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An overview on polarization in relativistic heavy ion collisions

OUTLINE

- Global polarization
- Spin in a relativistic fluid: theory
- Longitudinal polarization
- Spin tensor ?

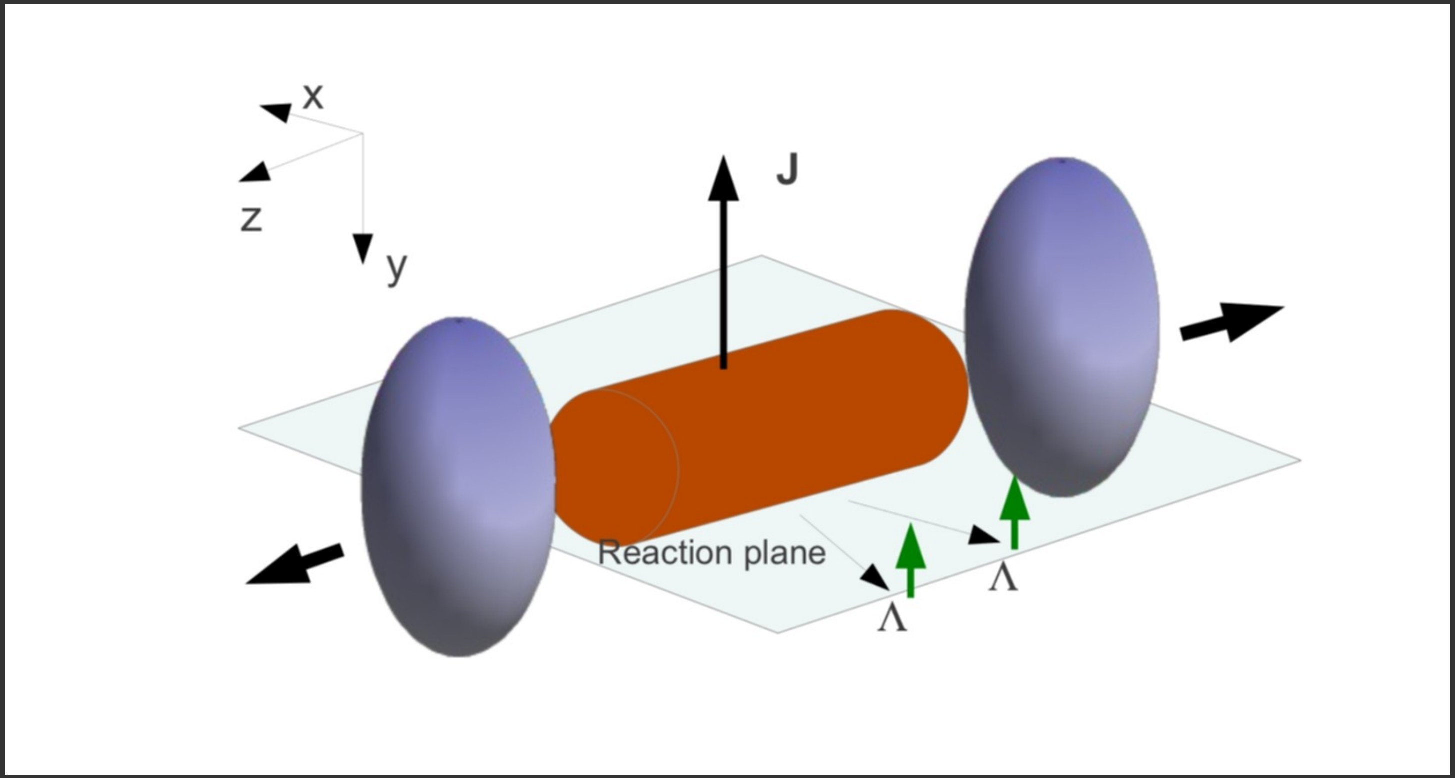
THOR meets THOR – Lisbon, June 10-14 2018

Heavy ion collisions

Peripheral collisions ➡ Angular momentum ➡ Global polarization w.r.t reaction plane

By parton spin-orbit coupling:: Z. T. Liang, X. N. Wang, Phys. Rev. Lett. 94 (2005) 102301

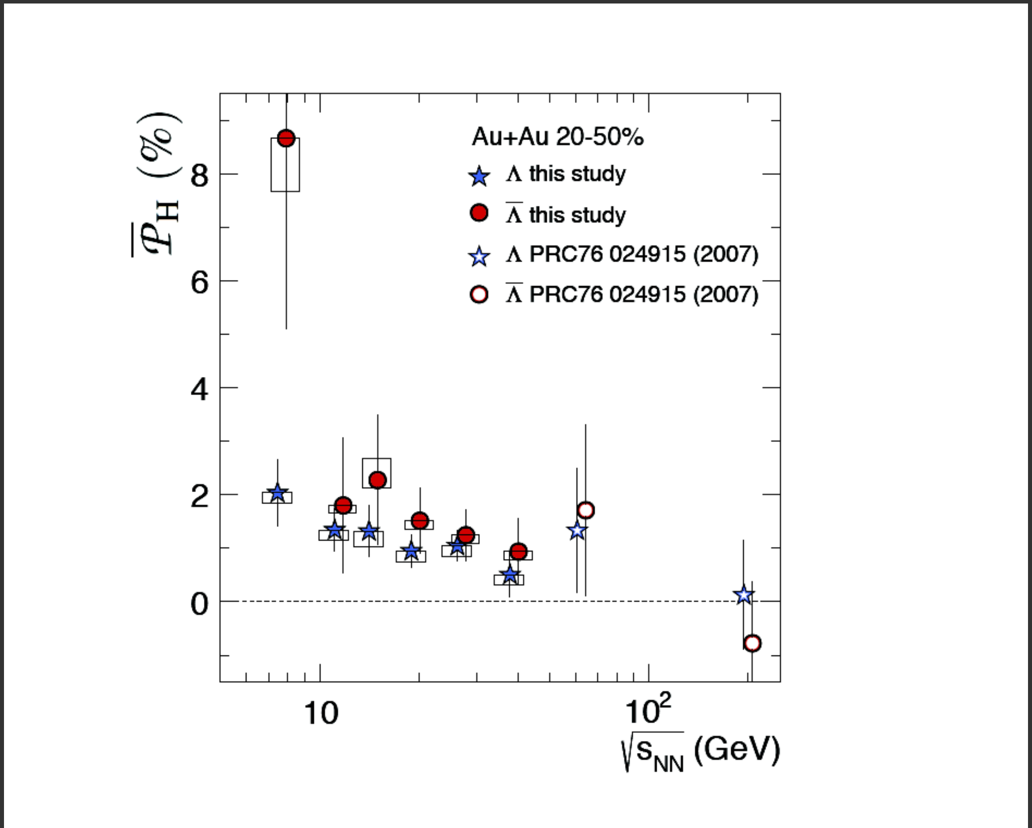
By local equilibration: F. B., F. Piccinini, Ann. Phys. 323 (2008) 2452; F. B., F. Piccinini, J. Rizzo, Phys. Rev. C 77 (2008) 024906 – Global polarization related to (thermal) vortiticy



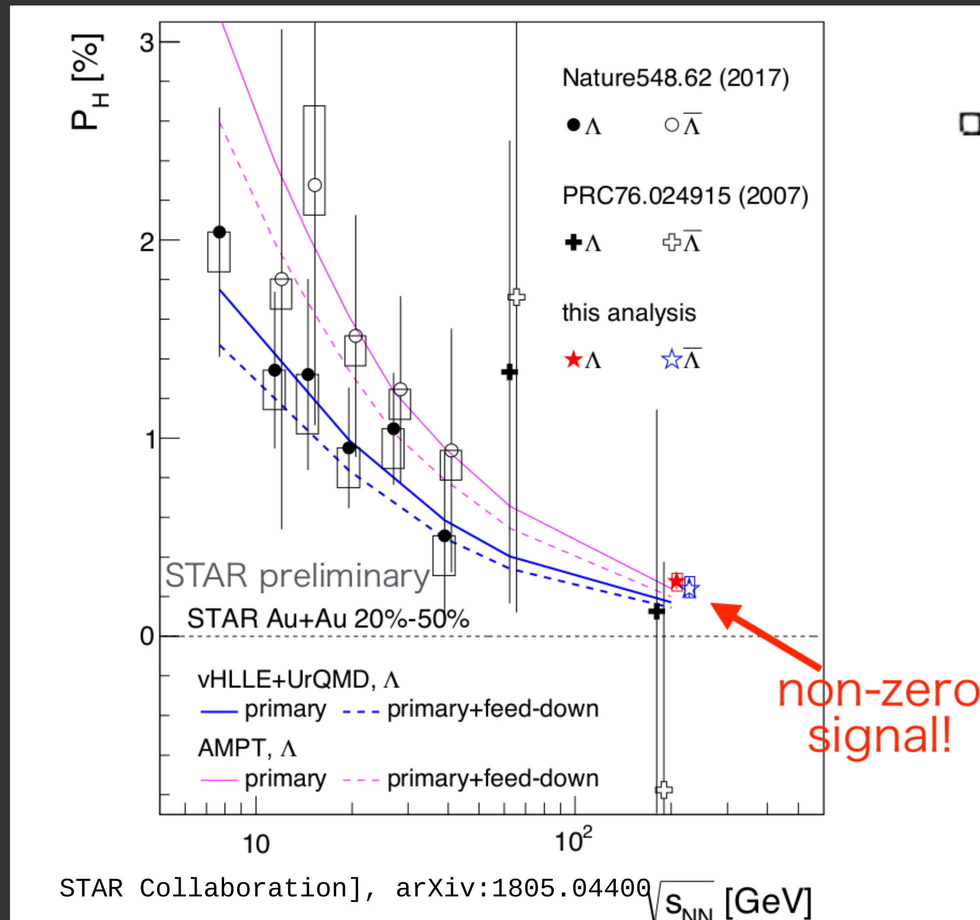
$$\frac{\hbar\omega}{KT} \approx \frac{c}{12\text{fm}200\text{MeV}} \approx 0.08 \qquad a \approx 10^{30} g \implies \frac{\hbar a}{cKT} \approx 0.06$$

Evidence of Λ polarization in relativistic heavy ion collisions

STAR collaboration, Nature 548 (2017) 62



Updated STAR plot presented in Quark Matter 2018



In agreement with most calculations using the formula

$$S^\mu(p) = \frac{1}{8m}\epsilon^{\mu\nu\rho\sigma}p_\sigma \frac{\int_\Sigma \mathrm{d}\Sigma_\tau p^\tau n_F(1-n_F)\partial_\nu\beta_\rho}{\int_\Sigma \mathrm{d}\Sigma_\tau p^\tau \ n_F}$$

F.B., V. Chandra, L. Del Zanna, E. Grossi, Ann. Phys. 338, 32 (2013)

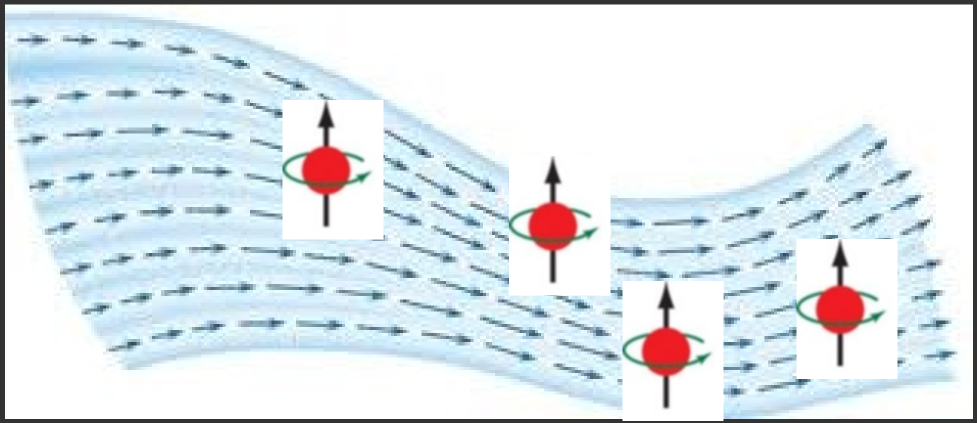
L. Csernai,
L. G. Pang, X. N. Wang,
C. Ko,
X. G. Wang, Q. Wang,
X. L. Xia, J. Liao,
A. Sorin, O. Teryaev,

Quantum effects in a relativistic fluid

- Chiral Magnetic Effect requires a quantum anomaly to work
- Spin is inherently a quantum phenomenon

$$\mathbf{j} = \frac{e^2}{2\pi^2}\frac{\mu_A}{\hbar^2}\mathbf{B} \qquad \mathbf{S} \simeq \frac{\hbar}{KT}\boldsymbol{\omega}$$

Spin from a relativistic fluid



How to approach the problem?
Conceptually analogous to the Cooper-Frye formula but it must be quantum from the outset, *classical* kinetic theory does not include spin

Polarization (2S+1)x(2S+1) matrix

$$\mathrm{tr}(\widehat{\rho} a^{\dagger}_{\sigma}(p) a_{\sigma'}(p))$$

Hydro: Local thermodynamic equilibrium plus corrections (Zubarev formalism)

$$\widehat{\rho} = \frac{1}{Z} \exp\left[- \int_{\Sigma} \mathrm{d}\Sigma_{\mu} \widehat{T}^{\mu\nu} \beta_{\nu} + \mathit{corrections} \right]$$

$$\beta = \frac{1}{T} u$$

$$\widehat{\rho} = \frac{1}{Z} \exp[-\beta(x)_{\mu} \widehat{P}^{\mu} + \frac{1}{2}(\partial_{\mu}\beta_{\nu}(x) - \partial_{\nu}\beta_{\mu}(x))\widehat{J}^{\mu\nu} + \text{terms vanishing at global equilibrium}]$$

$$\varpi_{\mu\nu} = \frac{1}{2}(\partial_{\mu}\beta_{\nu} - \partial_{\nu}\beta_{\mu})$$

Thermal vorticity

In principle, with these ingredients one should able to find the exact formula for the mean spin vector S^{μ}.

Formula which is based on an educated *ansatz* of the Wigner function of the Dirac field at global equilibrium:

$$S^\mu(p) = \frac{1}{8m}\epsilon^{\mu\nu\rho\sigma} p_\sigma \frac{\int_\Sigma \mathrm{d}\Sigma_\tau p^\tau n_F (1-n_F)\partial_\nu \beta_\rho}{\int_\Sigma \mathrm{d}\Sigma_\tau p^\tau \; n_F}$$

$$n_F = (\mathrm{e}^{\beta\cdot p - \xi} + 1)^{-1}$$

F.B., V. Chandra, L. Del Zanna, E. Grossi, Ann. Phys. 338, 32 (2013)
R. H. Fang, L. G. Pang, Q. Wang, X. N. Wang, Phys. Rev. C 94 (2016) 024904

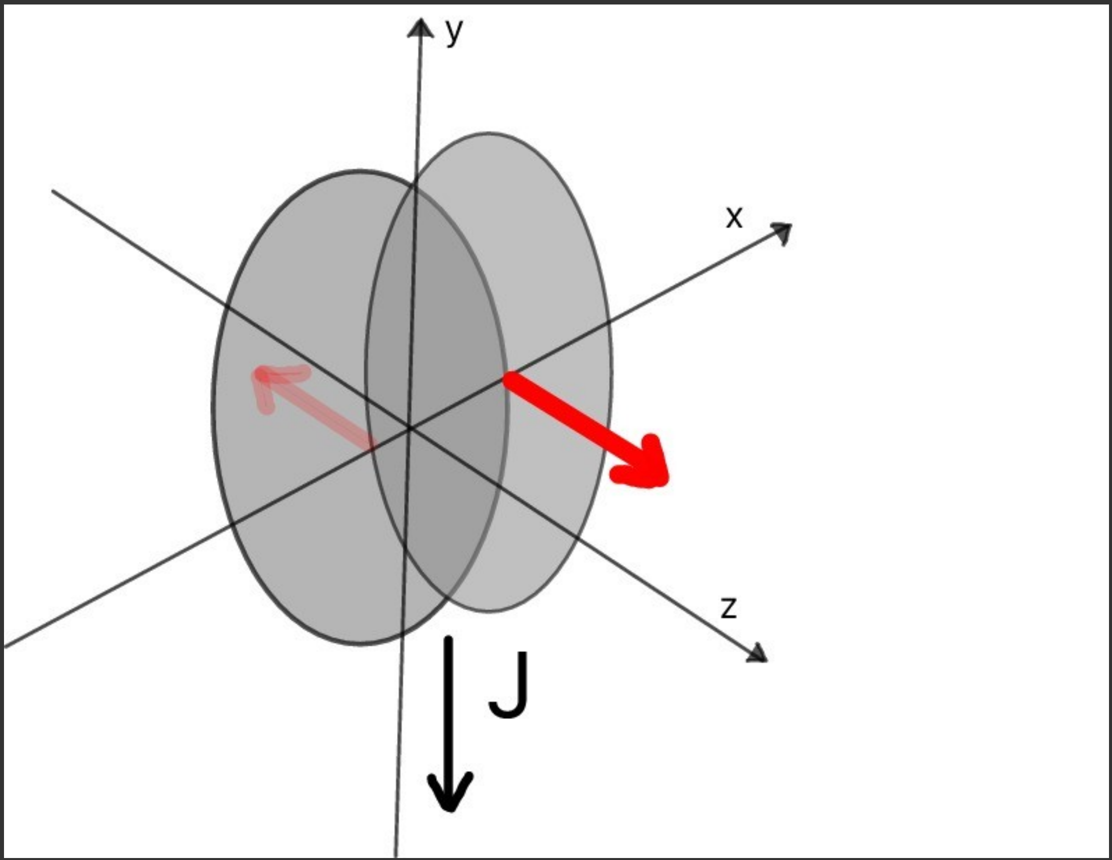
Simplest formula meeting the requirements:

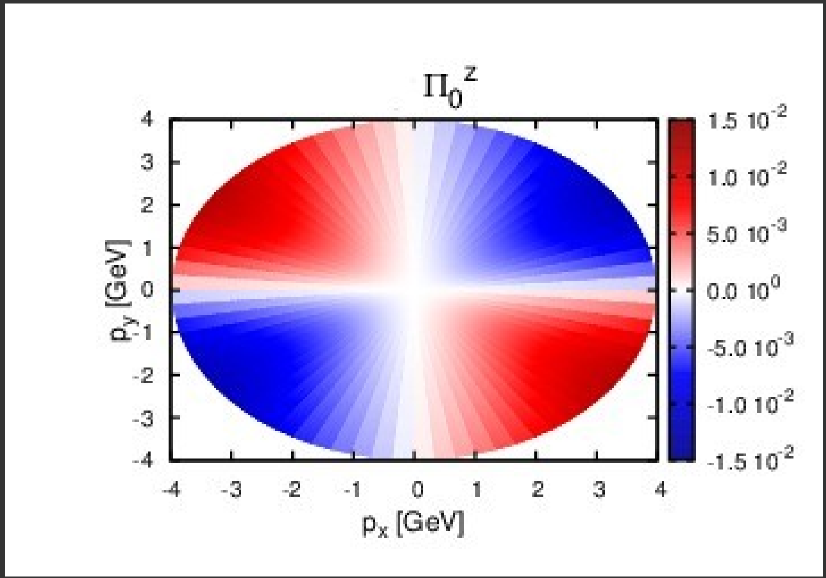
- 1st order in thermal vorticity
- Freeze-out integral
- Vanishing for a fully degenerate Fermi gas (i.e. $n_F = 1$)
- $S \cdot p = 0$

An exact solution at global equilibrium is still missing

Collective *longitudinal* polarization: quadrupole structure

F. B., I. Karpenko, Phys. Rev. Lett. 120 (2018) 012302





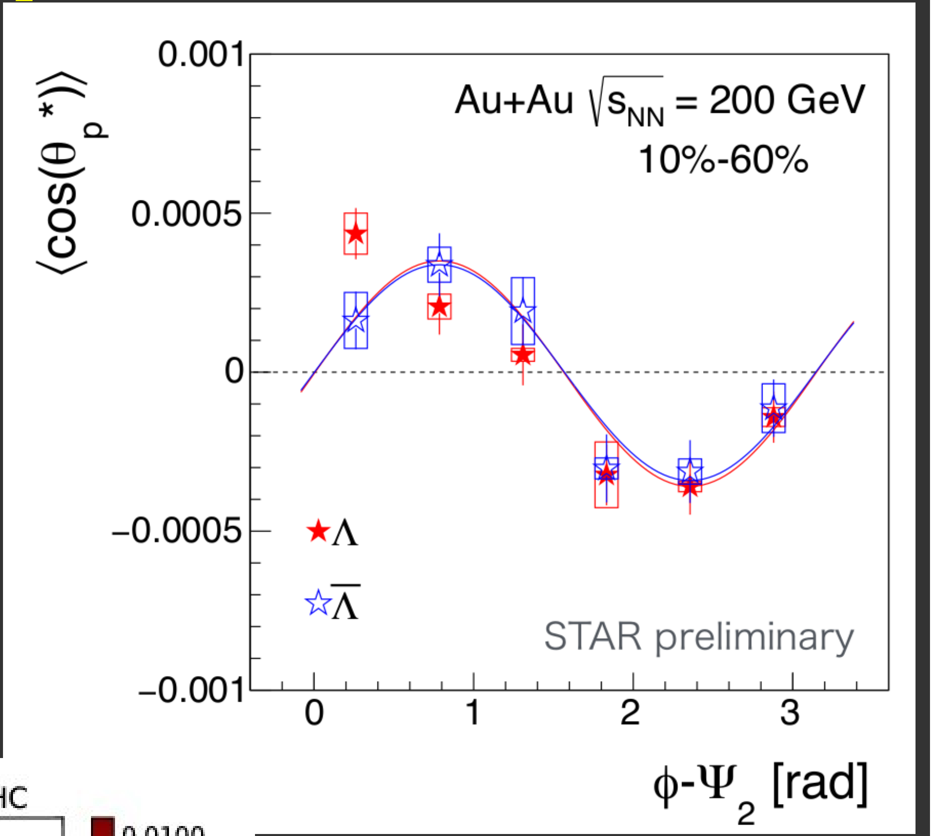
200 GeV: larger magnitude than S_J

Peripheral heavy ion collisions feature two discrete symmetries: reflection w.r.t. reaction plane and rotation by 180 around its perpendicular direction. This reflects into the quadrupole pattern of the global Λ polarization at midrapidity

$$S^z(\mathbf{p}_T, Y=0) = \frac{1}{2}\sum_{k=1}^{\infty} f_{2k}(p_T)\sin 2k\varphi$$

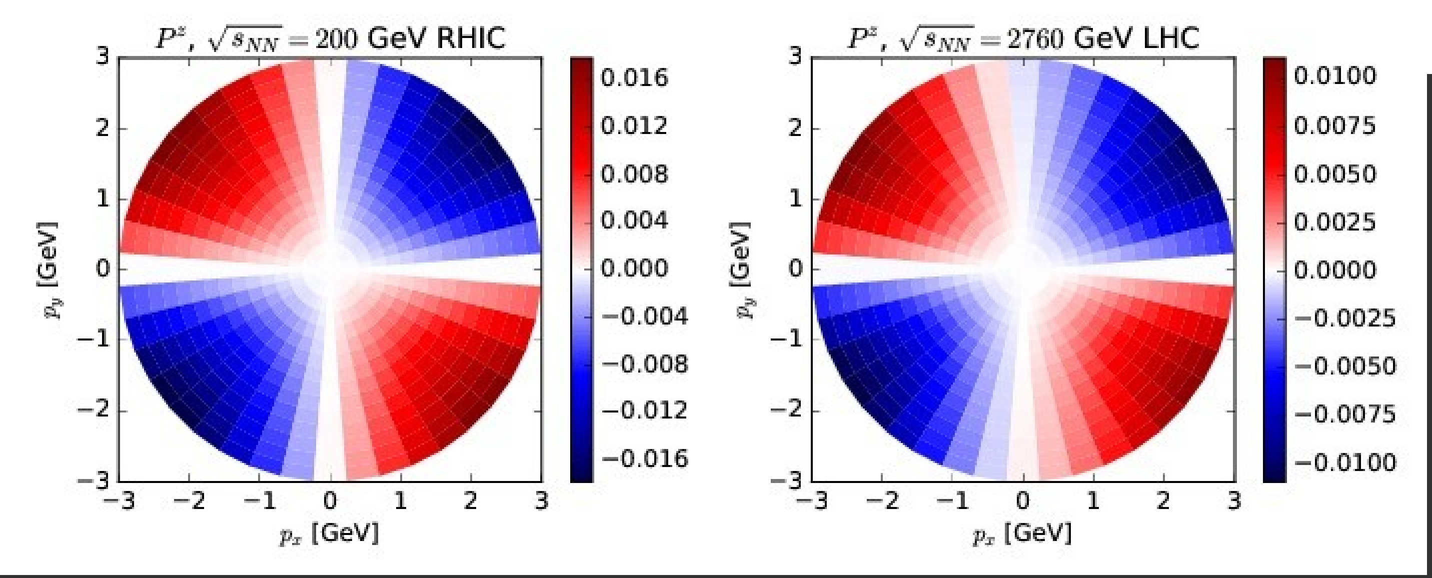
A *sign problem* for the longitudinal component

Quadrupolar structure of longitudinal polarization in the transverse momentum plane, as predicted.
Spectacular confirmation of hydro predictions...
yet with a flipped sign!



Z. Ye, T. Niida, Quark Matter 2018

- Hydro initial conditions? (polarization is a sensitive probe of the initial flow)
- Incomplete local thermodynamic equilibrium for the spin degrees of freedom (spin kinetic theory)?
- Effect of spin dissipative transport coefficients?
- Effect of initial state fluctuations?
- Effect of decays?
- Error in the calculation



F. B., I. Karpenko, Phys. Rev. Lett. 120 (2018) 012302
S. Voloshin, in SQM 2017

Same pattern found in AMPT+thermal vorticity calculation X. L. Xia, H. Li, Z. B. Tang and Q. Wang, 1803.00867

Acceleration-vorticity-grad T decomposition

$$\partial_\mu \beta_\nu = \partial_\mu \left(\frac{1}{T} \right) u_\nu + \frac{1}{T} \partial_\mu u_\nu$$

$$A^\mu = u \cdot \partial u^\mu$$
$$\omega^\mu = \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} \partial_\nu u_\rho u_\sigma$$

$$\begin{aligned} S^\mu(p) \int_\Sigma \mathrm{d}\Sigma_\tau p^\tau \, n_F &= \frac{1}{8m} \epsilon^{\mu\nu\rho\sigma} p_\sigma \int_\Sigma \mathrm{d}\Sigma \cdot p \, n_F (1 - n_F) \nabla_\nu (1/T) u_\rho \\ &+ \frac{1}{8m} \int_\Sigma \mathrm{d}\Sigma \cdot p \, n_F (1 - n_F) 2 \frac{\omega^\mu u \cdot p - u^\mu \omega \cdot p}{T} \\ &- \frac{1}{8m} \epsilon^{\mu\nu\rho\sigma} p_\sigma \int_\Sigma \mathrm{d}\Sigma \cdot p \, n_F (1 - n_F) \frac{1}{T} A_\nu u_\rho \end{aligned}$$

Grad T

Vorticity

Acceleration

In the rest frame of the particle:

$$\mathbf{S}^* \propto \frac{\hbar}{KT^2} \mathbf{u} \times \nabla T + \frac{\hbar}{KT} (\boldsymbol{\omega} - \boldsymbol{\omega} \cdot \mathbf{v} \mathbf{u}/c^2) + \frac{\hbar}{KT} \mathbf{A} \times \mathbf{u}/c^2$$

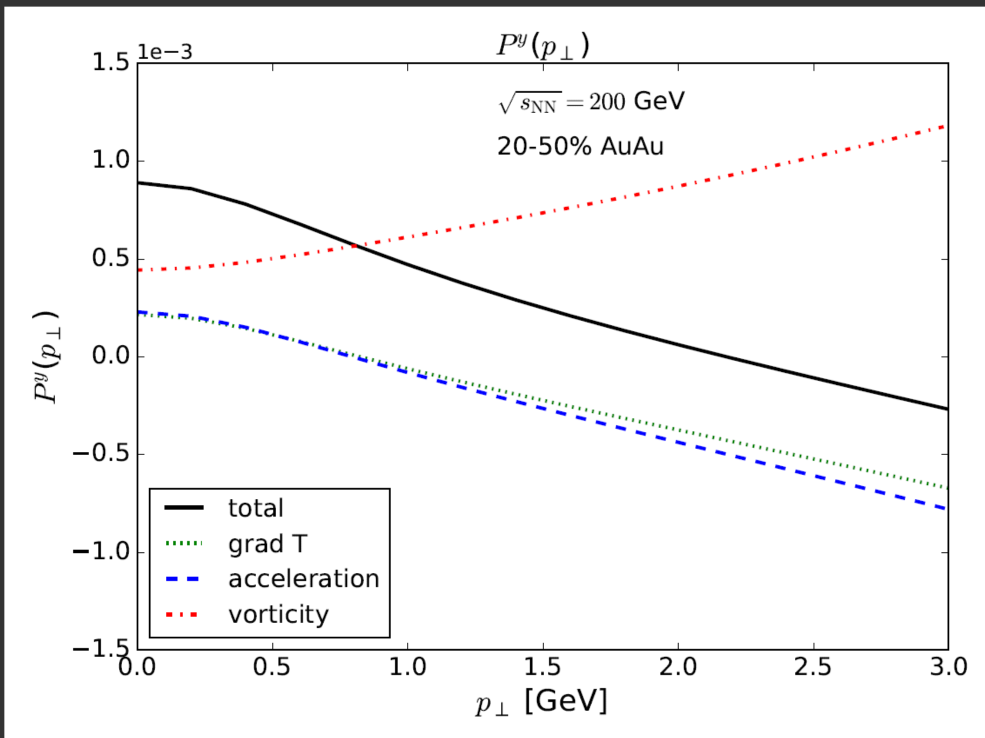
Thermal term (new effect)

Vorticous term (known)

Acceleration term (purely relativistic)

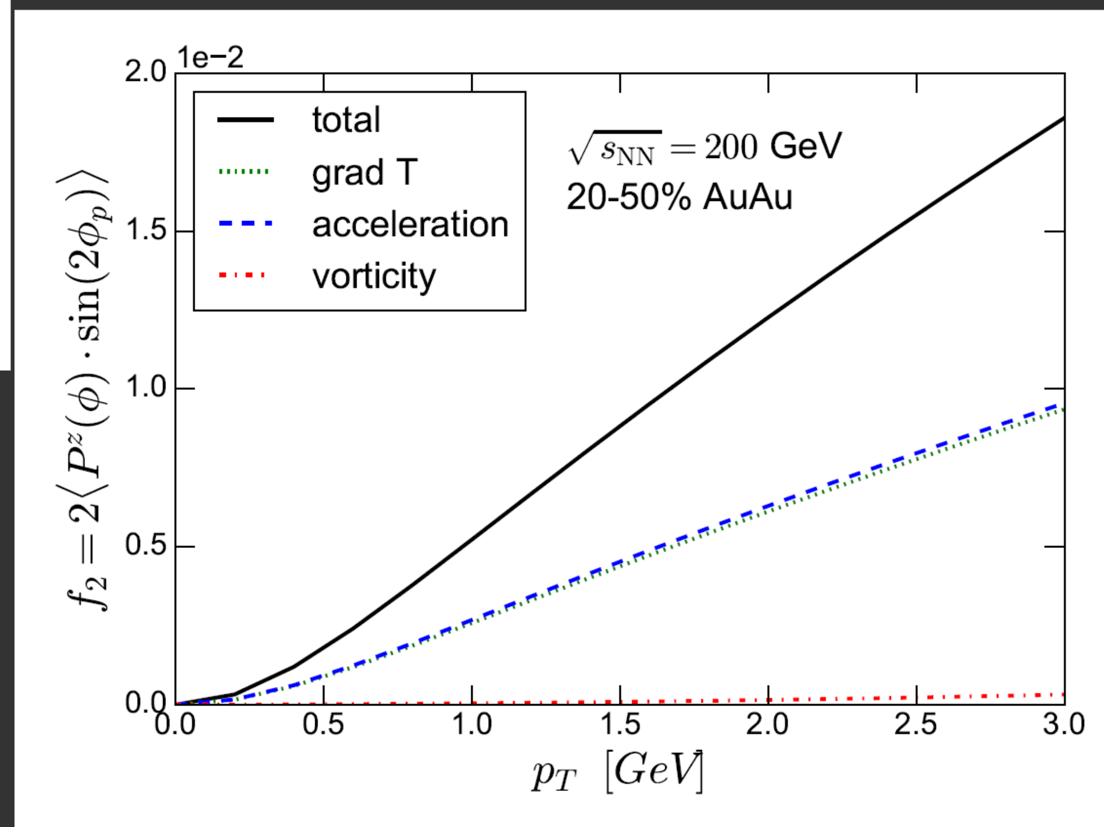
Are all these components needed?

I. Karpenko, this meeting



GLOBAL J- COMPONENT

LONGITUDINAL COMPONENT



A fascinating possibility: hydrodynamics with spin tensor

W. Florkowski et al., Phys. Rev. C 97 (2018) 041901,

In quantum field theory there are conserved currents arising from Noether theorem (canonical currents):

$$\partial_\mu \widehat{T}^{\mu\nu} = 0$$

$$\partial_\lambda \widehat{\mathcal{J}}^{\lambda,\mu\nu} = \partial_\lambda \left(\widehat{\mathcal{S}}^{\lambda,\mu\nu} + x^\mu \widehat{T}^{\lambda\nu} - x^\nu \widehat{T}^{\lambda\mu} \right) = \partial_\lambda \widehat{\mathcal{S}}^{\lambda,\mu\nu} + \widehat{T}^{\mu\nu} - \widehat{T}^{\nu\mu} = 0$$

Spin tensor

Pseudo-gauge transformation (F. W. Hehl, Rep. Mat. Phys. 9 (1976) 55)

$$\widehat{T}'^{\mu\nu} = \widehat{T}^{\mu\nu} + \frac{1}{2}\partial_\alpha \left(\widehat{\Phi}^{\alpha,\mu\nu} - \widehat{\Phi}^{\mu,\alpha\nu} - \widehat{\Phi}^{\nu,\alpha\mu} \right)$$

$$\widehat{\mathcal{S}}'^{\lambda,\mu\nu} = \widehat{\mathcal{S}}^{\lambda,\mu\nu} - \widehat{\Phi}^{\lambda,\mu\nu}$$

Leave P, J unchanged

Special case: Belinfante symmetrized stress-energy tensor, spin tensor vanishing.
Tacitly understood in relativistic hydrodynamics

$$\widehat{T}^{\mu\nu}_{\rm Bel} = \widehat{T}^{\mu\nu} + \frac{1}{2}\partial_\alpha \left(\widehat{\mathcal{S}}^{\alpha,\mu\nu} - \widehat{\mathcal{S}}^{\mu,\alpha\nu} - \widehat{\mathcal{S}}^{\nu,\alpha\mu} \right)$$

$$\widehat{\mathcal{S}}'^{\lambda,\mu\nu} = 0$$

Question: does it make any difference in our theoretical calculations ?

F. B., W. Florkowski, E. Speranza, in preparation

$$T^{\mu\nu} = \langle \widehat{T}^{\mu\nu}_{\rm can} \rangle \qquad\qquad T^{\mu\nu} = \langle \widehat{T}^{\mu\nu}_{\rm Bel} \rangle$$

ANSWER: it all depends on what we measure. In fact we measure spectra, not energy density. If their theoretical expression is not affected by the pseudo-gauge transformation, any tensor is good. In other words: spatial densities in the QGP are "objective" up to quantum corrections.

Polarization ultimately depends on

$$\mathrm{tr}(\widehat{\rho} a^{\dagger}_{\sigma}(p) a_{\sigma'}(p))$$

Does the density operator depend on pseudo-gauge transformations?

$$\widehat{\rho} = \frac{1}{Z} \exp\left[- \int_{\Sigma} \mathrm{d}\Sigma_{\mu} \widehat{T}^{\mu\nu}_{\rm Bel} \beta_{\nu}(x) \right]$$

Global thermodynamic equilibrium: NO

$$\widehat{\rho} = \frac{1}{Z} \exp\left[-b_{\mu} \widehat{P}^{\mu} + \frac{1}{2} \varpi_{\mu\nu} \widehat{J}^{\mu\nu} + \zeta \widehat{Q} \right]$$

Local thermodynamic equilibrium: YES, unless

$$\varpi_{\mu\nu} = \frac{1}{2} \partial_{\mu}\beta_{\nu} - \partial_{\nu}\beta_{\mu}$$

Hydrodynamics with spin tensor

The local thermodynamic equilibrium operator

$$\widehat{\rho} = \frac{1}{Z}\exp\left[-\int_\Sigma \mathrm{d}\Sigma_\mu \widehat{T}_{\rm C}^{\mu\nu}\beta_\nu(x) - \frac{1}{2}\varpi_{\lambda\nu}(x)\widehat{\mathcal{S}}_{\rm C}^{\mu,\lambda\nu}\right]$$

$$\varpi_{\mu\nu} \neq \frac{1}{2}\partial_\mu\beta_\nu - \partial_\nu\beta_\mu$$

6 additional thermodynamic fields to be evolved

$$\zeta = \mu/T \qquad \beta_\mu = \frac{1}{T}u_\mu$$

$$+ \qquad \varpi_{\mu\nu}$$

$$\partial_\mu T^{\mu\nu} = 0$$

$$\partial_\mu j^\mu = 0$$

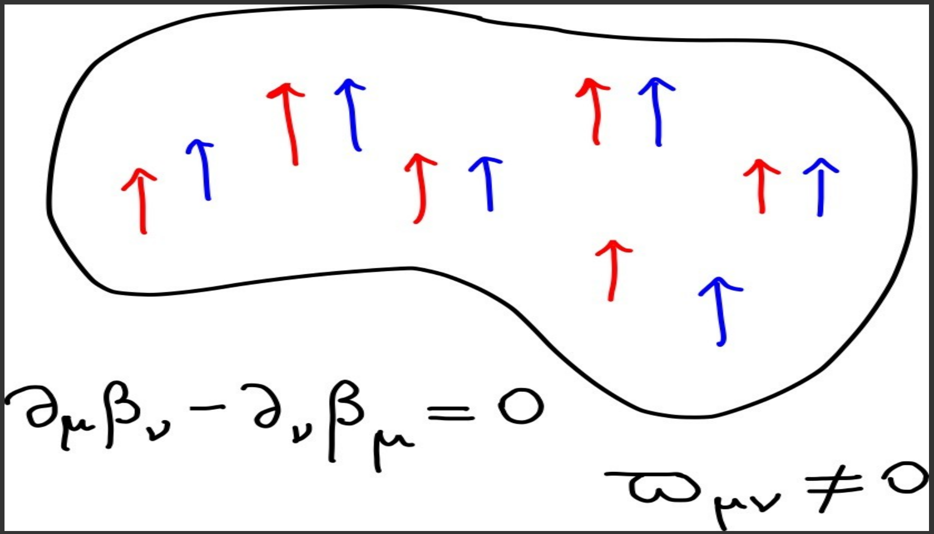
$$\partial_\mu \mathcal{S}^{\mu,\lambda\nu} = T^{\nu\lambda} - T^{\lambda\nu}$$

11 spin-hydrodynamical equations for 11 unknowns and 11 initial conditions

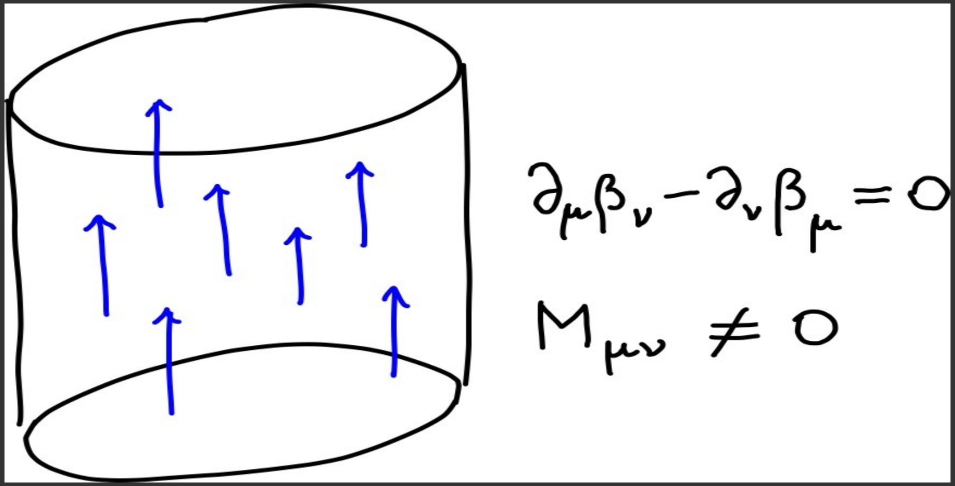
Need to determine the constitutive equations of the spin tensor!

This approach makes it possible to describe as local thermodynamic equilibrium

polarized C-even matter (QGP at high energy!)



NOTE! No need of a spin tensor to describe Einstein-De Haas/ Barnett effect because it is not a C-even system!



Conclusions

Polarization and Chirality have opened a new window in heavy ion physics. From a theory viewpoint, a new outlook, forcing us to rethink the foundations of relativistic hydrodynamics and kinetic theory in a fully quantum framework.

The study of the quantum features of Quark Gluon Plasma have exciting connections with fundamental physics problems even beyond QCD

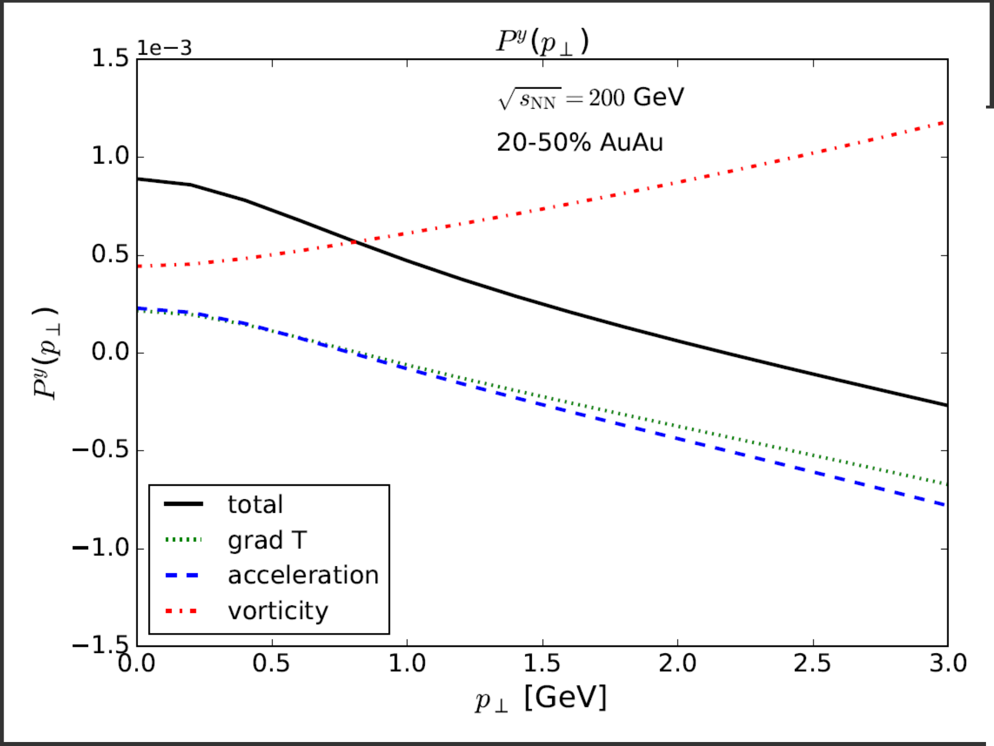
SPARE SLIDES

Can we forget about acceleration and grad T contributions?

$$S^\mu(p) = \frac{1}{8mT}\epsilon^{\mu\nu\rho\sigma}p_\sigma \frac{\int_\Sigma d\Sigma_\tau p^\tau n_F(1-n_F)\partial_\nu^\perp u_\rho}{\int_\Sigma d\Sigma_\tau p^\tau \; n_F}$$

Theory-wise: difficult to justify it. Acceleration and vorticity and grad T are so tightly related in relativity and relativistic hydrodynamics

Phenomenologically: all three contributions are relevant



I. Karpenko, this conference

LONGITUDINAL COMPONENT