\alpha}{n} \sim T \ll m$ case: NRQED can be used as a starting point
  - The potentials depend on T:

$$\delta\mathcal{L}_{pNRQED} = \int d^3\mathbf{x} \left(\frac{\alpha\pi T^2}{3m_e} \psi^\dagger \psi \right)$$

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$$+ \int d^3\mathbf{x_1}\, d^3\mathbf{x_2} N^\dagger(t,\mathbf{x_2})N(t,\mathbf{x_2}) \left[-\frac{4Z\alpha}{3m_e^2}\left(\log\left(\frac{\mu}{2\pi T}\right) - \log 2 \right.\right.$$

$$\left. +\gamma + \frac{3}{8v}\left(1+\frac{1}{3v^2}\right)\log\left(\frac{1+v}{1-v}\right) - \frac{1}{4v^2}\right)\delta^3(\mathbf{x_1}-\mathbf{x_2})$$

$$\left. + \frac{\alpha\rho(v)v^i v^j \partial^2_{ij}V_c(r)}{4\pi m_e^2 v^2} \right] \psi^\dagger(t,\mathbf{x_1})\psi(t,\mathbf{x_1})$$

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  - μ, factorization scale arising from IR divergences, which cancels against the one of ultrasoft contributions

$v \not\approx 1$: the $\frac{m\alpha}{n} \sim T \ll m$ case

- In the ultrasoft contributions,

$$1/(e^{|\beta^\mu k_\mu|} - 1) \to 1/|\beta^\mu k_\mu| - 1/2 + \cdots$$

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$$\delta E_{nlm} = \frac{\alpha \pi T^2}{3 m_e}$$

$$\delta \Gamma_{nlm} = \frac{Z^2 \alpha^3 T \sqrt{1 - v^2}}{3 n^2 v} \left(2 \log \left(\frac{1+v}{1-v} \right) + h_{lm}(v) \right)$$

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$$\delta E_{nlm} = \frac{\alpha\pi T^2}{3m_e} - \frac{\pi\alpha^3 T^2}{6m_e n^2} + \frac{4Z\alpha^2}{3m_e^2}\left(\gamma + \frac{3}{8v}\left(1 + \frac{1}{3v^2}\right) - \frac{1}{4v^2}\right)|\phi_n(\mathbf{0})|^2$$
$$-\frac{\alpha\rho(v)v^i v^j}{4\pi m_e^2 v^2}\langle n|\partial_{ij}^2 V_c(r)|n\rangle + \frac{2\alpha}{3\pi m_e^2}\sum_r |\langle n|\mathbf{p}|r\rangle|^2 (E_n - E_r)\left(\log\left(\frac{2\pi T}{|E_n - E_r|}\right) + \frac{5}{6}\right)$$

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Ultrarelativistic velocity ($v \sim 1$)

- Two cases analysed in detail:
  - The $T_+ \sim m\alpha/n \gg T_- \gg m\alpha^2/n^2$ case:
    - NRQED can be used as a starting point
    - Match to pNRQED + SCET
  - The $T_+ \sim m \gg m\alpha/n \gg T_- \gg m\alpha^2/n^2$ case:
    - QED must be used as a starting point
    - Match to NRQED + SCET
    - Match to pNRQED
  - Agreement with the $v \to 1$ limit of the $v \not\sim 1$ case

The $m\alpha/n \ll T \ll m$ case

- Results hold for both muonic hydrogen and heavy quarkonium [$\alpha \leftrightarrow C_f\alpha_s$, $m_\mu \leftrightarrow m_Q/2$, $m_D^2 = e^2T^2/3 \leftrightarrow m_D^2 = g^2T^2(N_c + N_f/2)$]
- Moderate velocity $(v \not\sim 1)$ presented, but results expected to hold for any v

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- One next matches to pNRQED, obtaining a v and T dependent potential

The $m\alpha/n \ll T \ll m$ case at $v = 0$

$$V(r,T) = -\frac{\alpha e^{-m_D r}}{r} - \alpha m_D + i\alpha T\phi(m_D r) + \mathcal{O}\left(\frac{\alpha T^2}{m_\mu}\right)$$

$$\phi(x) = 2\int_0^\infty \frac{dz z}{(z^2+1)^2}\left[\frac{\sin(zx)}{zx} - 1\right]$$

- It has an imaginary part ! (Laine, Philipsen, Romatschke, Tassler, 06; Escobedo, JS, 08; Brambilla, Ghiglieri, Vairo, Petreczky, 08)
- The disociation temperature becomes

$$T_d \sim m_\mu \alpha^{2/3}/\ln^{1/3}\alpha \, < \, m_\mu \alpha^{1/2}$$

where $m_\mu \alpha^{1/2}$ is the scale of the disociation temperature for the screening mechanism (Matsui, Satz, 86)

The $m\alpha/n \ll T \ll m$ case at $v \neq 0$

- $\Re V(r,T)$ calculated before (Chu, Matsui, 89)
- $V(r,T)$ is given by the Fourier transform of the longitudinal photon propagator $\Delta_{11}(k)$ at $k^0 = 0$ in the v-dependent HTL Lagrangian

$$\Delta_{11}(k) = \frac{1}{2}[\Delta_R(k) + \Delta_A(k) + \Delta_S(k)]$$

$\Delta_R^*(k) = \Delta_A(k)$, $\Delta_S(k)$ contains the imaginary part

- $\Delta_S(k)$ must be calculated through the following formula, which differs from the one of the $v = 0$ case (Carrington, Hou, Thoma, 97)

$$\Delta_S(k,u) = \frac{\Pi_S(k,u)}{2i\Im\Pi_R(k,u)}(\Delta_R(k,u) - \Delta_A(k,u))$$

$u = \gamma(1, \mathbf{v})$. Recall that in the real time formalism $\Pi_R = \Pi_{11} + \Pi_{12}$, $\Pi_S = \Pi_{11} + \Pi_{22},\ \Delta_R(k) = 1/(\mathbf{k}^2 - \Pi_R)$

The $m\alpha/n \ll T \ll m$ case at $v \neq 0$

- $\Pi_R(k,u)$ is a (complex) function of v and θ, $\mathbf{kv} = |\mathbf{k}| v\cos\theta$, that reduces to $-m_D^2$ when $v = 0$

$$\Pi_R(k,u) = -m_D^2(v,\theta) = -\left(a(z) + \frac{b(z)}{1-v^2}\right) \quad , \quad z = \frac{v\cos\theta}{\sqrt{1-v^2\sin^2\theta}}$$

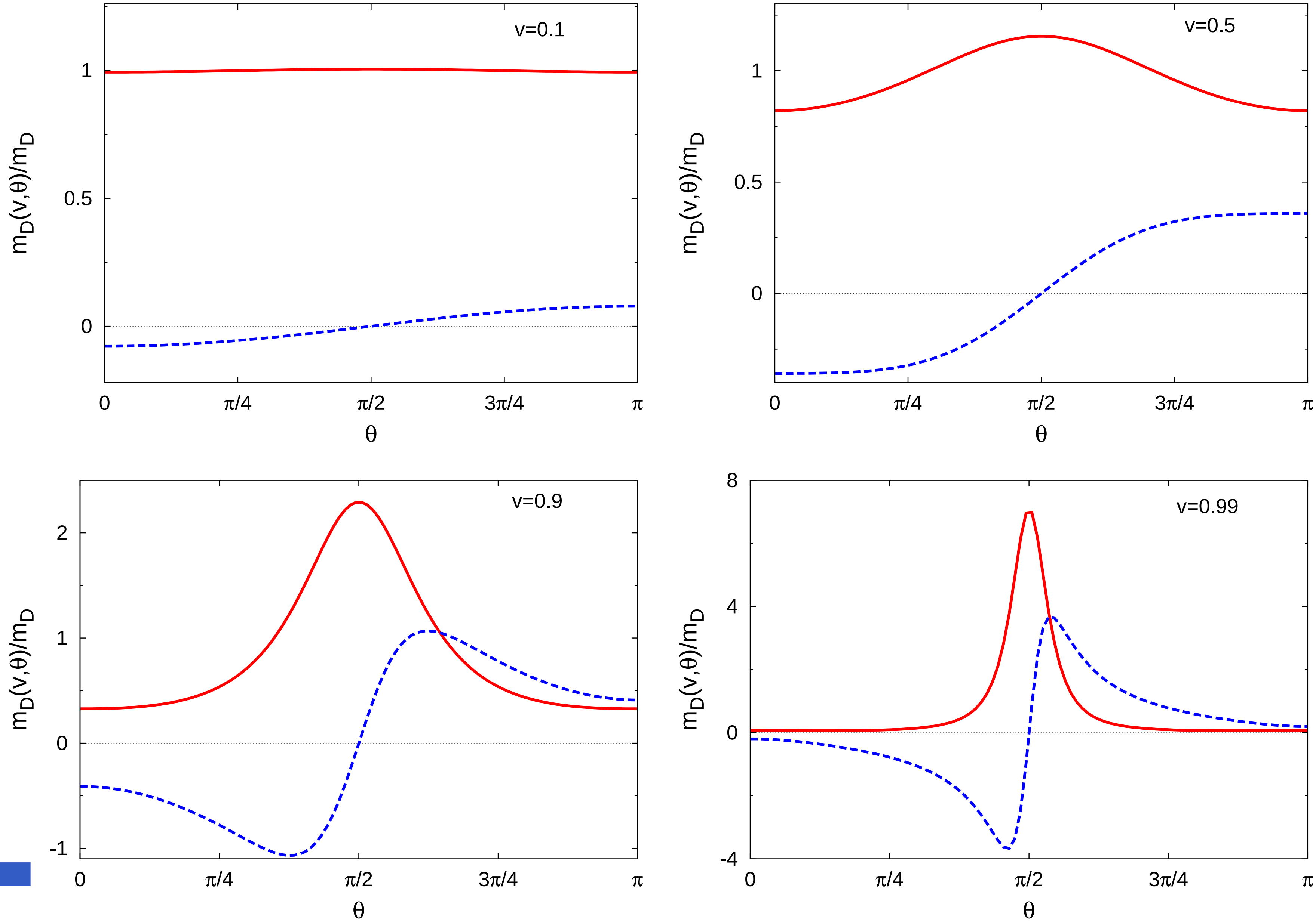
$$a(z) = \frac{m_D^2}{2}\left[z^2 - (z^2-1)\frac{z}{2}\ln\left(\frac{z+1+i\epsilon}{z-1+i\epsilon}\right)\right]$$

$$b(z) = (z^2-1)\left[a(z) - m_D^2(1-z^2)\left(1-\frac{z}{2}\ln\left(\frac{z+1+i\epsilon}{z-1+i\epsilon}\right)\right)\right]$$

- We obtain

$$\Pi_S(k,u) = \frac{i2\pi m_D^2 T(1-v^2)^{3/2}(1+\frac{v^2}{2}\sin^2\theta)}{|\mathbf{k}|(1-v^2\sin^2\theta)^{5/2}} = i\frac{2\pi m_D^2 T}{\mathbf{k}} f(v,\theta)$$

The $m\alpha/n \ll T \ll m$ case at $v \neq 0$



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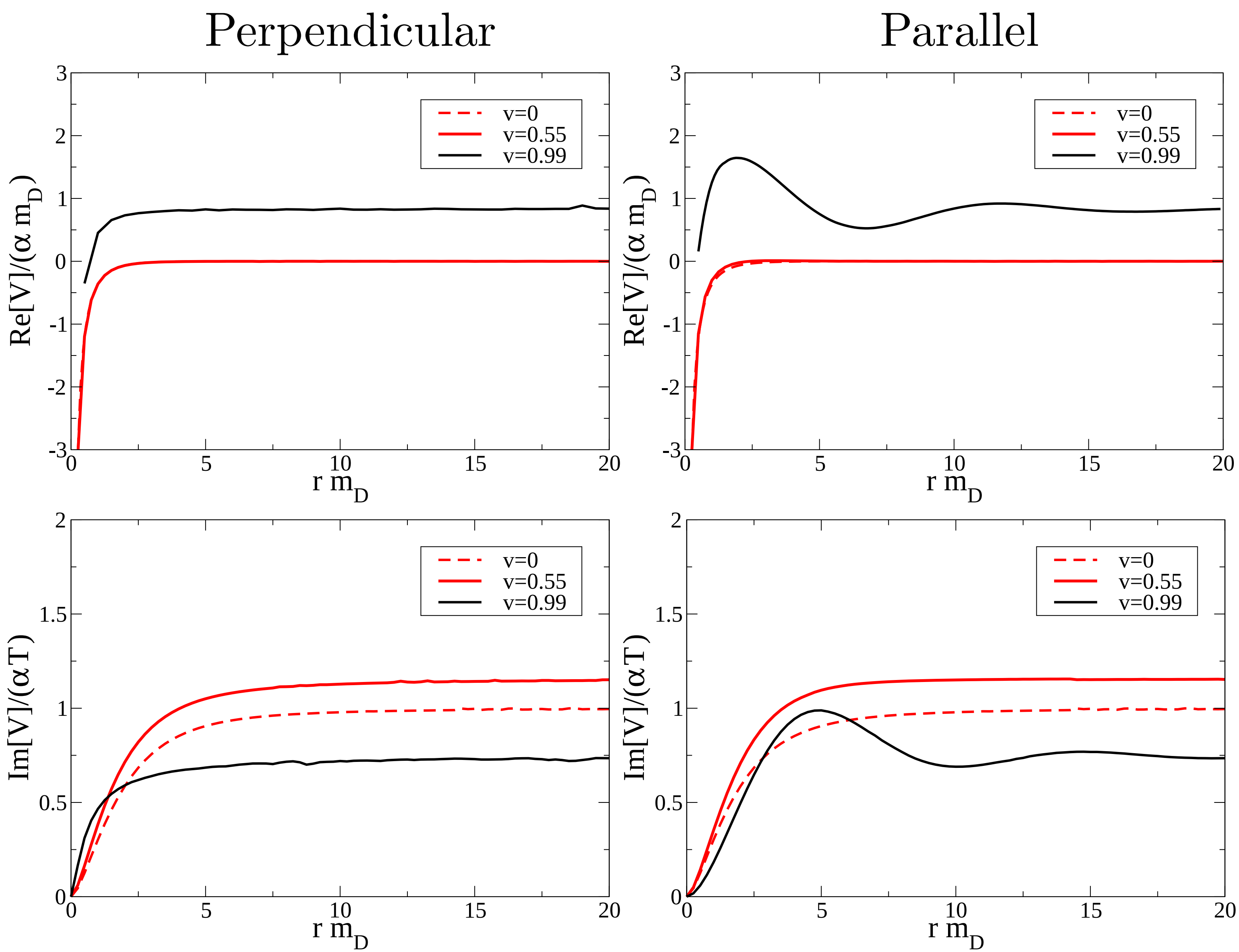


The $m\alpha/n \ll T \ll m$ case at $v \neq 0$

- The particular case $T \gg 1/r \sim 1/a_0 \gg m_D \gg E$ for 1S states

$$\Gamma_1^{s-wave} = 2\alpha_s C_F T m_D^2 \int_{-1}^{1} d\cos\theta f(v,\theta) \times$$

$$\times \int_0^\infty \frac{dkk}{(k^2 + m_D^2 g(z,v))(k^2 + m_D^2 g^*(z,v))} \left(1 - \frac{1}{(1 + \frac{k^2 a_0^2}{4})^2}\right)$$

$$\Gamma_1^{s-wave} = \frac{2\alpha_s C_F T m_D^2 a_0^2}{\sqrt{1-v^2}} \left[\log\left(\frac{2}{m_D a_0}\right) + \mathcal{O}(1)\right]$$

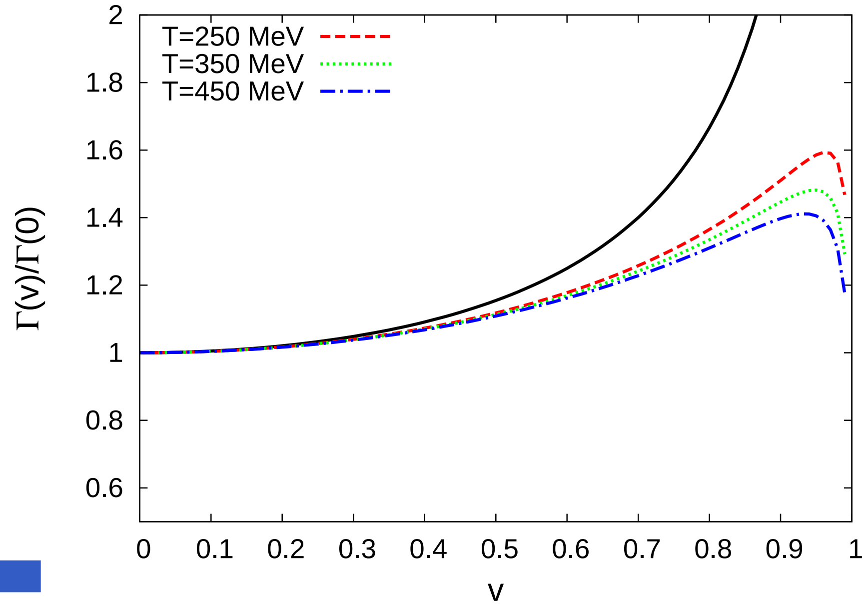
$$\frac{\Gamma_1^{s-wave}(v)}{\Gamma_1^{s-wave}(v=0)} \sim \frac{1}{\sqrt{1-v^2}}$$

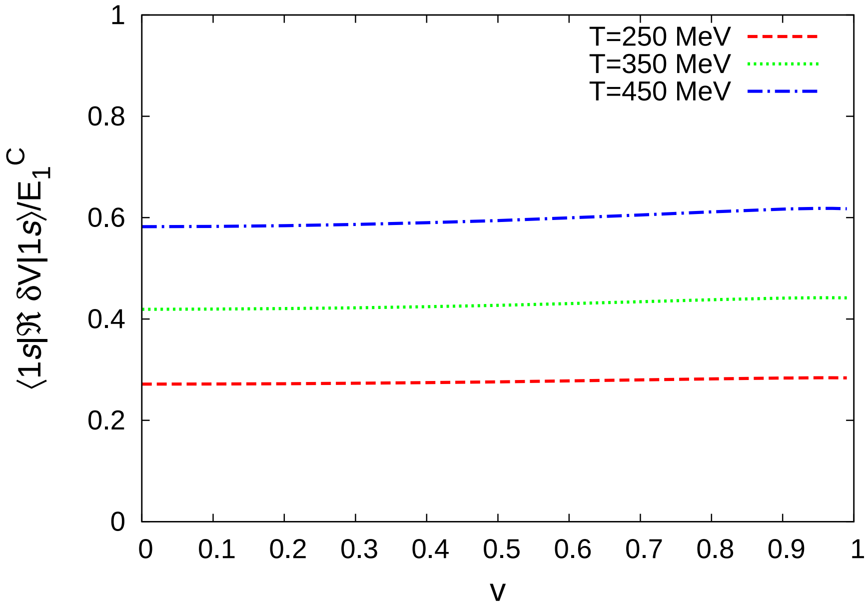
The $m\alpha/n \ll T \ll m$ case at $v \neq 0$

- The particular case $T \gg 1/r \sim 1/a_0 \gg m_D \gg E$ for 1S states
  - For $v \to 1$, new scales appear, analogous analysis leads to,

$$\Gamma \sim \alpha_s T \sqrt{1 - v^2} \quad , \quad \delta E \to \text{const.}$$

  - Numerical results:

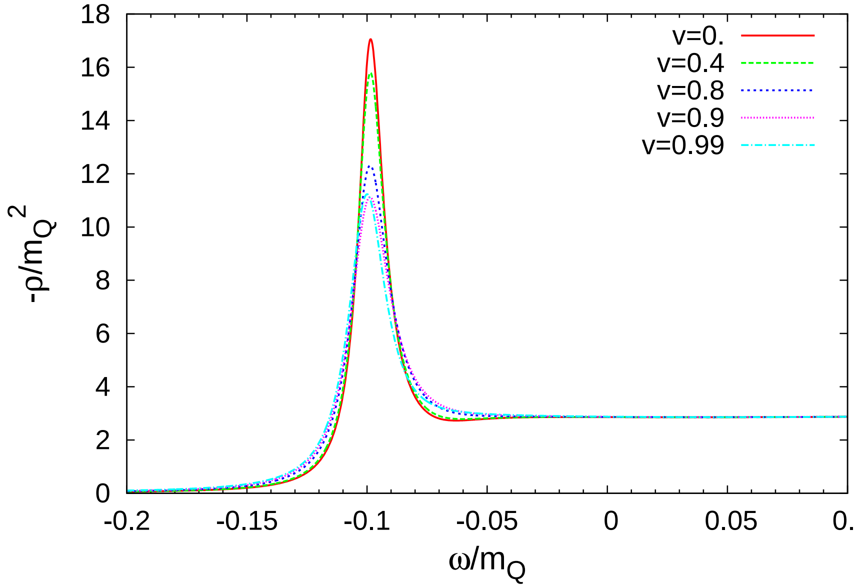




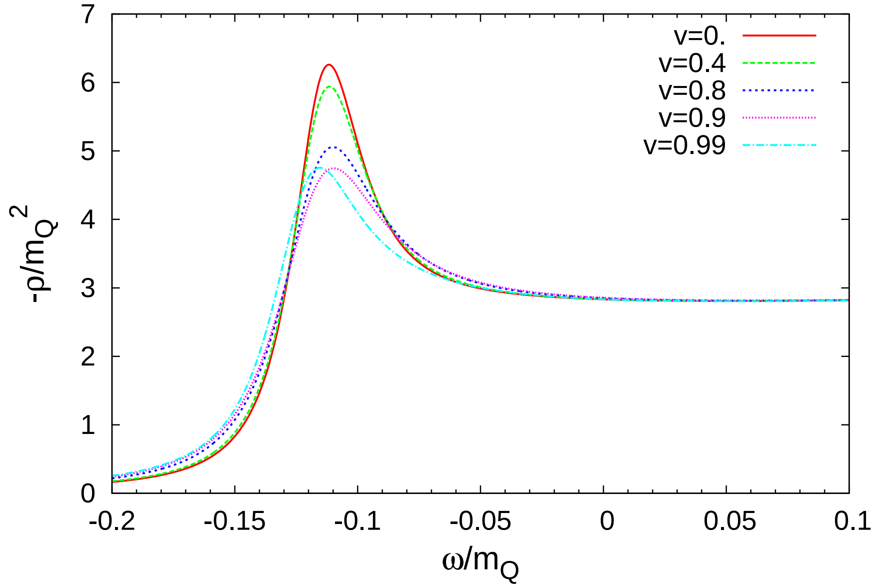
The $m\alpha/n \ll T \ll m$ case at $v \neq 0$

- The $\Upsilon(1S)$ spectral function at $v \neq 0$

$T = 250$ MeV



$T = 400$ MeV



Comparison with lattice results

- The $T \lesssim m\alpha/n$ case: compatible with Aarts, Allton, Kim, Lombardo, Oktay, Ryan, Sinclair, Skullerud, 2012, negative signal for $v \lesssim 0.2$.
- The $m\alpha/n \ll T \ll m$ case: compatible with Nonaka, Asakawa, Kitazawa, Kohno, 2011:

p	v	Γ_{lattice} (MeV)	$\Gamma \sim 1/\sqrt{1-v^2}$ (MeV)
0	0	106	input
6	0.6	135	132
7	0.65	134	139
8	0.67	128	142

Comparison with continuum approaches

- The $T \lesssim m\alpha/n$ case:
  - Weak coupling: compatible with LO calculation of Song, Park, Lee, Wong, 2008.
- The $m\alpha/n \ll T \ll m$ case:
  - Weak coupling: compatible with NLO calculation of Song, Park, Lee, Wong, 2008 and with Dominguez, Wu, 2009.
  - Strong coupling: spectral functions in qualitative agreement with the AdS/QCD calculations of Myers, Sinha, 2008; Fujita, Fukushima, Misumi, Murata, 2009.
  - Stability at ultrarelativistic velocities missed before. OK with Kopeliovich, Potashnikova, Schmidt, Siddikov, 2014.
- Consistent with arguments and results of Zhao, Rapp, 2008.

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- This suggests a non-trivial behavior of the yields as a function of the transverse momentum