Lattice QCD results for b hadron decays

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Royal Society meeting on Heavy Quarks

CKM fits

CKM mechanism: mixing of mass and weak eigenstates

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

Expansion based on empirical observation

$$|V_{us}| = 0.22 \ll 1 \qquad |V_{cb}| \approx |V_{us}|^2 \qquad |V_{ub}| \ll |V_{cb}|$$

$$\begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

In practice, go to next order $\bar{\rho} = \rho\left(1 - \frac{\lambda^2}{2}\right) \qquad \bar{\eta} = \eta\left(1 - \frac{\lambda^2}{2}\right)$

Table of quantities

quantity	process	LQCD matrix el.
ε	K	B
Δ	B	f
|	$B \rightarrow \pi\, l\, \nu$	f
|	$B \rightarrow \tau\, \nu$	f
|	B	$\mathcal{F}(w=1)$

Snapshot of recent work

f_B, f_{B_s}

ETM, PoS(LAT2009);
HPQCD, PRL 92 (2004);
FNAL/MILC, PoS(LAT2008);
HPQCD, PRD 80 (2009);
HPQCD, PRL 110 (2013);
ETMC, JHEP (2014);
ALPHA, PLB735 (2014);
RBC-UKQCD, arXiv:1404.4670;
Aoki, et al, arXiv:1406.6192

B_{B_d}, B_{B_s}

HPQCD, PRD 76 (2007);
RBC-UKQCD, PoS(LAT2007);
HPQCD, PRD 80 (2009);
RBC-UKQCD, PRD 82 (2010);
ETMC, JHEP (2014);
Aoki, et al, arXiv:1406.6192

$f_+^{B\to\pi}(q^2)$

HPQCD, PRD 73 (2006);
FNAL/MILC, PRD 79 (2009) 054507;
FNAL/MILC, PRD 80 (2010);
RBC-UKQCD, arXiv:1501.05373

$\mathcal{F}^{B\to D}(1)$

FNAL/MILC, NPB Proc Suppl (2005);
FNAL/MILC, PRD 85 (2012);
FNAL/MILC, PRL 109 (2012)

$\mathcal{F}^{B\to D^*}(1)$

FNAL/MILC, PRD 79 (2009);
FNAL/MILC, PRD 89 (2014)

$\hat{B}_K$

JLQCD, PRD 77 (2008);
HPQCD, PRD 73 (2006);
RBC-UKQCD, PRL 100 (2008);
Aubin et al., PRD 81 (2010);
BMW, PLB 705 (2011);
Bae et al, PRL 109 (2012)

$f_+^{K\to\pi}(0)$

RBC-UKQCD, PRL 100 (2008);
ETM, PRD 80 (2009);
RBC-UKQCD, EPJ C69 (2010)

f_π, f_K

NPLQCD, PRD 75 (2007);
HPQCD, PRL 100 (2008);
QCDSF, PoS(LAT2007);
PACS-CS, PoS(LAT2008);
PACS-CS, PRD 79 (2009);
RBC-UKQCD, PRD 78 (2008);
Aubin et al., PoS(LAT2008);
MILC, PoS(CD09);
MILC, RMP 82 (2010);
JLQCD/TWQCD, PoS(LAT2009);
ETM, JHEP 07 (2009);
BMW, PRD 82 (2010)

1995

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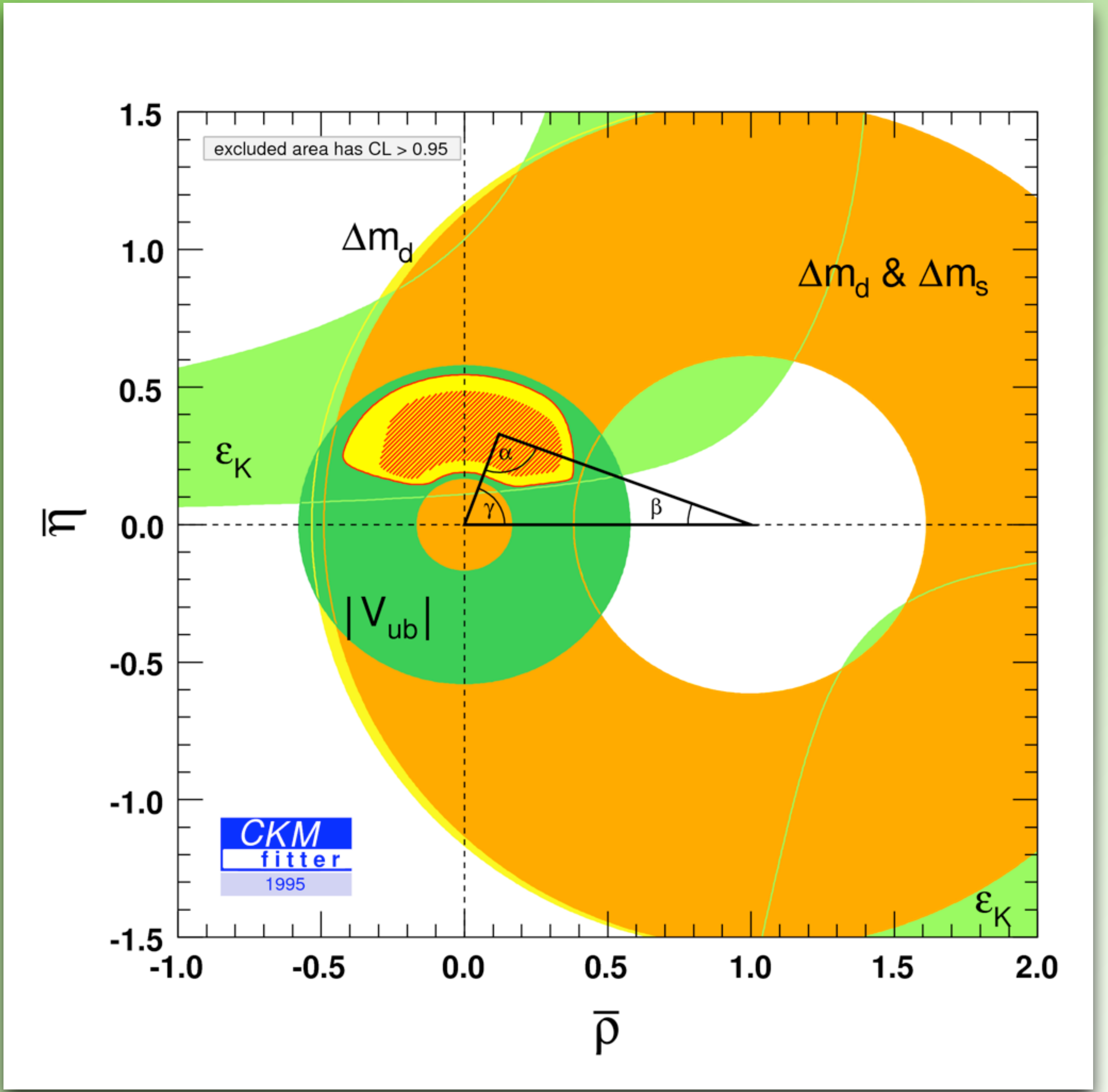

http://ckmfitter.in2p3.fr/

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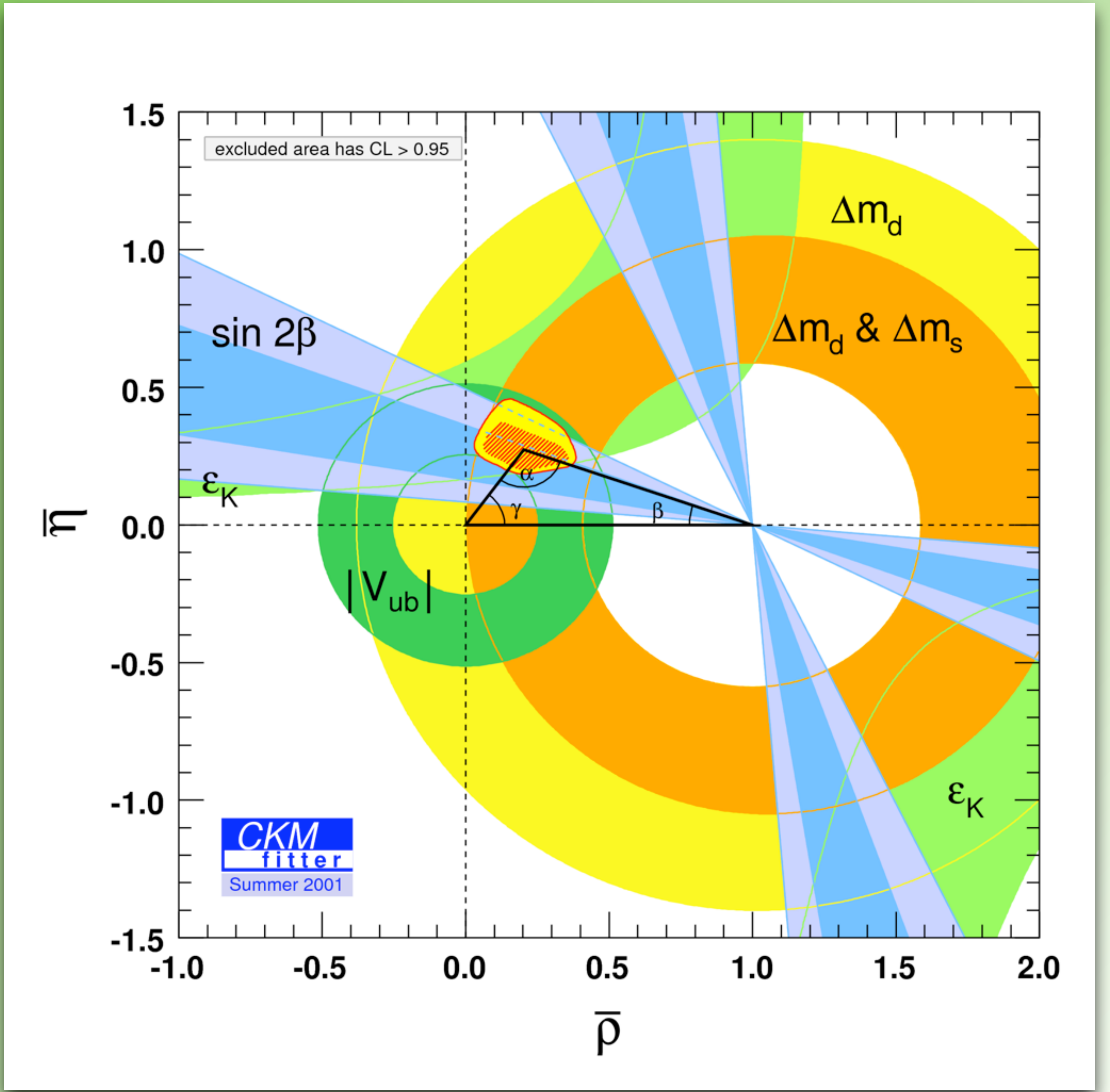

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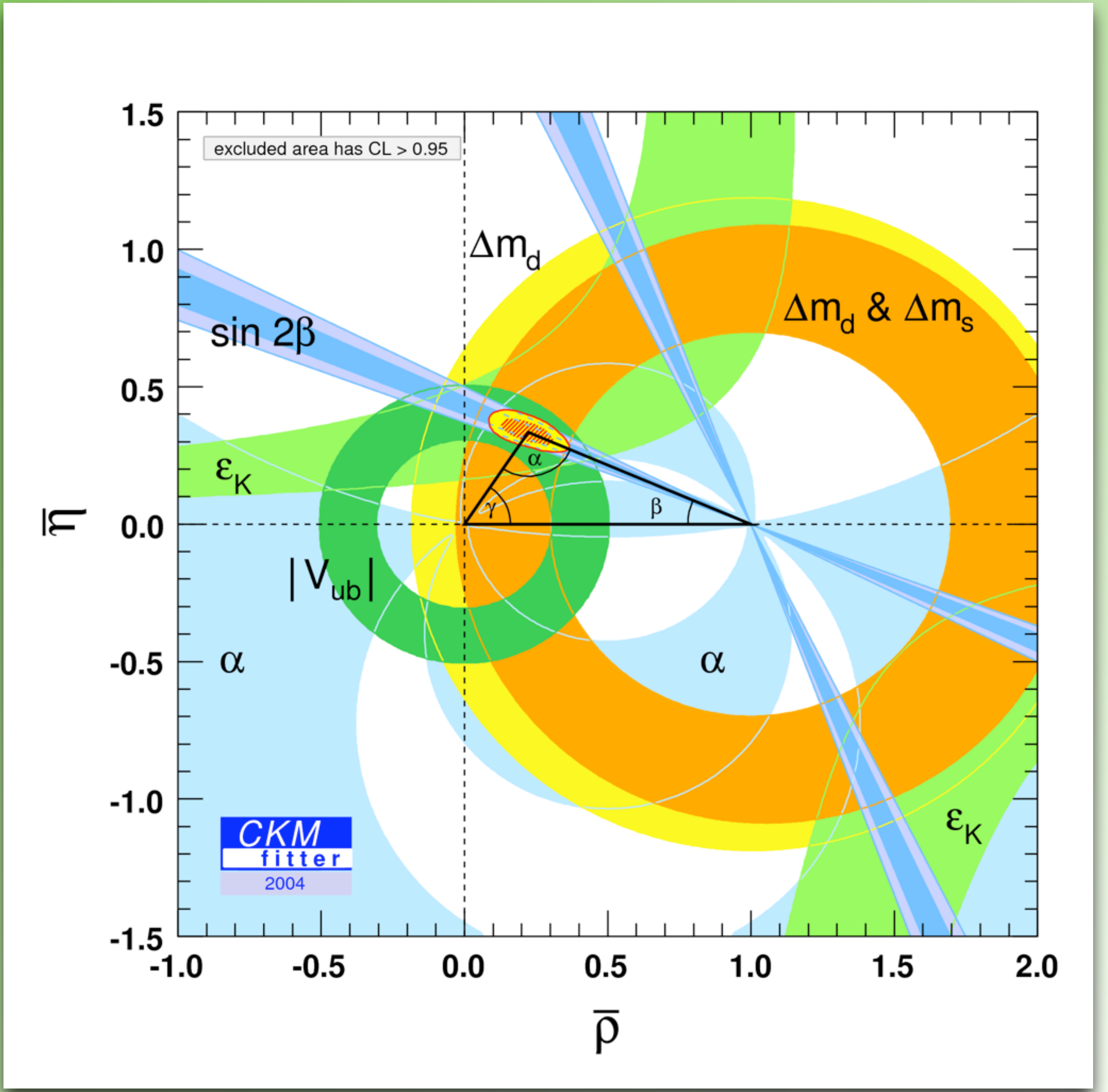

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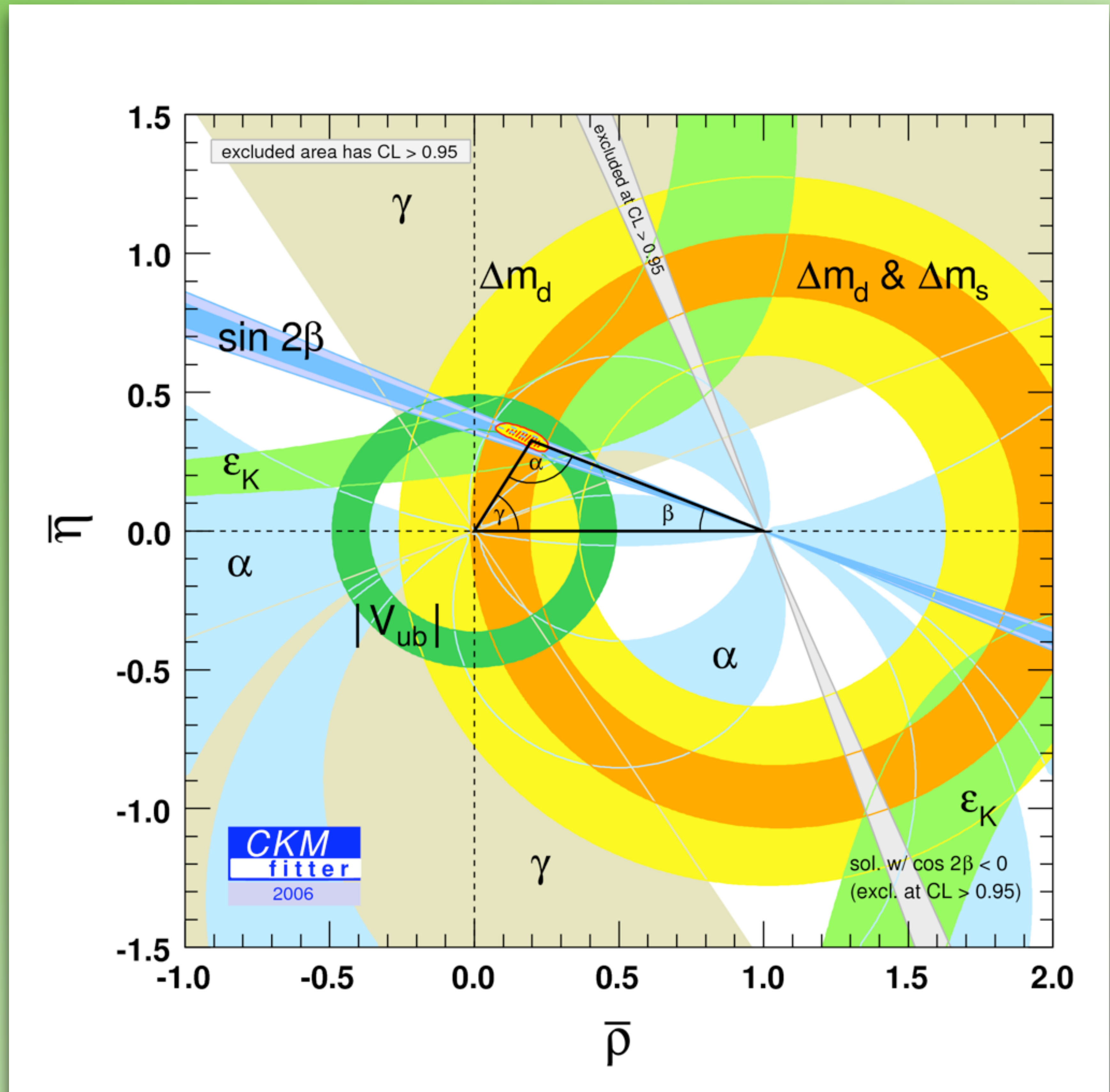

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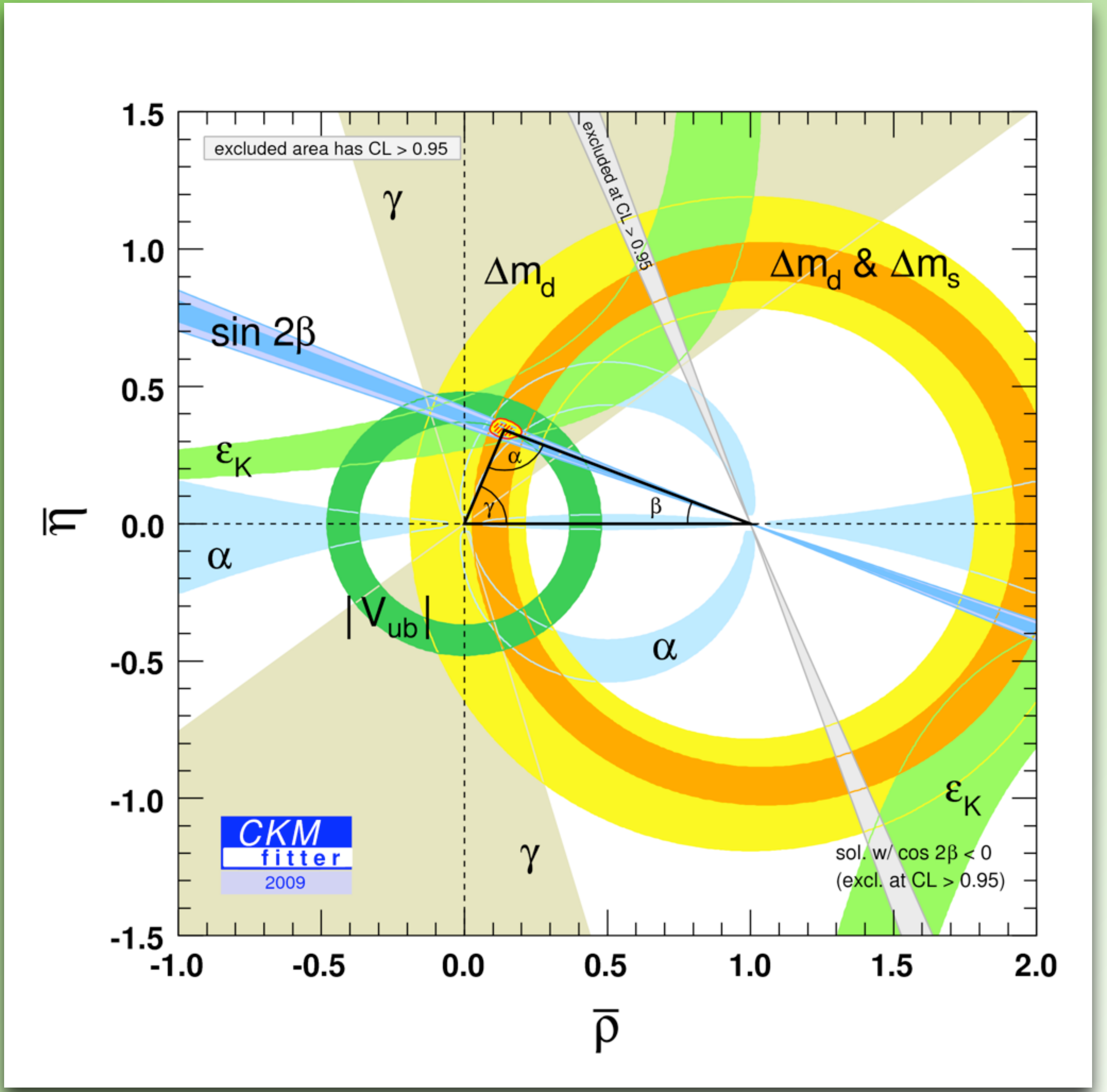

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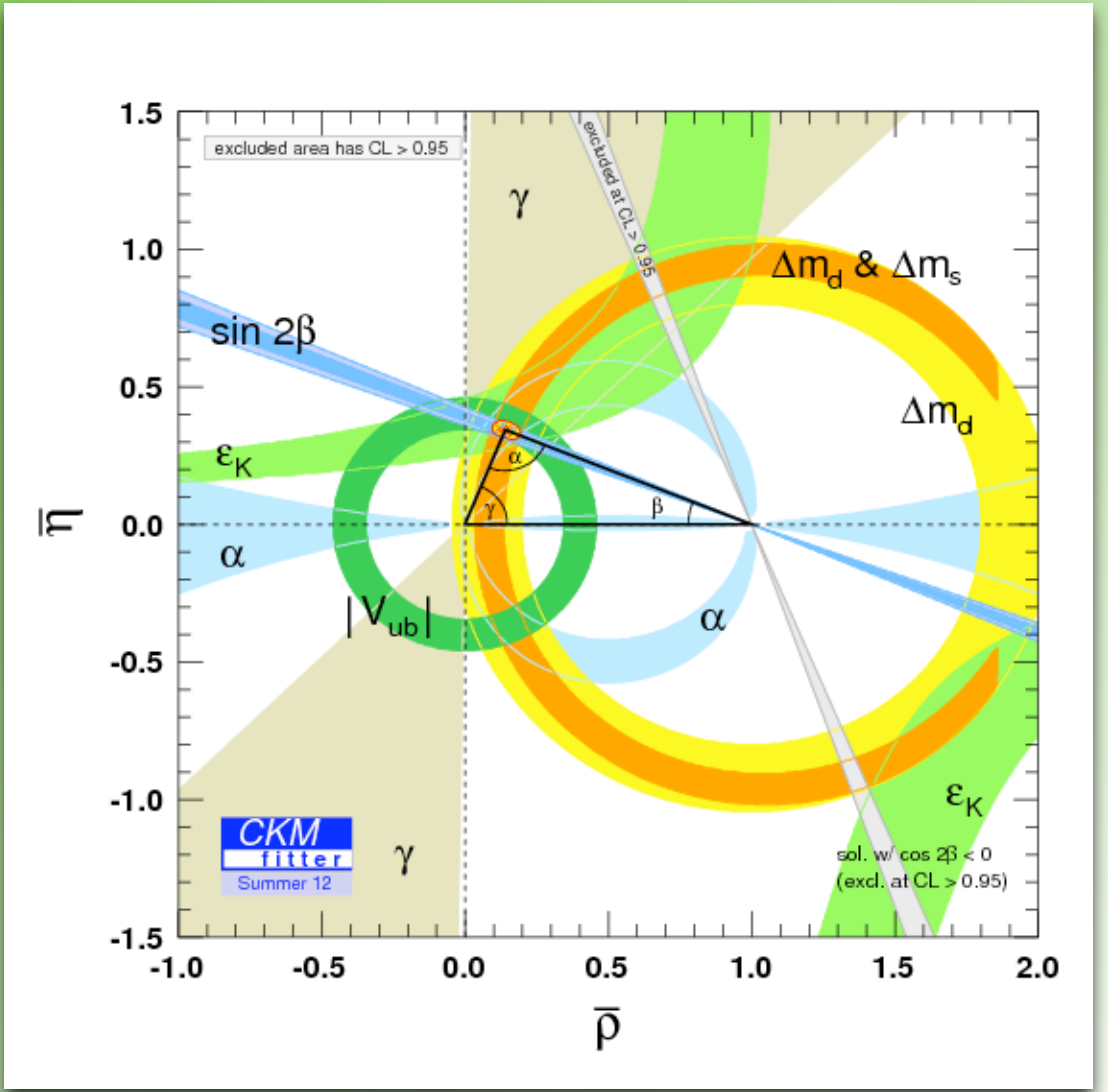

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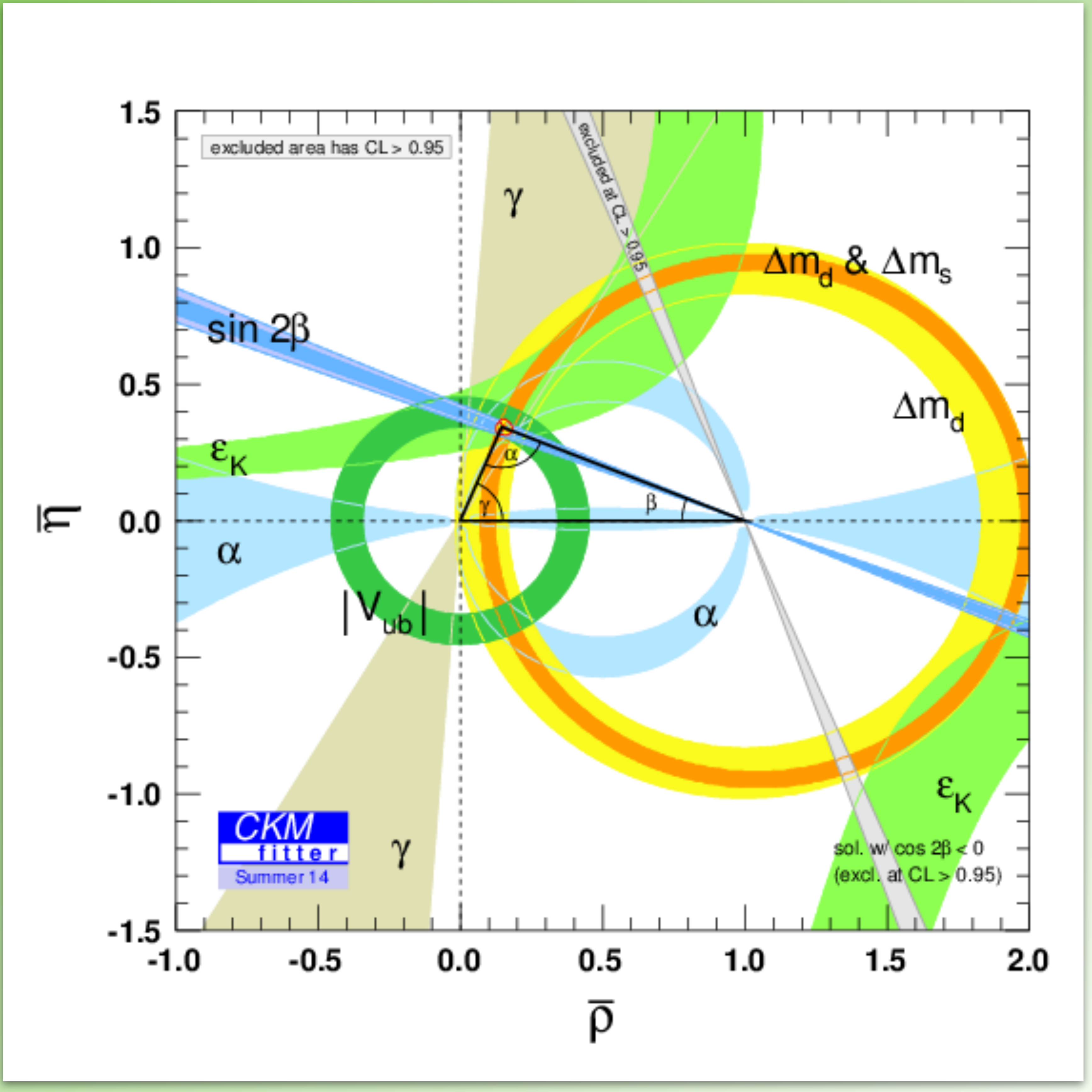


http://ckmfitter.in2p3.fr/

Progress constraining CKM

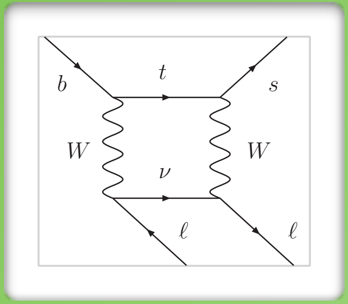
- Remarkable progress over past 20 years
- In addition to precise experimental results, “modern era of LQCD”
- Further reduction of errors requires further precision in LQCD
- Issues to confront
    - Matching error or m_b extrapolation error
    - Discretization errors
    - Electromagnetic effects

Rare decays

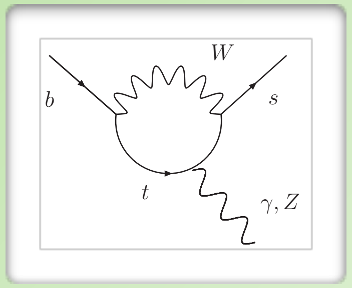
- $b \to s$ decays occur only at 1-loop level in Standard Model: Room for new physics?
- Following initial results from CDF, LHC experiments (esp LHC*b*) are making impressive measurements of rare, semileptonic decays
- There are a few tantalizing discrepancies with SM predictions
- Significant effort from theory remains to quantify and reduce SM uncertainties

Low energy description of $b \to s$ decays

$$\mathcal{H}_{\text{eff}}^{b\to s} = -\frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_i (C_i \mathcal{O}_i + C_i' \mathcal{O}_i')$$



In the Standard Model, i = 1, ..., 10, S, P with known Wilson coefficients C_i. Beyond SM, chirality-flipped operators are allowed and the $C_i^{(\prime)}$ depend on the model of new physics



Most important short-distance effects in $b \to sll$ come from 2-quark operators:

$$\mathcal{O}_9^{(\prime)} = \frac{e^2}{16\pi^2} \bar{s}\gamma^\mu P_{L(R)} b \, \bar{\ell}\gamma_\mu \ell \qquad \mathcal{O}_{10}^{(\prime)} = \frac{e^2}{16\pi^2} \bar{s}\gamma^\mu P_{L(R)} b \, \bar{\ell}\gamma_\mu \gamma_5 \ell$$

$$\mathcal{O}_7^{(\prime)} = \frac{m_b e}{16\pi^2} \bar{s}\sigma^{\mu\nu} P_{R(L)} b \, F_{\mu\nu}$$

Charmonium resonance effects arise from:

$$\mathcal{O}_1 = \bar{c}^\alpha \gamma^\mu P_L b^\beta \, \bar{s}^\beta \gamma^\mu P_L c^\alpha \qquad \mathcal{O}_2 = \bar{c}^\alpha \gamma^\mu P_L b^\alpha \, \bar{s}^\beta \gamma^\mu P_L c^\beta$$

Dramatis Personæ

- Pseudoscalar meson in final state

$$B \to K \mu^+\mu^-$$

- Vector meson in final state

$$B \to K^* \mu^+\mu^-$$

$$B_s \to \varphi \mu^+\mu^-$$

- Baryon in final state

$$\Lambda_b \to \Lambda \mu^+\mu^-$$

Dramatis Personæ

✤ Pseudoscalar meson in final state

$$B \to K\,\mu^+\mu^-$$

Simple & precise.

✤ Vector meson in final state

$$B \to K^*\,\mu^+\mu^-$$

$$B_s \to \varphi\,\mu^+\mu^-$$

Complicated but important. Loose ends still to be tied up.

✤ Baryon in final state

$$\Lambda_b \to \Lambda\,\mu^+\mu^-$$

Simple. Relatively uncertain, but growing more precise.

$B \to K$ form factors (pseudoscalar)

$$\langle K(k)|\bar{s}\gamma^\mu b|B(p)\rangle = \left[(p+k)^\mu - \frac{m_B^2 - m_K^2}{q^2}\, q^\mu\right] \boxed{f_+(q^2)} + \frac{m_B^2 - m_K^2}{q^2}\, q^\mu \boxed{f_0(q^2)}$$

$$\langle K(k)|\bar{s}\sigma^{\mu\nu} q_\nu b|B(p)\rangle = \frac{i\boxed{f_T(q^2)}}{m_B + m_K}\left[q^2(p+k)^\mu - (m_B^2 - m_K^2)q^\mu\right]$$

- "Golden" ("simple") matrix elements: QCD-stable $|i\rangle$ and $|f\rangle$ states
- Observables: differential branching fraction $d\Gamma/dq^2$, forward/backward asymmetry A_{FB} (zero in SM), and "flat term" F_H

$B \to V\,(K^*/\varphi)$ form factors (vector)

$$\langle V(k,\varepsilon)|\bar{q}\hat{\gamma}^\mu b|B(p)\rangle \;=\; \frac{2iV(q^2)}{m_B+m_V}\epsilon^{\mu\nu\rho\sigma}\varepsilon^*_\nu k_\rho p_\sigma$$

$$\langle V(k,\varepsilon)|\bar{q}\hat{\gamma}^\mu\hat{\gamma}^5 b|B(p)\rangle \;=2m_V A_0(q^2)\frac{\varepsilon^*\cdot q}{q^2}q^\mu+(m_B+m_V)A_1(q^2)\left(\varepsilon^{*\mu}-\frac{\varepsilon^*\cdot q}{q^2}q^\mu\right)$$
$$-A_2(q^2)\frac{\varepsilon^*\cdot q}{m_B+m_V}\left((p+k)^\mu-\frac{m_B^2-m_V^2}{q^2}q^\mu\right)$$

$$q^\nu\langle V(k,\varepsilon)|\bar{q}\hat{\sigma}_{\mu\nu}b|B(p)\rangle \;=\; 2T_1(q^2)\epsilon_{\mu\rho\tau\sigma}\varepsilon^{*\rho}p^\tau k^\sigma$$

$$-q^\nu\langle V(k,\varepsilon)|\bar{q}\hat{\sigma}_{\mu\nu}\hat{\gamma}^5 b|B(p)\rangle \;=iT_2(q^2)\left[\varepsilon^*_\mu(m_B^2-m_V^2)-(\varepsilon^*\cdot q)(p+k)_\mu\right]$$
$$+\,iT_3(q^2)(\varepsilon^*\cdot q)\left[q_\mu-\frac{q^2}{m_B^2-m_V^2}(p+k)_\mu\right]$$

$$A_{12}(q^2)=\frac{(m_B+m_V)^2(m_B^2-m_V^2-q^2)A_1(q^2)-\lambda A_2(q^2)}{16m_Bm_V^2(m_B+m_V)}$$

$$T_{23}(q^2)=\frac{m_B+m_V}{8m_Bm_V^2}\left[\left(m_B^2+3m_V^2-q^2\right)T_2(q^2)-\frac{\lambda T_3(q^2)}{m_B^2-m_V^2}\right]$$

with $\lambda \;=\; (t_+-t)(t_--t) \qquad t=q^2 \qquad t_\pm=(m_B\pm m_V)^2$

$\Lambda_b \to \Lambda$ form factors (baryon)

In general

$$\langle \Lambda | \bar{s}\gamma^\mu b | \Lambda_b \rangle = \bar{u}_\Lambda \left[f_1^V \gamma^\mu - f_2^V \frac{i\sigma^{\mu\nu} q_\nu}{m_{\Lambda_b}} + f_3^V \frac{q^\mu}{m_{\Lambda_b}} \right] u_{\Lambda_b}$$

$$\langle \Lambda | \bar{s}\gamma^\mu \gamma_5 b | \Lambda_b \rangle = \bar{u}_\Lambda \left[f_1^A \gamma^\mu - f_2^A \frac{i\sigma^{\mu\nu} q_\nu}{m_{\Lambda_b}} + f_3^A \frac{q^\mu}{m_{\Lambda_b}} \right] \gamma_5 u_{\Lambda_b}$$

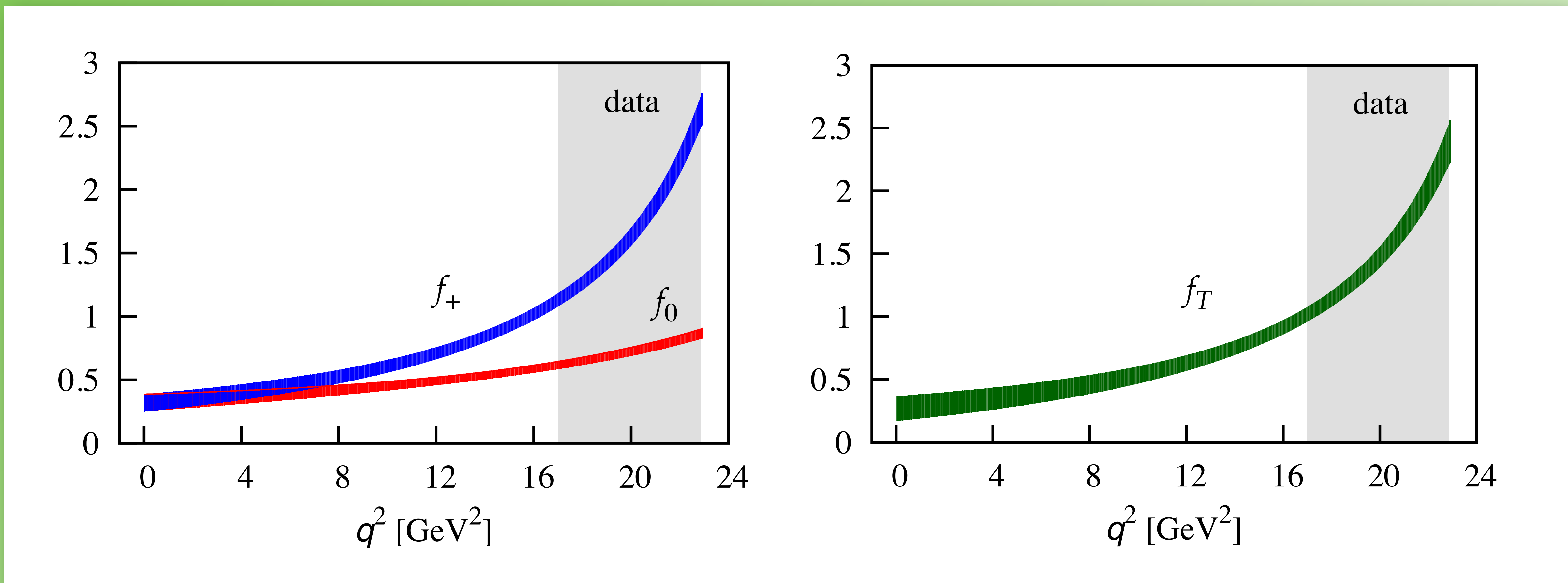
$$\langle \Lambda | \bar{s} i\sigma^{\mu\nu} q_\nu b | \Lambda_b \rangle = \bar{u}_\Lambda \left[f_1^{TV} \frac{\gamma^\mu q^2 - q^\mu \not{q}}{m_{\Lambda_b}} - f_2^{TV} \frac{q^\mu}{m_{\Lambda_b}} \right] u_{\Lambda_b}$$

$$\langle \Lambda | \bar{s} i\sigma^{\mu\nu} q_\nu \gamma_5 b | \Lambda_b \rangle = \bar{u}_\Lambda \left[f_1^{TA} \frac{\gamma^\mu q^2 - q^\mu \not{q}}{m_{\Lambda_b}} - f_2^{TA} \frac{q^\mu}{m_{\Lambda_b}} \right] \gamma_5 u_{\Lambda_b}$$

In the $m_b \to \infty$ limit

$$\langle \Lambda(p', s') | \, \bar{s}\Gamma Q \, | \Lambda_Q(v, 0, s) \rangle = \bar{u}(p', s') [F_1(p' \cdot v) + \not{v} F_2(p' \cdot v)] \Gamma \mathcal{U}(v, s)$$

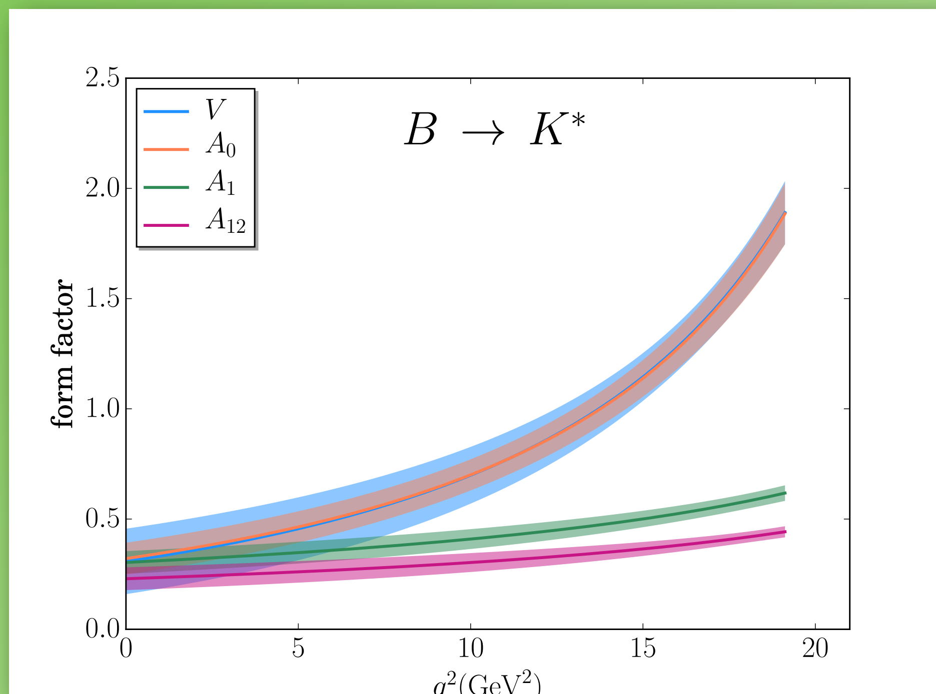
$B \to K$ form factors

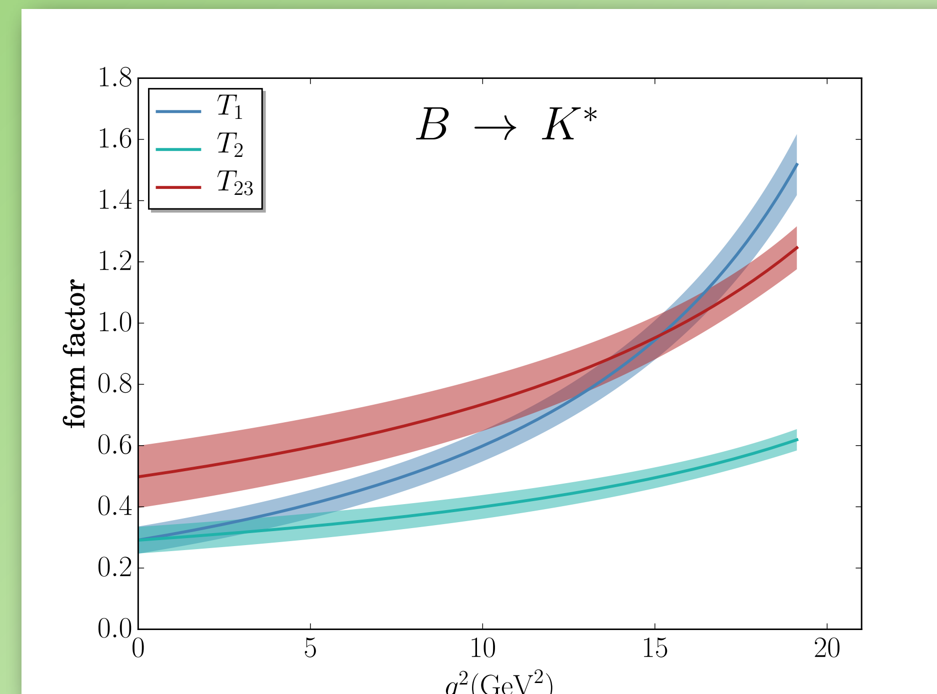


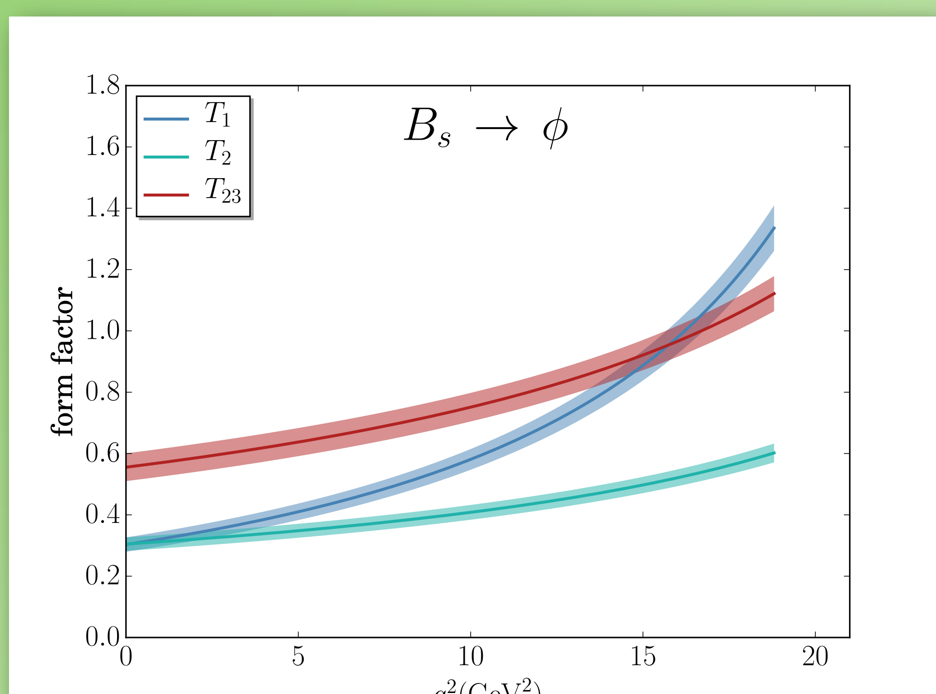
HPQCD Collaboration
(using NRQCD+HISQ valence on MILC n_f=2+1 asqtad)

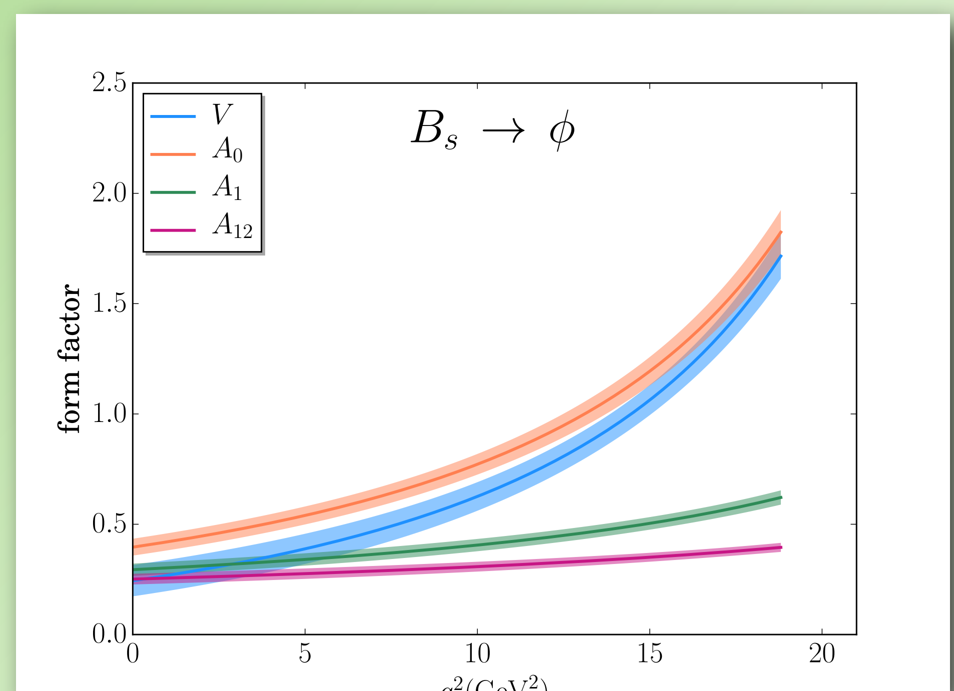
C. Bouchard *et al.*, arXiv:1306.0434, arXiv:1306:2384

$B \rightarrow V$ form factors







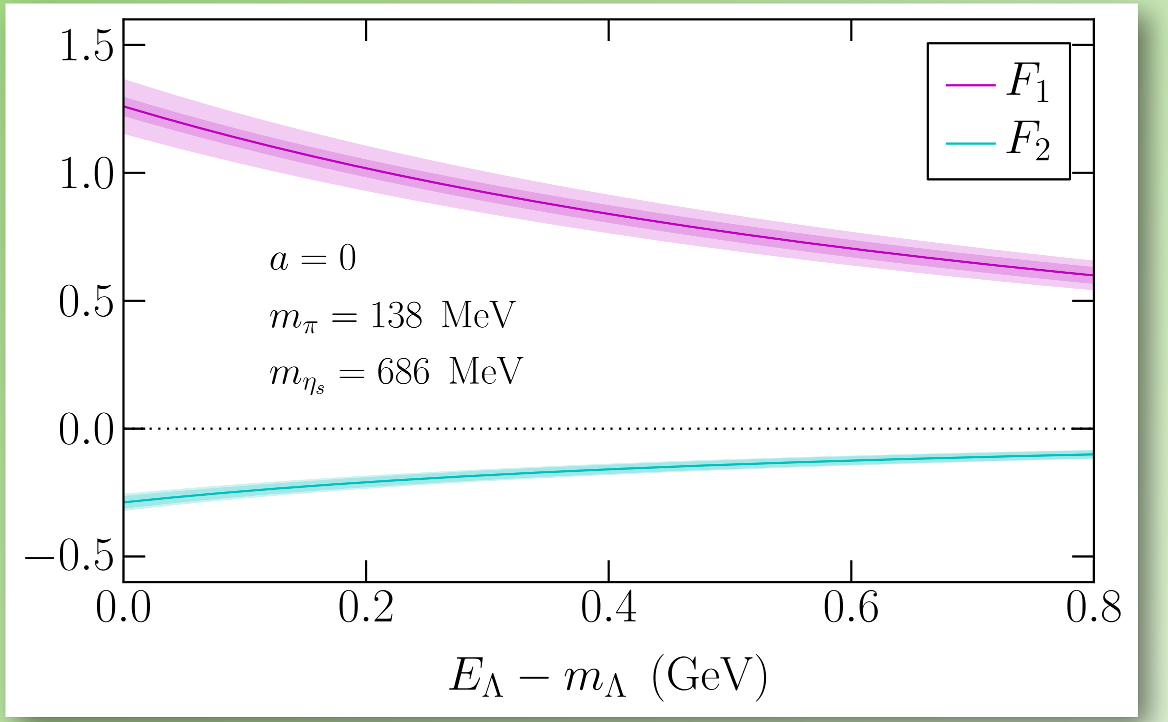


using NRQCD+asqtad valence on MILC n_f=2+1 asqtad

Horgan, Liu, Meinel, Wingate, arXiv:1310.3722

$\Lambda_b \to \Lambda$ form factors

Static limit, $m_b \to \infty$



(using Static+DWF on n_f=2+1 RBC-UKQCD)

In the static limit, 10 form factors reduce to 2

$$\langle \Lambda(p',s')|\,\bar{s}\Gamma Q\,|\Lambda_Q(v,0,s)\rangle = \bar{u}(p',s')\left[\boxed{F_1(p'\cdot v)} + \not{v}\boxed{F_2(p'\cdot v)}\right]\Gamma\,\mathcal{U}(v,s)$$

W Detmold, C-J D Lin, S Meinel, M Wingate, Phys Rev D87 (2013)

Physical m_b:

Preliminary results in S Meinel, Lattice 2013, arXiv:1301.2685

Form factor error budgets

$B \rightarrow K \mu^+\mu^-$

democratic mix of:	
statistical+discr.+chiral+inputs	4-6%
HQ operator matching	4%

$\Lambda_b \rightarrow \Lambda \mu^+\mu^-$

statistics (zero-higher recoil)	1-5%
HQ operator matching	6%
finite V	3%

(additional 8% included in BF. due to use of static approximation)

$B \rightarrow K^* \mu^+\mu^-$ ($B_s \rightarrow \varphi \mu^+\mu^-$)

statistical	4-8% (4-5%)
HQ operator matching	5%

$B \to K^*/\varphi$ complications

- K^* and φ are unstable resonances, while the present calculation so far assumes a narrow width approximation. Difficult to quantify associated quark mass and finite volume uncertainties.

- Quark mass effects not significant compared to present statistical uncertainties

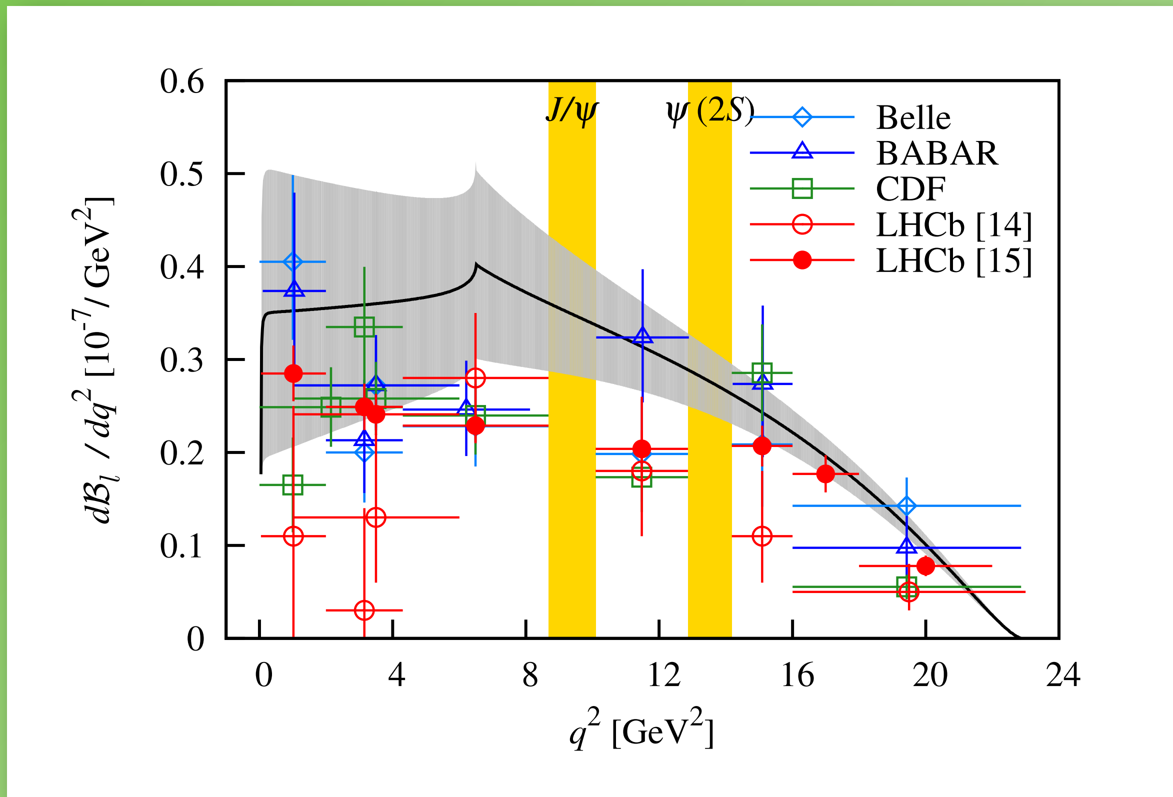
- Briceño, Hansen, Walker-Loud (arXiv:1406.5965) developed a framework for LQCD study of full $B \to K^*(K\pi/K\eta)\mu\mu$ and $B_s \to \varphi(\to KK)\mu\mu$ decays. Technically demanding to carry out.

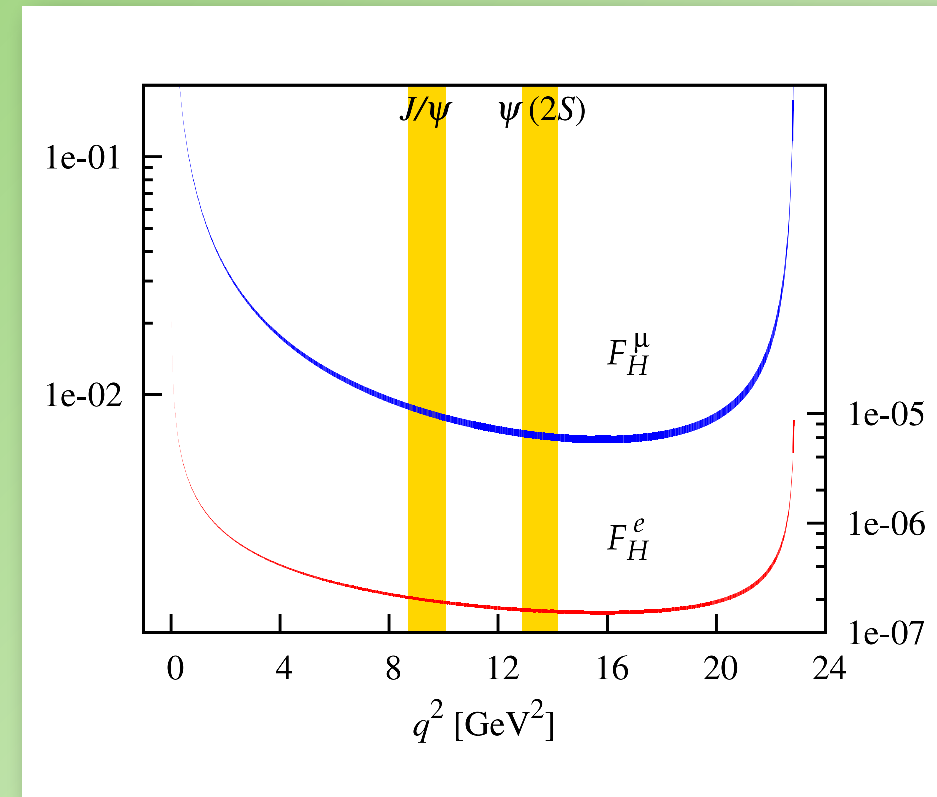
- Possible consistency check (assuming SM $b \to u$): $|V_{ub}|$ from $B \to \rho l \nu$ and $B_s \to K^* l \nu$

- Important to remember $B \to K^*/\varphi$ f.f. not of same standard as "simple" or "golden" or LQCD quantities

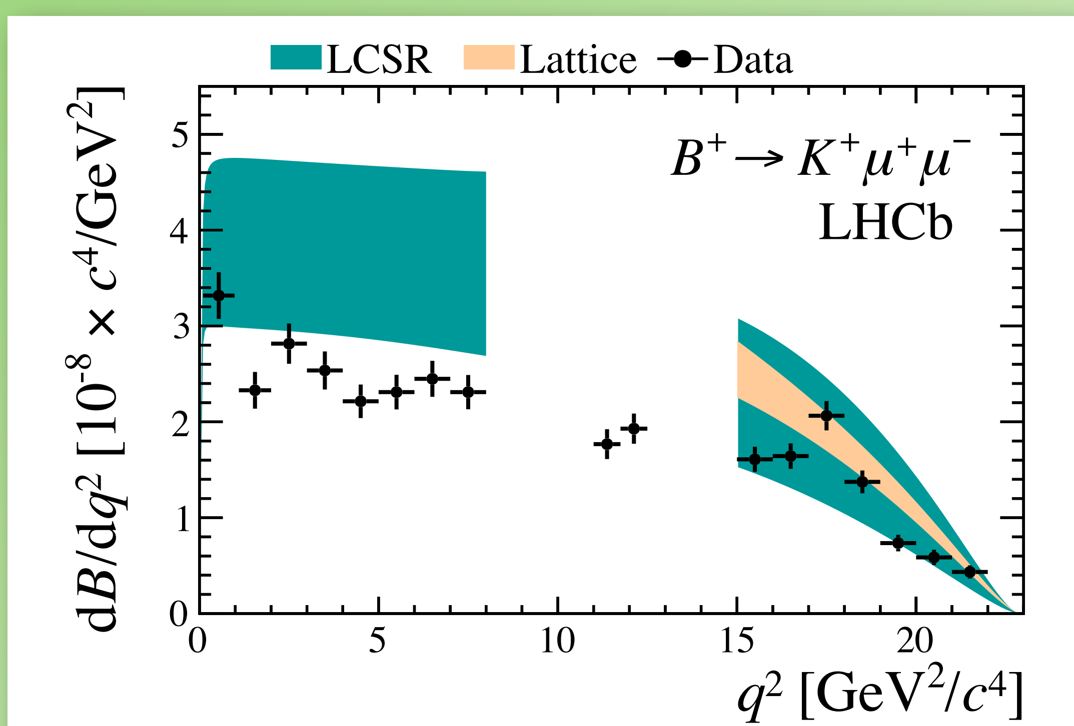
- Good first step for $B \to K^*/\varphi$ in modern LQCD era, with prospects for eventually removing assumptions, complements/improves upon LCSR

$B \to K\, l^+ l^-$



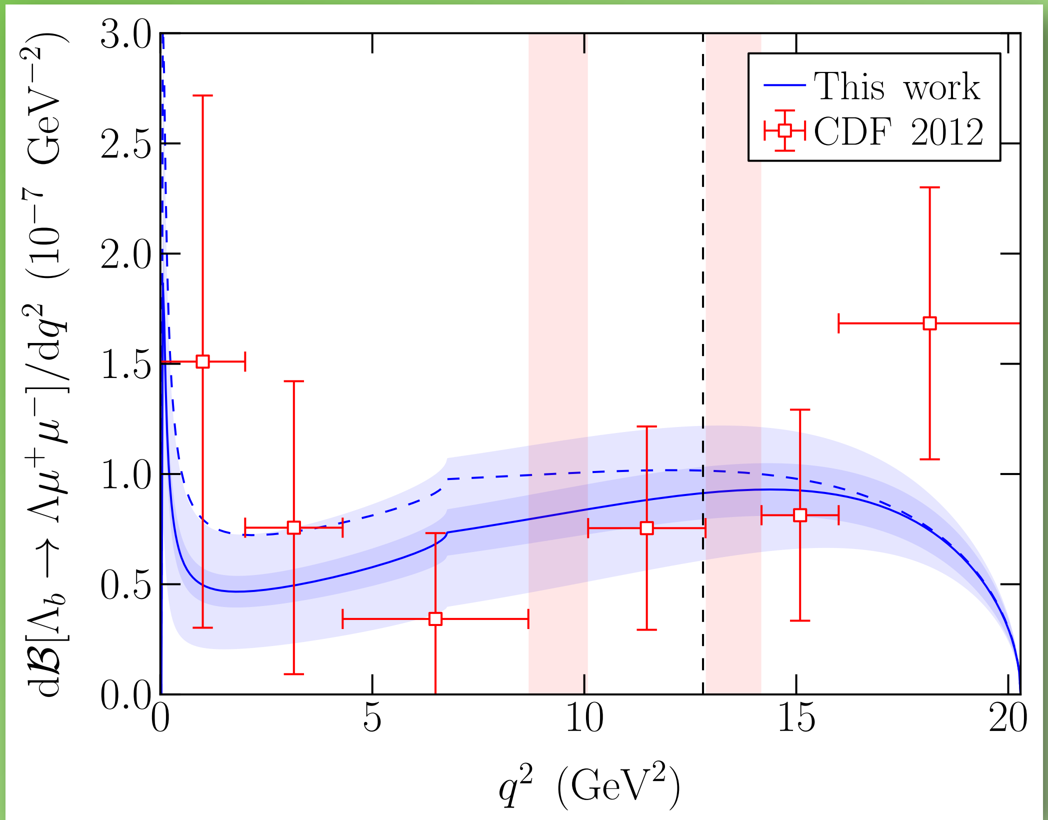


C. Bouchard *et al.*, PRL 111, (E) 112, arXiv:1306.0434v4, PRD 88, arXiv:1306:2384

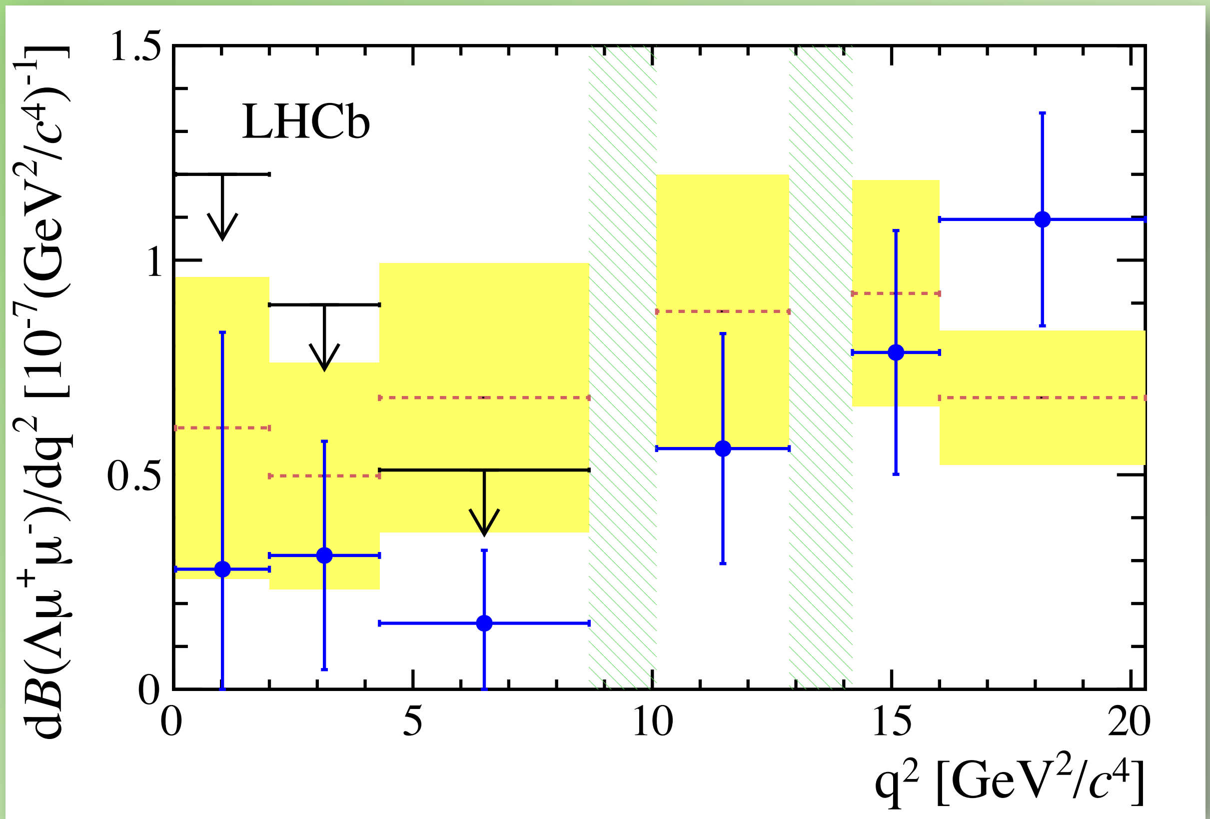


Resonant effect due to $\psi(4160)$
LHCb, PRL 111, arXiv:1307.7595
Fig. from LHCb, JHEP, arXiv:1403.8044

$\Lambda_b \to \Lambda\, l^+ l^-$



CDF: red; LQCD: blue

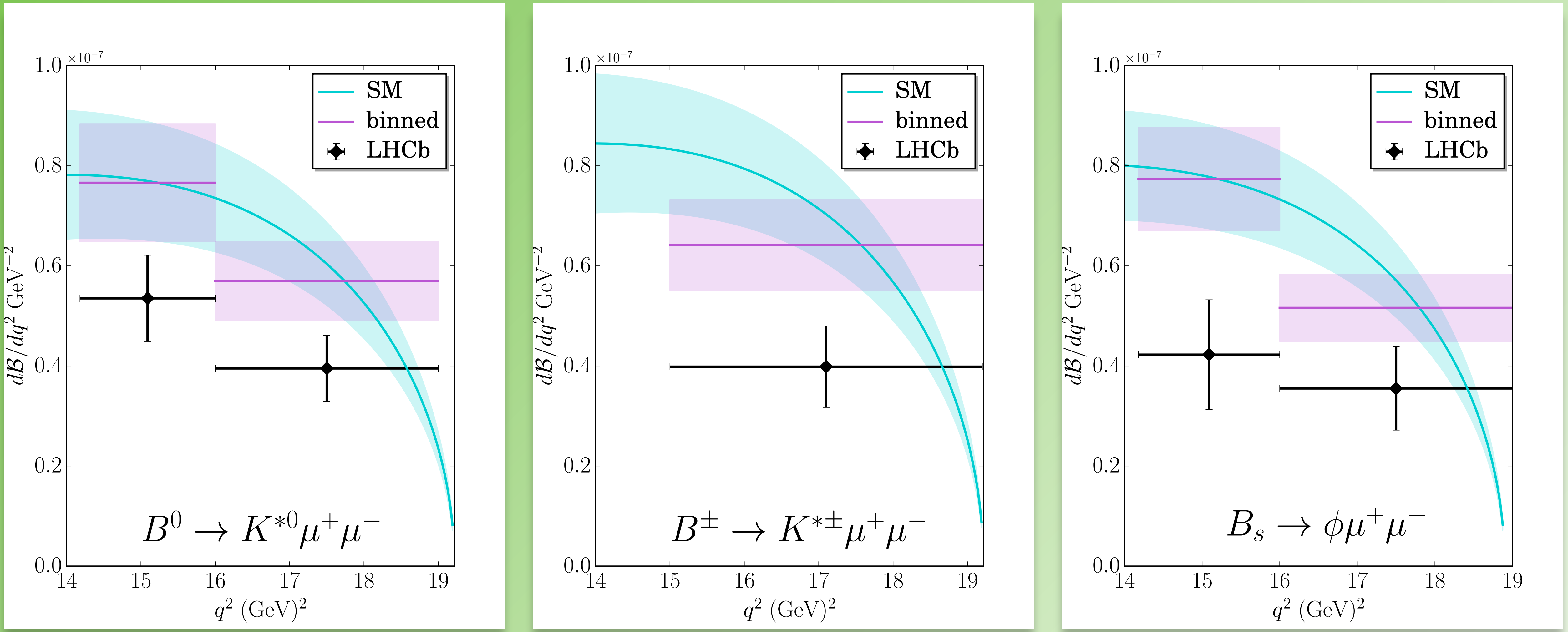


LHCb: blue; binned LQCD: red/yellow

Dominant theory error due to use of LO HQET (static action)
Improved calculation (w/ RHQ) underway (Meinel, Lattice 2013)

Detmold, Lin, Meinel, Wingate, Phys Rev D87 (2013)
CDF, public note 108xx, v0.1, http://www-cdf.fnal.gov/physics/new/bottom/bottom.html
LHC*b*, R Aaij, Phys. Lett. B 725 (2013) [arXiv:1306.2577]

Branching fractions

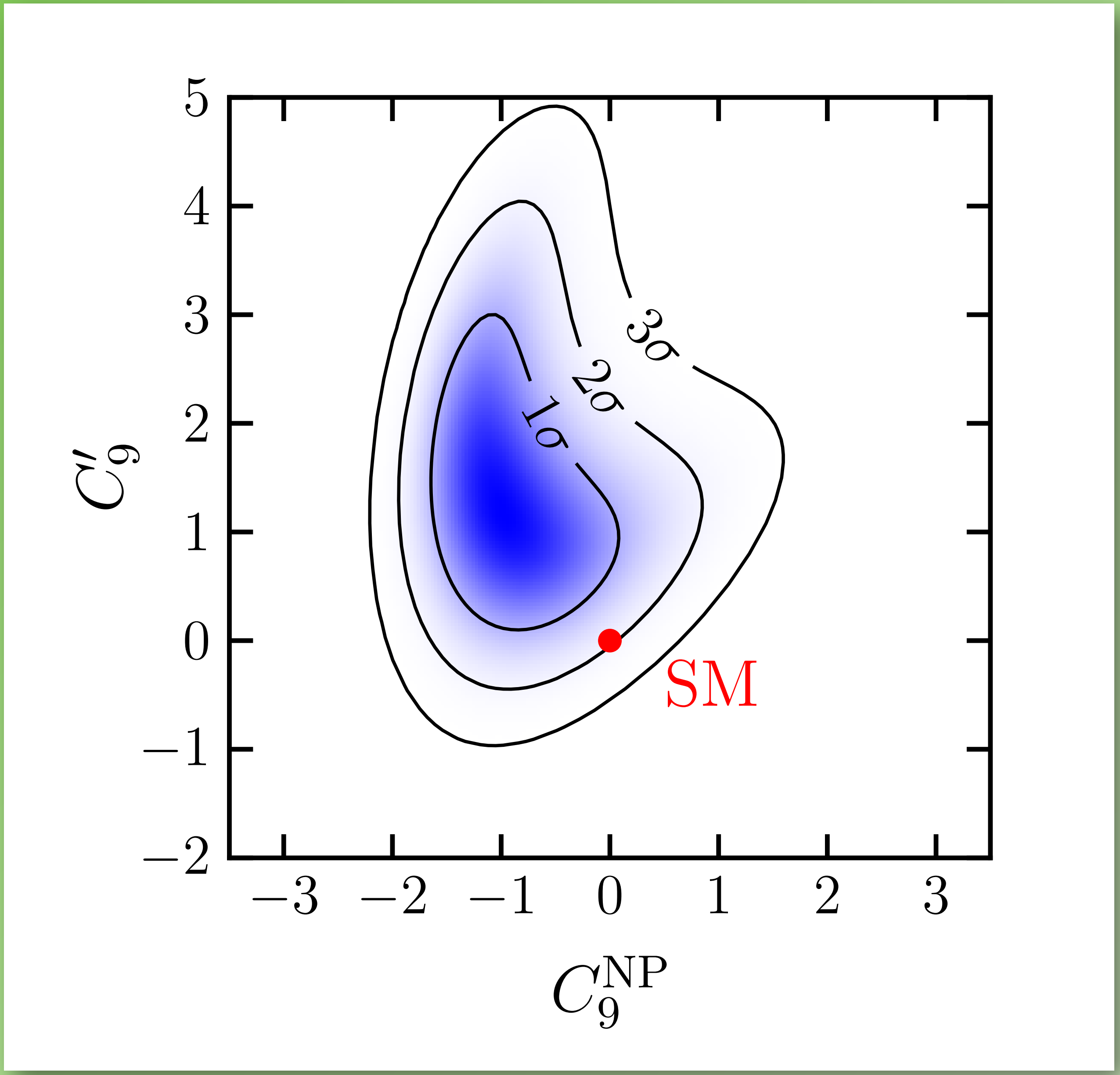


Theory:
Horgan, Liu, Meinel, Wingate, Phys. Rev. D (2013), arXiv:1310.3722
Horgan, Liu, Meinel, Wingate, Phys. Rev. Lett. (2014), arXiv:1310.3887
Horgan, Liu, Meinel, Wingate, Proc. of Science (LATTICE2014), arXiv:1501.00367

Experiment:
LHCb Collaboration, J. High Energy Phys (2013), arXiv:1304.6325
LHCb Collaboration, J. High Energy Phys (2014), arXiv:1403.8044
LHCb Collaboration, J. High Energy Phys (2013), arXiv:1305.2168

Fit to low recoil data

Best fit: $\quad C_9^{\mathrm{NP}} = -1.0 \pm 0.6 \qquad C_9' = 1.2 \pm 1.0$

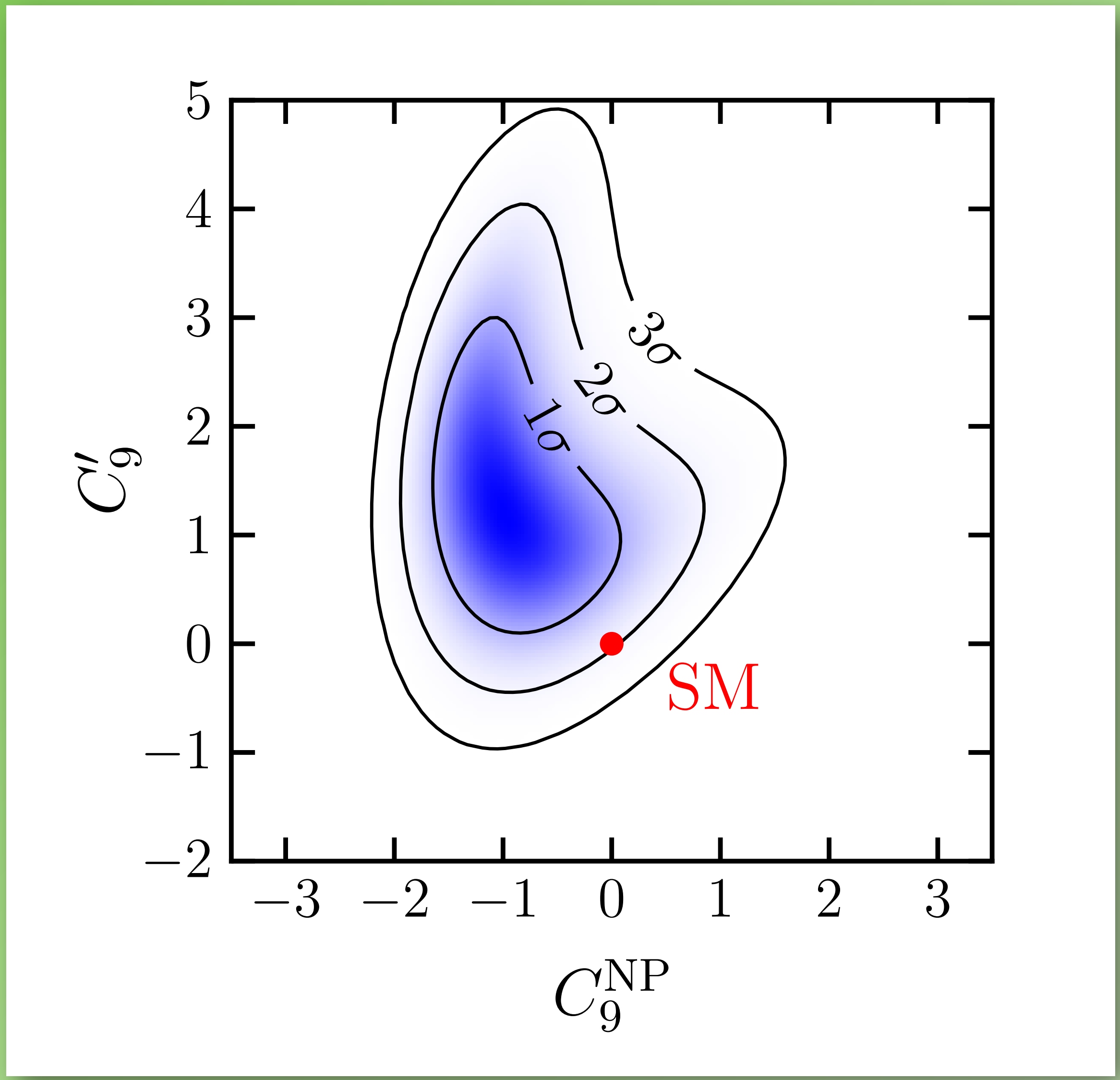


Likelihood function

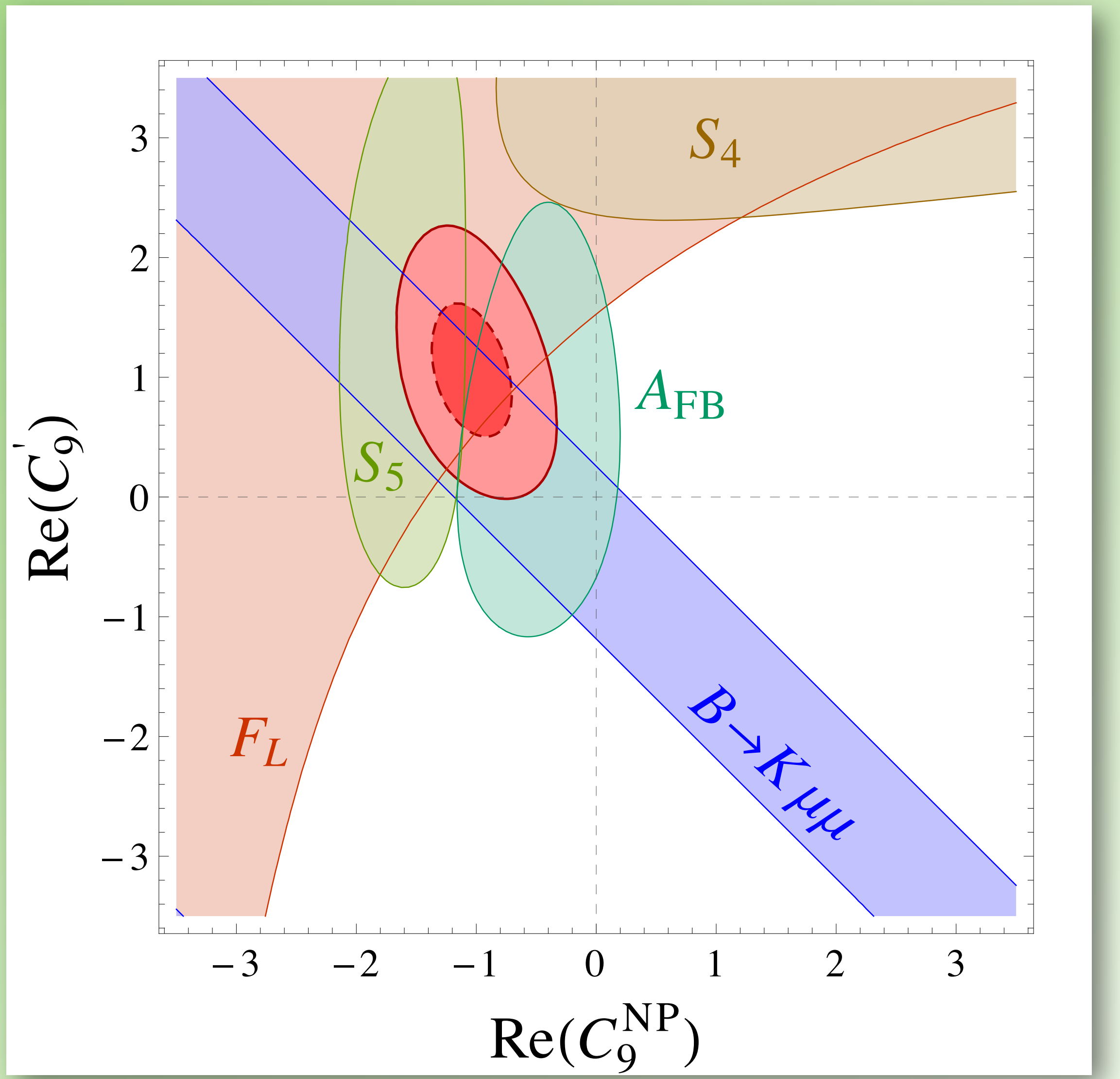
- C_9, C_9' assumed to be real
- Data in 2 highest q^2 bins
  - $B \to K^*\mu\mu$ (neutral mode): dB/dq^2, F_L, S_3, S_4, S_5, A_{FB}
  - $B_s \to \varphi\mu\mu$: dB/dq^2, F_L, S_3
- Theory correlations between observables & bins taken into account

Horgan, Liu, Meinel, Wingate, PRL 112, arXiv:1310.3887

2 complementary fits



Horgan, Liu, Meinel, Wingate,
arXiv:1310.3887



Altmannshofer & Straub,
arXiv:1308.1501

Low q^2 discrepancy

Descotes-Genon, Matias, Virto [PRD88, 074002, (2013), arXiv:1307.5683]

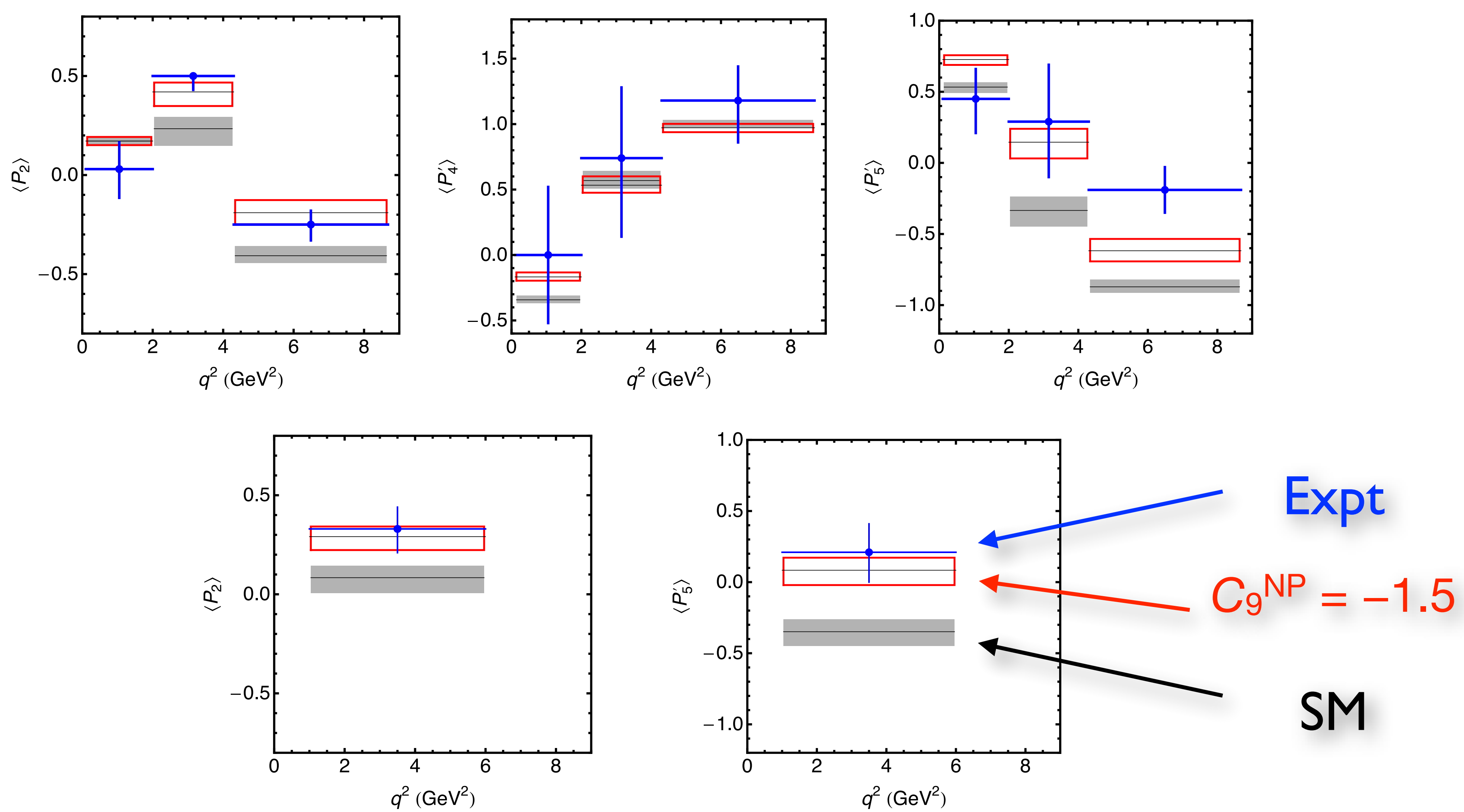
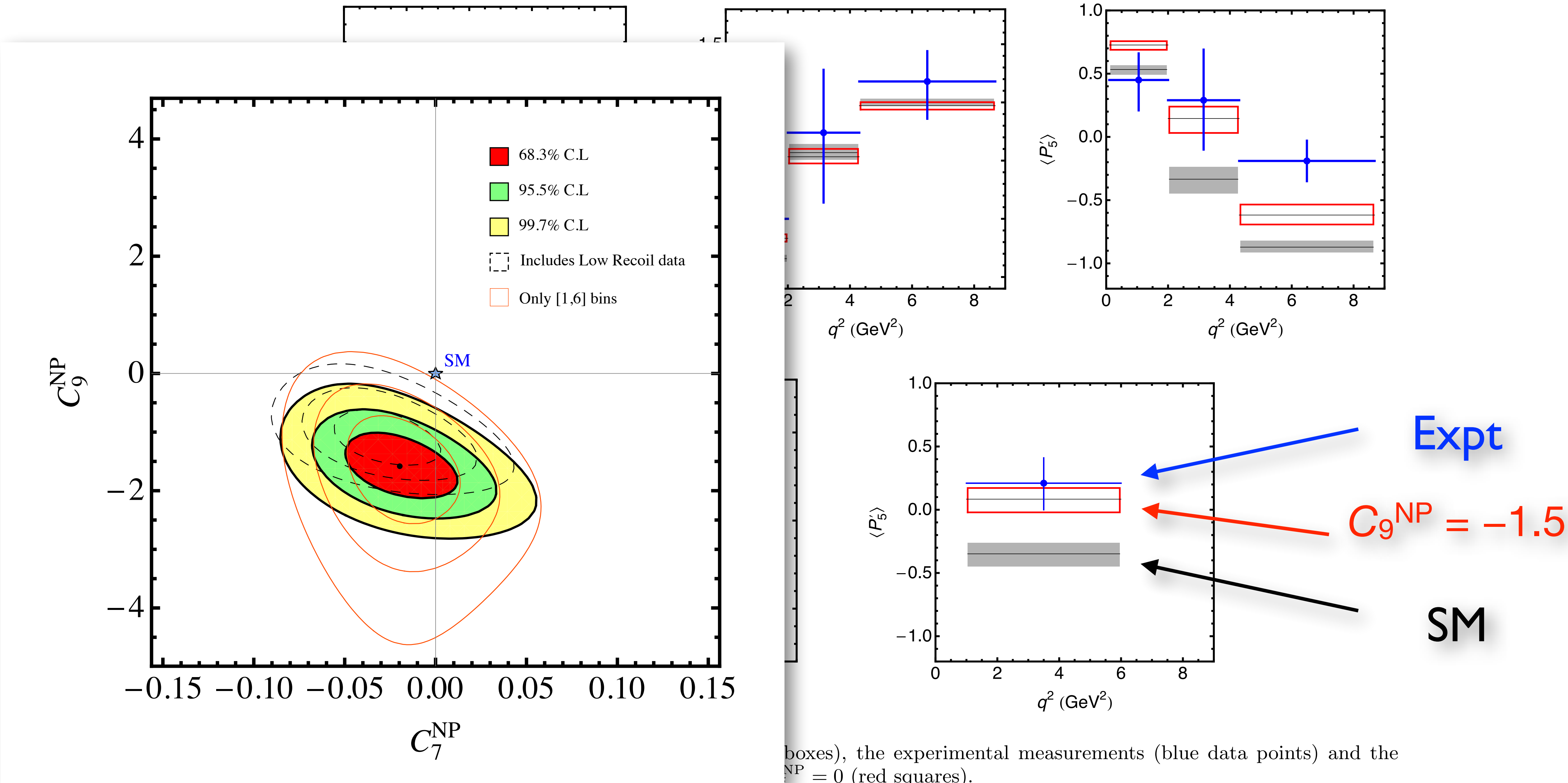


FIG. 2: Comparison between the SM predictions (gray boxes), the experimental measurements (blue data points) and the predictions for the scenario with $\mathcal{C}_9^{\rm NP} = -1.5$ and other $\mathcal{C}_i^{\rm NP} = 0$ (red squares).

Agree with negative NP contribution to C_9. They do not find $C_9' \neq 0$

Low q^2 discrepancy

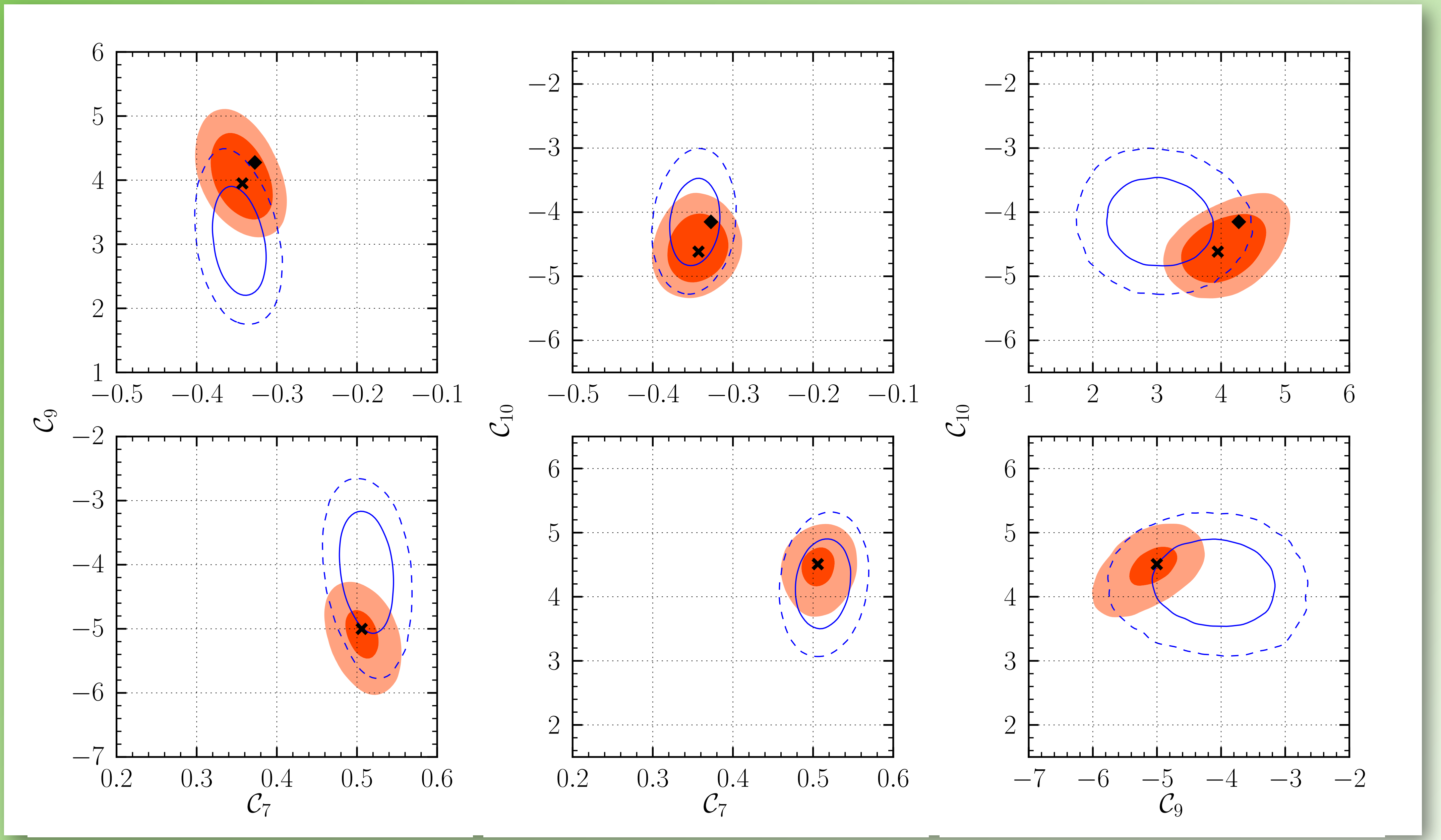
Descotes-Genon, Matias, Virto [PRD88, 074002, (2013), arXiv:1307.5683]



Agree with negative NP contribution to C_9. They do not find $C_9' \neq 0$

But another fit...

Consistent with SM (and also with negative NP contribution to C_9)



Orange: full fit. Blue: selection fit

Beaujean, Bobeth, van Dyk, [Eur. Phys. J. C 74 (2014), arXiv:1310.2478]

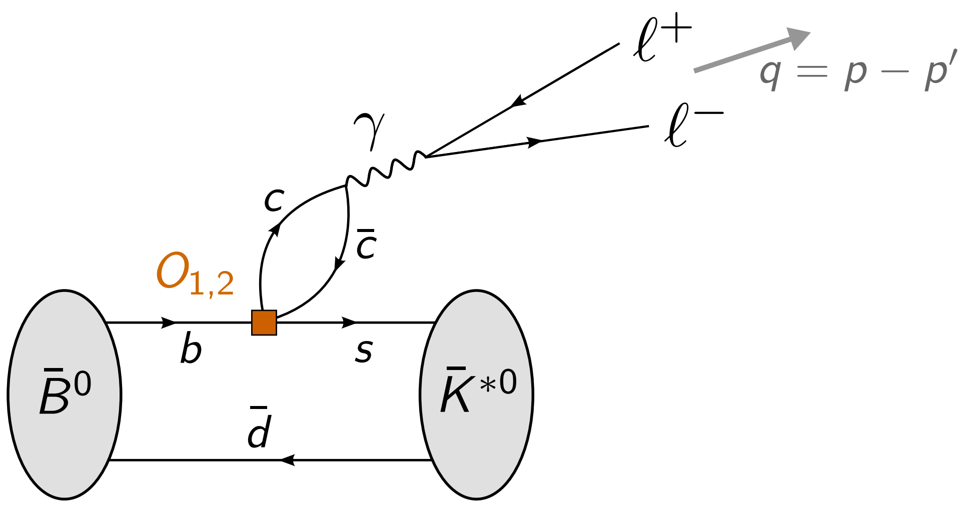
Matrix elements of nonlocal operators

$$\mathcal{M} = \frac{G_F\,\alpha}{\sqrt{2}\pi} V_{tb}V_{ts}^* \Big[(\mathcal{A}_\mu + \mathcal{T}_\mu)\bar{u}_\ell\gamma^\mu v_\ell + \mathcal{B}_\mu \bar{u}_\ell \gamma^\mu \gamma_5 v_\ell\Big]$$

$$\mathcal{A}_\mu = -\frac{2m_b}{q^2}\, q^\nu \langle \bar{K}^* |\, \bar{s}\, i\sigma_{\mu\nu}(C_7 P_R + C_7' P_L) b\, |\bar{B}\rangle$$

$$+ \langle \bar{K}^* |\, \bar{s}\gamma_\mu (C_9 P_L + C_9' P_R) b\, |\bar{B}\rangle$$

$$\mathcal{B}_\mu = \langle \bar{K}^* |\, \bar{s}\gamma_\mu (C_{10} P_L + C_{10}' P_R) b\, |\bar{B}\rangle$$



$$\mathcal{T}_\mu = \frac{-16 i\pi^2}{q^2} \sum_{i=1\ldots6;8} C_i \int \mathrm{d}^4 x\ e^{iq\cdot x} \langle \bar{K}^* |\, \mathsf{T}\, O_i(0)\, j_\mu(x)\, |\bar{B}\rangle$$

Affects all *b* ➞ *sll* decays, regardless of initial/final hadrons

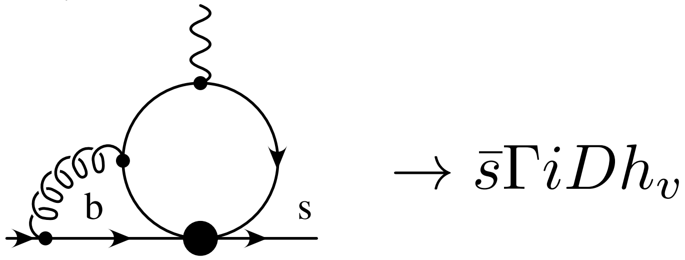
OPE at large q^2

$$\mathcal{T}_\mu = -T_7(q^2)\frac{2m_b}{q^2}\, q^\nu \langle \bar{K}^*|\bar{s}\, i\sigma_{\mu\nu}P_R b|\bar{B}\rangle + T_9(q^2)\langle \bar{K}^*|\bar{s}\gamma_\mu P_L b|\bar{B}\rangle$$

$$+O\left(\frac{\alpha_s\Lambda}{m_b}, \frac{\Lambda^2}{m_b^2}, \frac{m_c^4}{q^4}\right)$$

Grinstein & Pirjol, PRD 70, 114005 (2004)

- First correction in expansion (m_c^2/q^2) simply augments C_7^{eff} and C_9^{eff} : Buras, Misiak, Münz, Pokorski (BMMP) ➙ Grinstein, Pirjol (GP)
- Order $\alpha_s\Lambda/m_b$ corrections calculable on lattice
- Local duality: bin observables in q^2
- Duality violations estimated to be small (~2% in model): Beylich, Buchalla, Feldmann, [Eur. Phys. J C71, 1635 (2011), arXiv:1101.5118]
- On the other hand, Lyon & Zwicky [arXiv:1406.0566] claim charmonium resonances can have a much larger effect, even on binned observables: "complete breakdown of factorization"



Lepton flavor non-universality?

$$\mathcal{R}(D^{(*)}) = \frac{\mathcal{B}(\bar{B} \to D^{(*)}\tau\bar{\nu})}{\mathcal{B}(\bar{B} \to D^{(*)}\ell\bar{\nu})}$$

$$\mathcal{R}(D) = 0.440 \pm 0.058 \pm 0.042$$
$$\mathcal{R}(D^*) = 0.332 \pm 0.024 \pm 0.018$$

$\boxed{\approx 1 - 2\sigma}$

BaBar, PRL 109 (2012)
FNAL/MILC, PRL109 (2012)

$$\mathcal{R}_K = \frac{\langle \mathcal{B}(B^+ \to K^+\mu^+\mu^-)\rangle_{q^2\,\mathrm{bin}}}{\langle \mathcal{B}(B^+ \to K^+e^+e-)\rangle_{q^2\,\mathrm{bin}}}$$

$$\mathcal{R}_K = 0.745^{+0.090}_{-0.074} \pm 0.036$$

$1 < q^2 < 6\ (\mathrm{GeV}^2)$

$\boxed{2.6\sigma}$

LHCb, PRL 113 (2014)

[Lepton flavor violation?

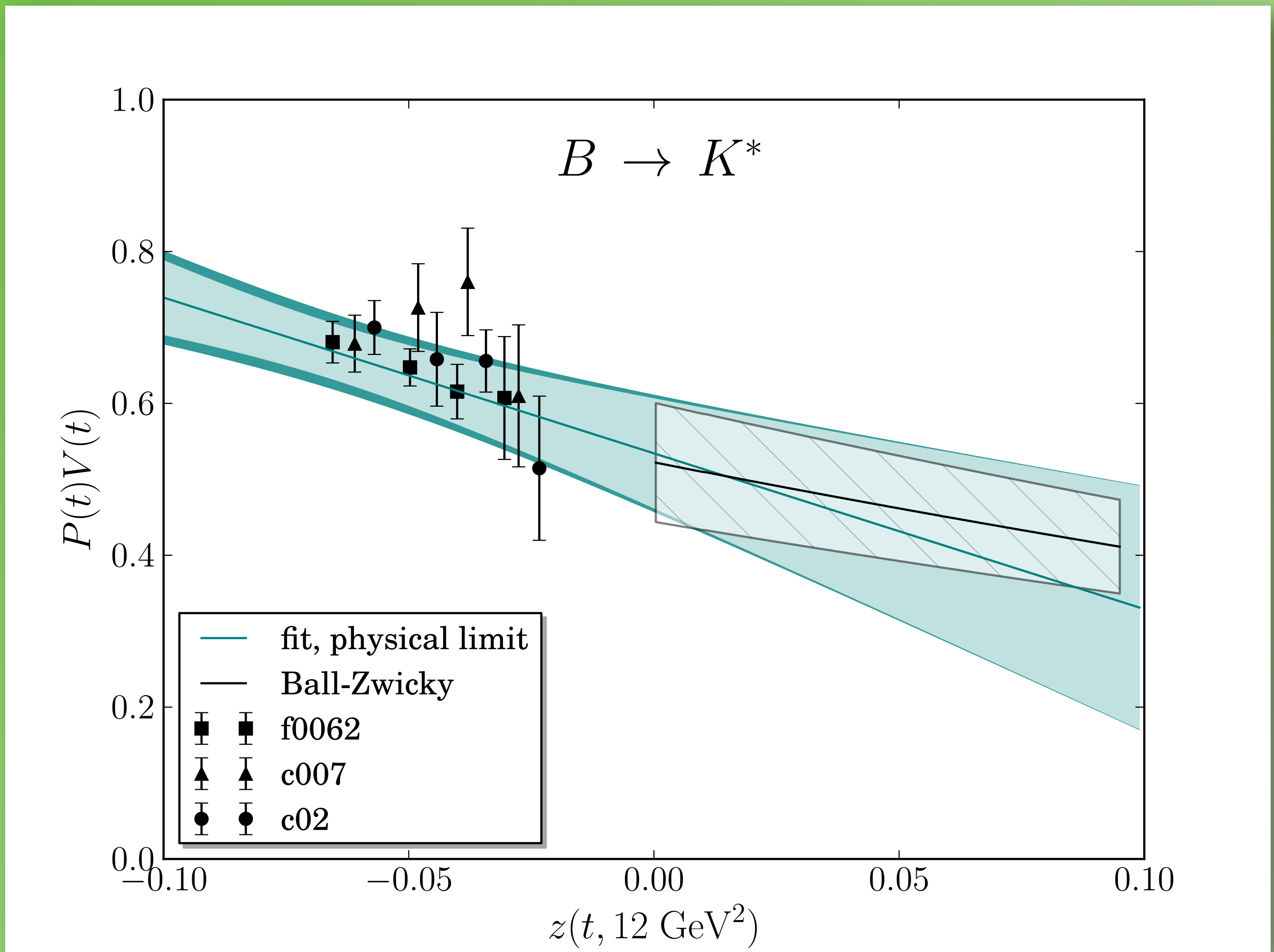
$$\mathcal{B}(h \to \tau\mu) = (0.89^{+0.40}_{-0.37})\%$$

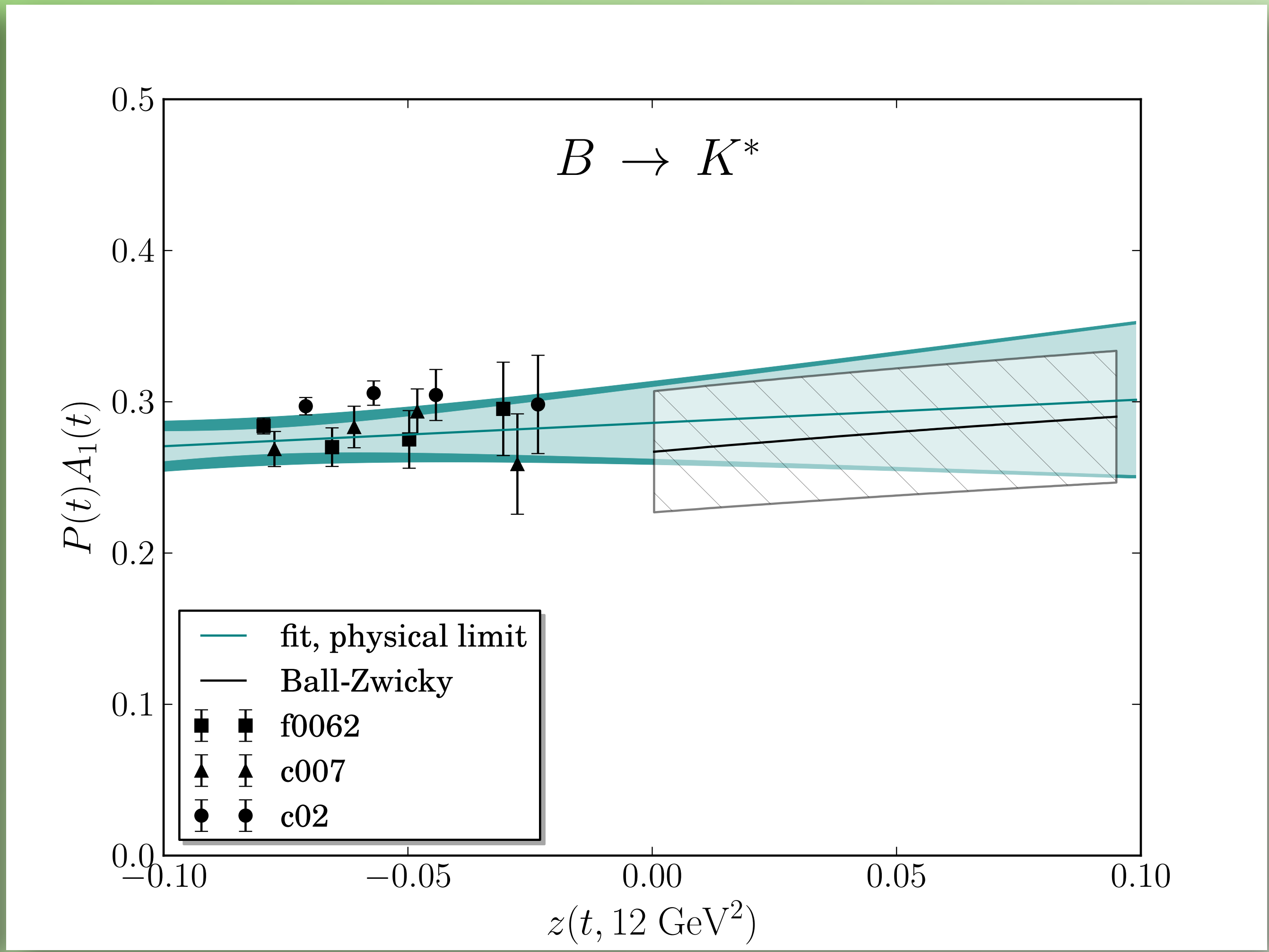
$\boxed{2.4\sigma}$ CMS, CMS-PAS-HIG-14005]

Conclusions

- LQCD vital to constraining CKM parameters
- From 2013: $b \to s$ form factors from unquenched LQCD
  - Direct access to low-recoil region
  - Measured differential branching fractions are consistently smaller than SM results in high q^2 bin(s)
  - Improvement for $\Lambda_b \to \Lambda\mu\mu$ is underway
  - Fully-controlled calculation for $B \to K^*\mu\mu$ and $B_s \to \varphi\mu\mu$ will take time
- Interesting hints in LHC flavor physics data!

$B \to K^*$ form factors





using NRQCD+asqtad valence on MILC n_f=2+1 asqtad

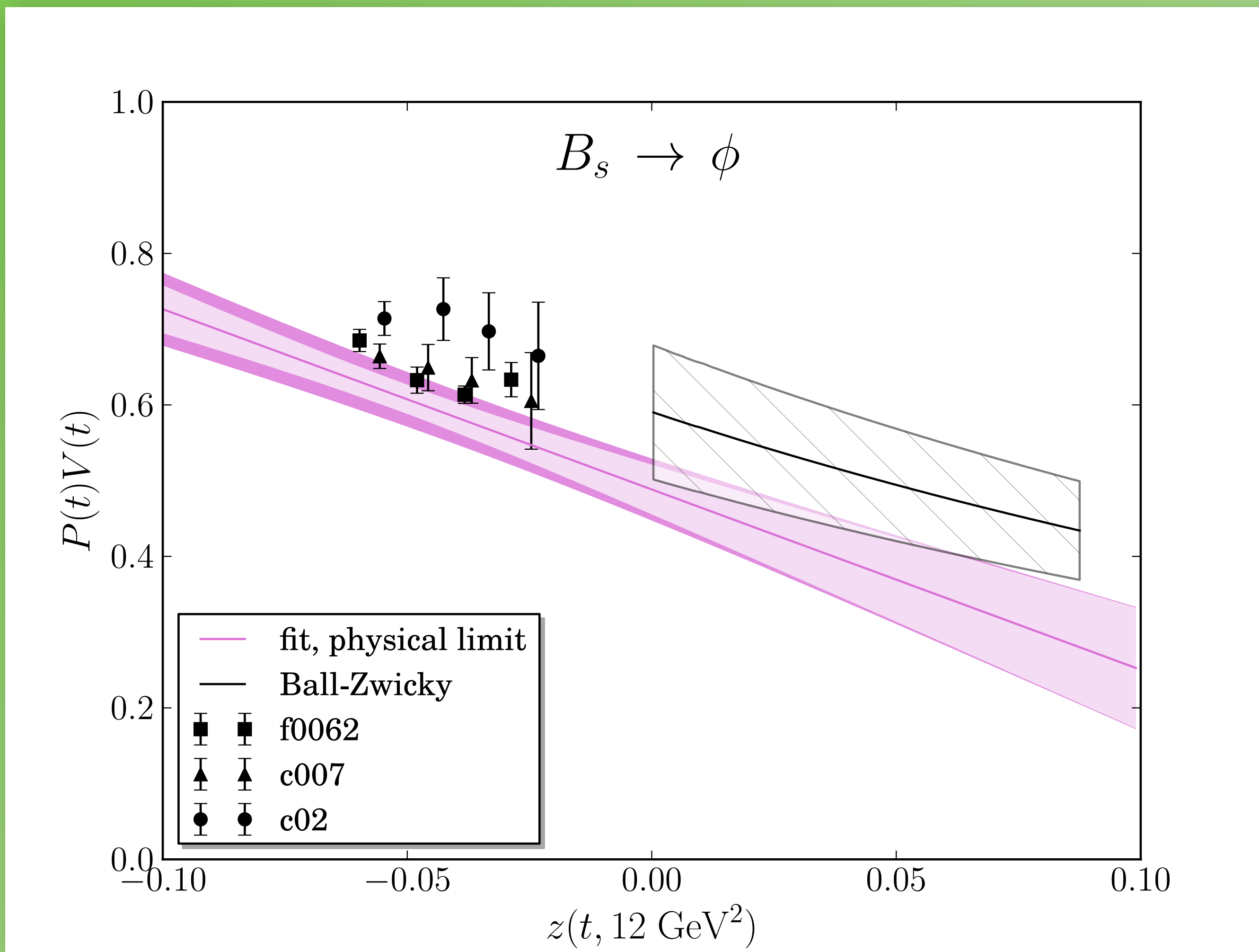
Horgan, Liu, Meinel, Wingate, arXiv:1310.3722

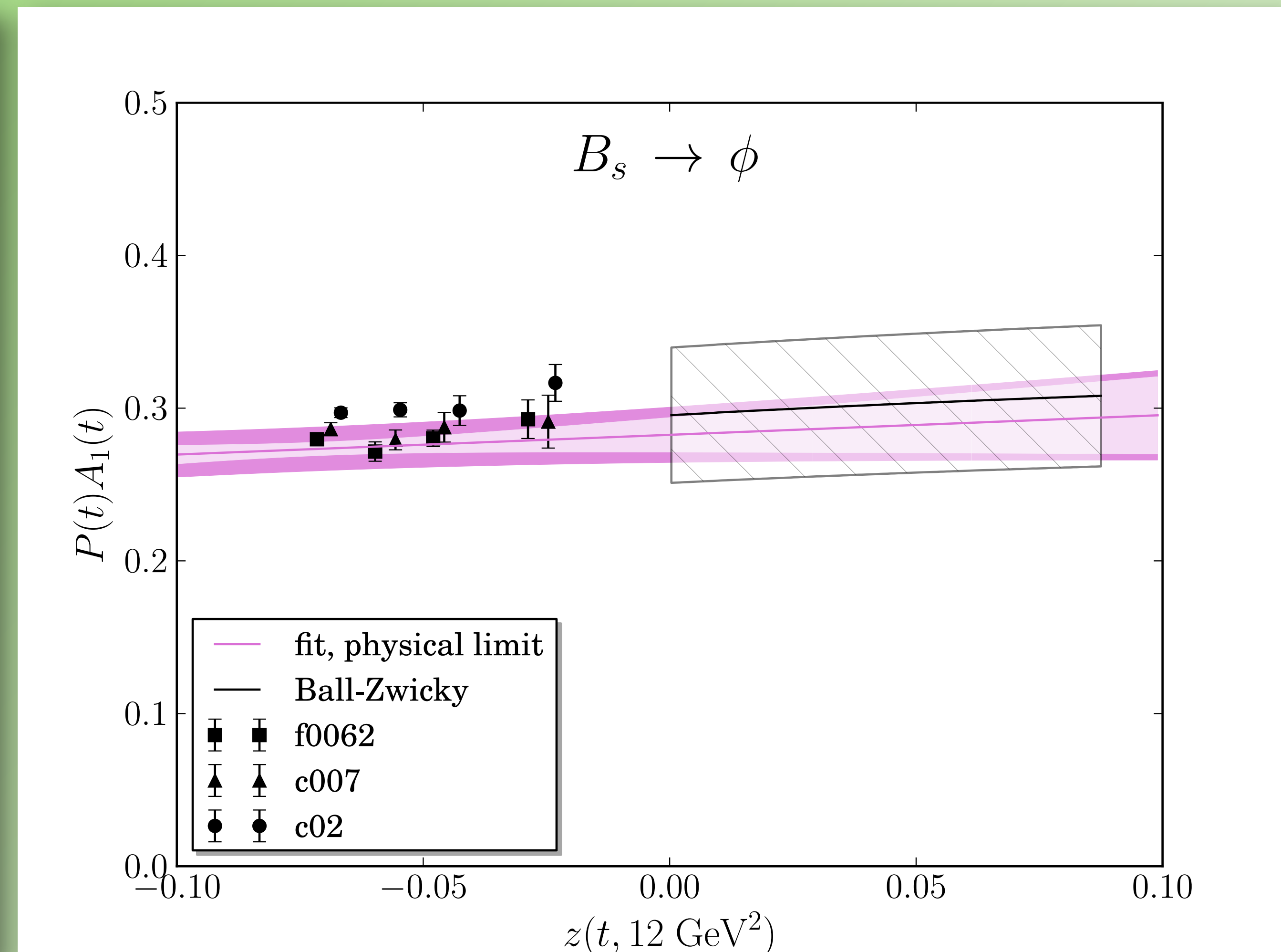
$$z = \frac{\sqrt{t_+ - t} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - t} + \sqrt{t_+ - t_0}}$$

$$t = q^2$$
$$t_\pm = (m_B \pm m_F)^2$$

$$F(t) = \frac{1}{1 - t/m_{\rm res}^2} \sum_n a_n z^n$$

$B_s \to \varphi$ form factors





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