Non-extensive fragmentation functions for high-energy collisions

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Outline

- Motivation
  - Motivation#1 The non-extensive phenomena: Tsallis–Pareto distributions
  - Motivation#2 Spectra fit in high-energy collisions

- Non-extensive fragmentation function parametrization in e^+e^-
  - A statistical model for hadron production in e^+e^- collisions
  - A non-extensive, Tsallis-like fragmentation function parametrization
  - Validity of scaling and comparison to other FFs.

- Discussion
  - Hadronization in the non-extensive statistical approach
  - Connection to the ‘Tsallis thermometer’

Motivation for the non-extensive hadronization models

Motivation #1
Statistical & thermodynamical point of view

The non-extensive statistical approach

- Extensive Boltzmann – Gibbs statistics

$$S_{12} = S_1 + S_2$$
$$E_{12} = E_2 + E_2$$

$\rightarrow$ $$S_B = -\sum_i p_i \ln p_i$$

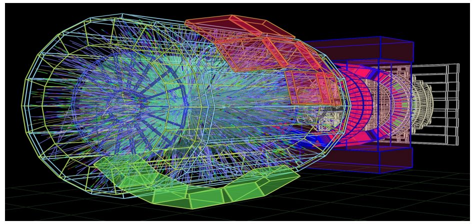
The non-extensive statistical approach

- Extensive Boltzmann – Gibbs statistics

$$S_{12} = S_1 + S_2 \qquad E_{12} = E_2 + E_2 \;\Rightarrow\; S_B = -\sum_i p_i \ln p_i$$

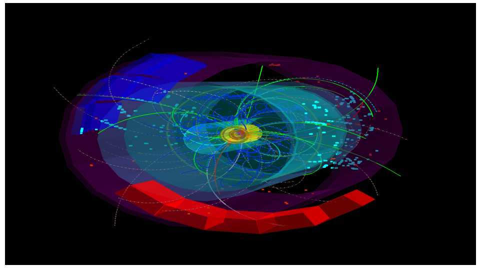
- Non-extensivity → generalized entropy

$$\hat{L}_{12}(S_{12}) = \hat{L}_1(S_1) + \hat{L}_2(S_2), \qquad L_{12}(E_{12}) = L_1(E_1) + L_2(E_2) \;\Rightarrow\; S_T = \frac{1}{1-q}\sum_i (p_i^q - p_i)$$



- Tsallis entropy

$$S_{12} = S_1 + S_2 + (q-1)S_1 S_2 \;\Rightarrow\; \hat{L}(S) = \frac{1}{q-1}\ln\left(1 + (q-1)S\right)$$



from here: Tsallis – Pareto distribution

$$f(\varepsilon) = \left[1 + (q-1)\frac{\varepsilon}{T}\right]^{-\frac{1}{q-1}}$$

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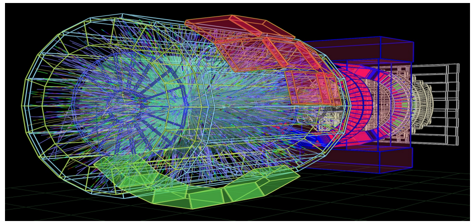
The non-extensive statistical approach

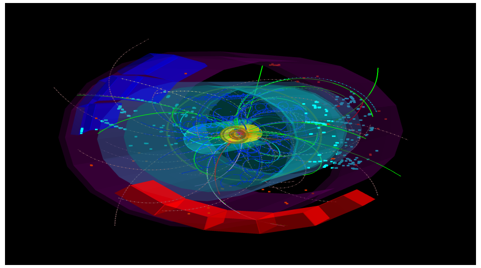
- Tsallis – Pareto distribution

$$f(\varepsilon) = \left[1 + (q-1)\frac{\varepsilon}{T}\right]^{-\frac{1}{q-1}}$$

$$q = \frac{\langle S'(E)^2 + S''(E)\rangle}{\langle S'(E)\rangle^2}$$

$$\frac{1}{T} = \langle S'(E)\rangle$$





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The non-extensive statistical approach

- Tsallis – Pareto distribution

$$f(\varepsilon) = \left[1 + (q-1)\frac{\varepsilon}{T}\right]^{-\frac{1}{q-1}}$$

$$q = \frac{\langle S'(E)^2 + S''(E)\rangle}{\langle S'(E)\rangle^2}$$

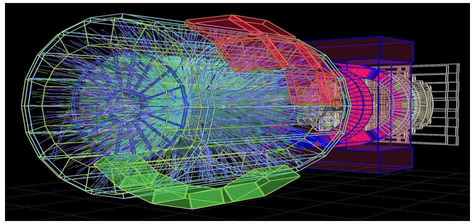
$$\frac{1}{T} = \langle S'(E)\rangle$$

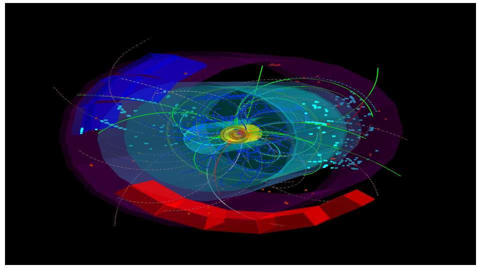
$$q = \frac{\langle n(n-1)\rangle}{\langle n\rangle^2}.$$

$$q = 1 + \frac{\Delta T^2}{T^2} - \frac{1}{C}$$

$$T = \frac{E}{\langle n\rangle}$$

$$T = \frac{\int \epsilon f_{TS}(\epsilon)}{\int f_{TS}(\epsilon)} = \frac{DT}{1-(q-1)(D+1)}$$





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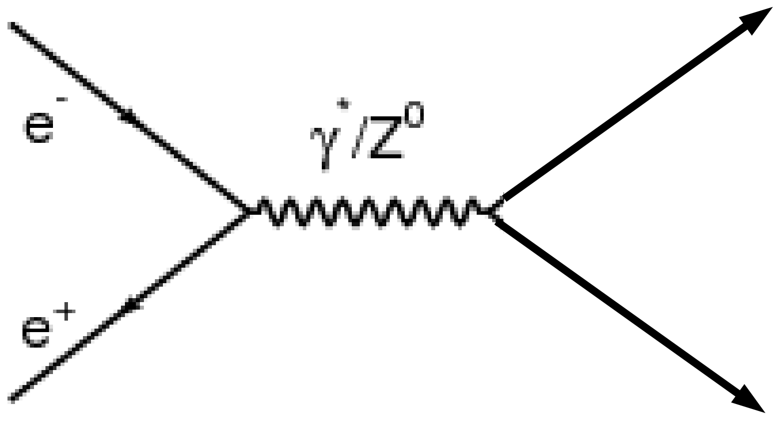
Motivation #2
High-energy (particle) physics point of view

Modeling hadronization in $e^{+}e^{-}$ collisions

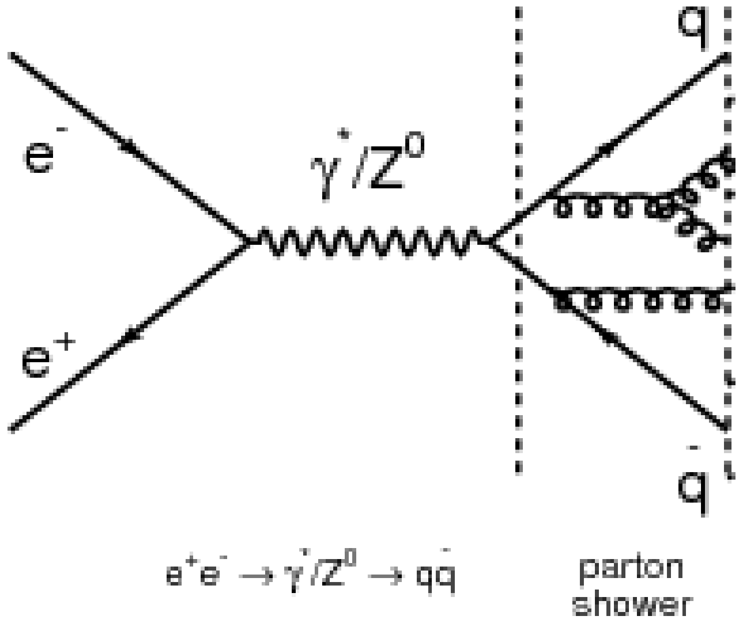
Final state processes & hadronization



$e^{+}e^{-} \rightarrow \gamma^{*}/Z^{0} \rightarrow q\bar{q}$

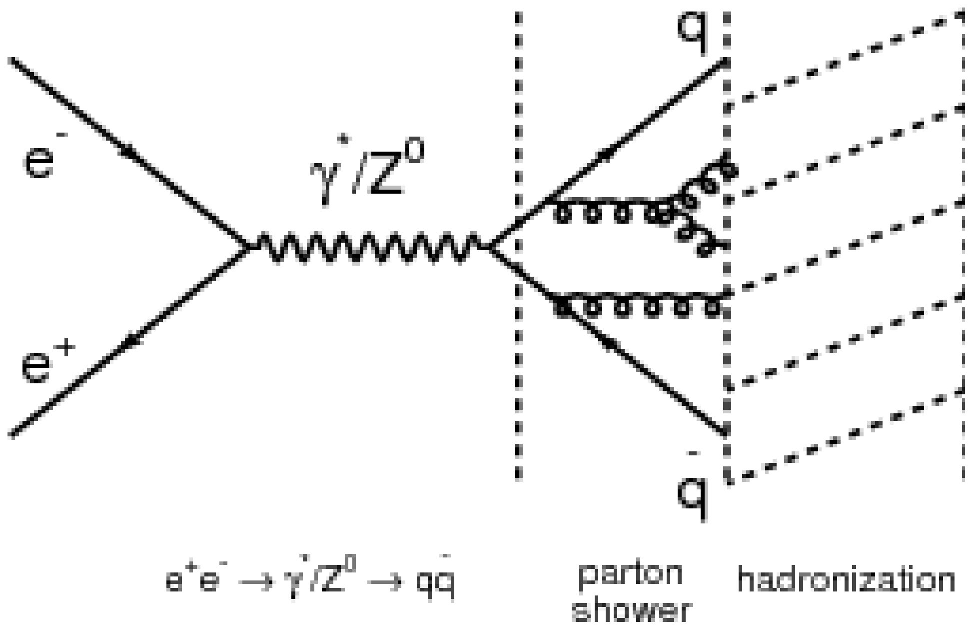
Modeling hadronization in e^+e^- collisions

Final state processes & hadronization



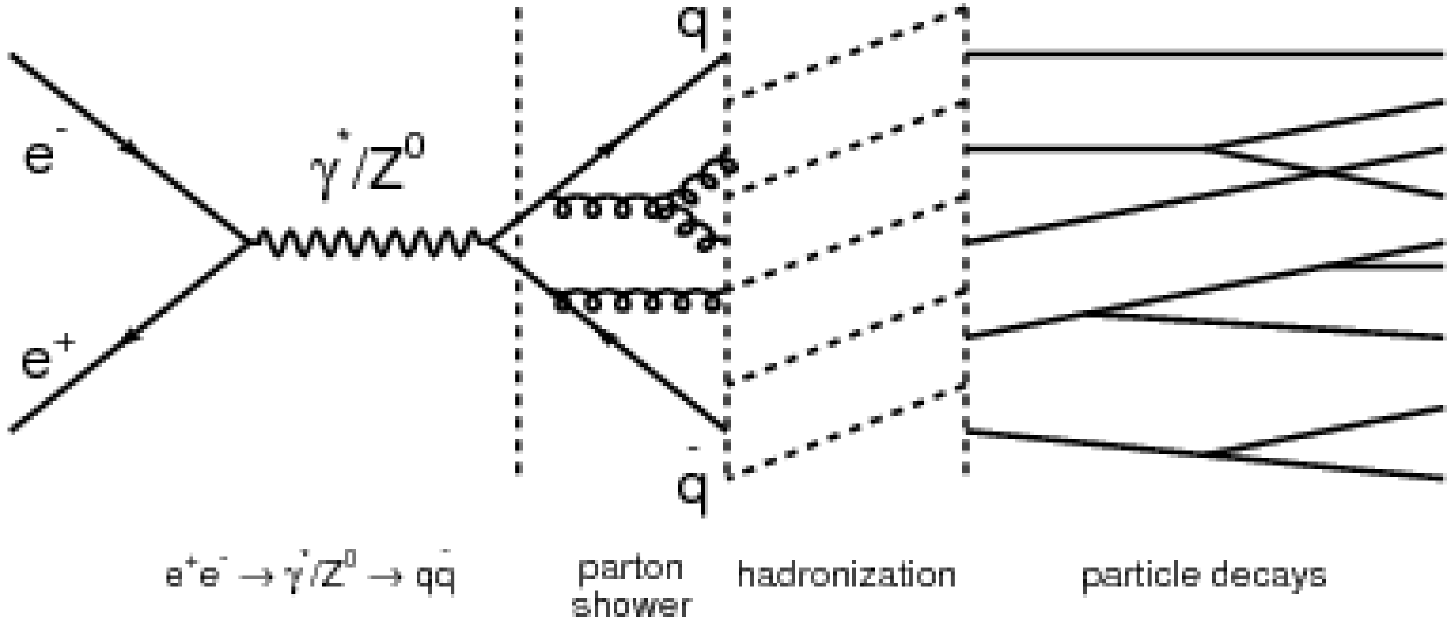
Modeling hadronization in e^+e^- collisions

Final state processes & hadronization



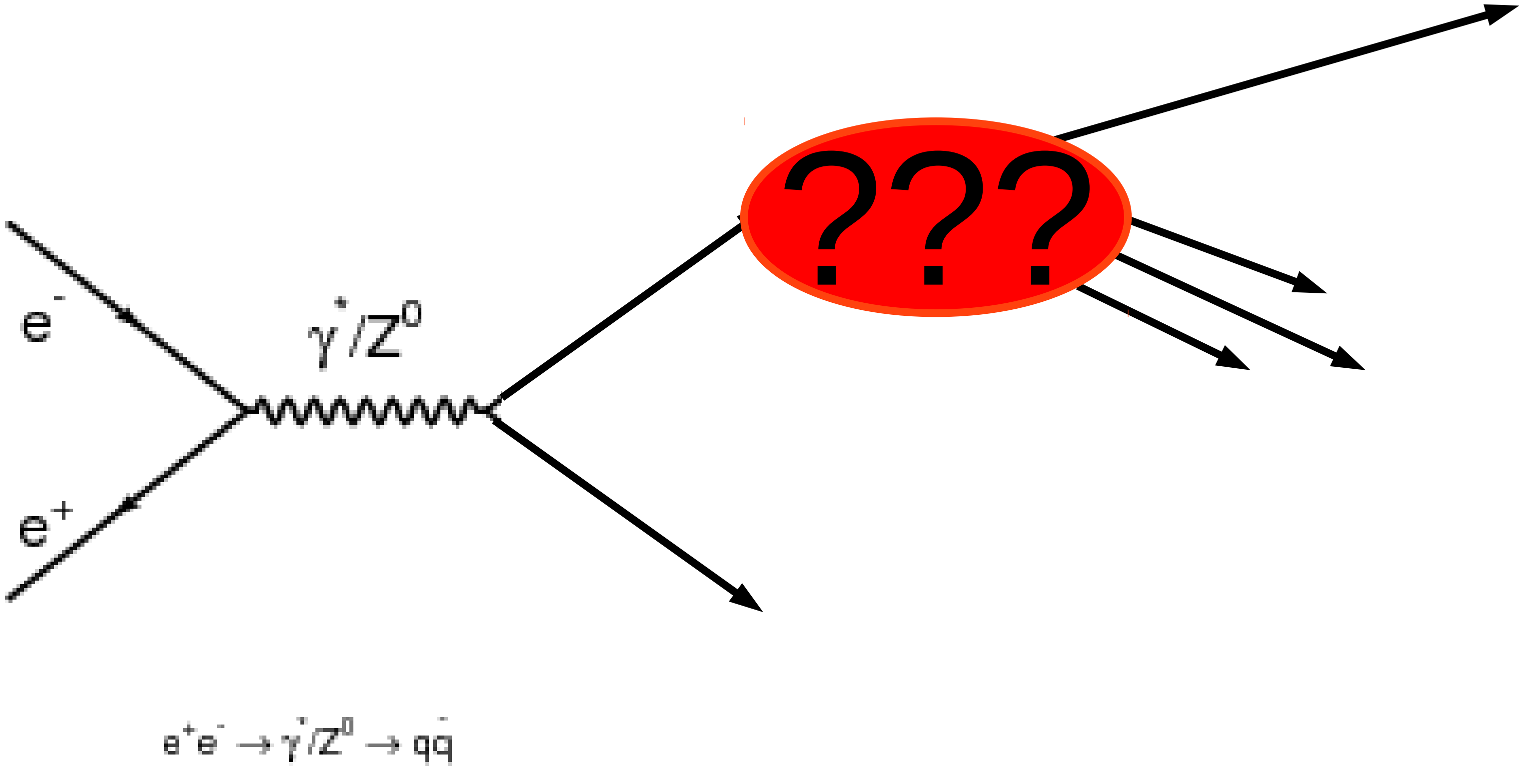
Modeling hadronization in e^+e^- collisions

Final state processes & hadronization



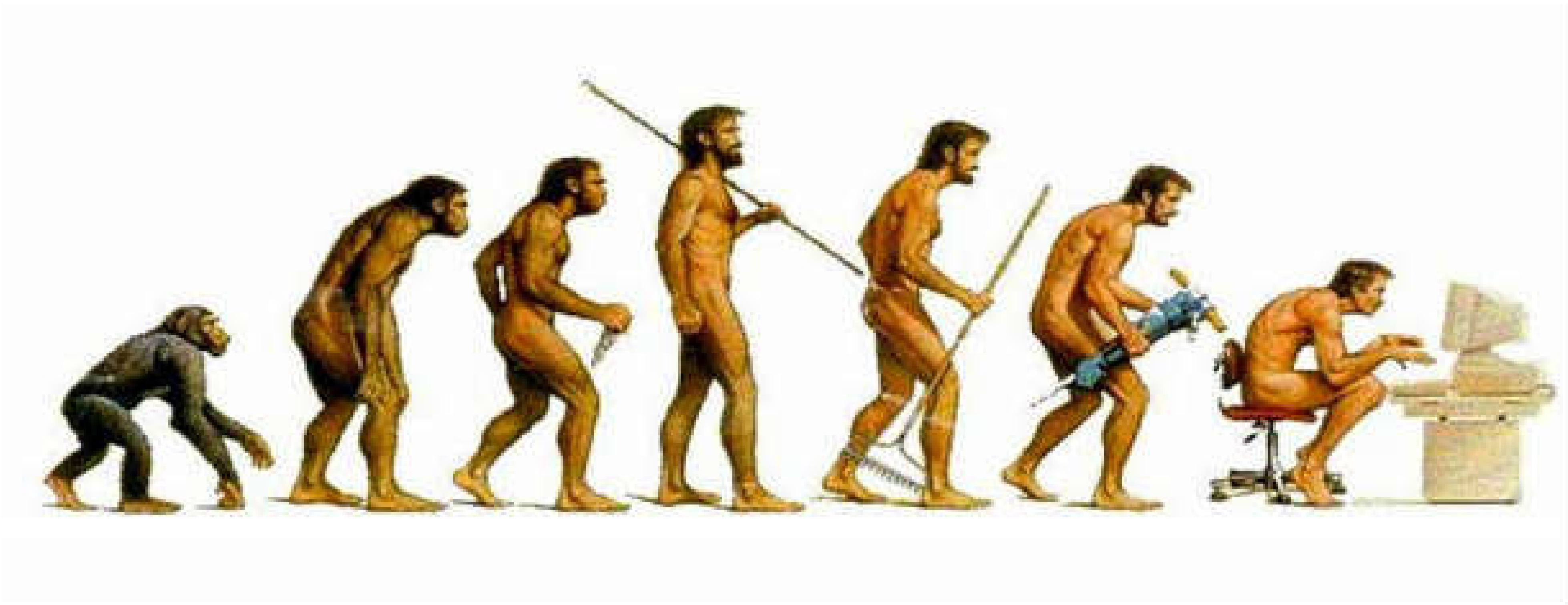
Modeling hadronization in e^+e^- collisions

Hadronization in the phenomenologial picture



Modeling hadronization in e^+e^- collisions

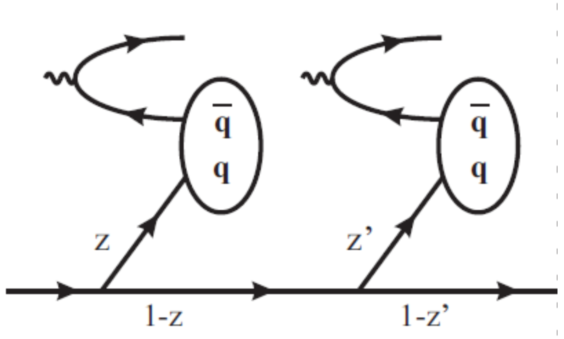
The evolution ...



Hadronization models – history

The evolution of hadronization models

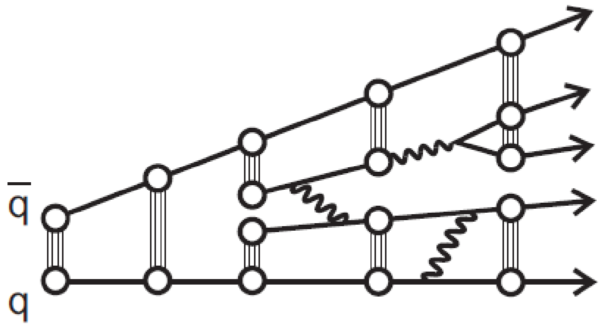
Feynman-Field



$$f(z) \propto \left[z \left(1 - \frac{1}{z} - \frac{\epsilon}{1-z} \right)^2 \right]^{-1}$$

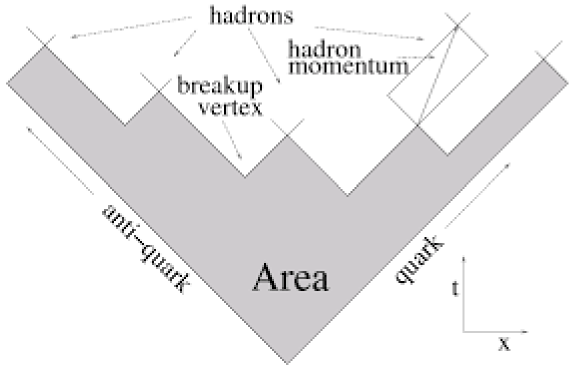
pQCD models

pair production



Non-pQCD models

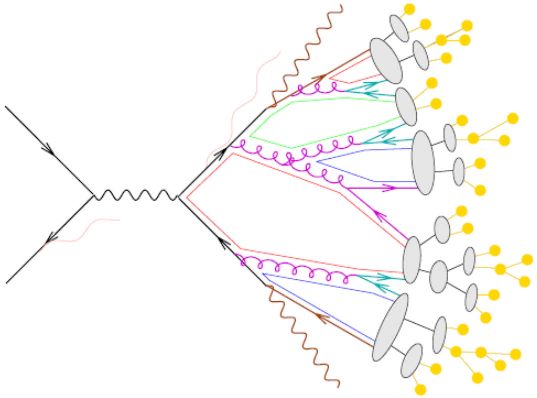
Lund model



$$f(z) \propto z^{-1}(1-z)^a \cdot \exp\left(\frac{-b\, m_T^2}{z}\right)$$

PYTHIA/HIJING

cluster model



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Hadronization models – FF comparison

Name	Feynman-Field polynomial	String (Lund) model	Non-extensive (Tsallis-like)
Formula	$D_i^h(z,Q^2) = N_i^h z^{\alpha_i^h}(1-z)^{\beta_i^h}$	$f(z) \propto z^{-1}(1-z)^a \cdot \exp\left(\frac{-b\, m_T^2}{z}\right)$	$D_i^h(z,Q) = N_i^h(1-z)\left[1-\frac{q_i^h-1}{T_i^h}\log(1-z)\right]^{-\frac{1}{q_i^h-1}}$
Physical motivation	propagator-like, power-law spectra	propagator-like + string model power-law + exponential	Non-extensive phenomena Tsallis-Pareto spectra
Physical meaning of parameters	No: spectra power, disagree with the theory	String tension + slope, but no for spectra power	Depending the statistical framework q (non-extensivity), T
Number of parameters	3/channel	3/channel	(2+1)/channel (normalized)
Evolution	DGLAP	DGLAP for power law	DGLAP

Motivation for the non-extensive formula

Fragmentation model families

$$\frac{d\sigma(e^-e^+ \to hX)}{dz} = \sum_i \sigma_0^i(s) D_i^h(z,Q)$$

$$f(z) \propto \left[z \left(1 - \frac{1}{z} - \frac{\epsilon}{1-z} \right)^2 \right]^{-1}$$

$$f(z) \propto z^{-1}(1-z)^a \cdot \exp\left(\frac{-b\, m_T^2}{z} \right)$$

$$D_i^h(z,Q) = N_i^h (1-z) \left[1 - \frac{q_i^h - 1}{T_i^h} \log(1-z) \right]^{-\frac{1}{q_i^h - 1}}$$

$$D_q^h(z) = N_q^h z^{\alpha_q^h} (1-z)^{\beta_q^h} \left(1 + \gamma_q^h (1-z)^{\delta_q^h} \right)$$

$$D_i^h(z,Q^2) = N_i^h z^{\alpha_i^h} (1-z)^{\beta_i^h}$$

Motivation for the non-extensive formula

Fragmentation model families

$$\frac{d\sigma(e^-e^+ \to hX)}{dz} = \sum_i \sigma_0^i(s) D_i^h(z,Q)$$

$$D_i^h(z,Q^2) = N_i^h z^{\alpha_i^h}(1-z)^{\beta_i^h}$$

$$f(z) \propto z^{-1}(1-z)^a \cdot \exp\left(\frac{-b\, m_T^2}{z}\right)$$

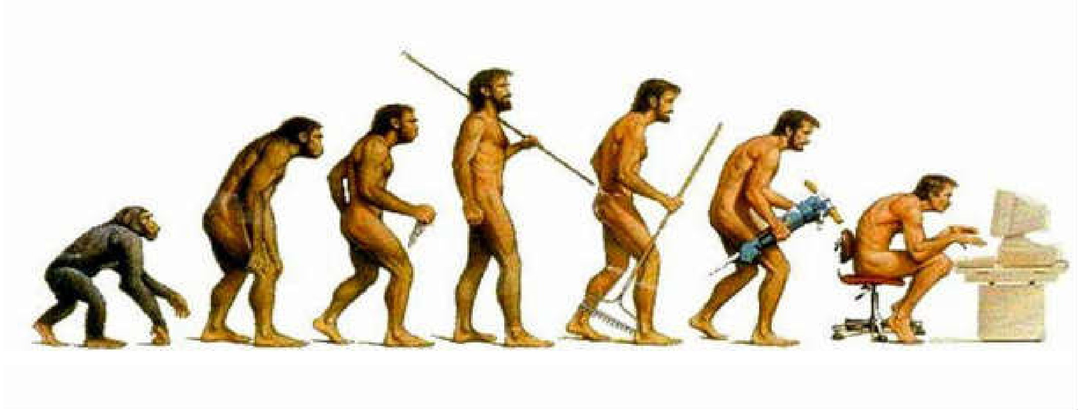
$$D_i^h(z,Q) = N_i^h(1-z)\left[1 - \frac{q_i^h - 1}{T_i^h}\log(1-z)\right]^{-\frac{1}{q_i^h-1}}$$

q → 1

$$\lim_{q\to 1} D_i^h(z,Q) = N_i^h(1-z)^{1+\frac{1}{T_i^h}}$$

Motivation for the non-extensive formula

Can we make the next evolution step?



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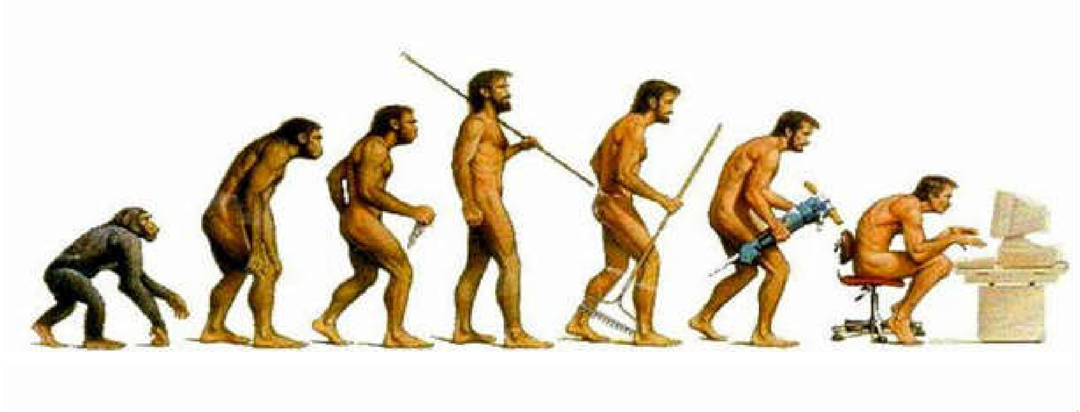
Motivation for the non-extensive formula

Can we make the next evolution step?



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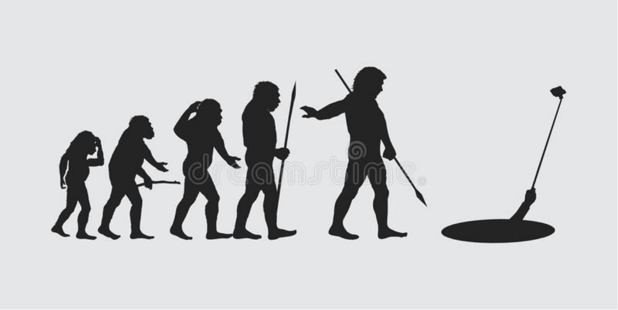

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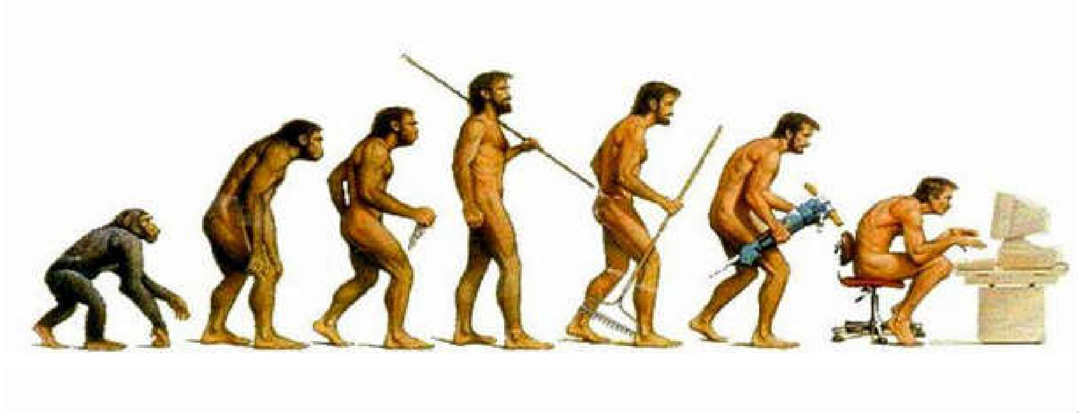

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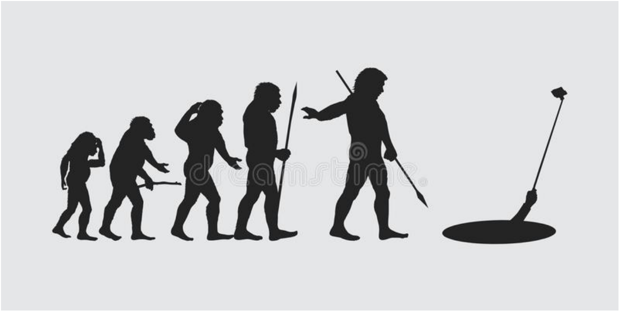

Motivation for the non-extensive formula

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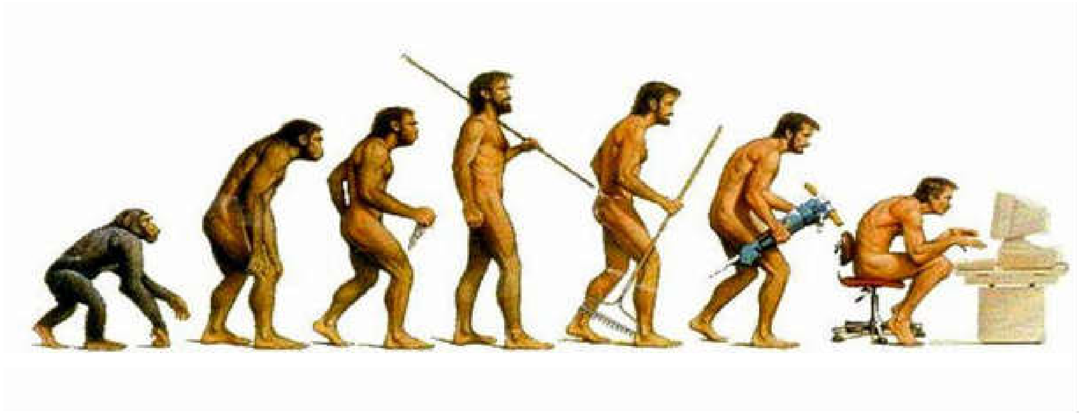

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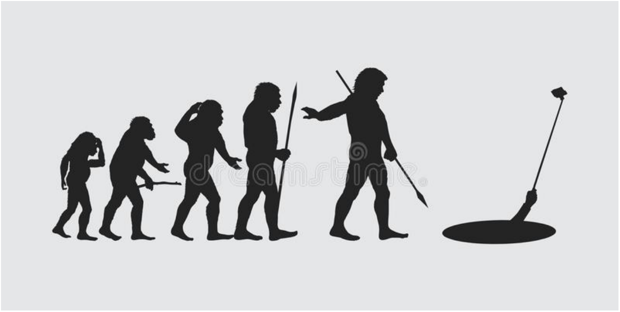
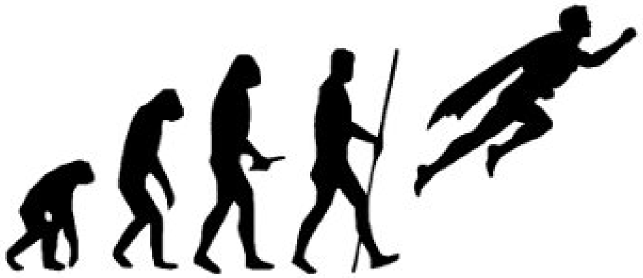




Fragmentation function parametrization in the non-extensive statistical approach

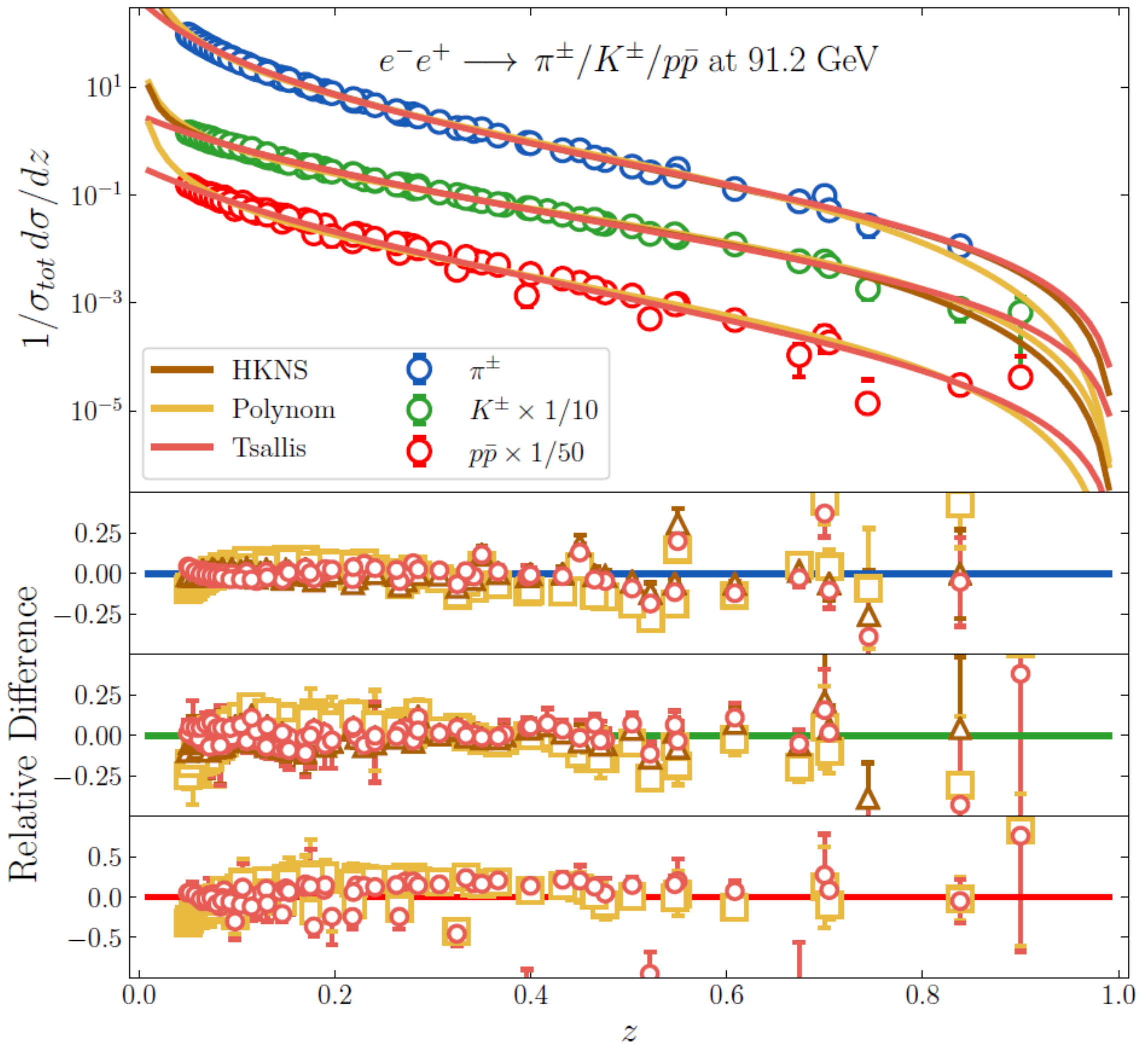
Fit the non-extensive formula in e^+e^- collisions

LO pQCD parton model

$$\frac{d\sigma(e^-e^+ \rightarrow hX)}{dz} = \sum_i \sigma_0^i(s) D_i^h(z,Q)$$

measure

parameters



Fit identified pion data in e^+e^- collisions to get FF parameters.

Comparison to KKP, HKNS FFs

Fit the non-extensive formula in e^+e^- collisions

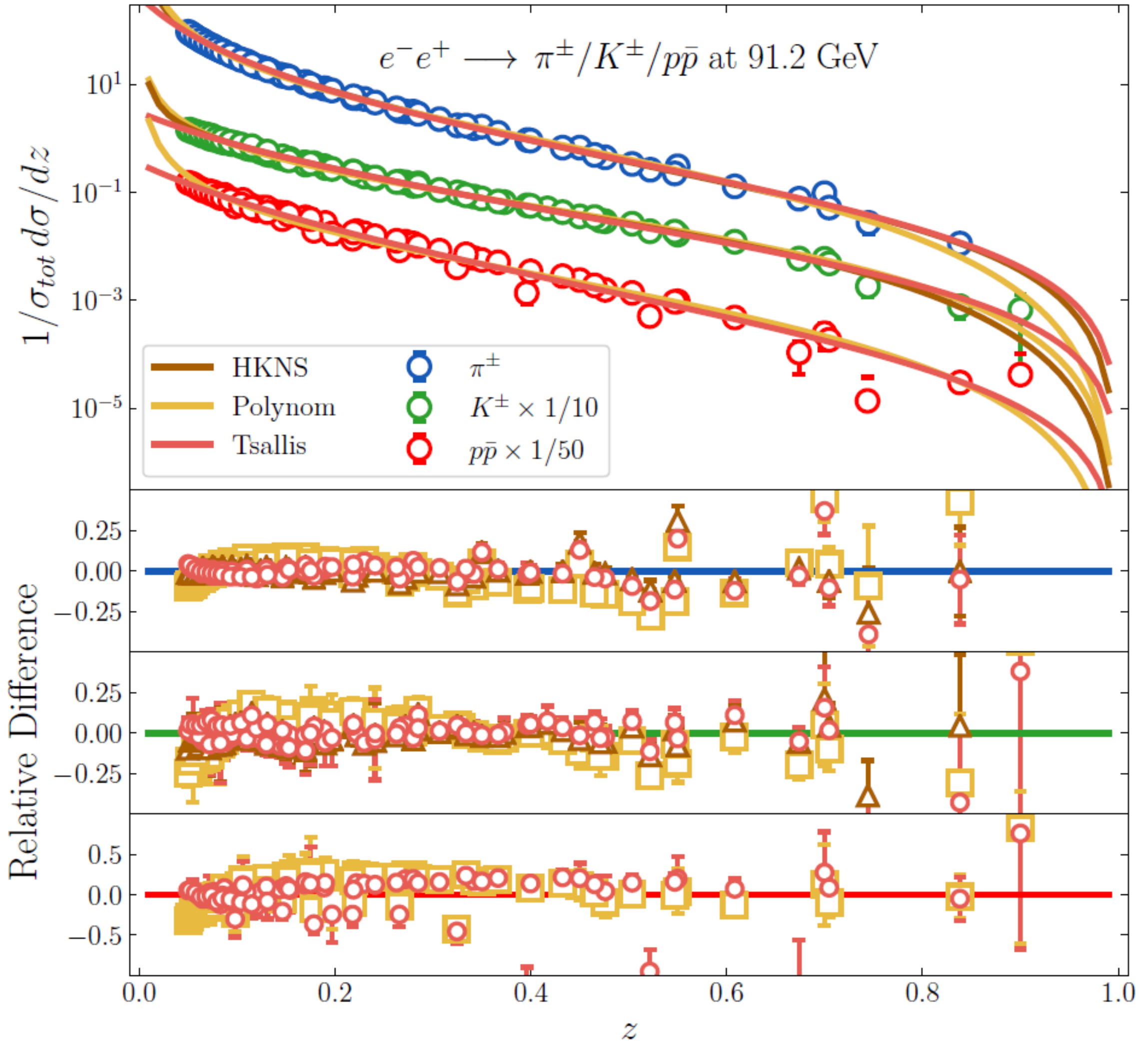
LO pQCD parton model

Partonic channels q→h are fitted at initial Q scale in LO.

$$F^h(z,Q) = \frac{1}{\sigma_{tot}} \frac{d\sigma(e^-e^+ \to hX)}{dz}$$

via minimizing the merit function, using the data

$$\chi^2 = \sum_i \frac{\left(F^h(x_i,Q^2) - y_i\right)^2}{(\sigma_i)^2}$$



Fit the non-extensive formula in e^+e^- collisions

LO pQCD parton model

Partonic channels q→h are fitted at initial Q scale in LO.

Need to reduce the number of fit parameters by the symmetries:

Here we fit (charge-averaged) pions

$$D_i^{\pi^0}(z,Q) = \frac{1}{2}\left[D_i^{\pi^+}(z,Q) + D_i^{\pi^-}(z,Q)\right]$$

$$\pi^+ = |u\bar{d}\rangle$$
$$\pi^- = |\bar{u}d\rangle$$

$$D_q^{\pi^-}(z,Q) = D_{\bar{q}}^{\pi^+}(z,Q)$$
$$D_g^{\pi^-}(z,Q) = D_g^{\pi^+}(z,Q)$$

$$D_u^{\pi^+} = D_{\bar{d}}^{\pi^+},$$
$$D_d^{\pi^+} = D_{\bar{u}}^{\pi^+} = D_s^{\pi^+} = D_{\bar{s}}^{\pi^+},$$
$$D_c^{\pi^+} = D_{\bar{c}}^{\pi^+},$$
$$D_b^{\pi^+} = D_{\bar{b}}^{\pi^+},$$
$$D_g^{\pi^+}.$$

For charge-average pions

Fit the non-extensive formula in e^+e^- collisions

LO pQCD parton model

Partonic channels q→h are fitted at initial Q scale in LO.

Need to reduce the number of fit parameters by the symmetries:

Isospin, (anti)particle, neglect top, sea/valence contributions

(2×6+1)×3=69 → 3×(2×2+1)=15

$$\pi^+ = |u\bar{d}\rangle$$
$$\pi^- = |\bar{u}d\rangle$$
$$D_q^{\pi^-}(z,Q) = D_{\bar{q}}^{\pi^+}(z,Q)$$
$$D_g^{\pi^-}(z,Q) = D_g^{\pi^+}(z,Q)$$

$$D_u^{\pi^+} = D_{\bar{d}}^{\pi^+},$$
$$D_d^{\pi^+} = D_{\bar{u}}^{\pi^+} = D_s^{\pi^+} = D_{\bar{s}}^{\pi^+},$$
$$D_c^{\pi^+} = D_{\bar{c}}^{\pi^+},$$
$$D_b^{\pi^+} = D_{\bar{b}}^{\pi^+},$$
$$D_g^{\pi^+}.$$

For charge-average pions

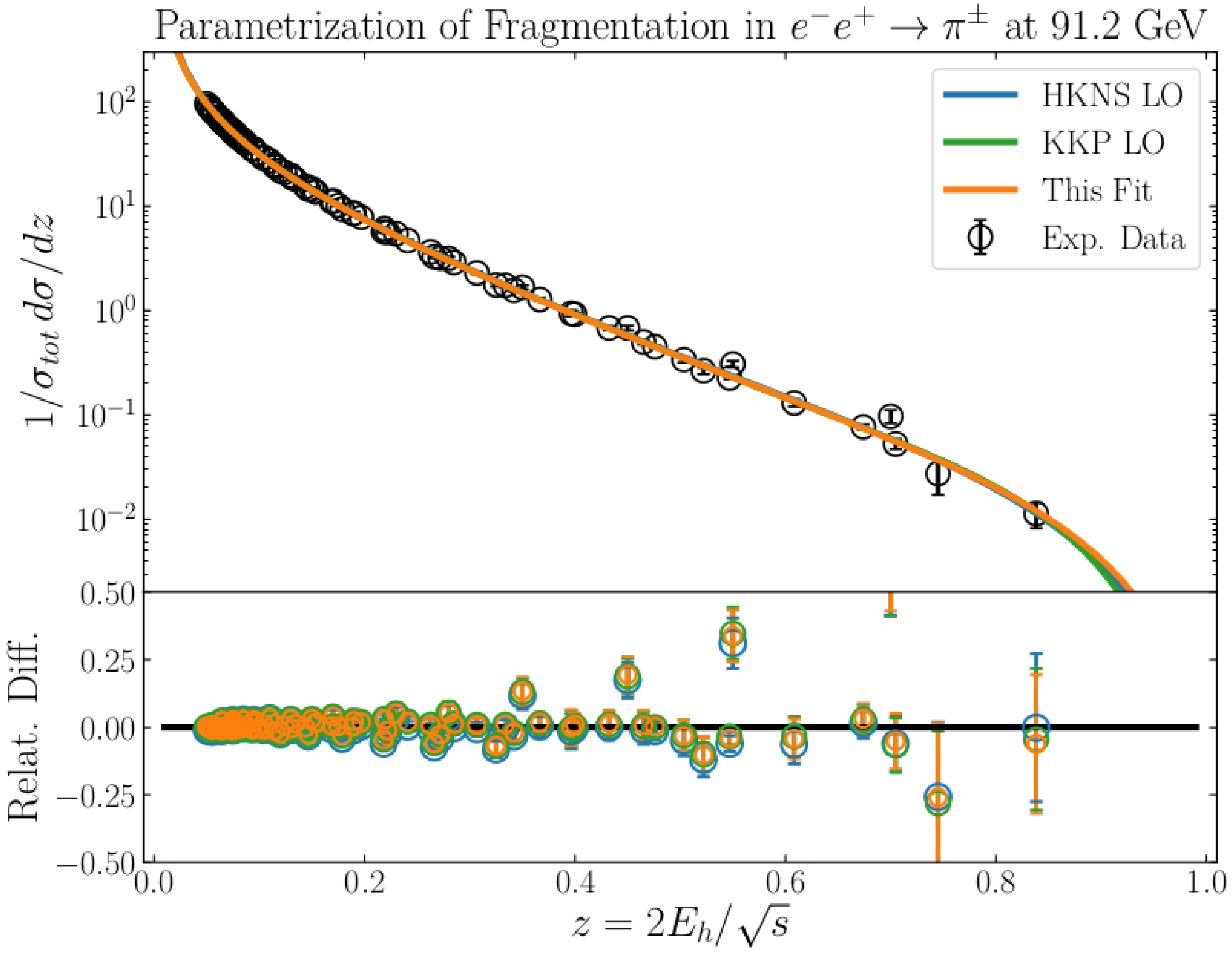
Fit the non-extensive formula in e^+e^- collisions

LO pQCD parton model

Partonic channels q→h are fitted at initial Q scale in LO.

We used the symmetries of sea & valence channels, up to beauty.

$$D_i^{\pi^0}(z,Q) = \frac{1}{2}\left[D_i^{\pi^+}(z,Q) + D_i^{\pi^-}(z,Q)\right]$$



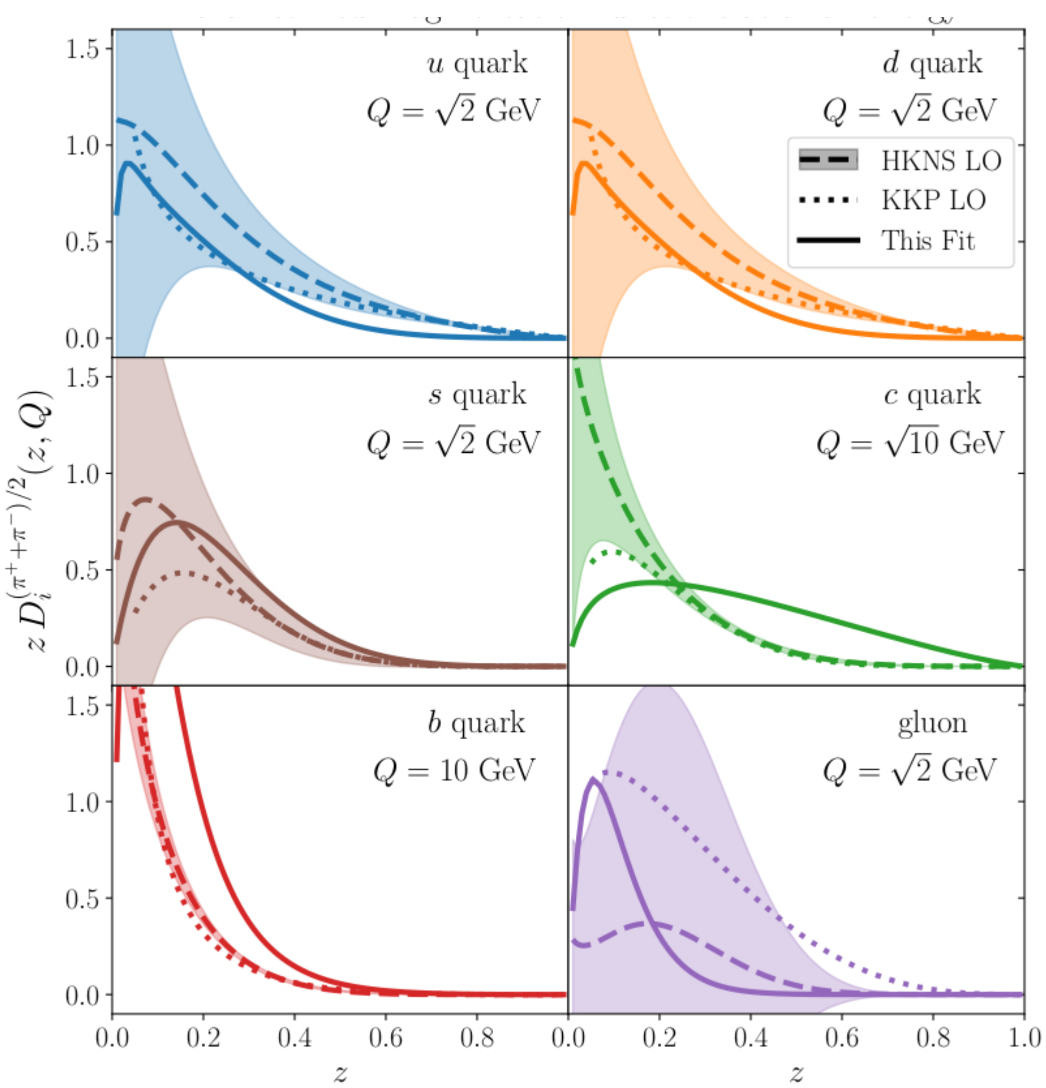
Fit the non-extensive formula in e^+e^- collisions

LO pQCD parton model

Partonic channels q→h are fitted at initial Q scale in LO.

Parameters (10+5)

$i \to \pi^+$	q	$1/T$	N
u	1.455	62.72	207.1
$\bar{u} = s$	1.063	4.850	13.68
c	2.238	25.89	14.97
b	1.211	21.45	151.6
g	1.059	17.05	53.81



Fit the non-extensive formula in e^+e^- collisions

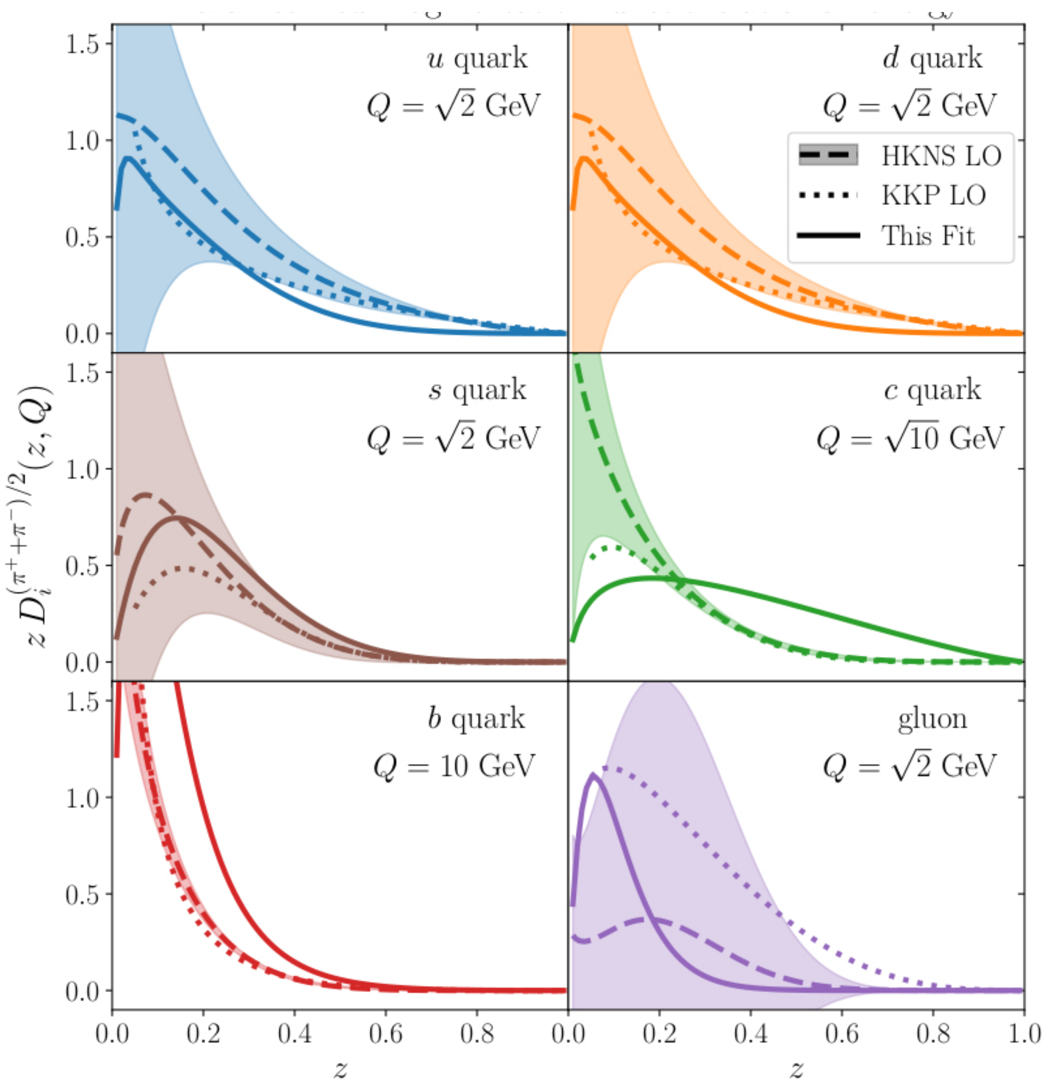
Pion LO FF parametrization

All FFs have similar high-z trend, especially for valence quarks

Non-extensive FFs have a clear maxima at low z (< 2GeV) values.

KKP has low-z cut, HKNS presents uncertainties

Sea kvark & gluon channels has more difference.



Fit the non-extensive formula in e^+e^- collisions

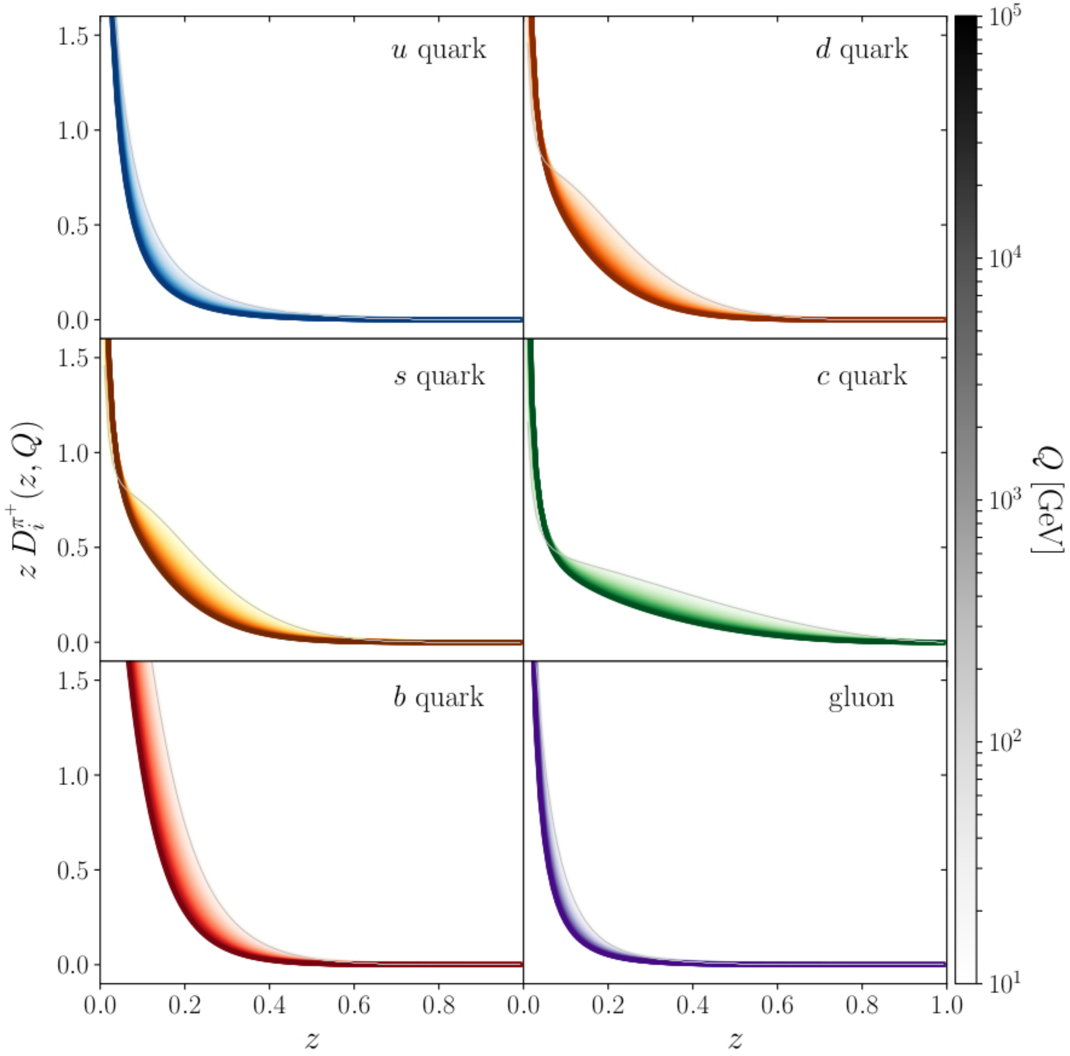
Parameter scale evolution

DGLAP evolution is given in LO:

$$\frac{dD_i^h(z,Q^2)}{d\log Q^2} = \sum_j \int_z^1 \frac{dx}{x} P_{ij}(z/x,Q^2) D_i^h(x,Q^2)$$

This is converted fitted by a simply formula for the parameters

$$D_i^h(z,Q) = N_i^h (1-z) \left[1 - \frac{q_i^h - 1}{T_i^h} \log(1-z)\right]^{-\frac{1}{q_i^h - 1}}$$



Fit the non-extensive formula in e^+e^- collisions

Parameter scale evolution

DGLAP evolution is given in LO:

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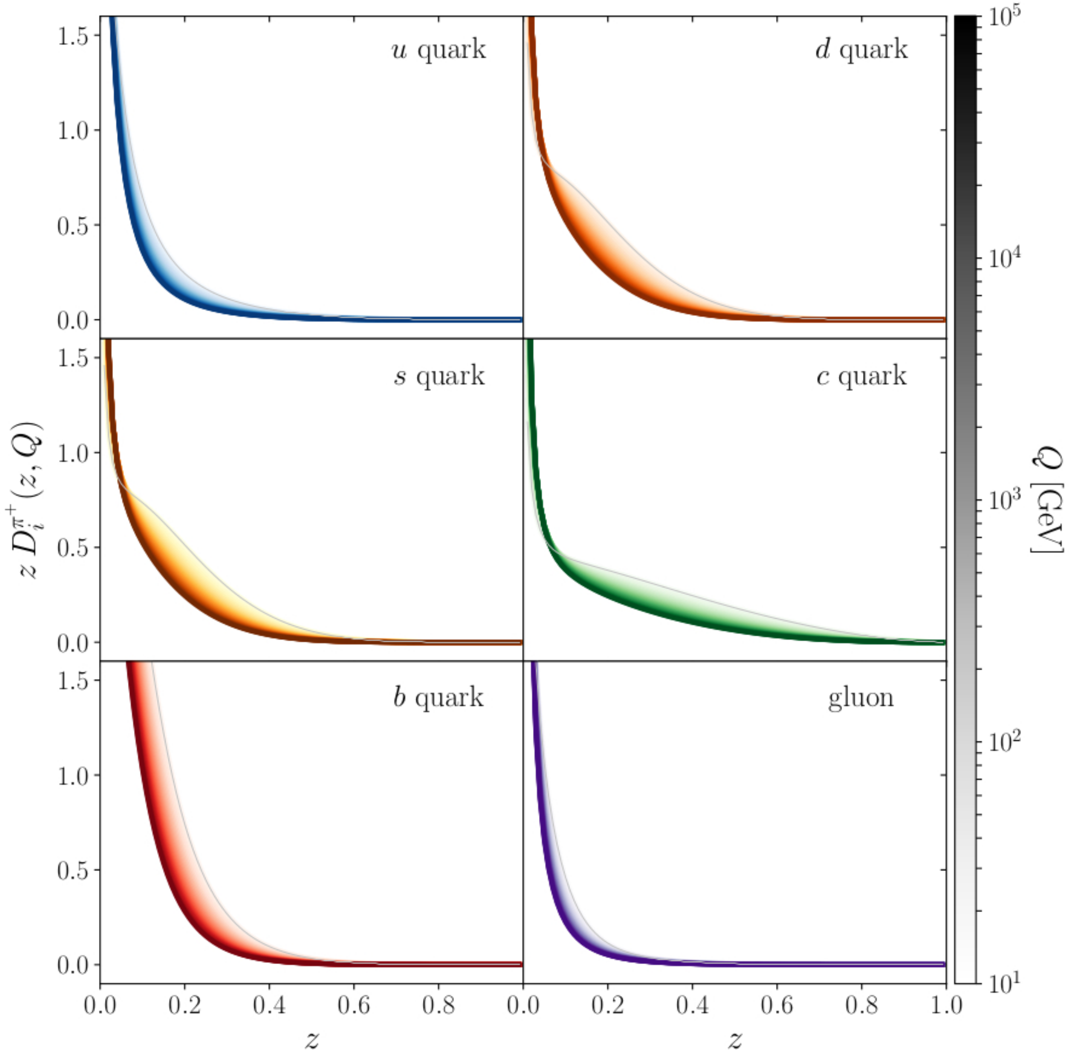
This is converted fitted by a simply formula for the parameters

$$D_i^h(z,Q) = N_i^h (1-z) \left[1 - \frac{q_i^h - 1}{T_i^h} \log(1-z)\right]^{\frac{1}{q_i^h - 1}}$$

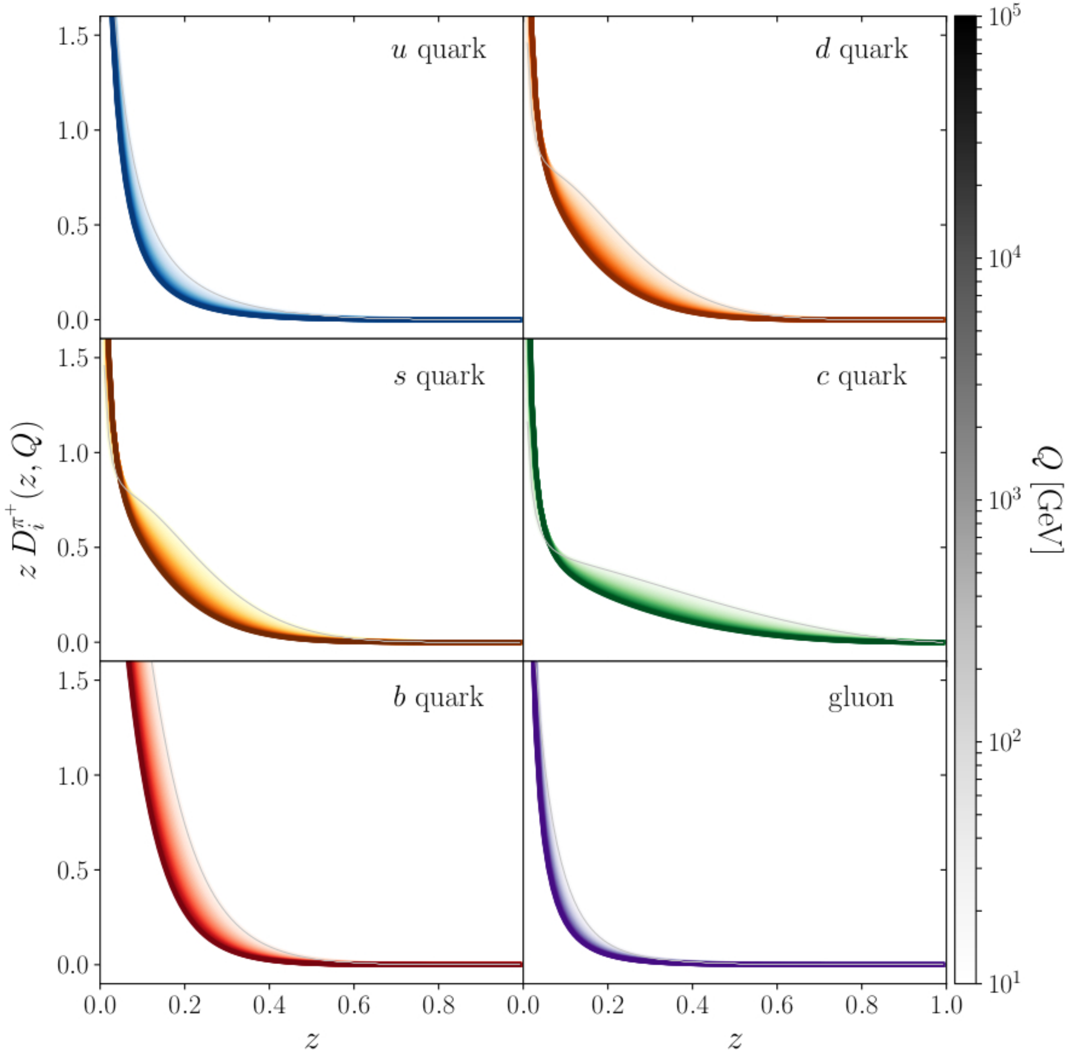


Fit the non-extensive formula in e^+e^- collisions

Parameter scale evolution

$$D_i^h(z,Q) = N_i^h (1-z)\left[1 - \frac{q_i^h - 1}{T_i^h}\log(1-z)\right]^{\frac{1}{q_i^h - 1}}$$

DGLAP is converted, fitted by a simply formula for the *q*,*T*, & *N*



Fit the non-extensive formula in e^+e^- collisions

Parameter scale evolution

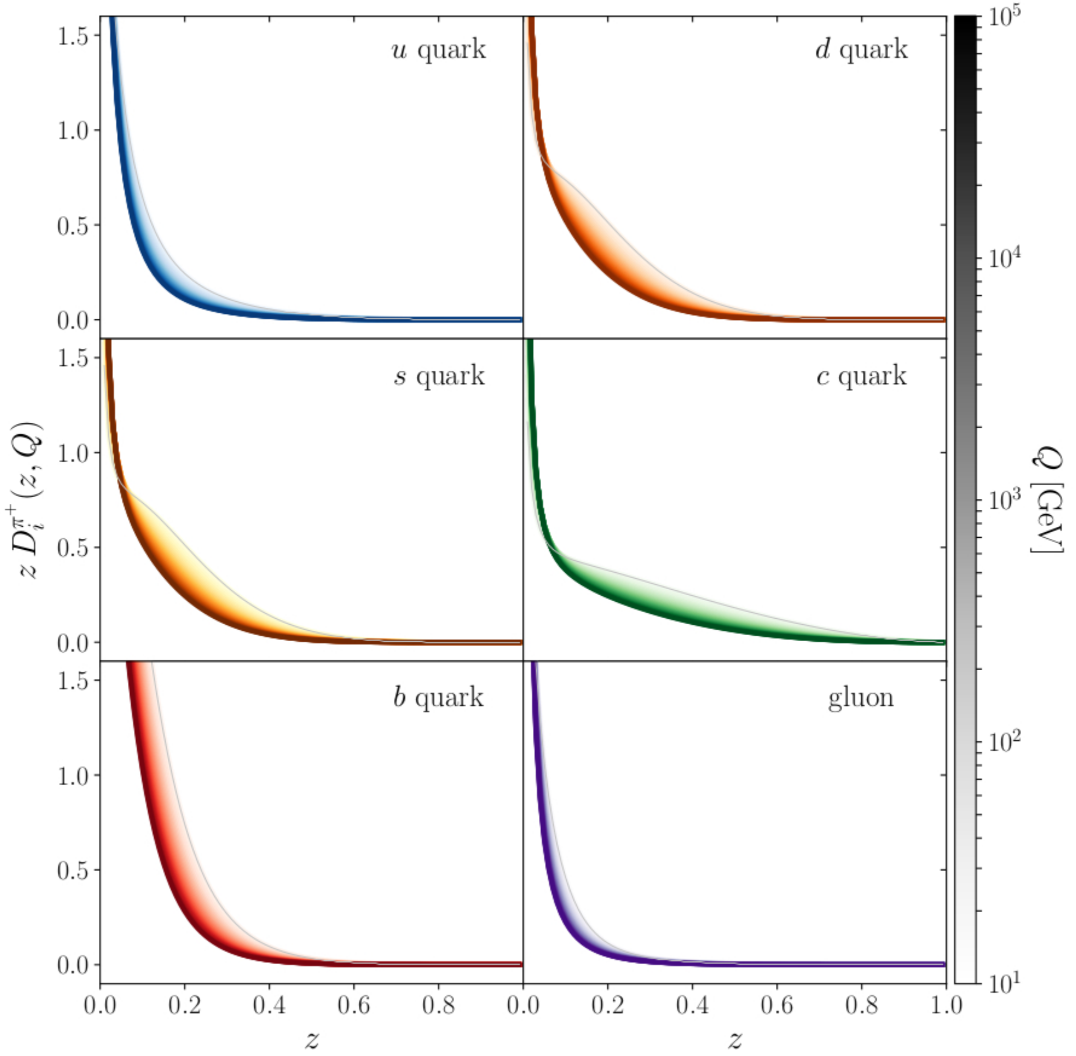
$$D_i^h(z,Q) = N_i^h (1-z) \left[1 - \frac{q_i^h - 1}{T_i^h} \log(1-z)\right]^{\frac{1}{q_i^h - 1}}$$

DGLAP is converted, fitted by a simply formula for the *q*,*T*, & *N*

$$N_i^h = a_{N_i^h} + b_{N_i^h}\,\bar{s} + c_{N_i^h}\,\bar{s}^2 + d_{N_i^h}\,\bar{s}^3,$$
$$q_i^h = a_{q_i^h} + b_{q_i^h}\,\bar{s} + c_{q_i^h}\,\bar{s}^2 + d_{q_i^h}\,\bar{s}^3,$$
$$T_i^h = a_{T_i^h} + b_{T_i^h}\,\bar{s} + c_{T_i^h}\,\bar{s}^2 + d_{T_i^h}\,\bar{s}^3,$$

where

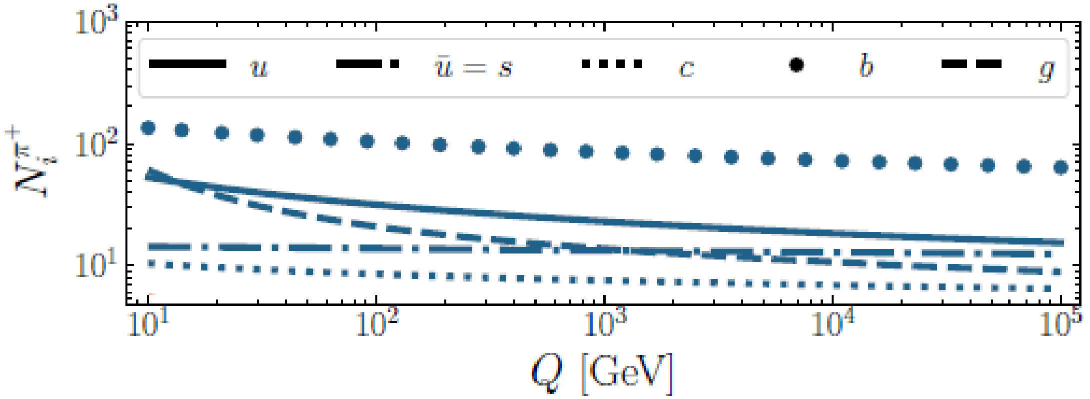
$$\bar{s} = \log\left[\frac{\log(Q^2/\Lambda^2)}{\log(Q_0^2/\Lambda^2)}\right]$$



Fit the non-extensive formula in e^+e^- collisions

Parameter scale evolution

$$D_i^h(z,Q) = N_i^h (1-z)\left[1 - \frac{q_i^h - 1}{T_i^h}\log(1-z)\right]^{-\frac{1}{q_i^h - 1}}$$



DGLAP is converted, fitted by a simply formula for the *q*,*T*, & *N*

$$N_i^h = a_{N_i^h} + b_{N_i^h}\,\bar{s} + c_{N_i^h}\,\bar{s}^2 + d_{N_i^h}\,\bar{s}^3,$$

$$q_i^h = a_{q_i^h} + b_{q_i^h}\,\bar{s} + c_{q_i^h}\,\bar{s}^2 + d_{q_i^h}\,\bar{s}^3,$$

$$T_i^h = a_{T_i^h} + b_{T_i^h}\,\bar{s} + c_{T_i^h}\,\bar{s}^2 + d_{T_i^h}\,\bar{s}^3,$$

where

$$\bar{s} = \log\left[\frac{\log(Q^2/\Lambda^2)}{\log(Q_0^2/\Lambda^2)}\right]$$

Fit the non-extensive formula in e^+e^- collisions

Parameter scale evolution

$$D_i^h(z,Q) = N_i^h(1-z)\left[1 - \frac{q_i^h - 1}{T_i^h}\log(1-z)\right]^{\frac{1}{q_i^h - 1}}$$

DGLAP is converted, fitted by a simply formula for the *q*,*T*, & *N*

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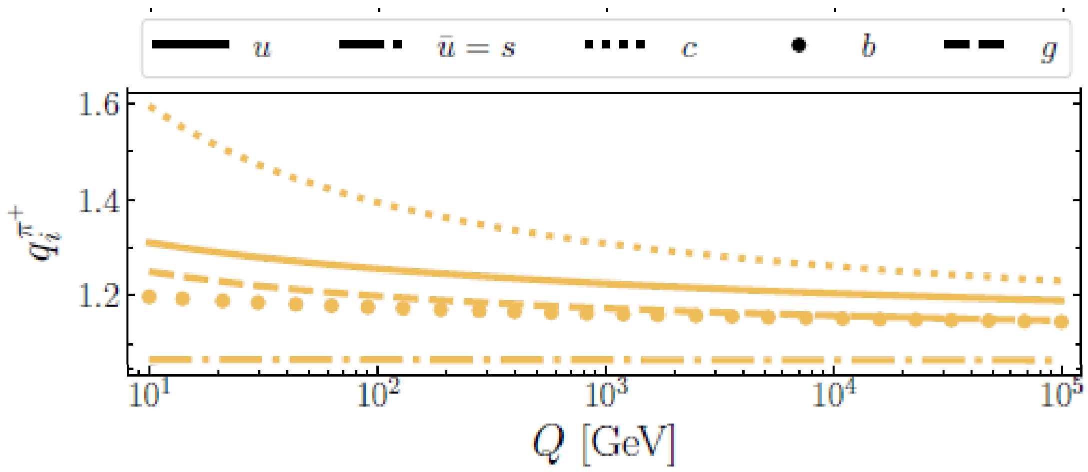

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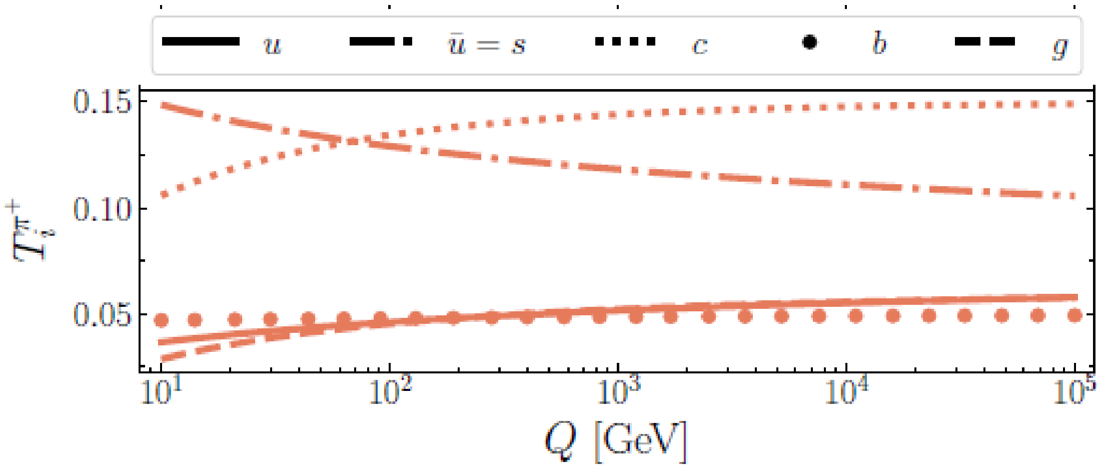

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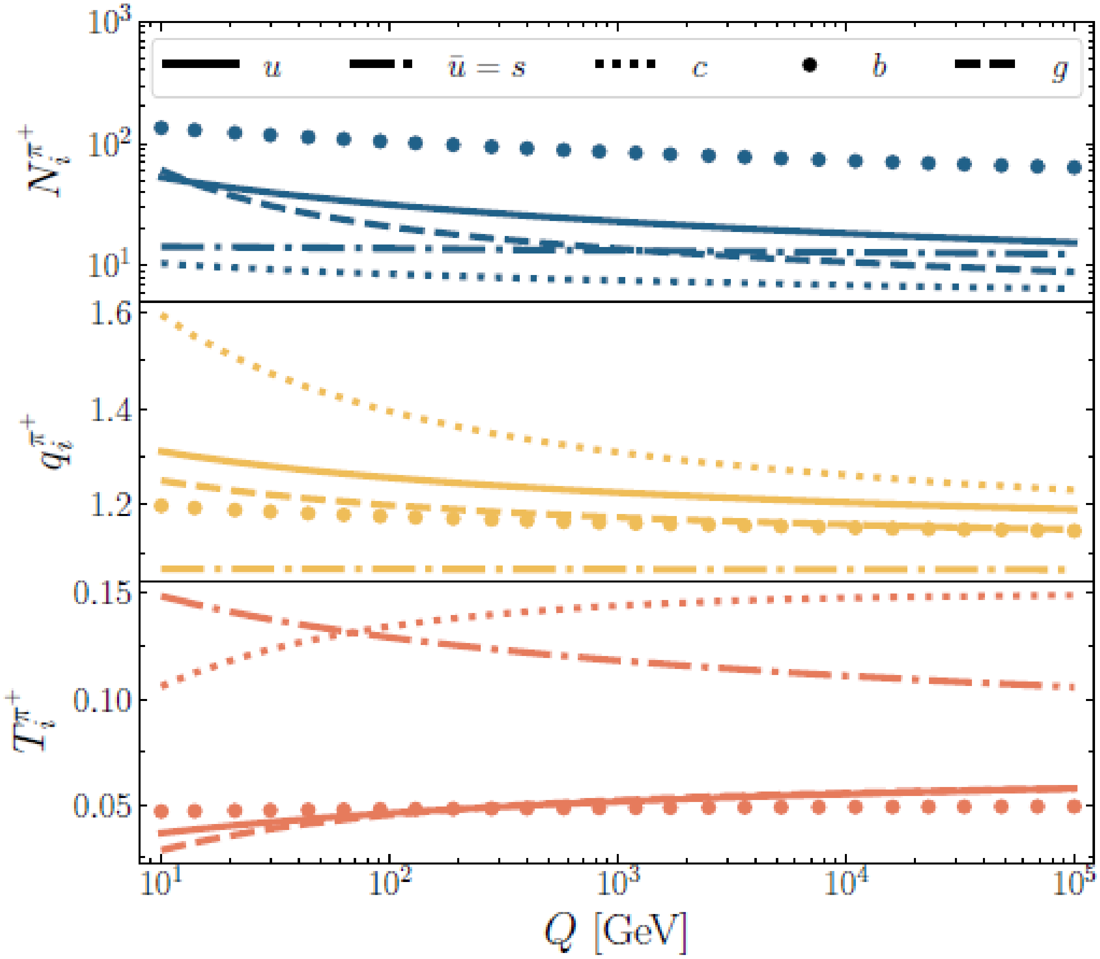


Self-tests of the non-extensive formula in e^+e^- collisions

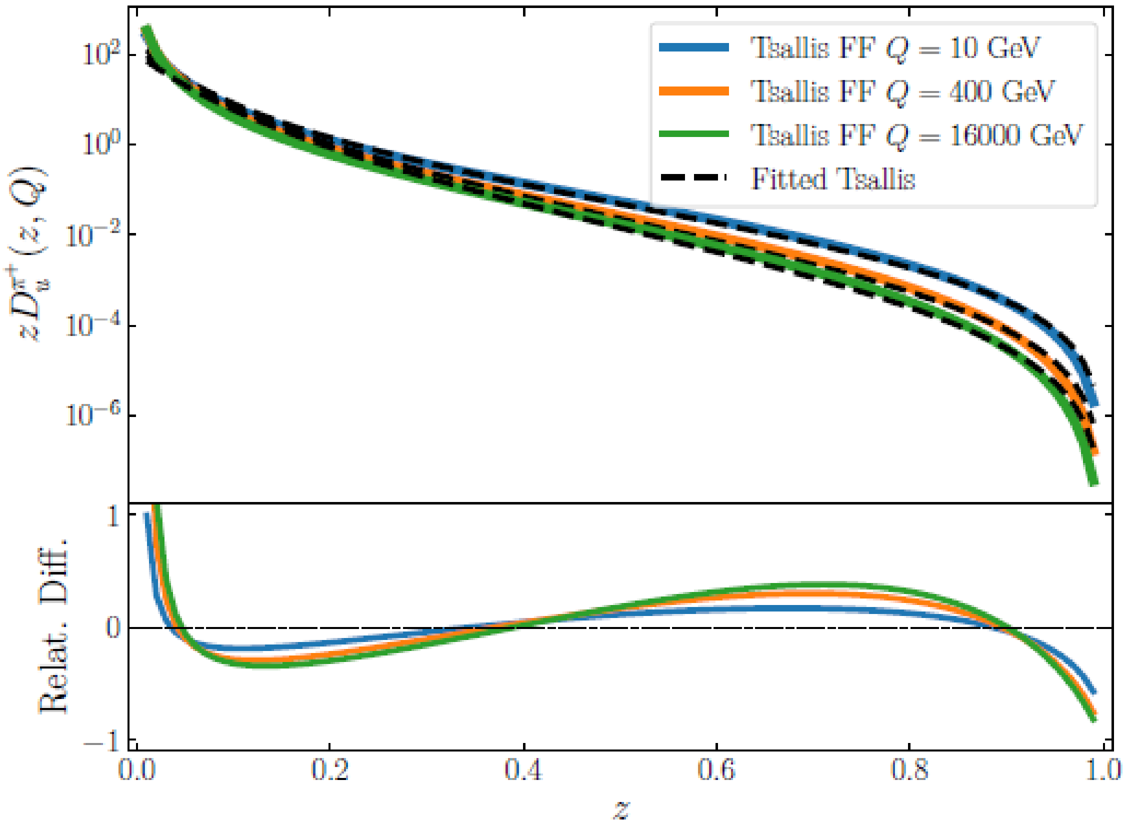
Test of scale evolution

DGLAP evolution is given in LO:

$$\frac{dD_i^h(z,Q^2)}{d\log Q^2} = \sum_j \int_z^1 \frac{dx}{x} P_{ij}(z/x,Q^2) D_i^h(x,Q^2)$$

The real DGLAP scaling can be compared to the fit of the full, parametrized formula

$$D_i^h(z,Q) = N_i^h(Q)\left[1 - \frac{q_i^h(Q)-1}{T_i^h(Q)}\log(1-z)\right]^{-\frac{1}{q_i^h(Q)-1}}$$



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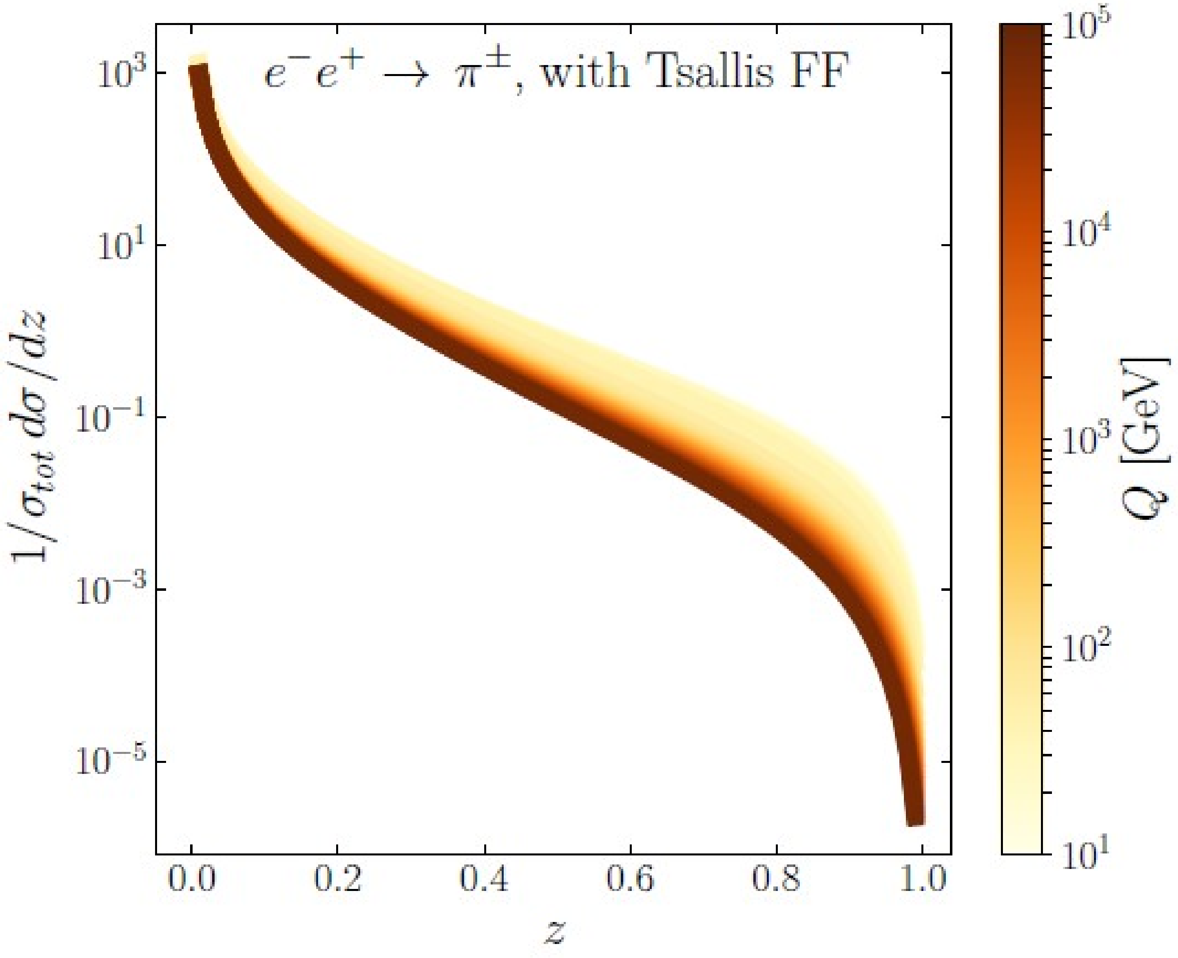
The real DGLAP scaling can be compared to the fit of the full, parametrized formula →

In full agreement with our earlier works' scaling ansatz ~log(log(Q))



Self-tests of the non-extensive formula in e^+e^- collisions

Channel contribution test

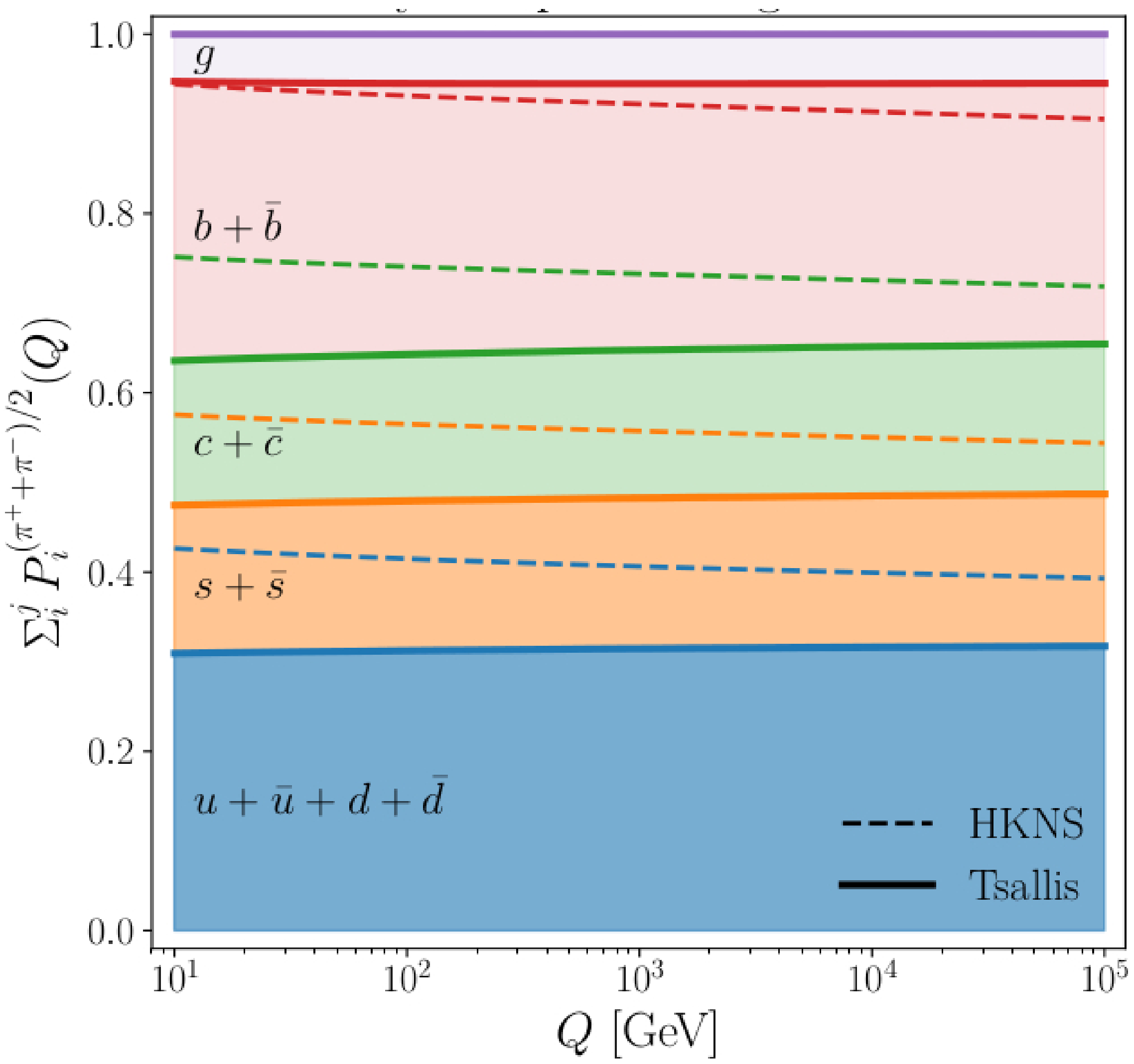
DGLAP evolution is given in LO:

$$\frac{dD_i^h(z,Q^2)}{d\log Q^2} = \sum_j \int_z^1 \frac{dx}{x} P_{ij}(z/x,Q^2) D_i^h(x,Q^2)$$

Using the sum rule,

$$1 = \sum_h \int_0^1 dz\, z D_i^h(z,Q)$$

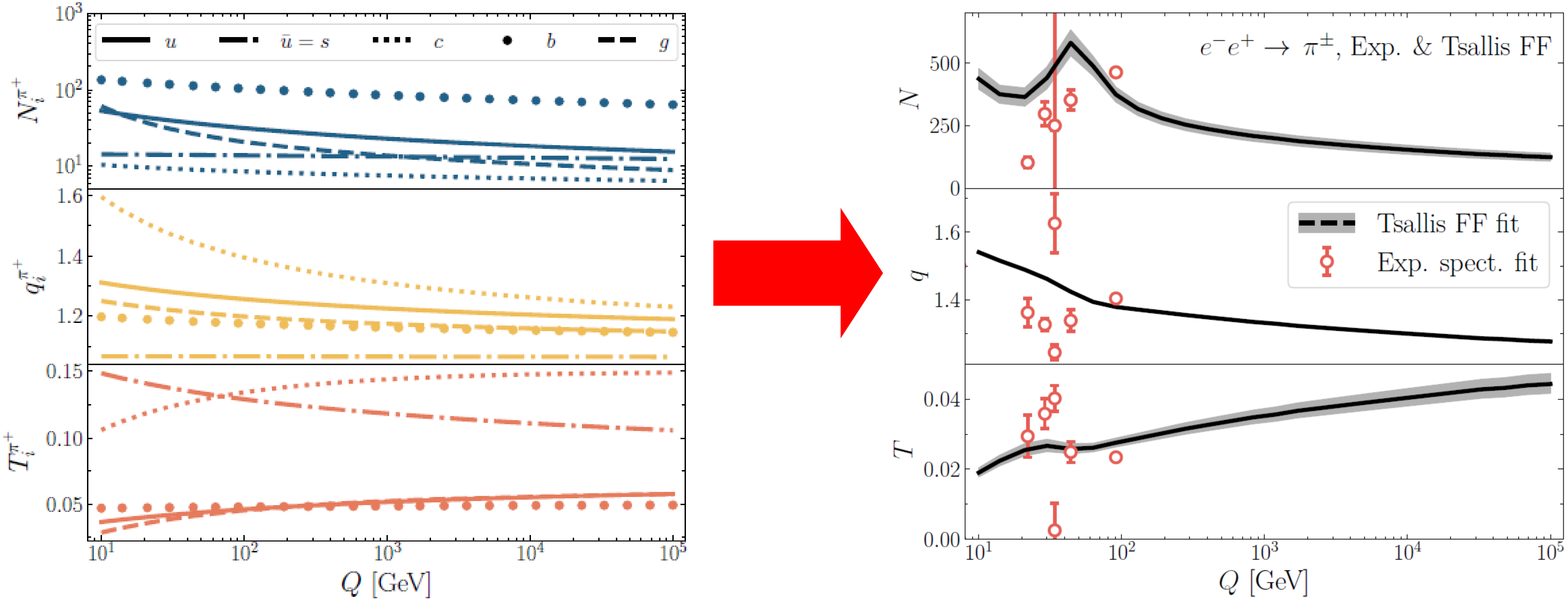
We calculated the evolution of the channels contribution (probability) to form charge-averaged pion.



Discussion & comparison to data

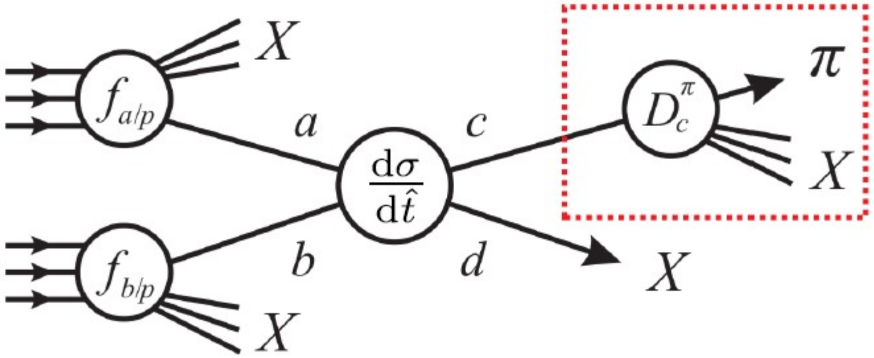
Comparing non-extensive FF with e^+e^- data

Scale evolution in channels → full formula + errors + pion data



Comparing non-extensive FF with pp data

Test of non-extensive FFs within the kTpQCD_v20 model



Comparing non-extensive FF with pp data

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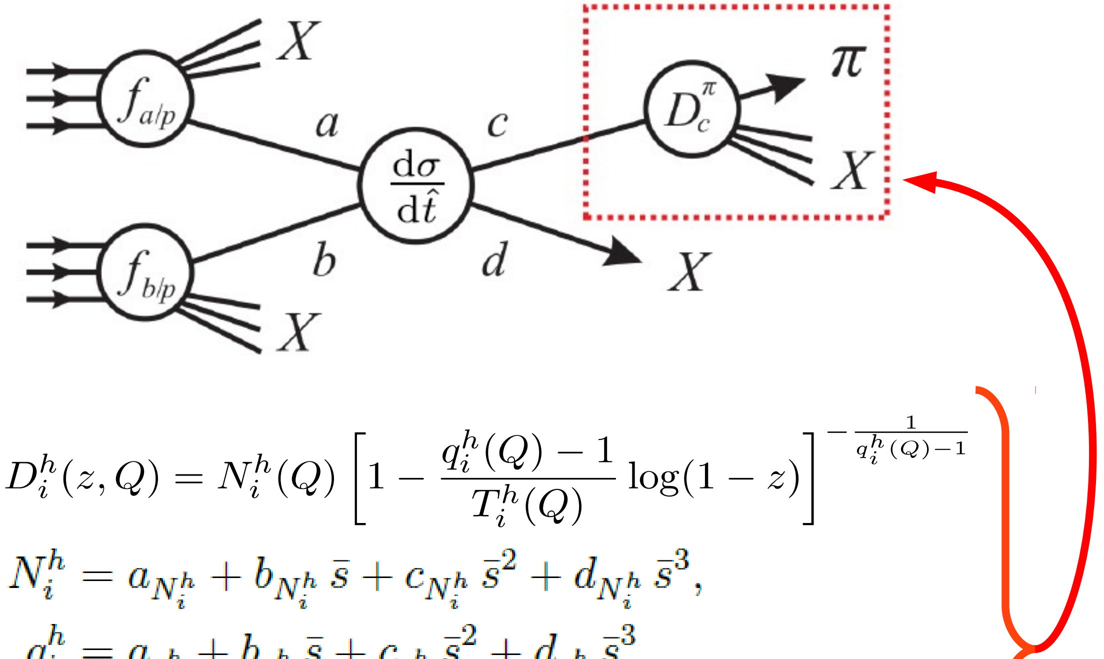

$$D_i^h(z,Q) = N_i^h(Q)\left[1 - \frac{q_i^h(Q)-1}{T_i^h(Q)}\log(1-z)\right]^{-\frac{1}{q_i^h(Q)-1}}$$

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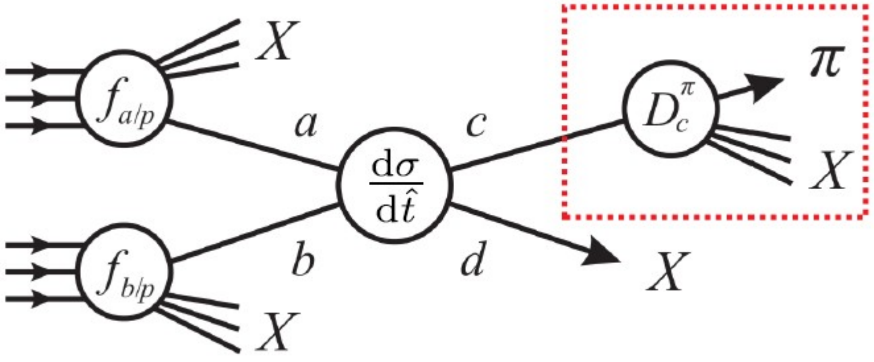
$$q_i^h = a_{q_i^h} + b_{q_i^h}\,\bar{s} + c_{q_i^h}\,\bar{s}^2 + d_{q_i^h}\,\bar{s}^3,$$

$$T_i^h = a_{T_i^h} + b_{T_i^h}\,\bar{s} + c_{T_i^h}\,\bar{s}^2 + d_{T_i^h}\,\bar{s}^3,$$

$$\bar{s} = \log\left[\frac{\log(Q^2/\Lambda^2)}{\log(Q_0^2/\Lambda^2)}\right] \qquad Q_0 = \begin{cases} 1\ \text{GeV}, & \text{for } u,d,s,g \\ m_c & \text{for } c \\ m_b & \text{for } b \end{cases}$$

Comparing non-extensive FF with pp data

Test of non-extensive FFs within the kTpQCD_v20 model



$$D_i^h(z,Q) = N_i^h(Q)\left[1 - \frac{q_i^h(Q)-1}{T_i^h(Q)}\log(1-z)\right]^{-\frac{1}{q_i^h(Q)-1}}$$

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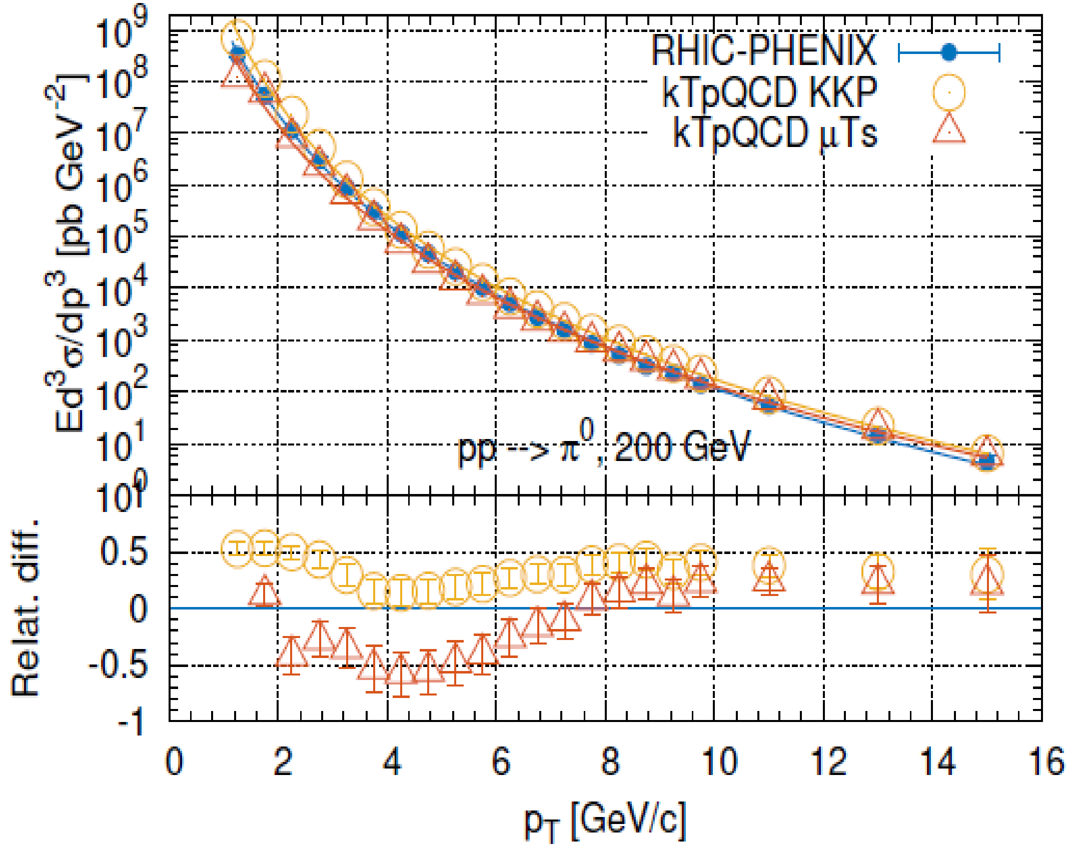


Comparing non-extensive FF with e^+e^- data

'Tsallis thermometer' → full formula + errors + pion data

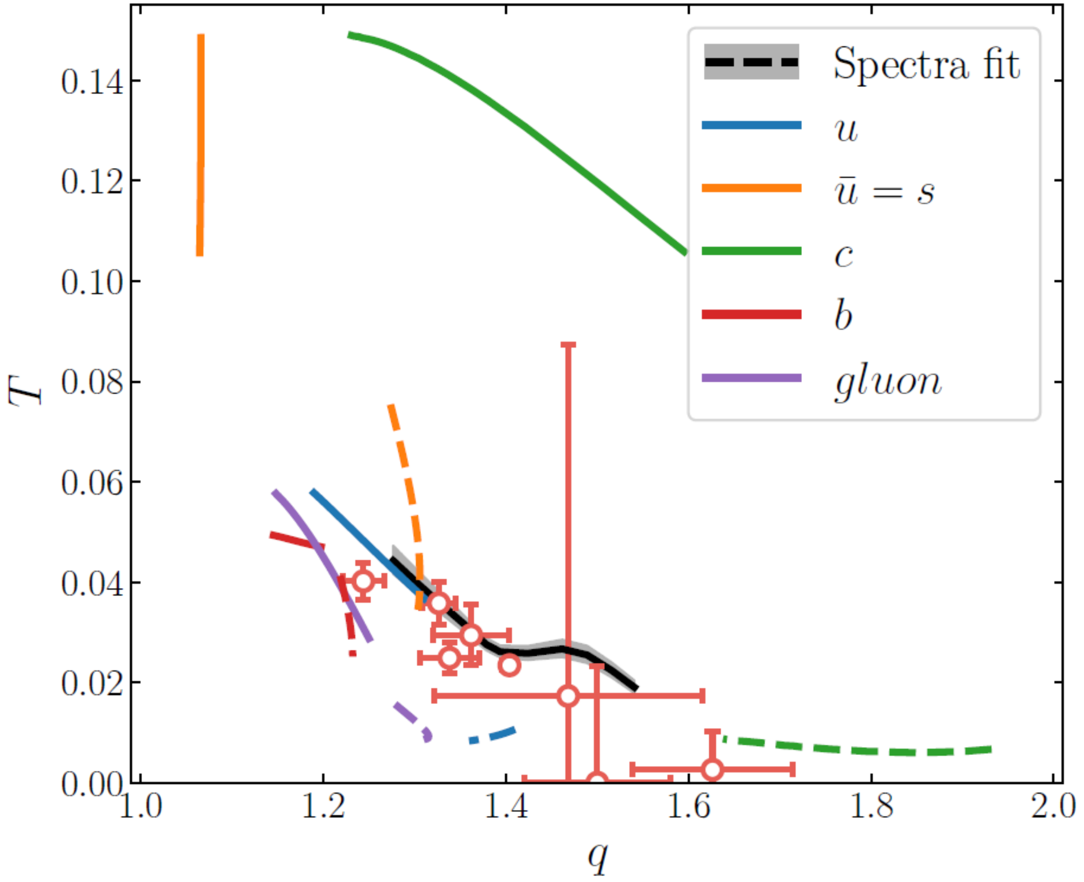
Parameters from channels (color)

$i \to \pi^+$	q	$1/T$	N
u	1.455	62.72	207.1
$\bar{u} = s$	1.063	4.850	13.68
c	2.238	25.89	14.97
b	1.211	21.45	151.6
g	1.059	17.05	53.81

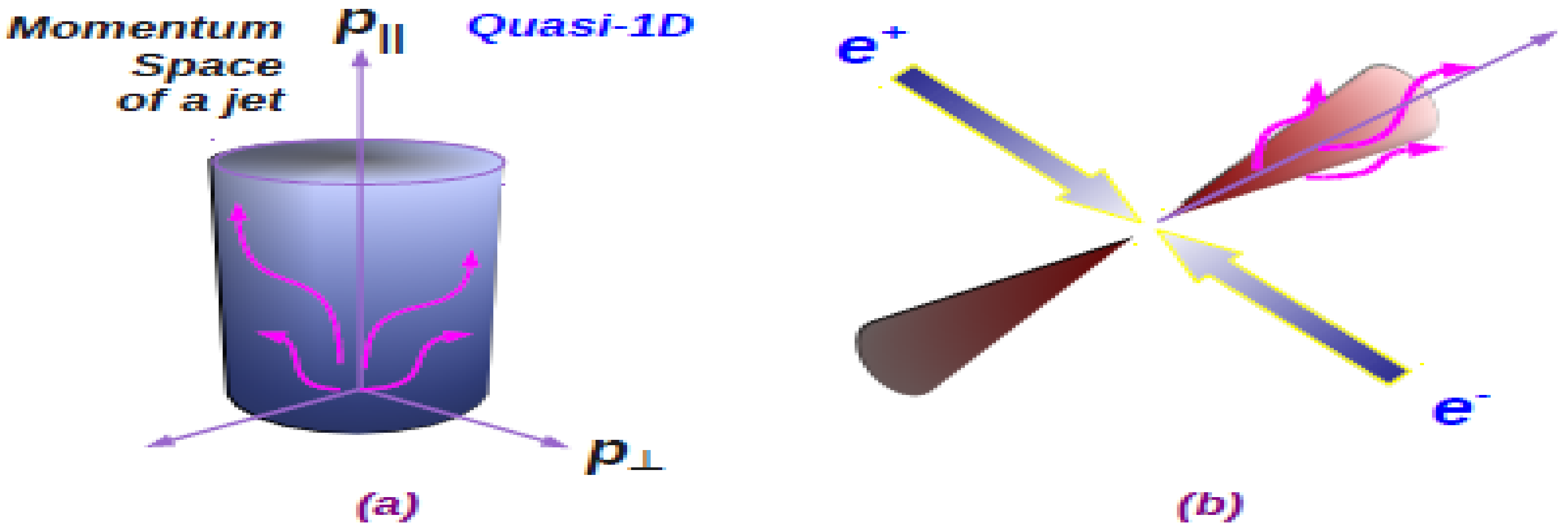
Overall parameters (black)

Non-extensivity: $q = 1.39$

T parameter: $1/T = 40$



Comparing non-extensive FF with e^+e^- data



K. Ürmössy, G.G. Barnaföldi, T.S. Bíró:

- Microcanonical Jet-Fragmentation in pp at LHC energies:
  Phys. Lett. B701 (2011) 111
- Generalized Tsallis distribution in e^+e^- collisons
  Phys. Lett. B718 (2012) 125

Comparing non-extensive FF with e^+e^- data

Tsallis thermodynamics

$$\frac{1}{\sigma}\frac{d\sigma}{dx} = A x^{D-1}\left(1 + \frac{q-1}{T/(\sqrt{s}/2)}x\right)^{-1/(q-1)}$$

$$q = 1 + 1/(\alpha + D + 1)$$

$$T = (\sqrt{s}/2)\beta/(D(\alpha + D + 1))$$

Microcanonical Tsallis

$$\frac{1}{\sigma}\frac{d\sigma}{dx} = \frac{A\,x^{D-1}(1-x)}{\left(1 - \frac{q-1}{T/(\sqrt{s}/2)}\ln(1-x)\right)^{1/(q-1)}}$$

$$q = 1 + 1/(\alpha + D + 1)$$

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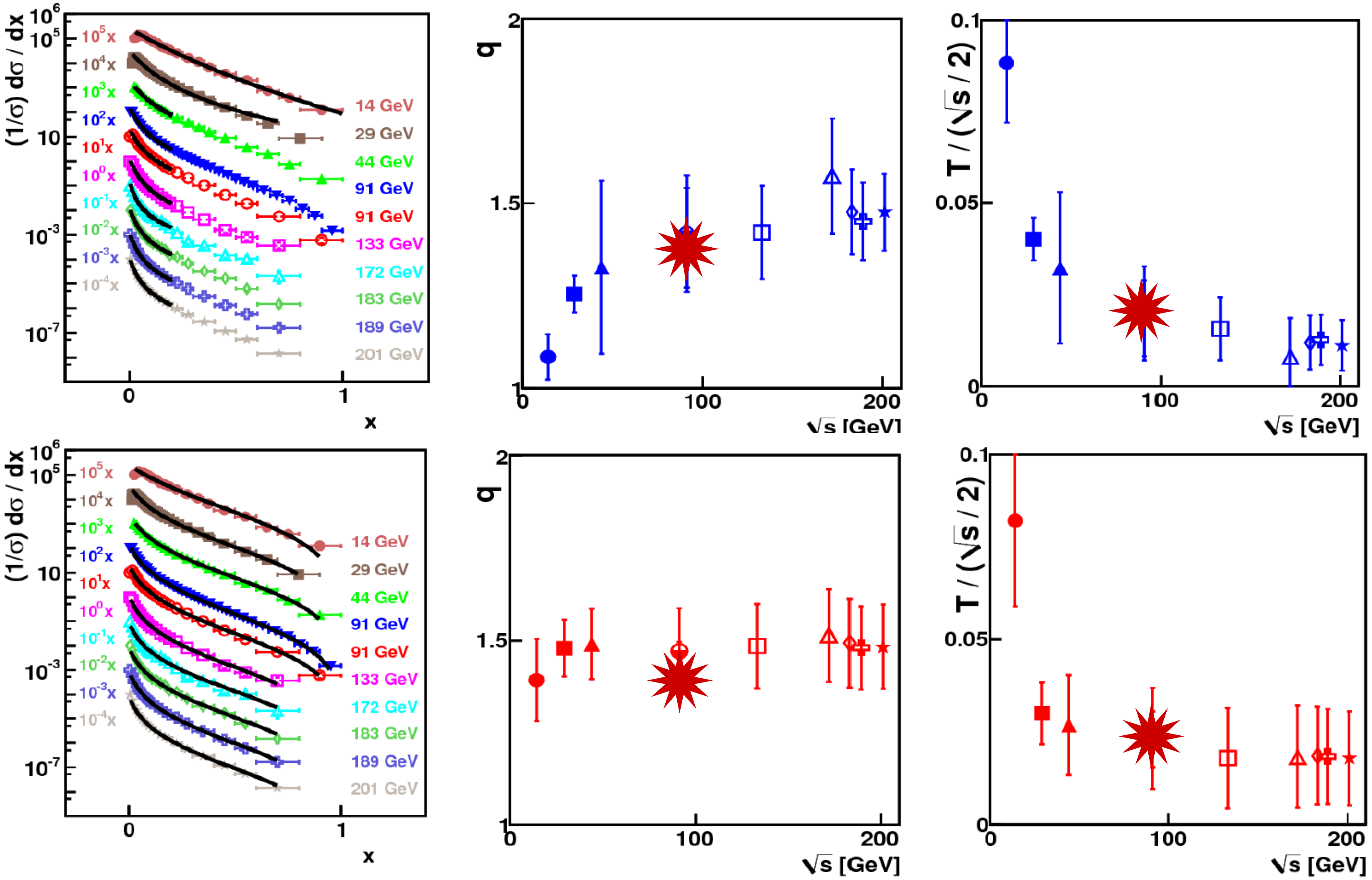
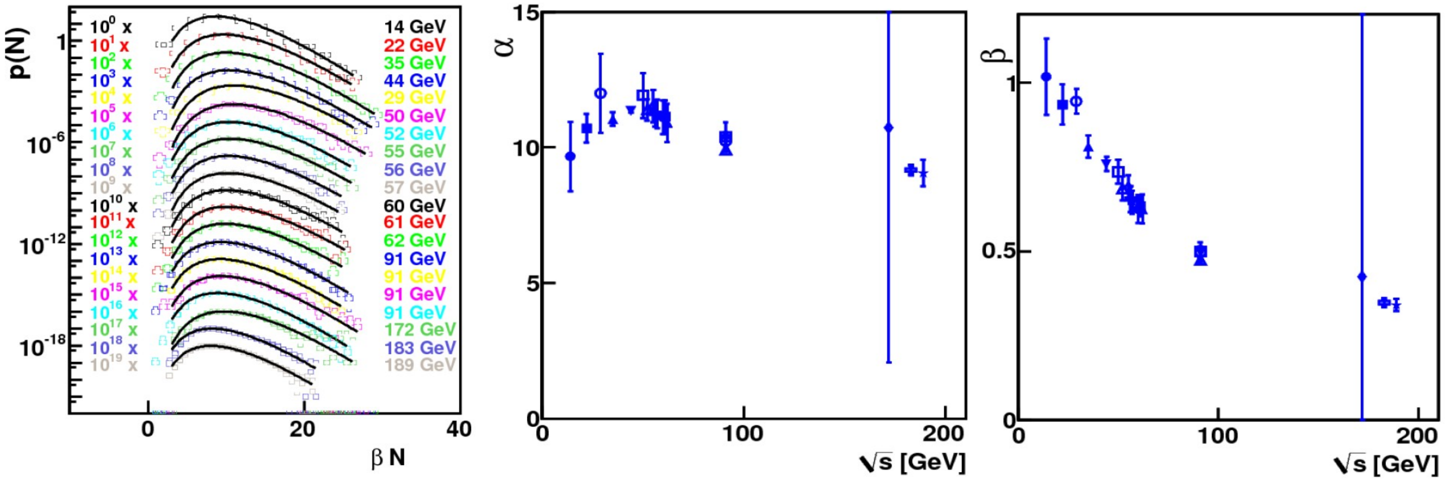


Comparing non-extensive FF with e^+e^- data

Tsallis thermodynamics

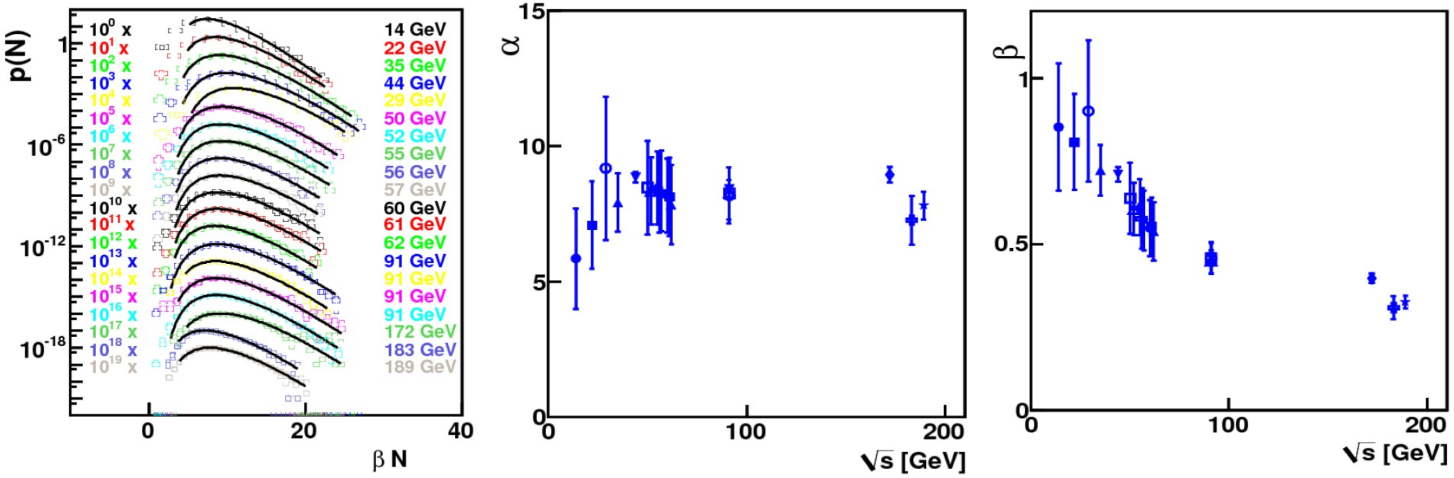
$$p(N) = A_m\, N^{\alpha-1} e^{-\beta N}$$



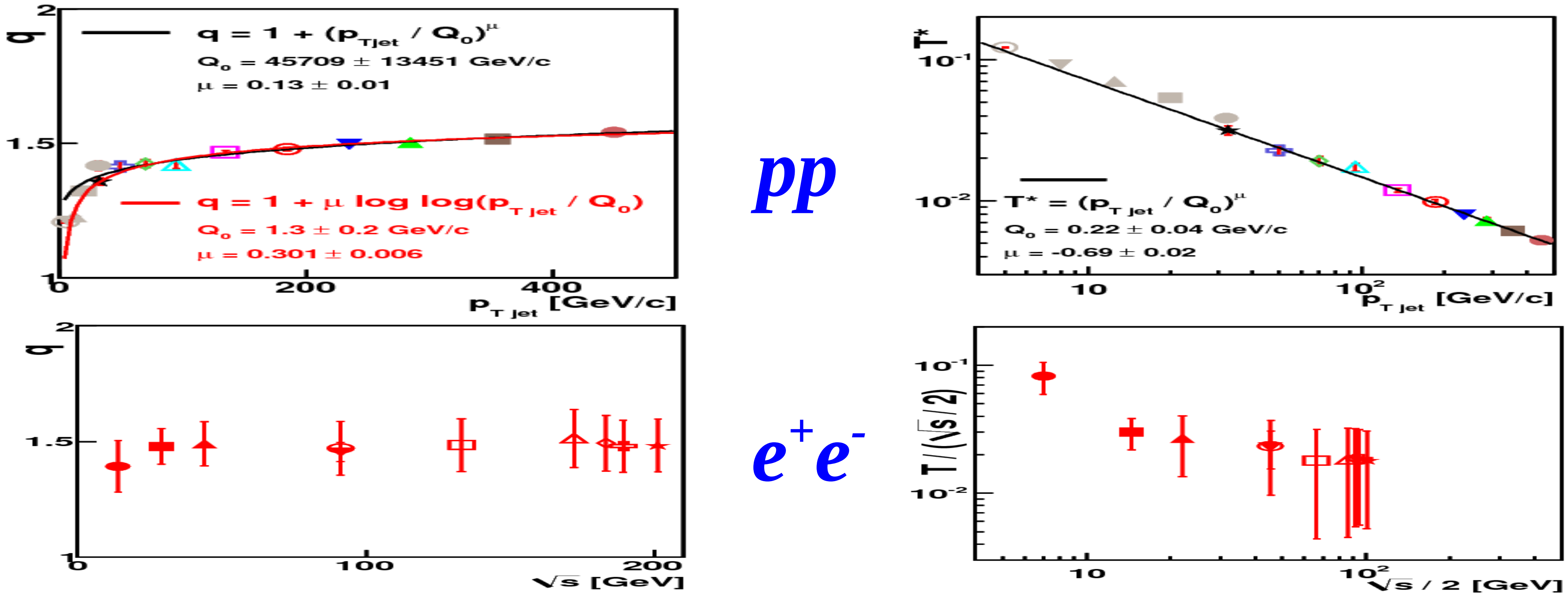
Microcanonical Tsallis

$$p(N) = A_m\,(N - N_0)^{\alpha-1} e^{-\beta(N-N_0)}$$

$$N_0 = 1 + 2/D$$



Comparing non-extensive FF with e^+e^- & *pp* data



K Ürmössy, GGB, TS Biró,
PLB 710 (2011) 111, PLB 718 (2012) 125.

- Energy dependence (hard)
  - Parameters *q* seem to increase & saturate at high energies
  - Parameter *T* is decreasing & saturate with increasing energy

Summary

Aim: non-extensive fragmentation function parametrization

→ Pion FFs are available for tests, fit on one dataset so far

So far we have:

- Non-extensive phenomena motivated Tsallis-like distribution with physical meaning of the FF parameters
- Scale evolution is fully observed in (q,T,N) for channels & overall
  - Other models: in comparison to HKNS, AKK present similar trends
  - Data: comparisons & tests with other pion data fits well
  - Better low-z behavior: even applying in pp spectra.
  - Model: similarities with jet (1D) themodynamics & multiplicities

Next: More data to the fits and apply for kaon & protons

BACKUP

The non-extensive statistical approach

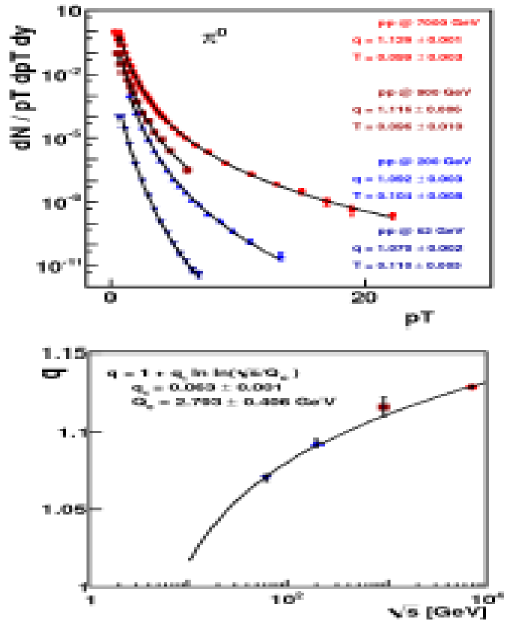
Hadron spectra in *pp* collisions can be described by the *Tsallis distribution*:

$$\frac{dN}{d^3 p} \propto \left[1 + \frac{q-1}{T}(m_T - m)\right]^{-1/(q-1)} .$$

***π* spectra in *pp* collisions depends similarly on √s and on the multiplicity**

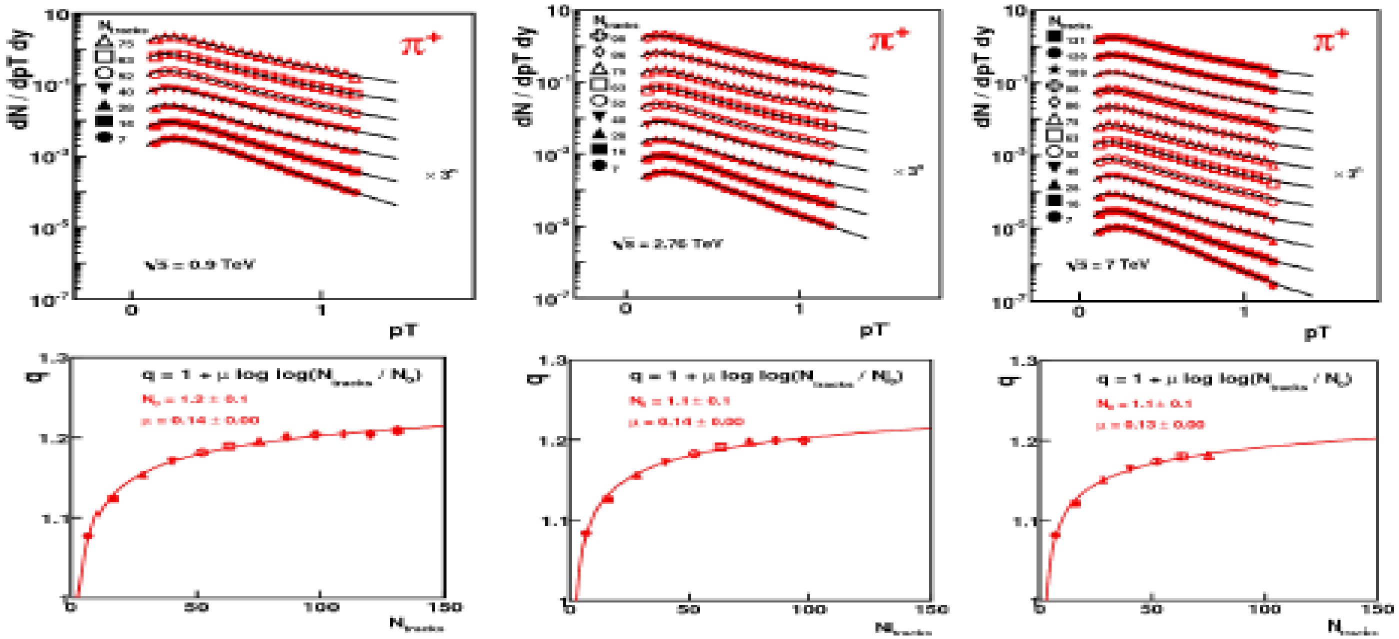
$$q(s) = 1 + q_1 \ln\ln(\sqrt{s}/Q_0) ,$$
$$q(N) = 1 + \mu \ln\ln(N/N_0) .$$

√s = fix



N = fix

arXiv:1405.3963, 1501.02352, 1501.05959



The non-extensive statistical approach

Hadron spectra in pp collisions can be described by the Tsallis distribution:

$$\frac{dN}{d^3\mathbf{p}} \propto \left[1 + \frac{q-1}{T}(m_T - m)\right]^{-1/(q-1)} .$$

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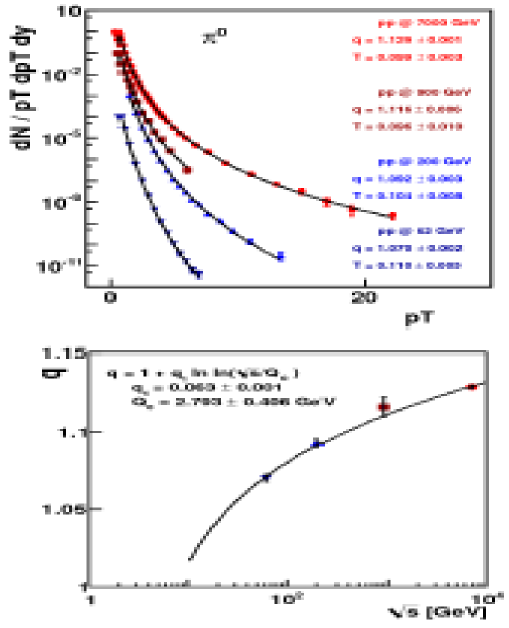

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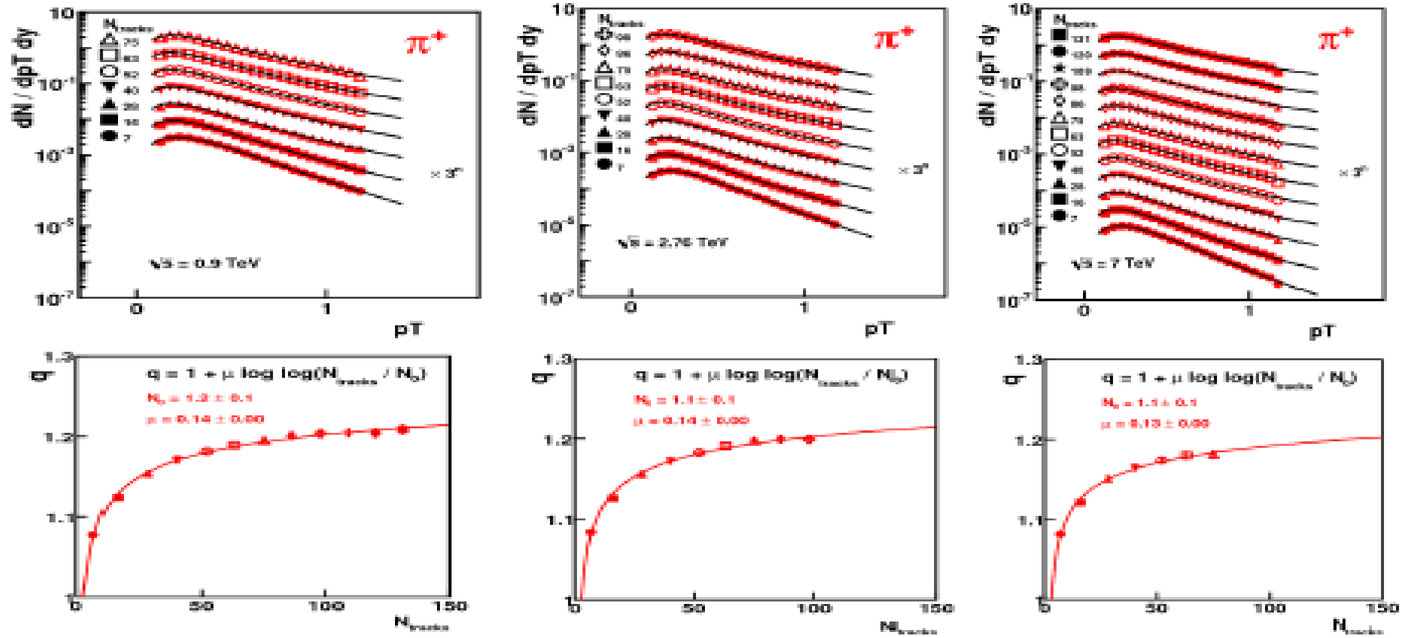


In pp: the Tsallis thermometer on the *T-(q-1)* plane

- Parameter space

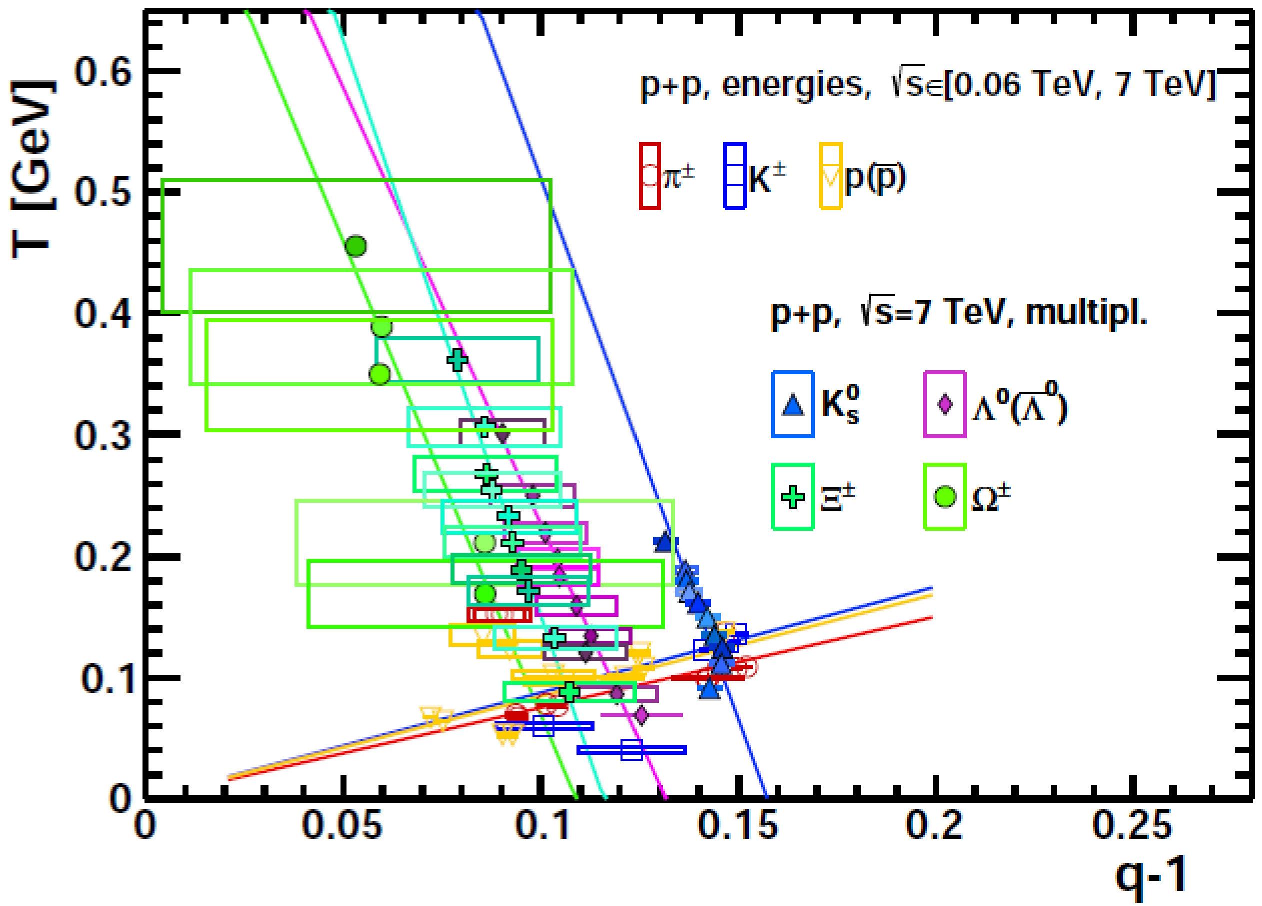
  (i) c.m. energy makes changes along q-axis

  (ii) multiplicity vary the parameter T

  (iii) the mass hierarchy can be seen clearly with bunches

  (iv) valid for pp

  (v) T & q are connected



In pA: the Tsallis thermometer on the *T-(q-1)* plane

- Parameter space

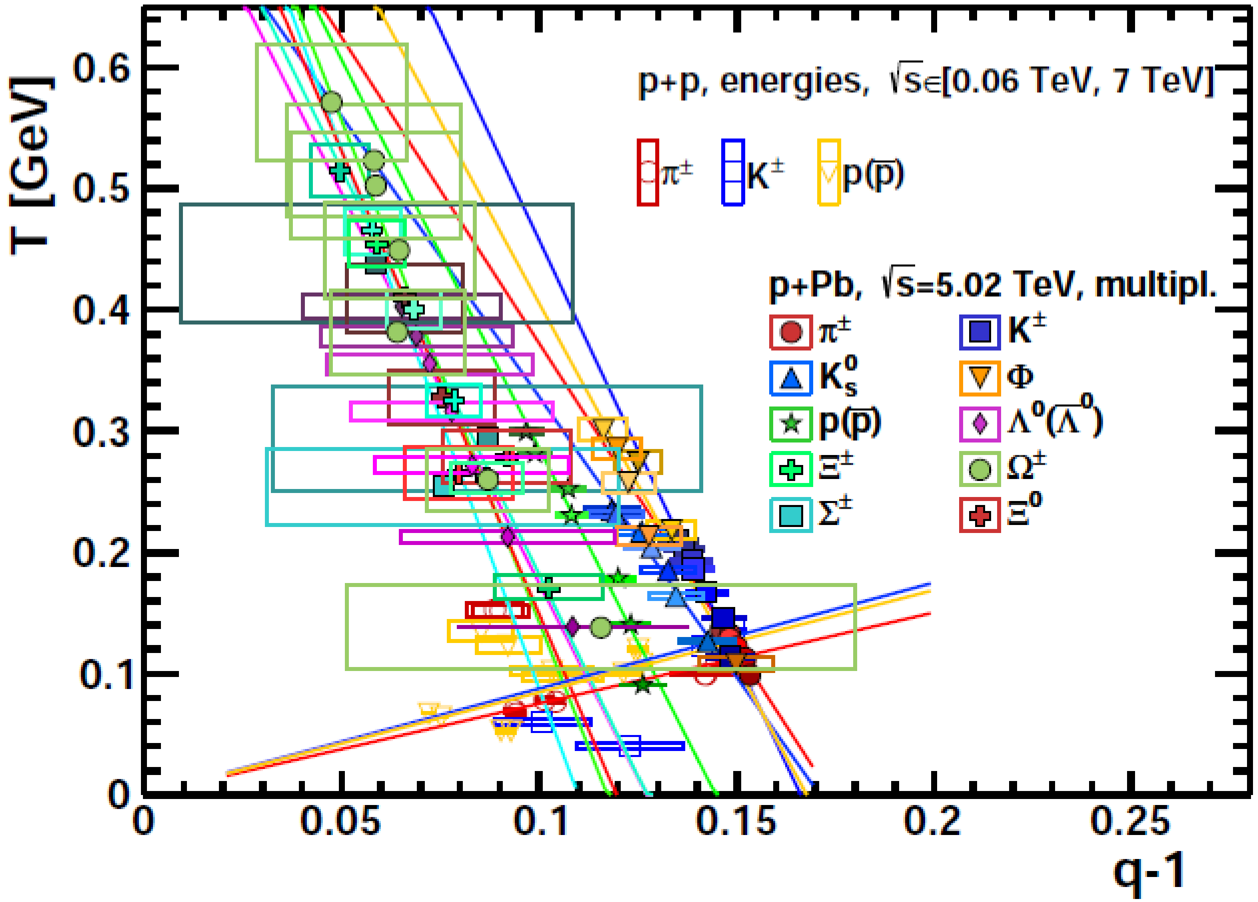
  (i) c.m. energy makes changes along q-axis

  (ii) multiplicity vary the parameter T

  (iii) the mass hierarchy can be seen clearly with bunches

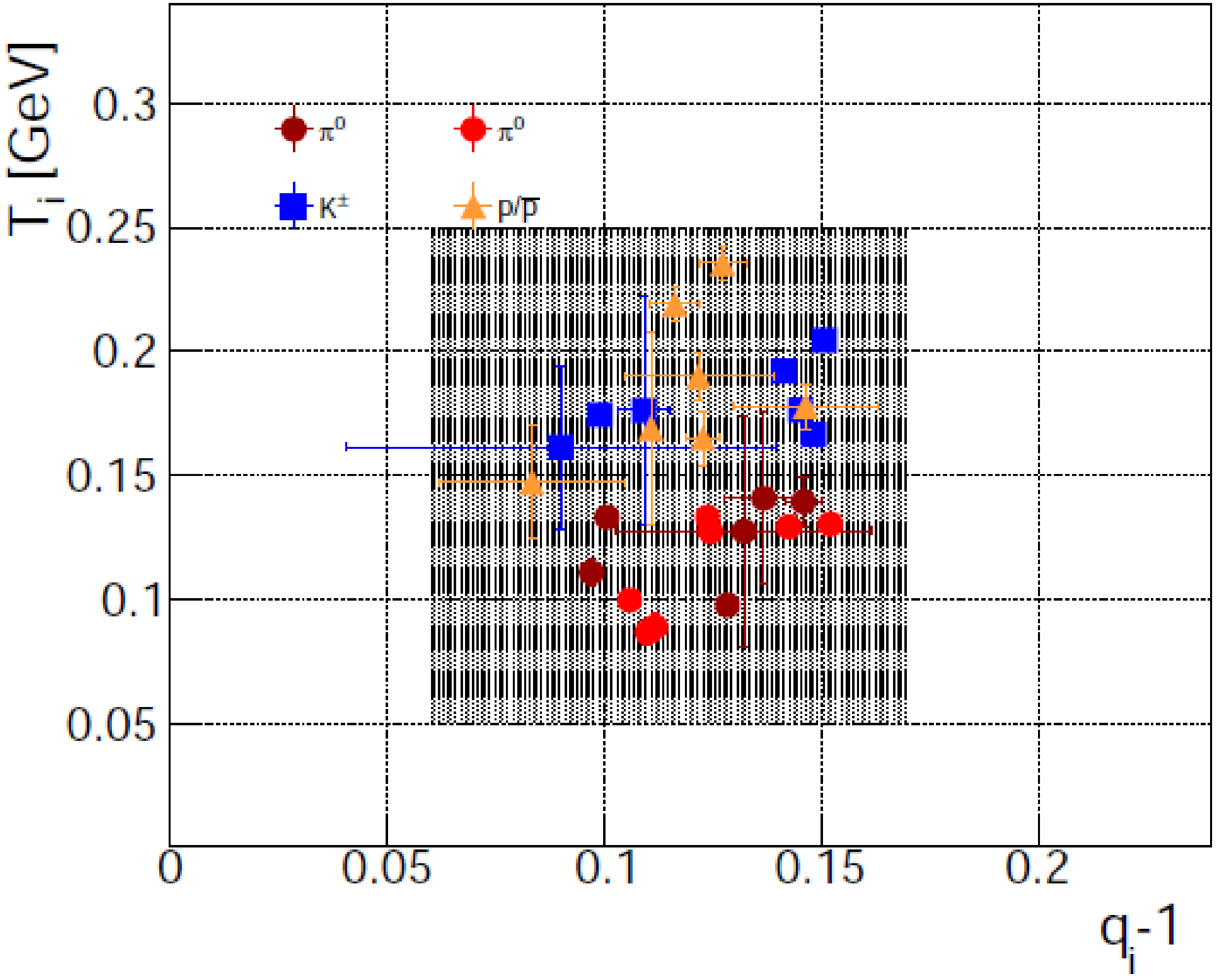
  (iv) valid for pp & pA

  (v) T & q are connected



In pp: the Tsallis thermometer on the *T-(q-1)* plane

- Measurements in pp
  - – parameter space is compact, especially in q
  - – c.m. energy makes changes along q-axis
  - – T & q are connected



In pp: the Tsallis thermometer on the *T-(q-1)* plane

- Theory in pp

  Both are overlapping

  – PYTHIA8 deviates where no statistics in the tail.

  – kTpQCD_v20 is a pQCD code is misses the low p_T body part.