

Anisotropic (v_1 and v_2) flow and freeze-out in relativistic heavy-ion collisions in the energy range of BES /FAIR/NICA



L.Bravina,

in collaboration with

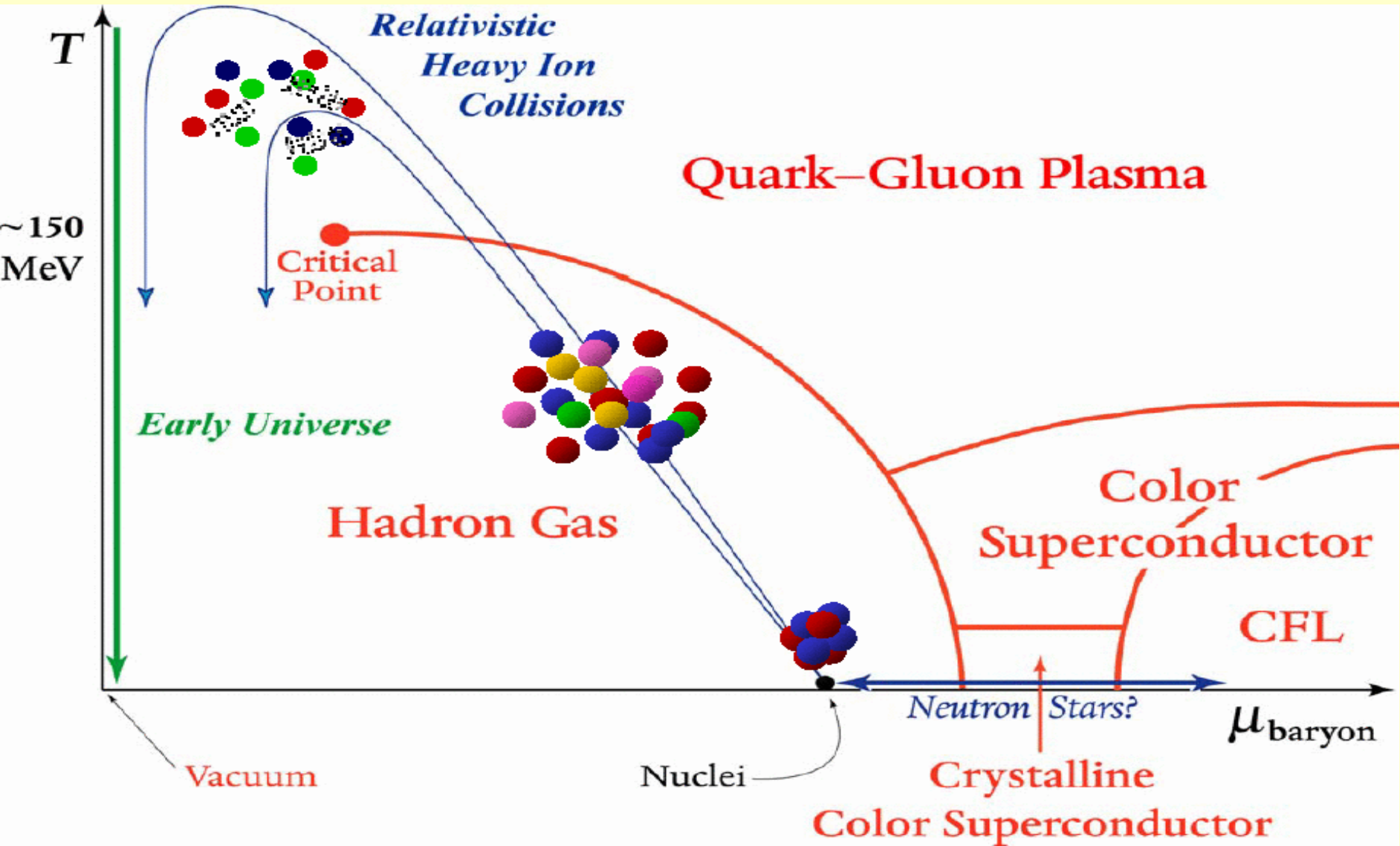
E. Zabrodin, S. Sivoklokov, Yu. Kvasyuk, A. Vityuk

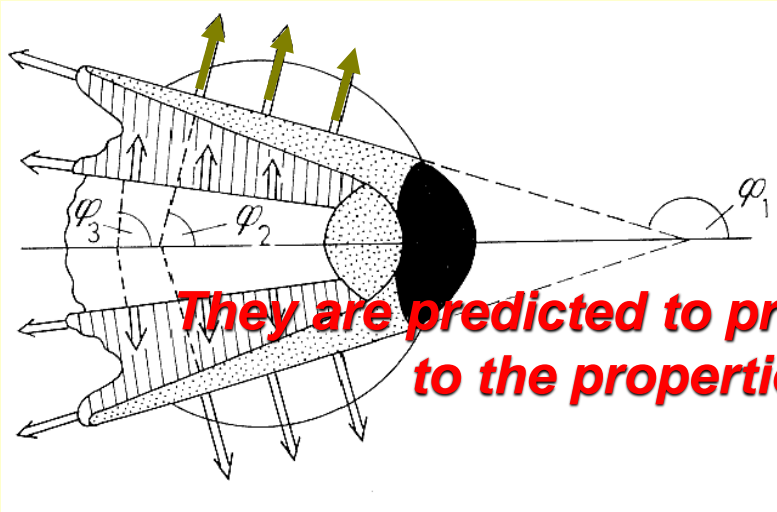


Content

- *Motivation. Why flow?*
- *Flow: how to quantify this phenomenon*
- *Connection to Equation of State*
- *Transverse flow at FAIR/NICA and AGS energies*
- *Features of transverse flow at energies between AGS and RHIC*
- *Transverse flow and freeze-out*
- *Summary and perspectives*

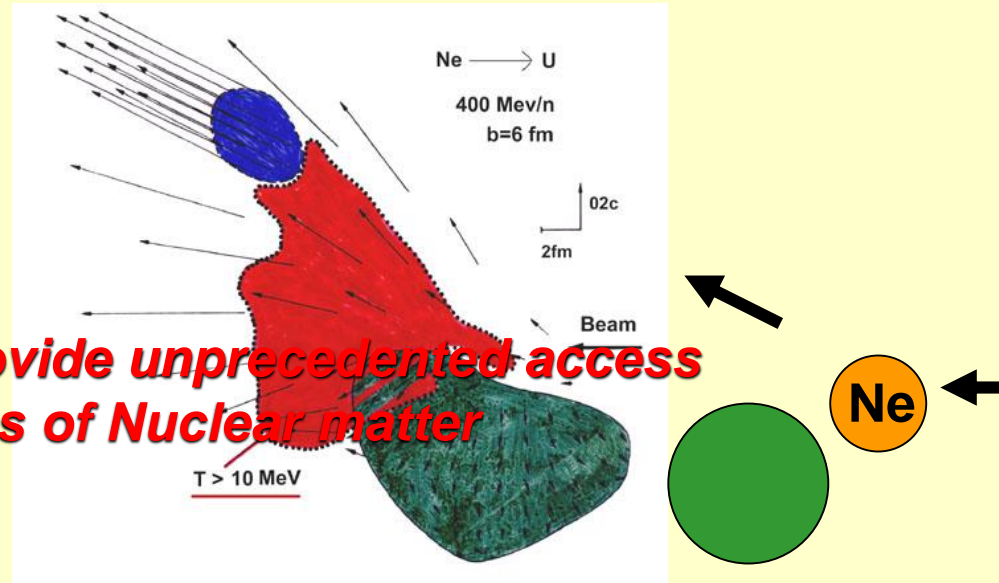
Present status of the Equation of State





They are predicted to provide unprecedented access to the properties of Nuclear matter

W. Scheid, H. Muller, and W. Greiner,
PRL **32**, 741 (1974)



H. Stöcker, J.A. Maruhn, and W. Greiner,
PRL **44**, 725 (1980)

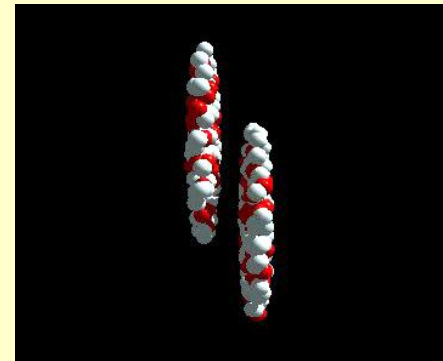
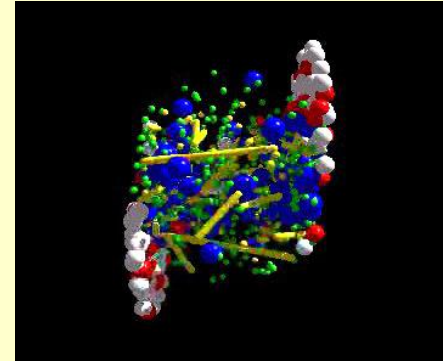
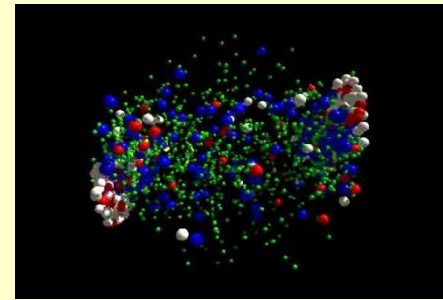
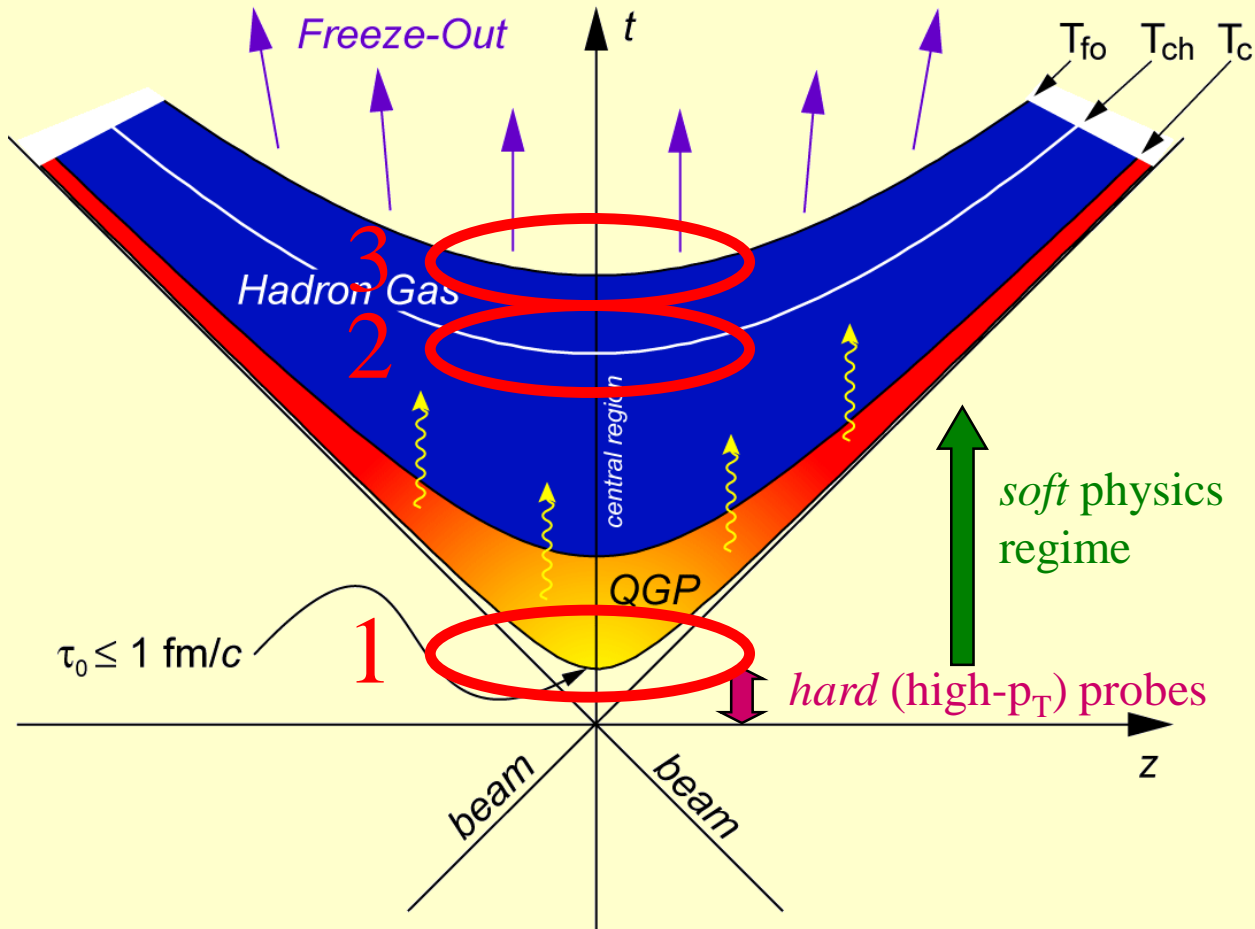
M.I. Sobel, P.J. Siemens, J.P. Bondorf, and H.A. Bethe, Nucl. Phys. A **251**, 502 (1975)

G.F. Chapline, M.H. Johnson, E. Teller, and M.S. Weiss, PRD **8**, 4302 (1973)

E. Glass Gold et al. Annals of Physics **6**, 1 (1959)

The idea to use collective flow to Probe the properties of nuclear matter is long-standing

Motivation



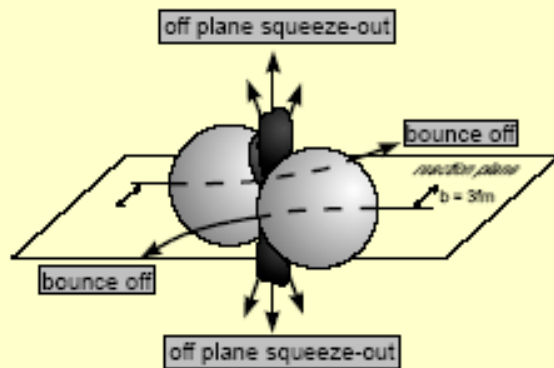
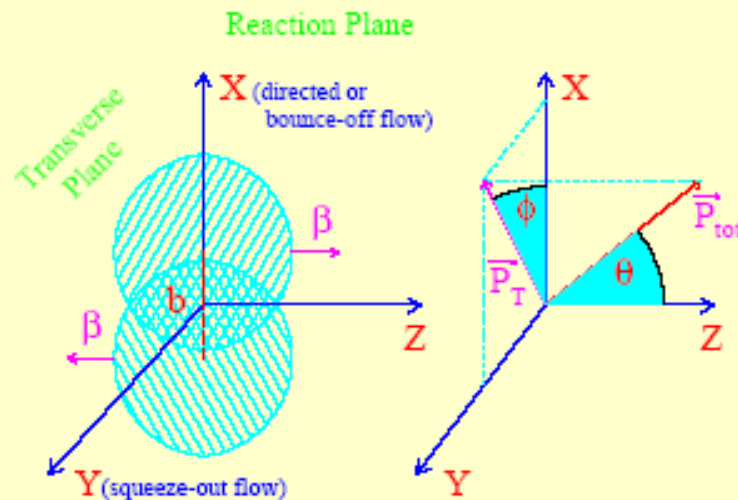
Chemical freezeout ($T_{ch} \geq T_c$): inelastic scattering ceases

Kinetic freeze-out ($T_{fo} \leq T_{ch}$): elastic scattering ceases

Flow:

**We have to quantify
this
phenomenon**

Definitions. Non-central Collisions ($b > 0$)



Flow Decomposition:

Transverse flow = Radial
+ Bounce-off + Squeeze-out

S. Voloshin and Y. Zhang, ZPC 70 (1996) 665

Modern analysis:

Transverse flow =
Radial + Directed + Elliptic + ...
{isotropic} {anisotropic}

$$E \frac{d^3 N}{d^3 p} = \frac{1}{2\pi} \frac{d^2 N}{p_T dp_T dy} \left(1 + \sum_{n=1}^{\infty} 2v_n \cos(n\phi') \right)$$

Directed flow:

$$v_1 = \left\langle \frac{p_x}{p_T} \right\rangle \equiv \langle \cos(\phi') \rangle$$

Elliptic flow:

$$v_2 = \left\langle \left(\frac{p_x}{p_T} \right)^2 - \left(\frac{p_y}{p_T} \right)^2 \right\rangle \equiv \langle \cos(2\phi') \rangle$$

Distributions

Rapidity dependence

$$v_n(y, \Delta p_t, \Delta b) = \frac{\int_{\Delta p_t} \int_{\Delta b} \cos(n\phi) \frac{d^3N}{dydbdp_t} dp_t db}{\int_{\Delta p_t} \int_{\Delta b} \frac{d^3N}{dydbdp_t} dp_t db}$$

Transverse momentum dependence

$$v_n(p_t, \Delta y, \Delta b) = \frac{\int_{\Delta y} \int_{\Delta b} \cos(n\phi) \frac{d^3N}{dydbdp_t} dy db}{\int_{\Delta y} \int_{\Delta b} \frac{d^3N}{dydbdp_t} dy db}$$

$n=1,2,\dots$

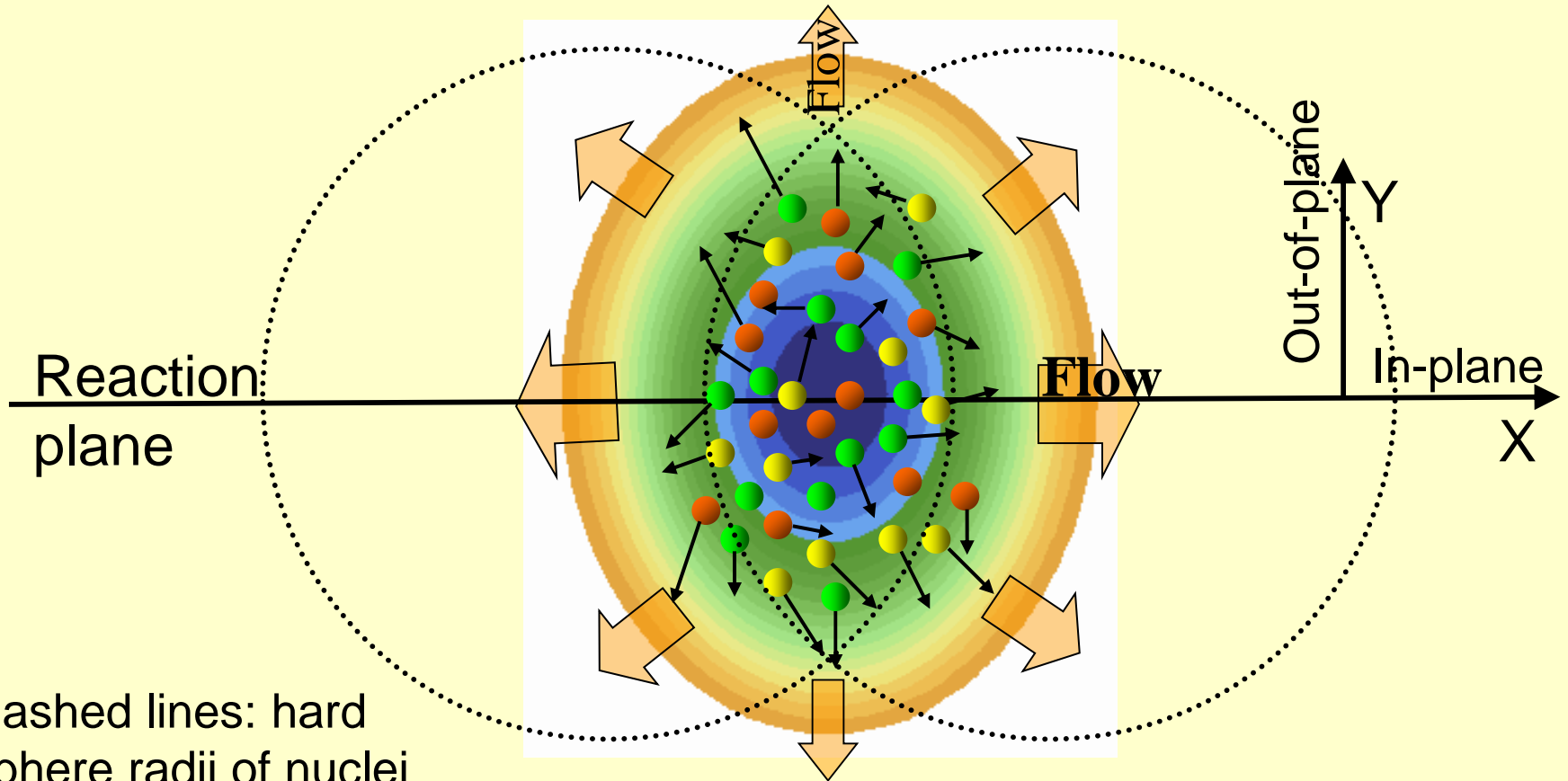
Centrality dependence

$$v_n(b, \Delta p_t, \Delta y) = \frac{\int_{\Delta p_t} \int_{\Delta y} \cos(n\phi) \frac{d^3N}{dydbdp_t} dp_t dy}{\int_{\Delta p_t} \int_{\Delta y} \frac{d^3N}{dydbdp_t} dp_t dy}$$

Motivation:
connection to
Equation of State

Flow (in the transverse plane)

A mid-peripheral collision

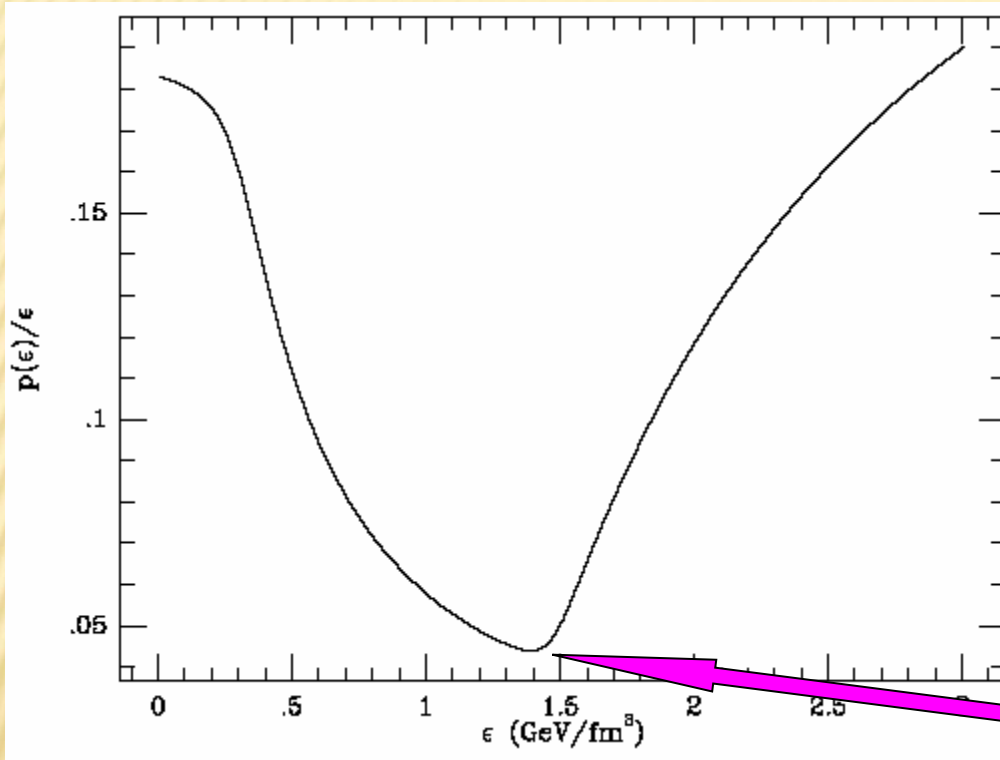


Dashed lines: hard
sphere radii of nuclei

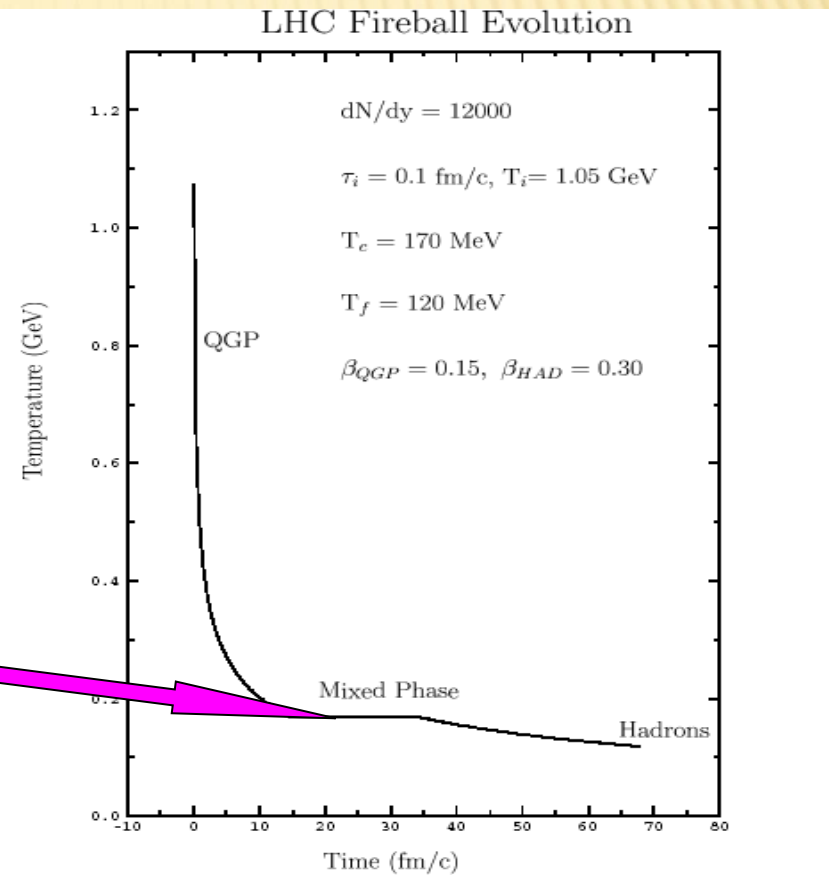
Re-interactions → **FLOW**

Re-interactions among what? **Hadrons, partons or both?**
In other words, what equation of state?

DISAPPEARANCE OF DIRECTED FLOW



Hung and Shuryak, PRL 75 (1995) 4003



Braun-Munzinger, NPA 661 (1999) 261c

In case of first order phase transition

$$\frac{dP}{d\epsilon} = c_s^2 = 0$$

V1 OF NUCLEONS AND FRAGMENTS AT LOWER ENERGIES

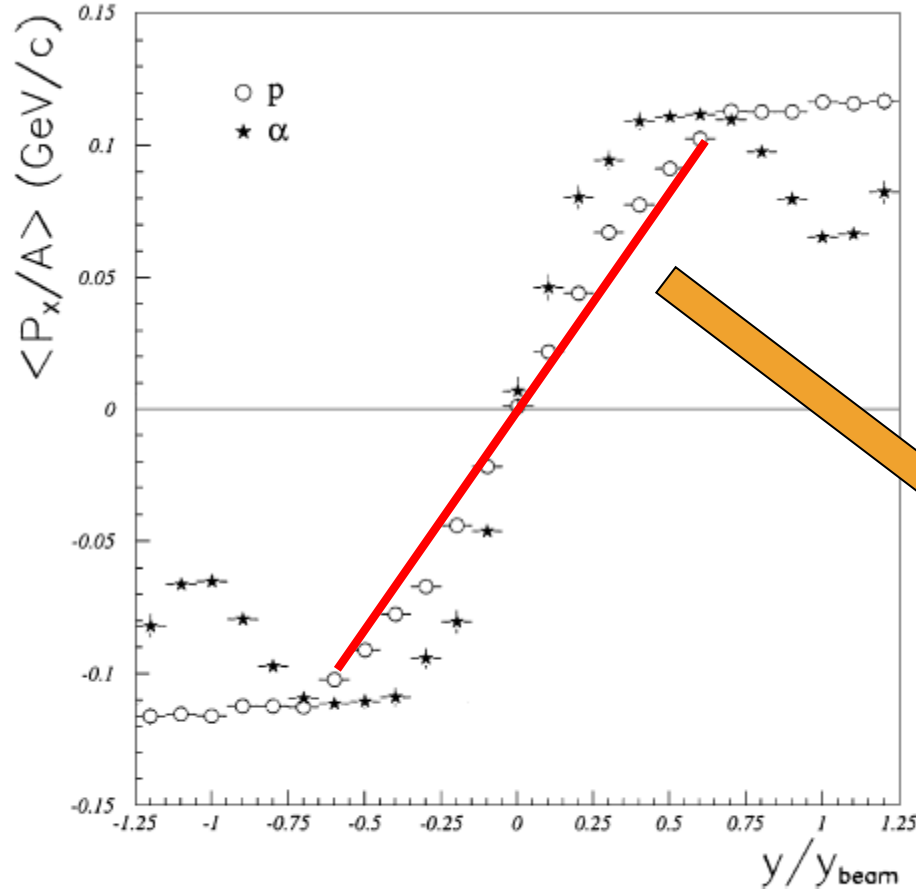


Figure 1 Average in-plane transverse momentum versus normalized rapidity in the reaction Au+Au at 800.4 MeV. The points at $y/y_{beam} < 0$ are reflected.

Plastic Ball Collaboration
introduced a slope parameter

$$F = \frac{d\langle p_x \rangle / A}{dy_n}, \quad y_n = y / y_{max}$$

$$F_y = \frac{d\langle p_x \rangle / A}{dy}$$

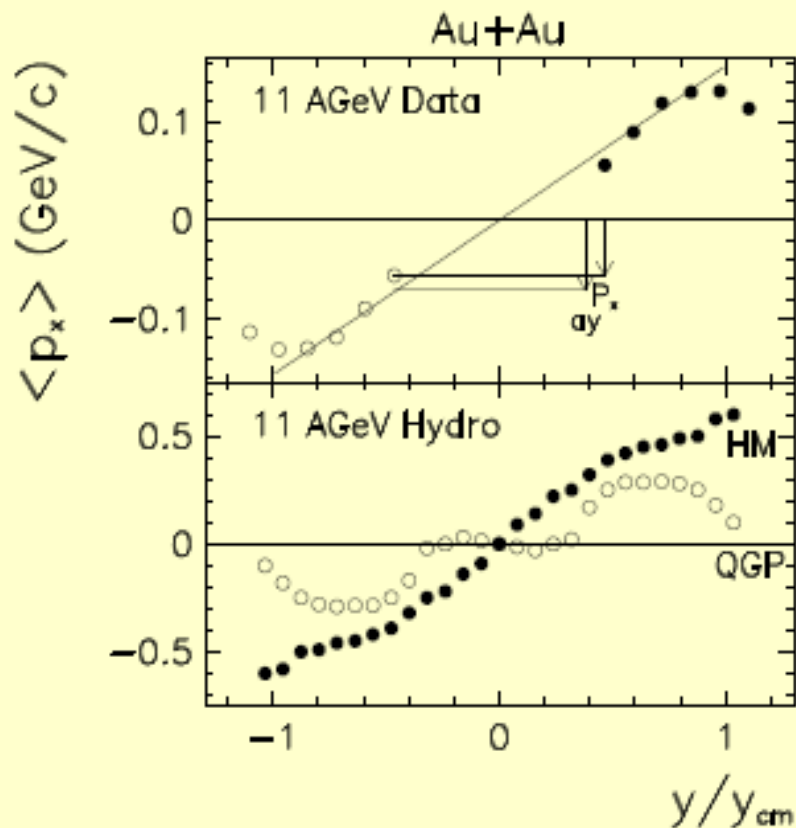
Directed flow of nucleons
and fragments has **linear**
slope in normal direction

=> **normal flow**

Transverse flow between AGS and SPS

SOFTENING OF DIRECTED FLOW

L.P. Csernai, D. Röhrich, PLB 458 (1999) 454



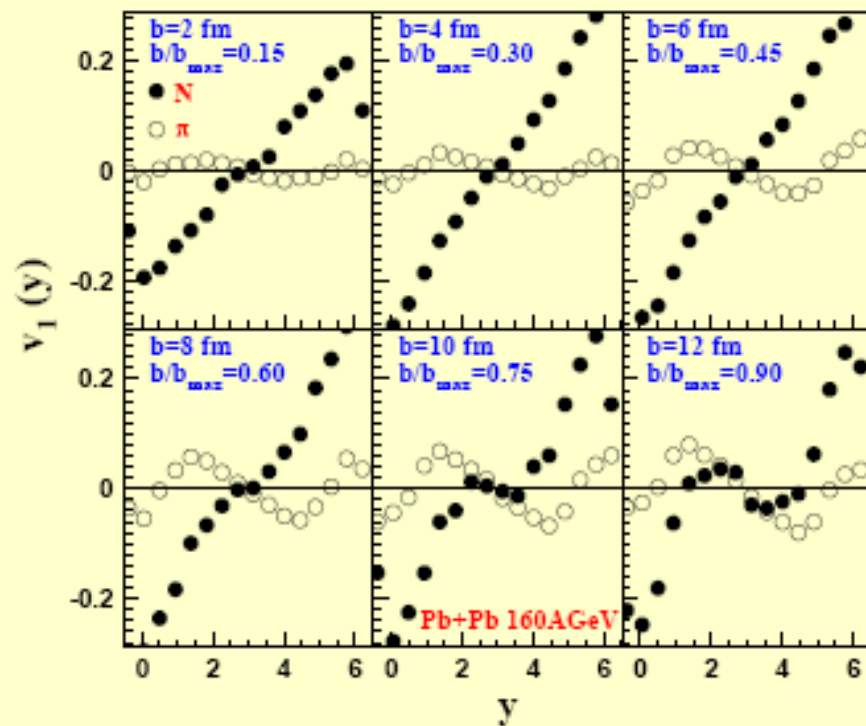
Transition to the **Quark-Gluon Plasma**
 → decrease in pressure → softening
 of the directed flow

L. Bravina, PLB 334, 49 (1995)

H. Liu, S. Panitkin, N. Xu, PRC 59, 348 (1999)

R.J.M. Snellings *et al.*, PRL 84, 2803 (2000)

L. Bravina *et al.*, PRC 61, 064902 (2000)



Wiggle structure: The effect is more pronounced in peripheral and light-ion collisions, therefore, it cannot be explained by the softening of the **EOS** because of the formation of strings

Beam energy scan results for v1 (STAR)

S. Singha et al. (STAR Collab.), PoS CPD2017 (2018) 004

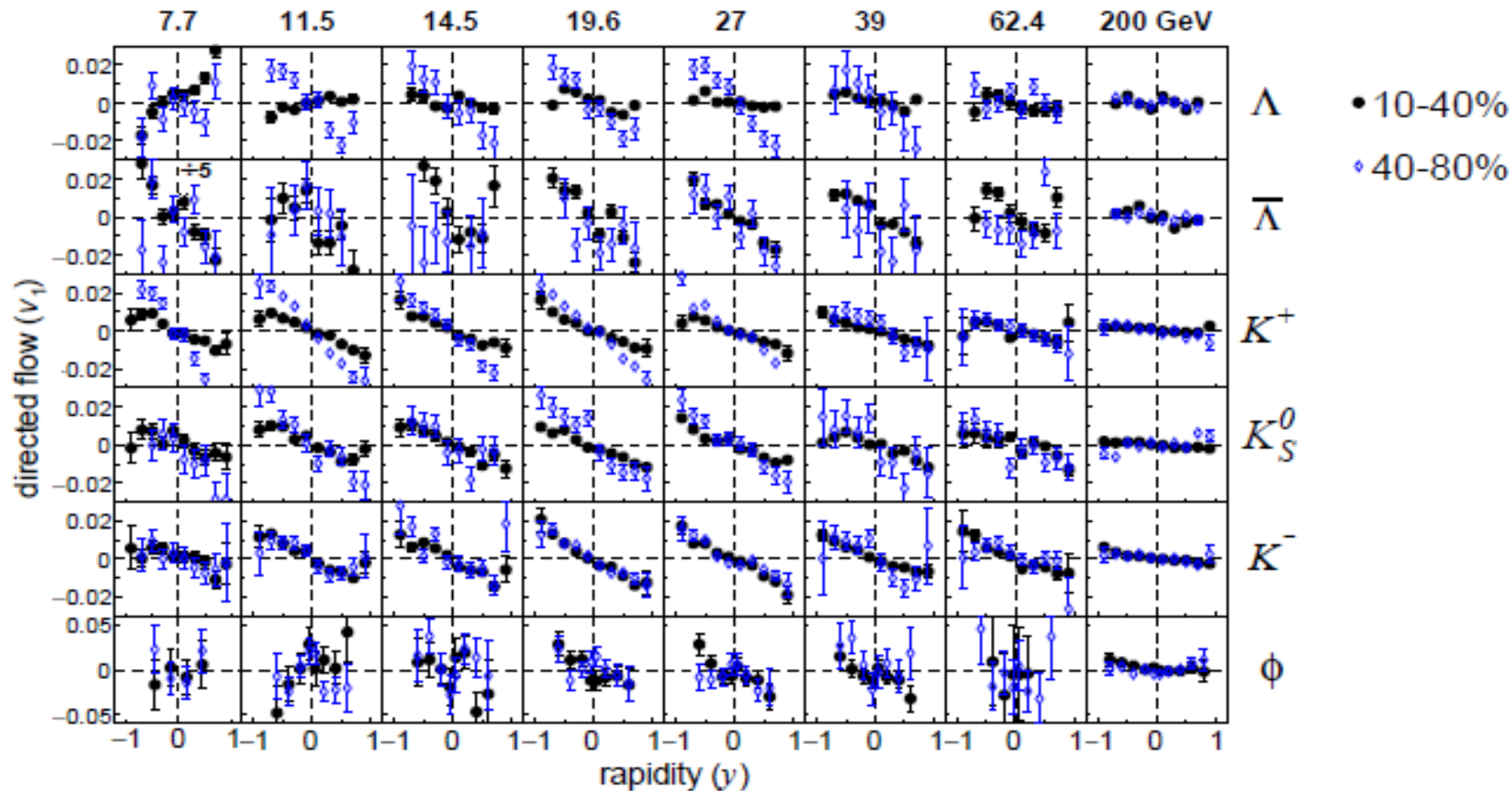
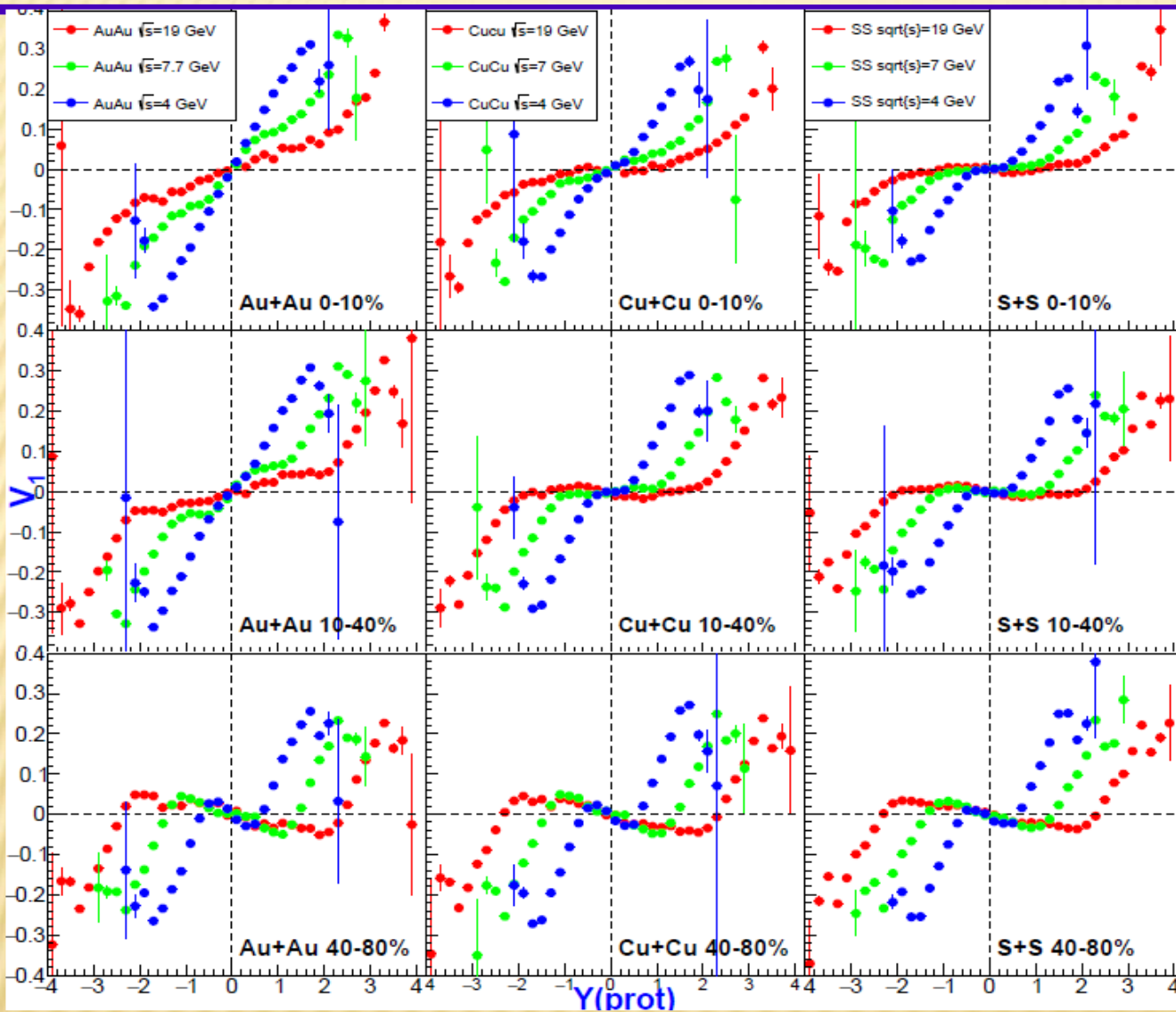


Figure 1: (Color online) Rapidity dependence of directed flow (v_1) for Λ , $\bar{\Lambda}$, K^+ , K_S^0 , K^- and ϕ in 10-40% and 40-80% Au+Au collisions at $\sqrt{s_{NN}} = 7.7, 11.5, 14.5, 19.6, 27, 39, 62.4$ and 200 GeV.

Directed flow of protons in light and heavy systems



QGSM

Blue - $\sqrt{s} = 4$ GeV

Green - 7.7 GeV

Red - 19 GeV

- Softening and development of antiflow at midrapidity with increasing impact parameter

- In central events – “normal” flow with decreasing CM energy

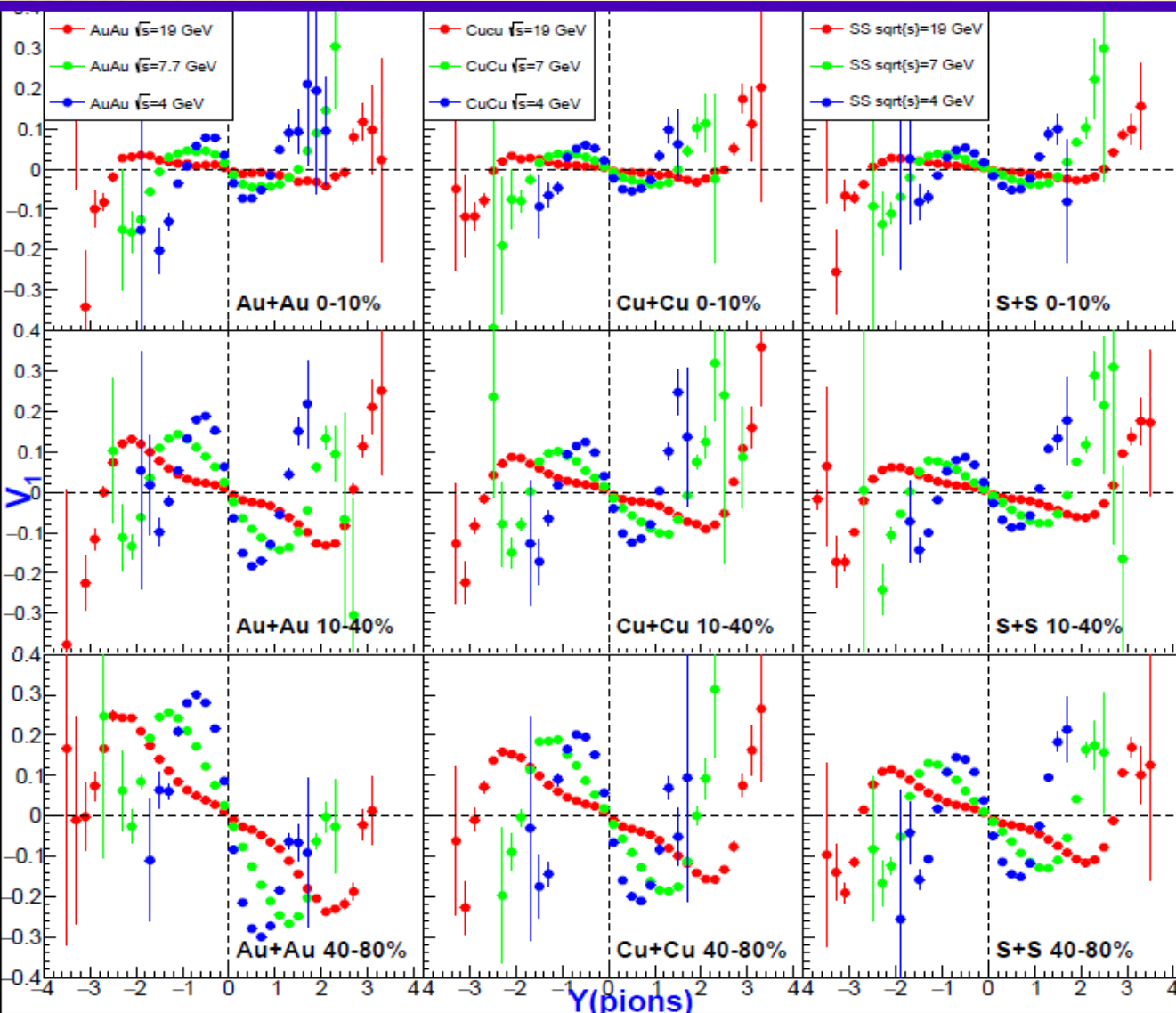
- Softening of v_1 at midrapidity is stronger for small colliding systems, whereas in case of QGP formation the effect should be opposite

Au + Au

Cu + Cu

S + S

Directed flow of pions in light and heavy systems



QGSM

Blue - $\sqrt{s} = 4 \text{ GeV}$
 Green - 7.7 GeV
 Red - 19 GeV

- Development of antiflow at midrapidity at any impact parameters
- The antiflow slope at midrapidity is stronger for smaller colliding systems
- The slope decreases with rising CM energy of collisions

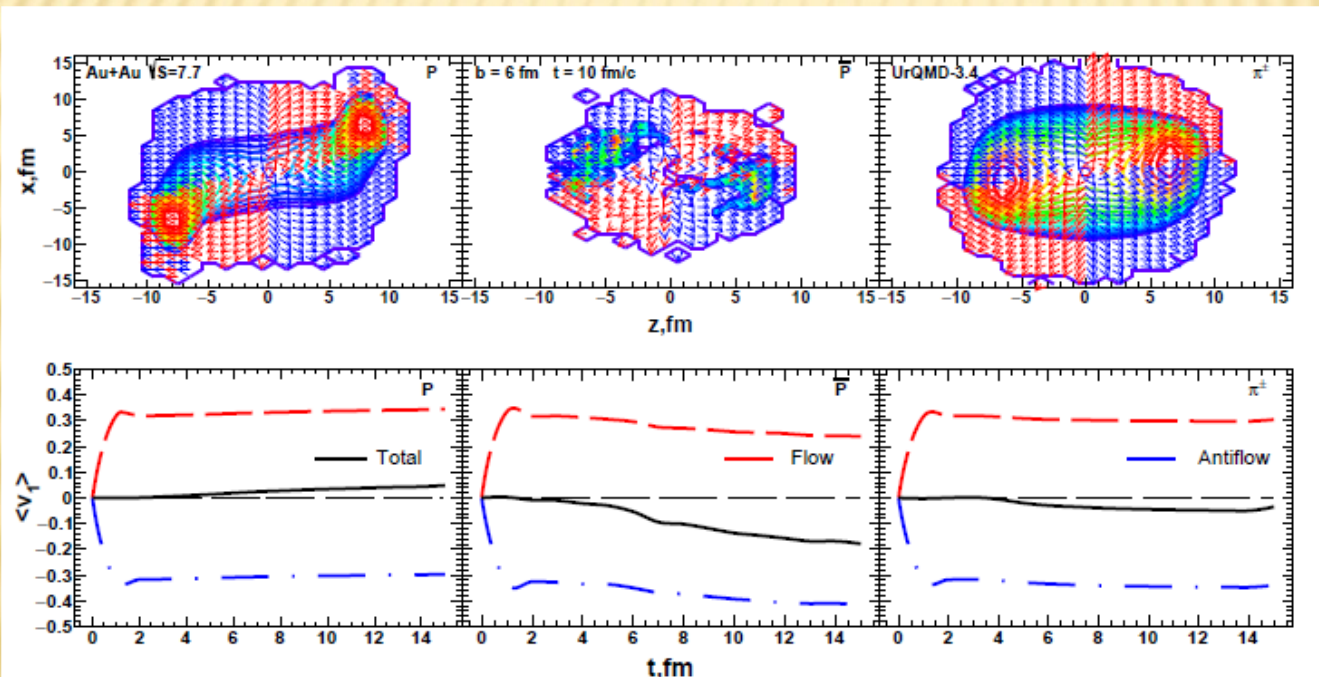
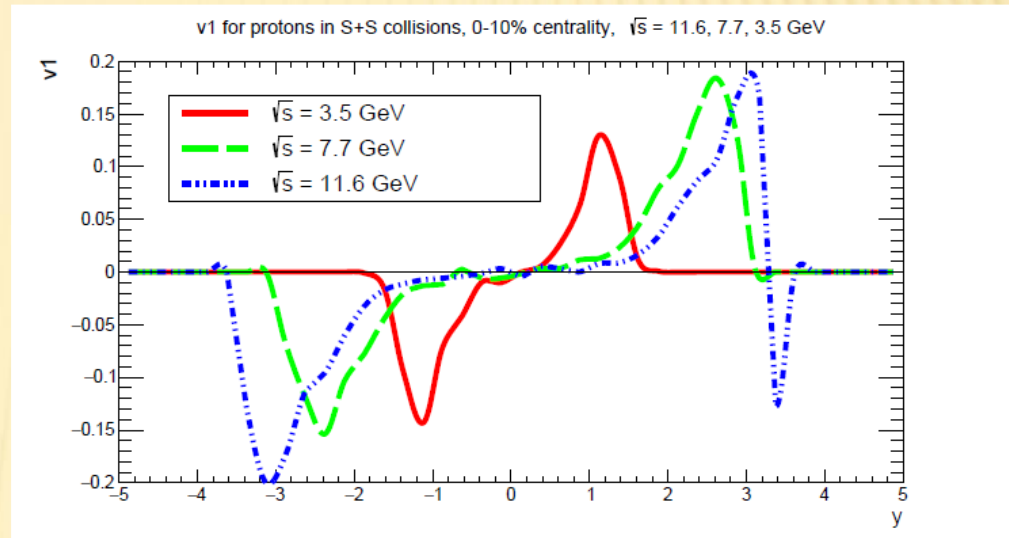
Au + Au

Cu + Cu

S + S

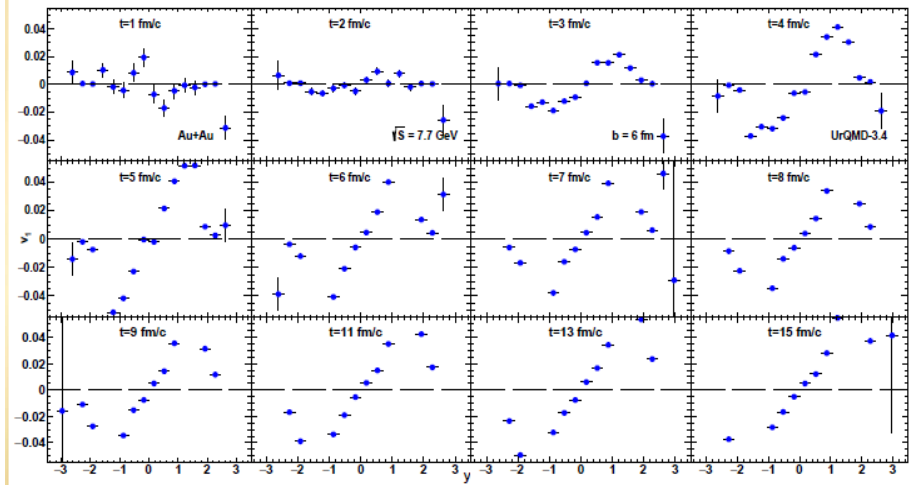
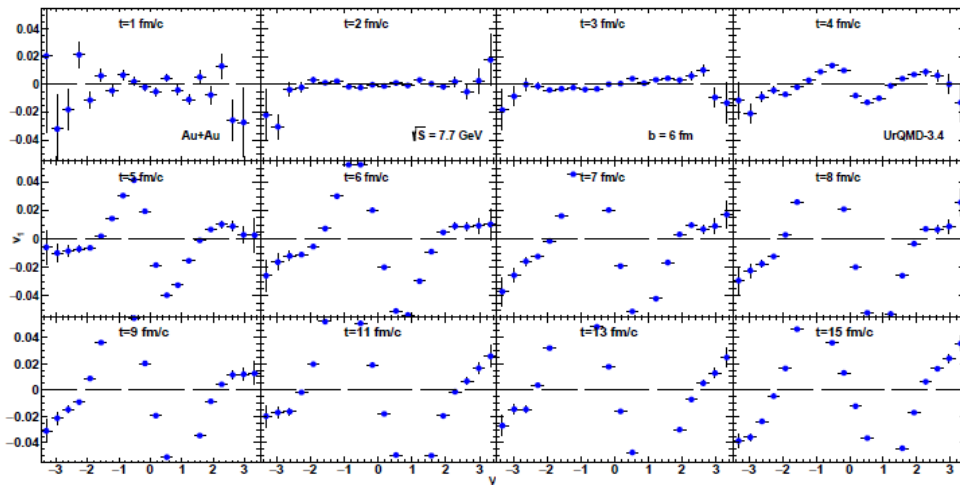
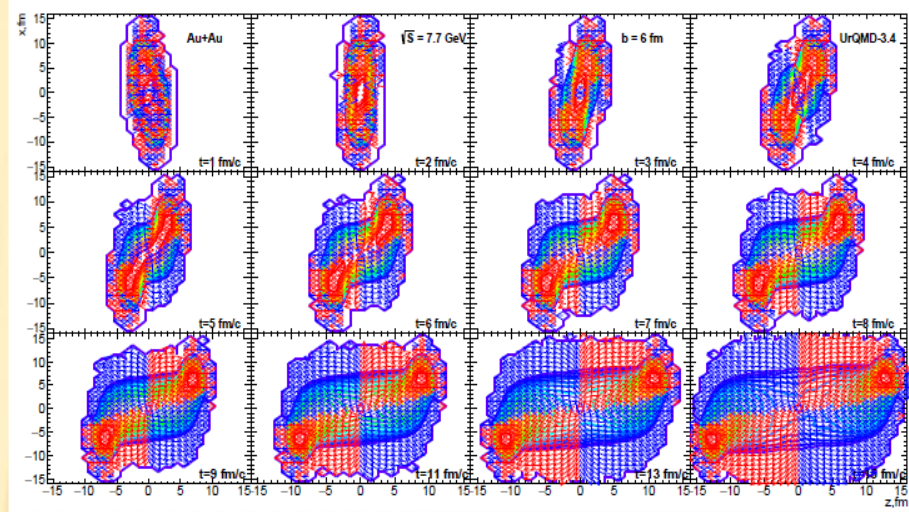
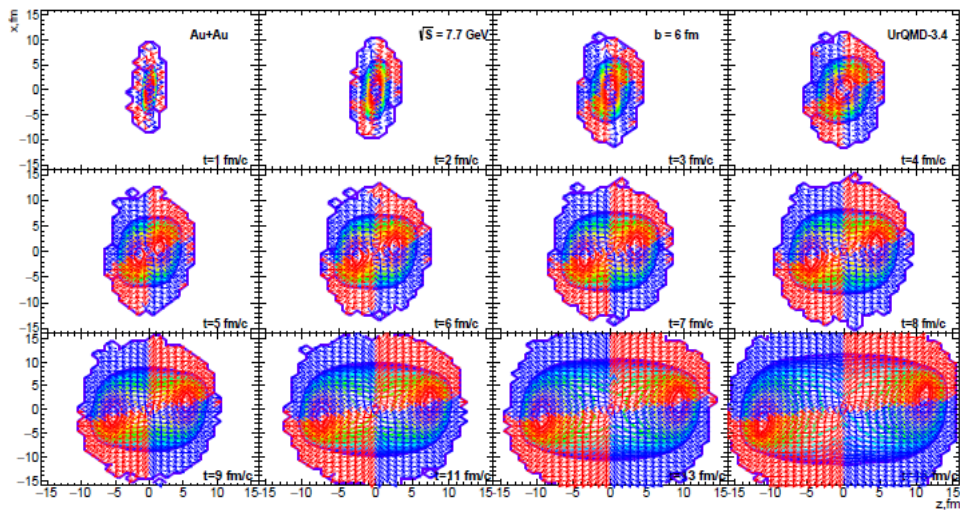
Directed flow in HI collisions at FAIR/NICA energies

Origin of changing of proton directed flow from antiproton to normal flow with decrease of CM energy in microscopic transport models



Directed flow =
Normal flow –
Antiflow

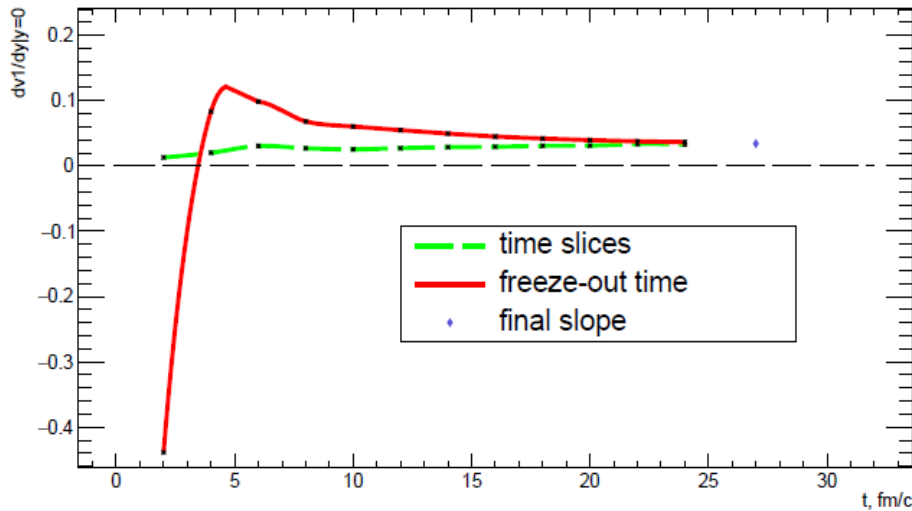
Time development of directed flow



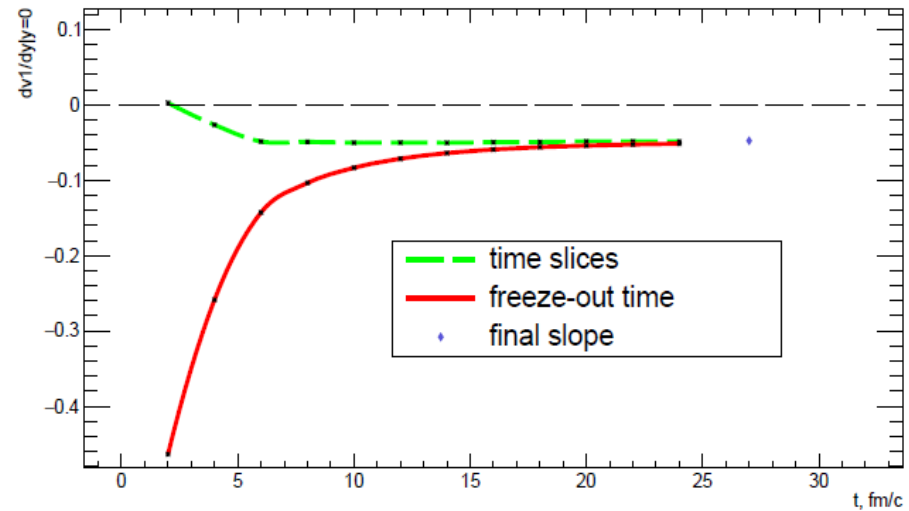
It appears that V_1 of both pions and protons at midrapidity is formed not earlier than 5 – 6 fm/c

Time development of directed flow

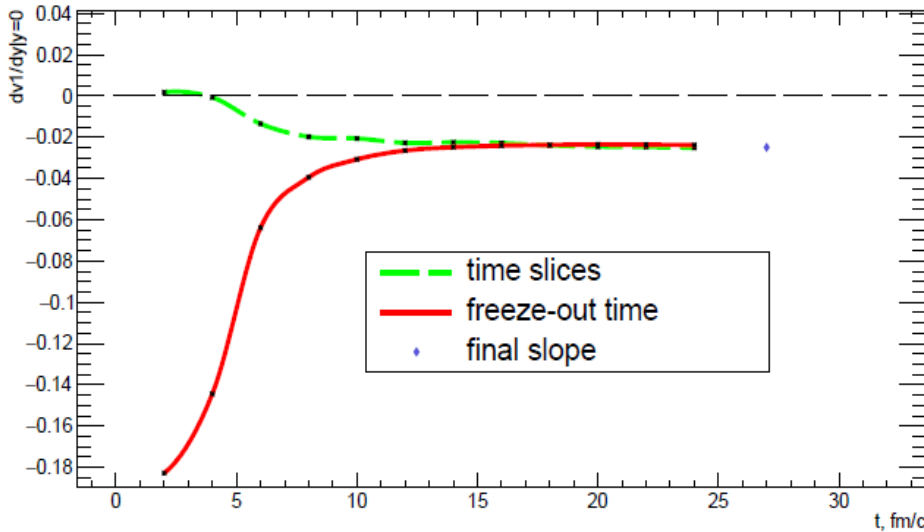
Protons, Au+Au, $\sqrt{s}=11.6$, $b=6$



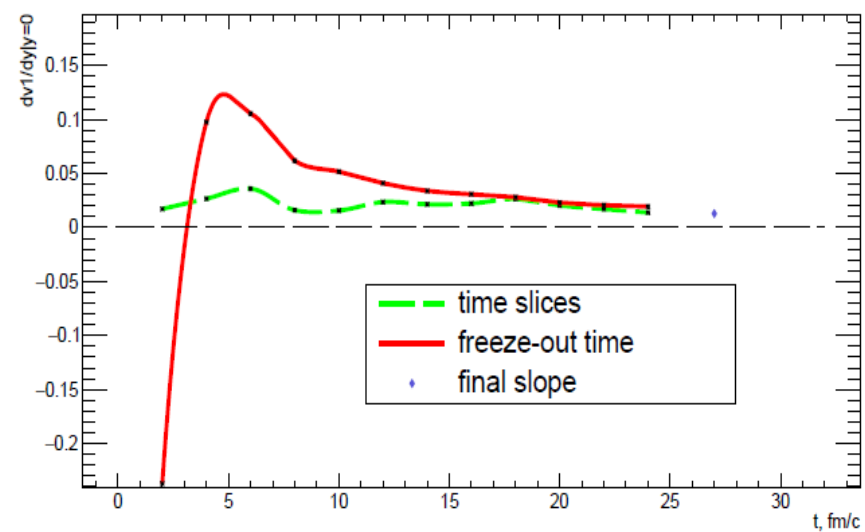
Pions, Au+Au, $\sqrt{s}=11.6$, $b=6$



Kaons, Au+Au, $\sqrt{s}=11.6$, $b=6$



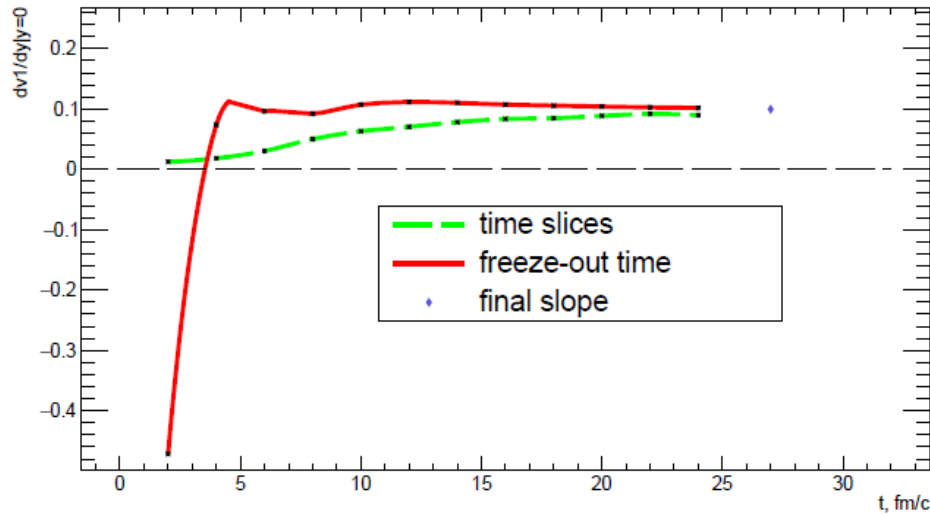
Lambdas, Au+Au, $\sqrt{s}=11.6$, $b=6$



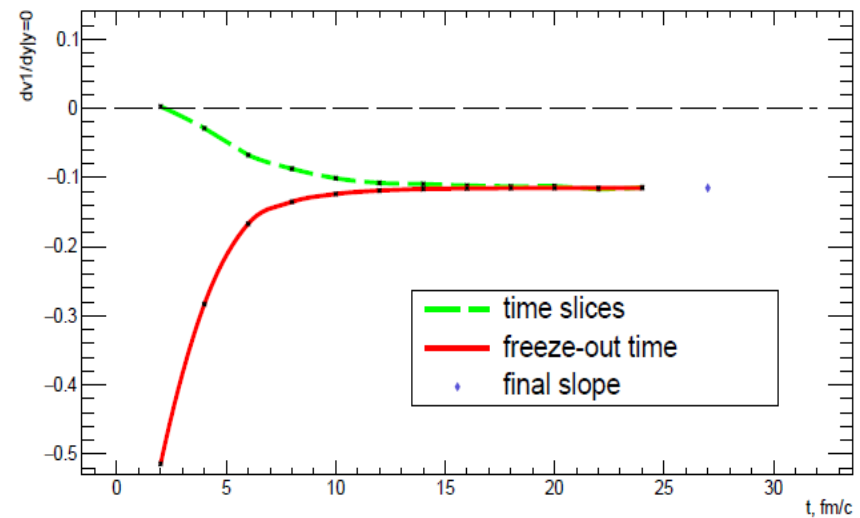
V_1 of both mesons and baryons at midrapidity is formed at approximately 6 – 10 fm/c

Influence of resonances on the development of V_1

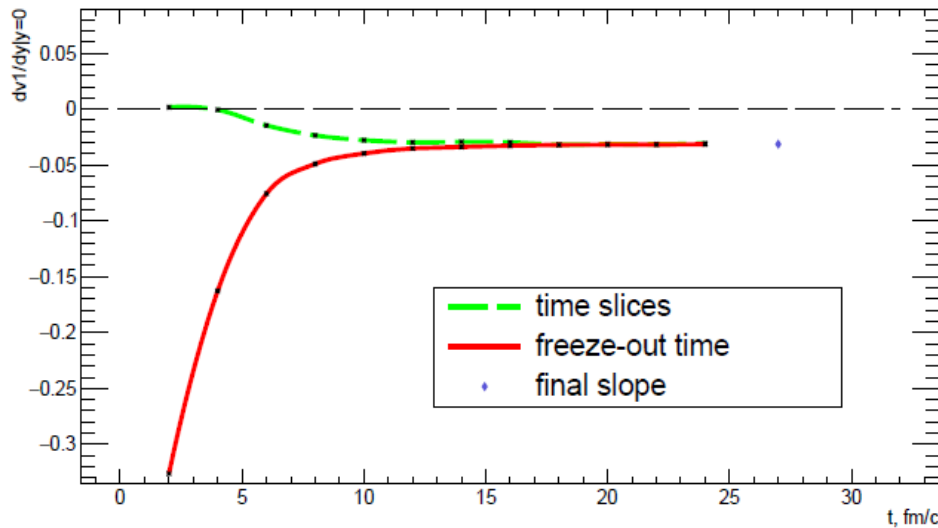
Protons, Au+Au, $\sqrt{s}=11.6$, $b=6$, no resonance decays



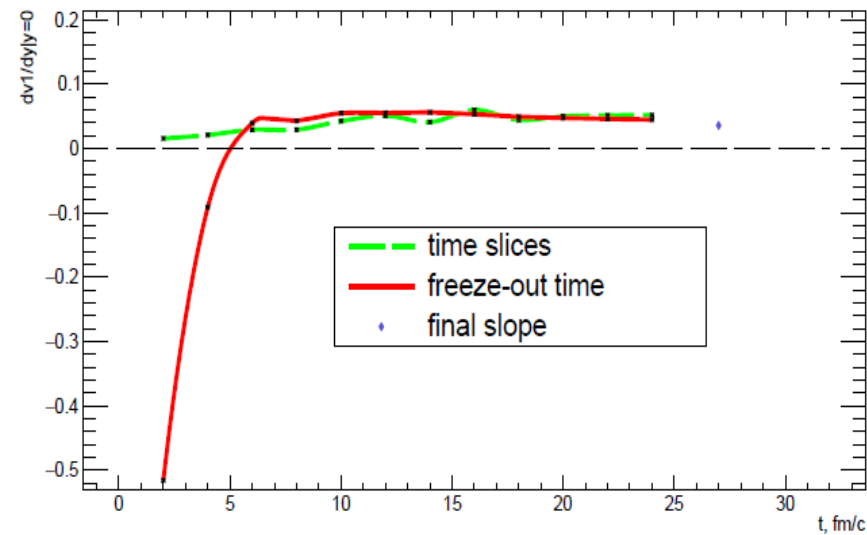
Pions, Au+Au, $\sqrt{s}=11.6$, $b=6$, no resonance decays



Kaons, Au+Au, $\sqrt{s}=11.6$, $b=6$, no resonance decays

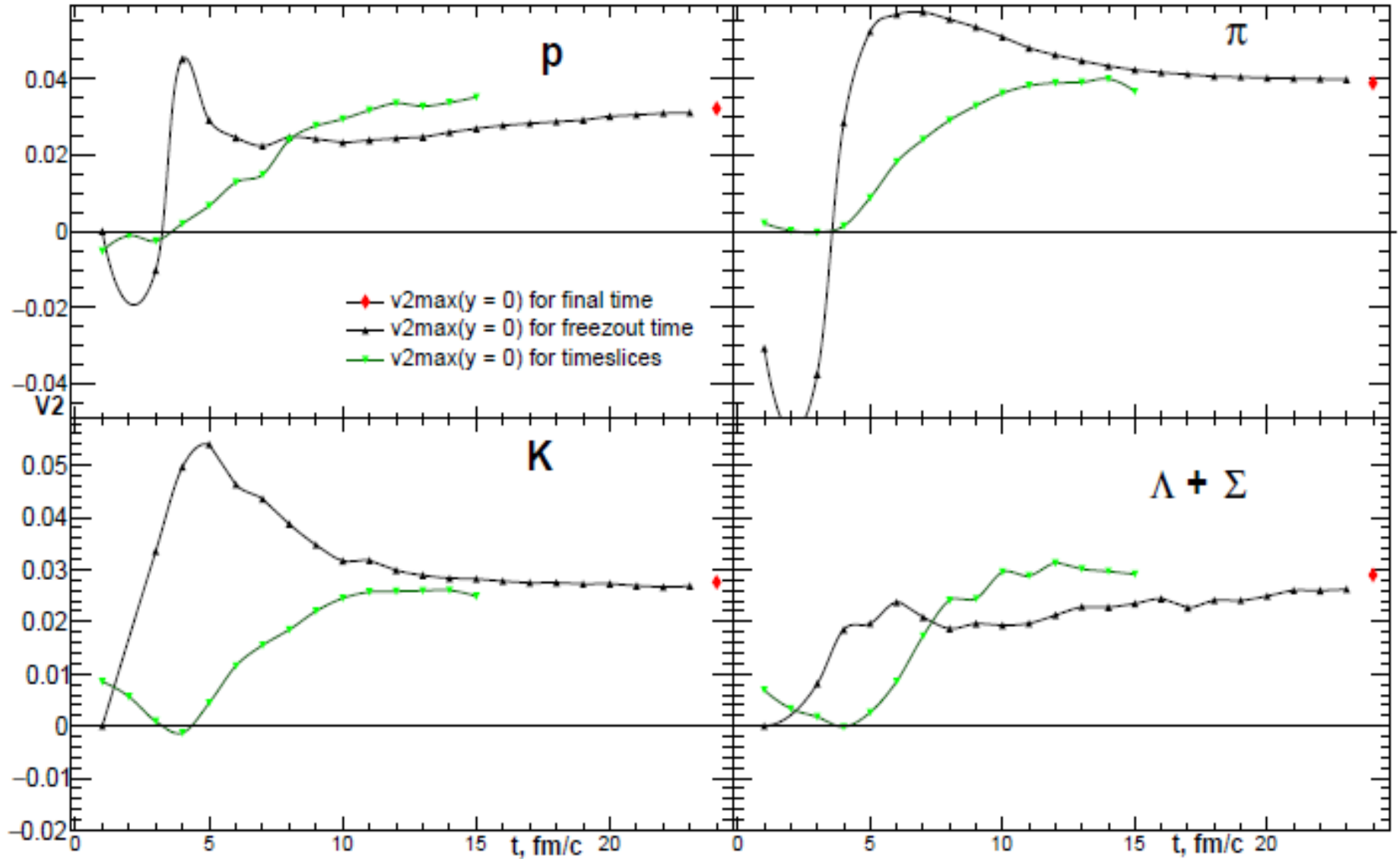


Lambdas, Au+Au, $\sqrt{s}=11.6$, $b=6$, no resonance decays



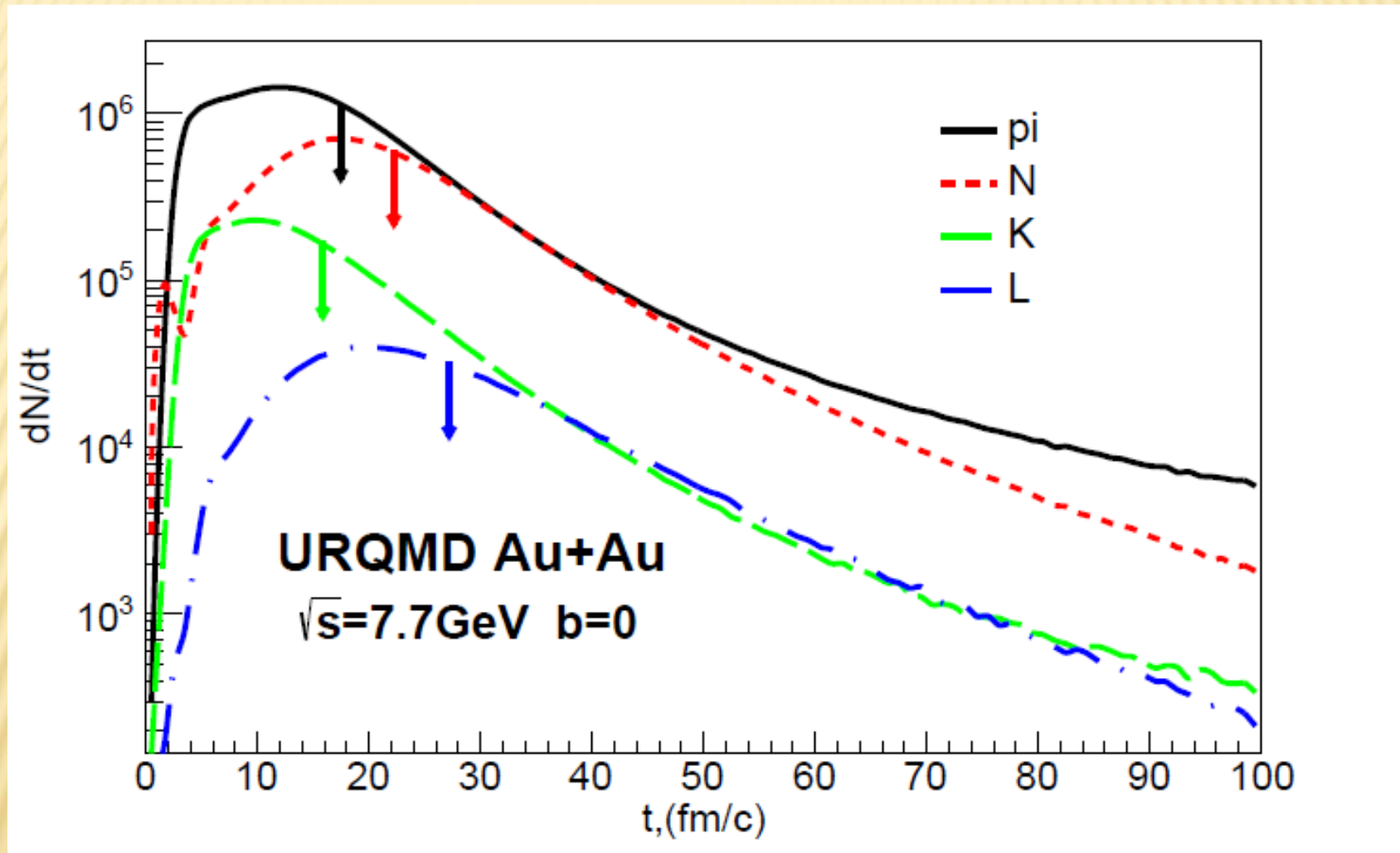
● Difference is seen only for Lambdas

Time development of elliptic flow



● V_2 of both mesons and baryons at midrapidity is formed after 10 fm/c

Sequential freeze-out of hadrons at NICA energies

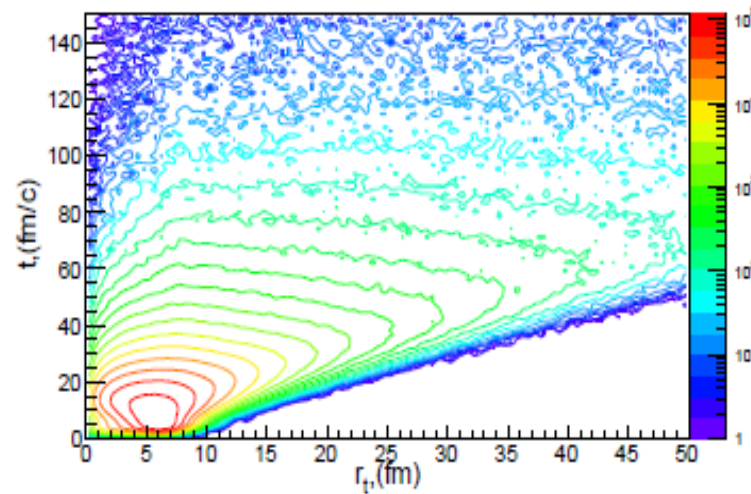


- There is no sharp freeze-out for different hadrons
- The order of freeze-out is as follows: mesons (kaons and pions), nucleons and lambdas

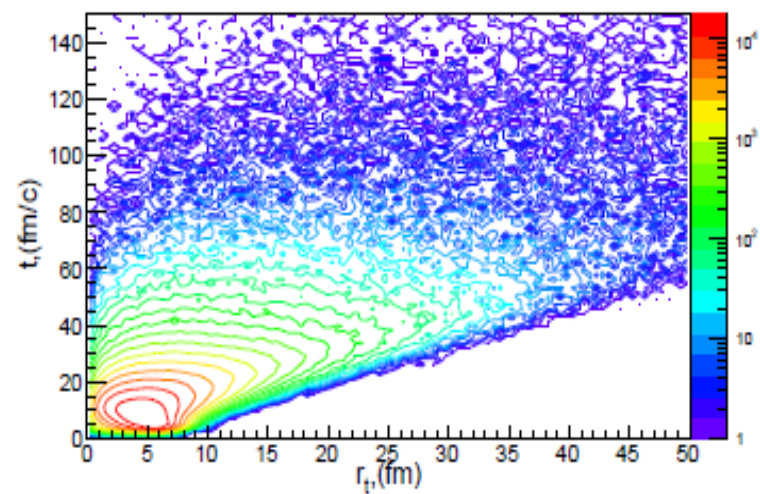
Freeze-out of hadrons at NICA energies

Au+Au @ 7.7 GeV ; $b = 0$ fm

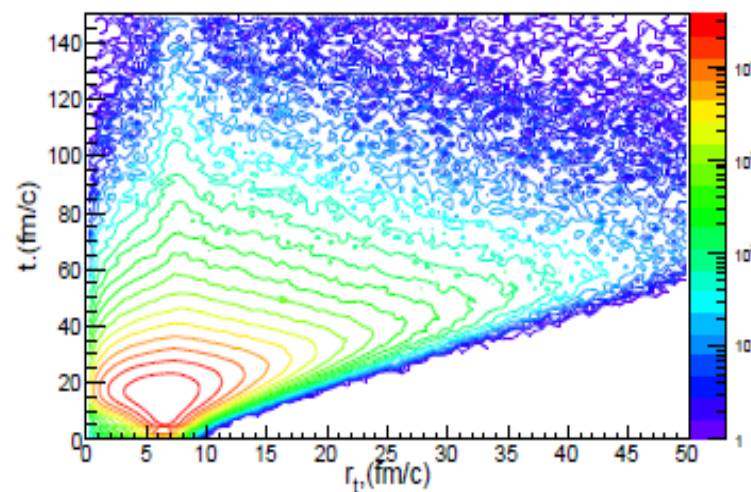
π^+ transverse radius spectra, URQMD Au+Au ($b=0$) $\sqrt{s}=7.7$ GeV



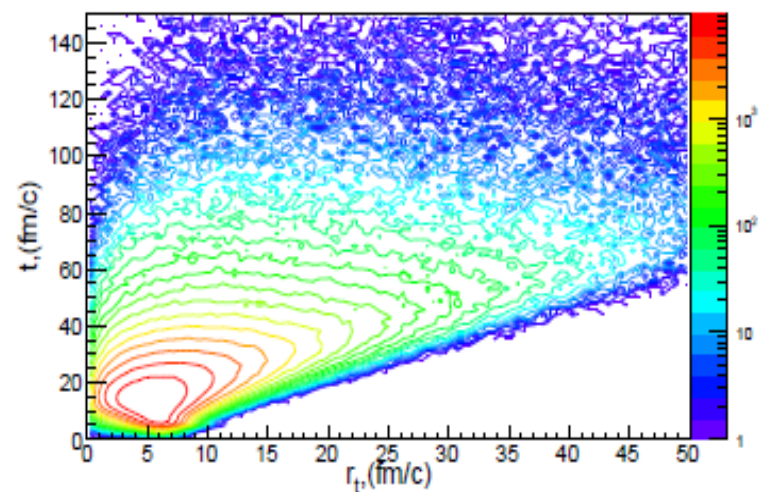
K^+ transverse radius spectra, URQMD Au+Au ($b=0$) $\sqrt{s}=7.7$ GeV



p transverse radius spectra, URQMD Au+Au ($b=0$) $\sqrt{s}=7.7$ GeV



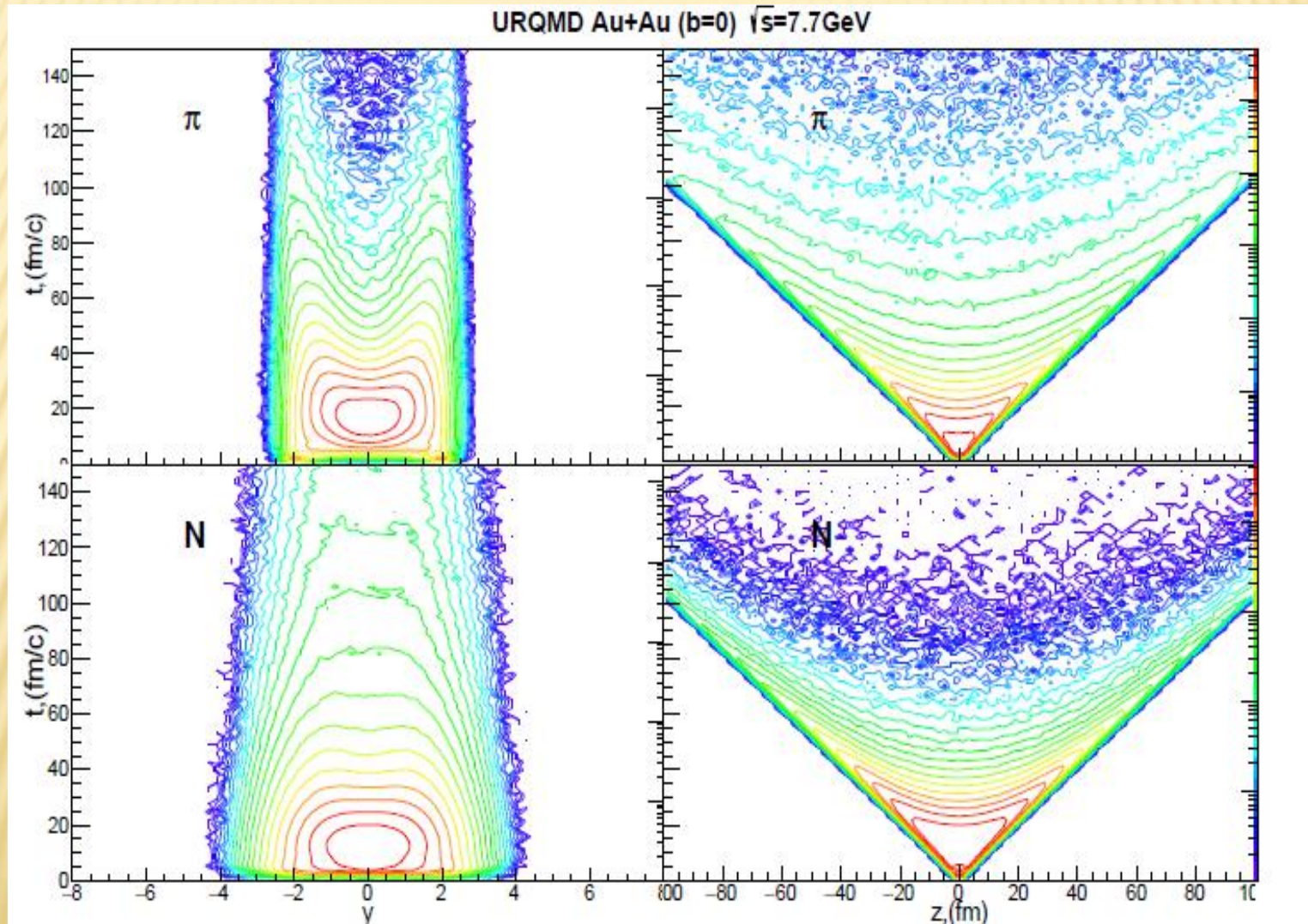
$\Lambda + \Sigma^0$ transverse radius spectra, URQMD Au+Au ($b=0$) $\sqrt{s}=7.7$ GeV



● Baryons are emitted longer and from larger areas than mesons

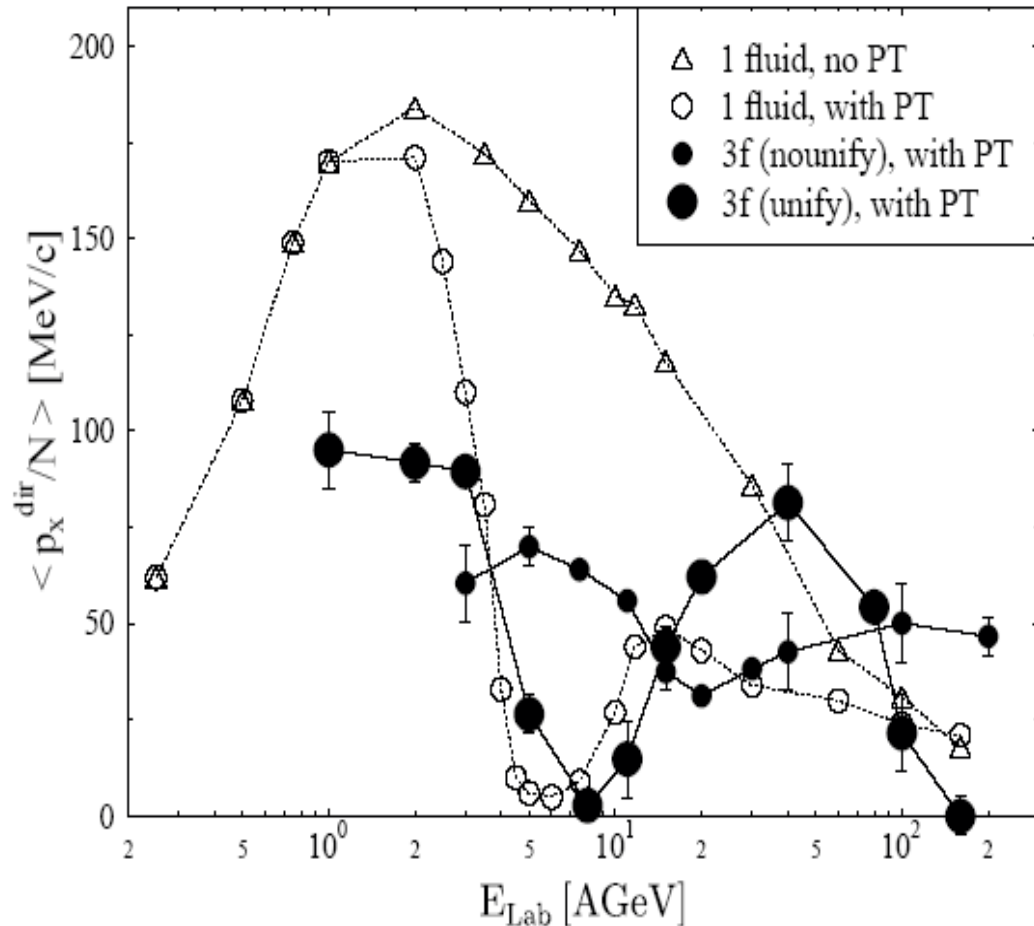
Freeze-out of hadrons at NICA energies

Au+Au @ 7.7 GeV ; $b = 0$ fm



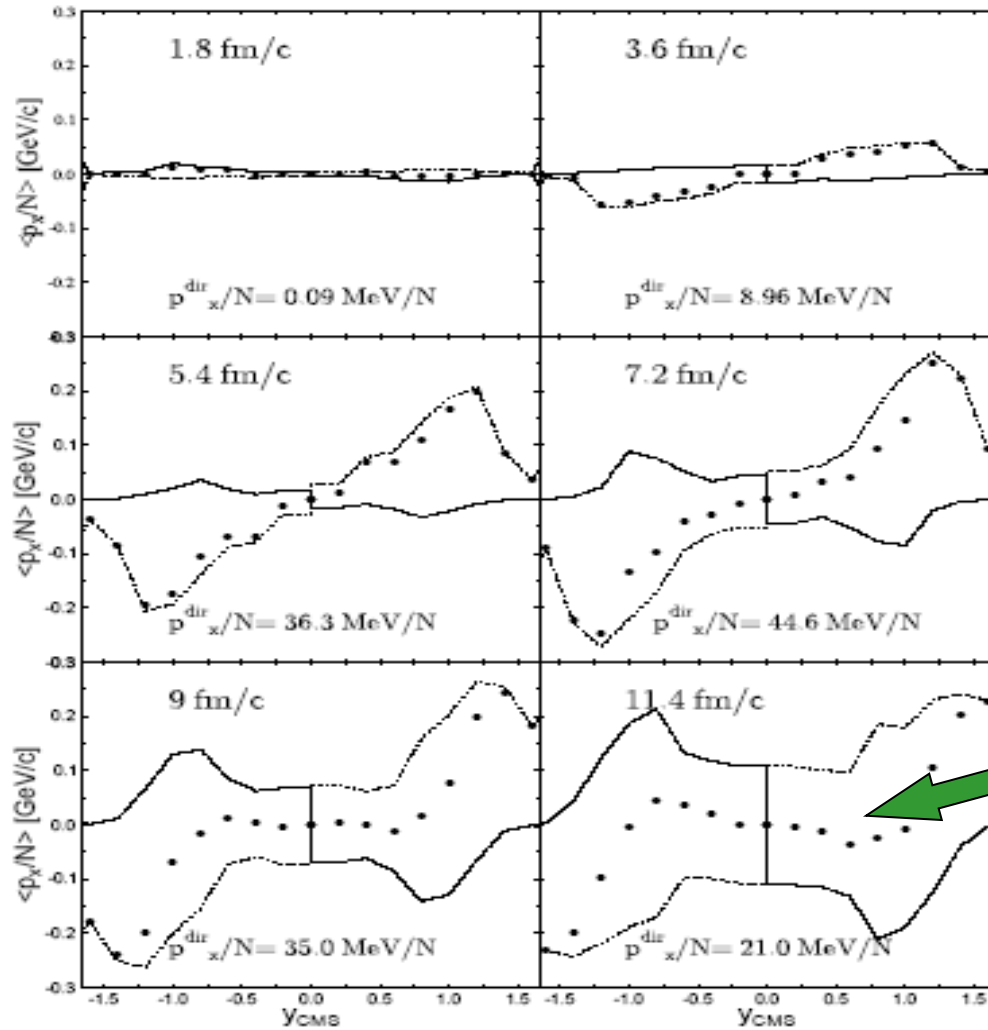
● Baryons are emitted longer and from larger areas than mesons

Directed flow of nucleons. 3-fluid hydro



The model predicts a local minimum in the excitation function of directed flow at energies between 10 and 20 AGeV (so far not been observed)

Directed flow of nucleons. 3-fluid hydro



The antiflow component is a source of the reduction of directed flow at midrapidity

NB! This is a very rare case when the **antiflow behavior** is reproduced in hydrodynamic model

Directed flow of hadrons in 3-fluid hydro *THESEUS*

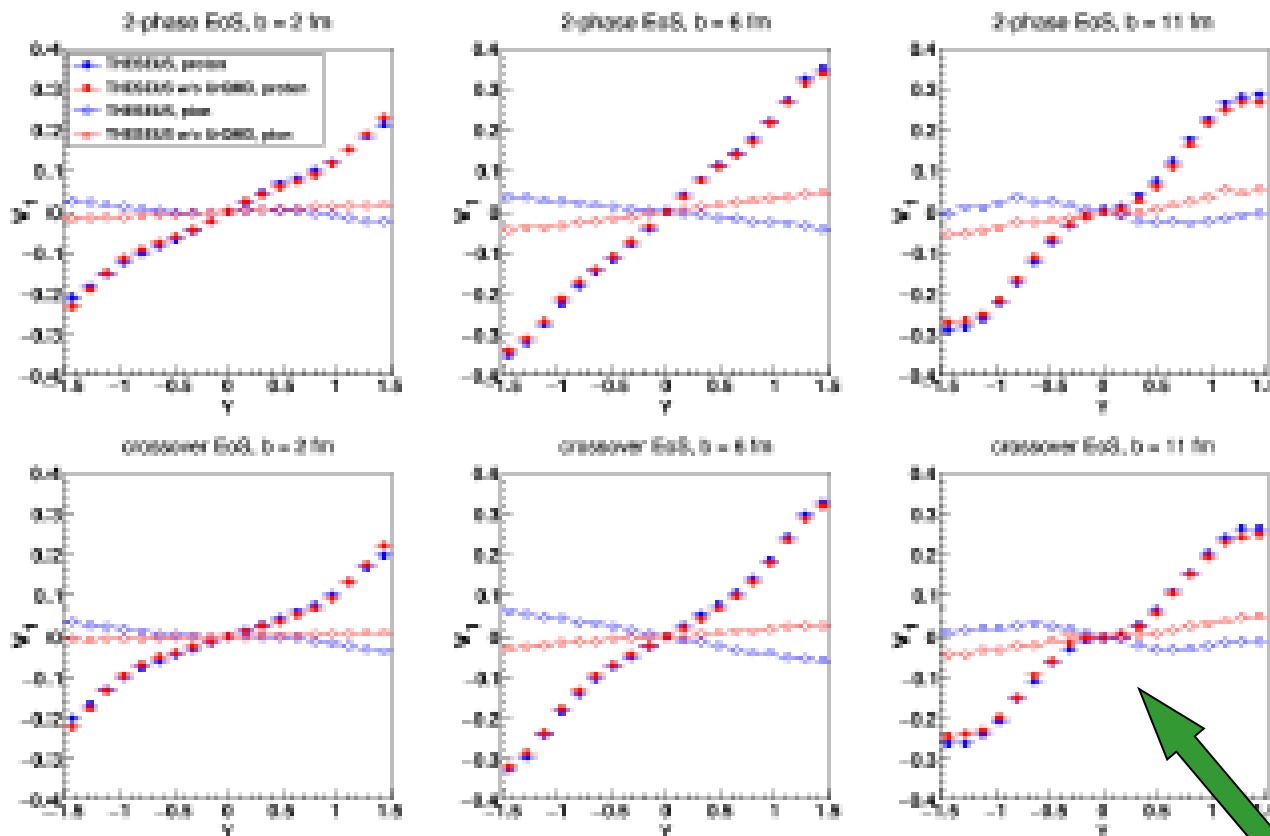


Figure 5. Two upper rows: Directed flow (v_1) of protons (full symbols) and pions (open symbols) for central ($b = 2$ fm), semicentral ($b = 6$ fm) and peripheral ($b = 11$ fm) Au+Au collisions at $E_{\text{lab}} = 8$ A GeV. The upper row is for the 2-phase EoS while the lower row shows results for the crossover EoS. In each panel we show the direct comparison of THESEUS with (blue symbols) and without (red symbols) UrQMD afterburner.

Remarkable is the effect of turning pion flow to antiflow due to hadronic re-scattering in the dense baryonic medium.

Directed flow of hadrons in 3-fluid hydro THESEUS

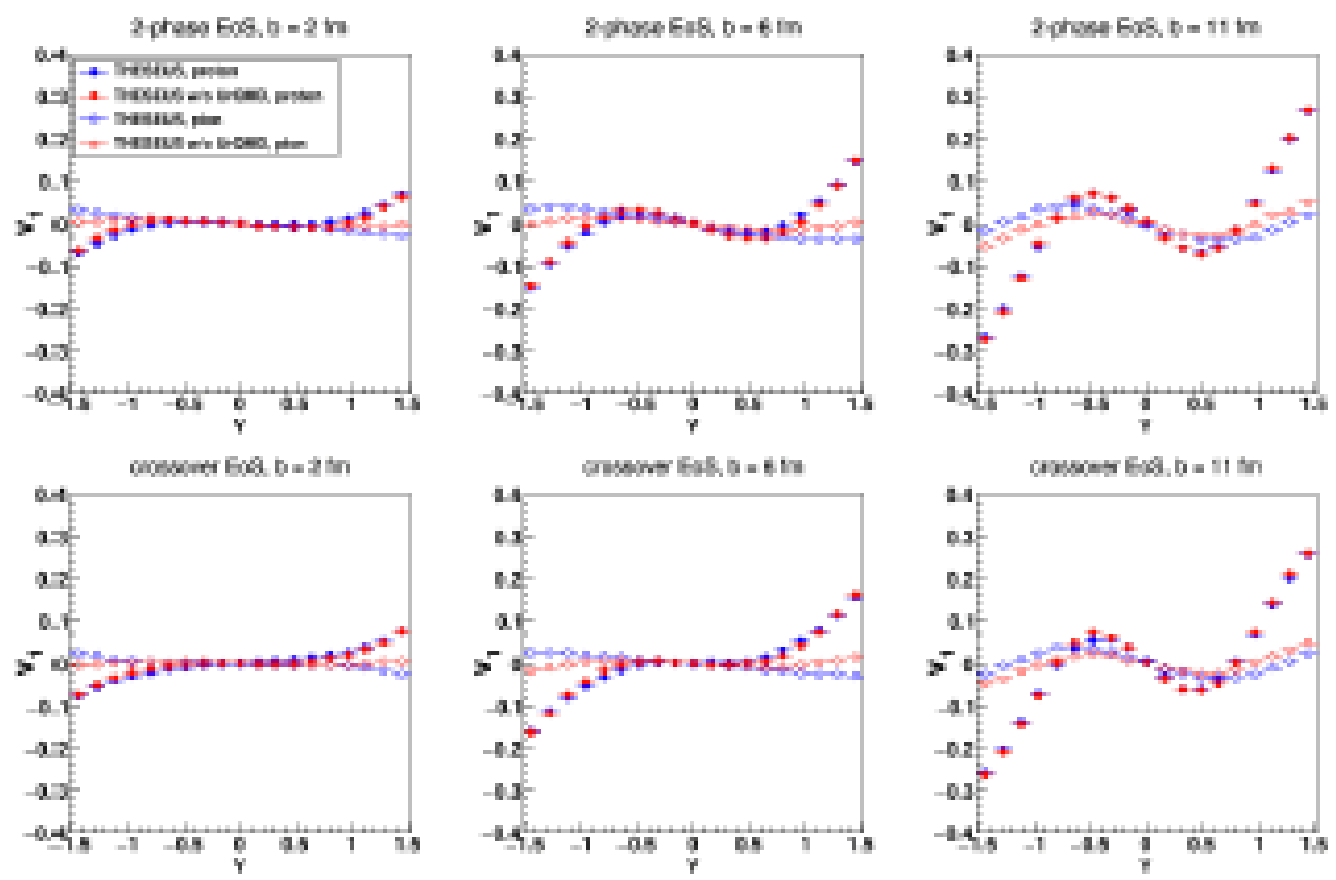
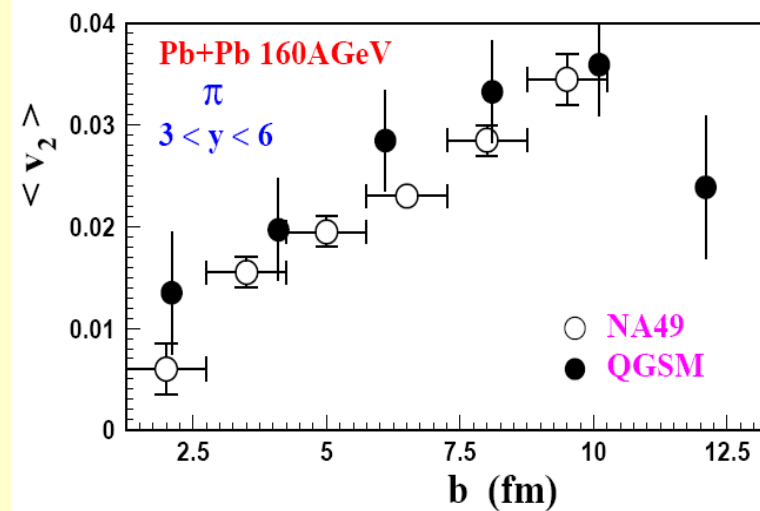
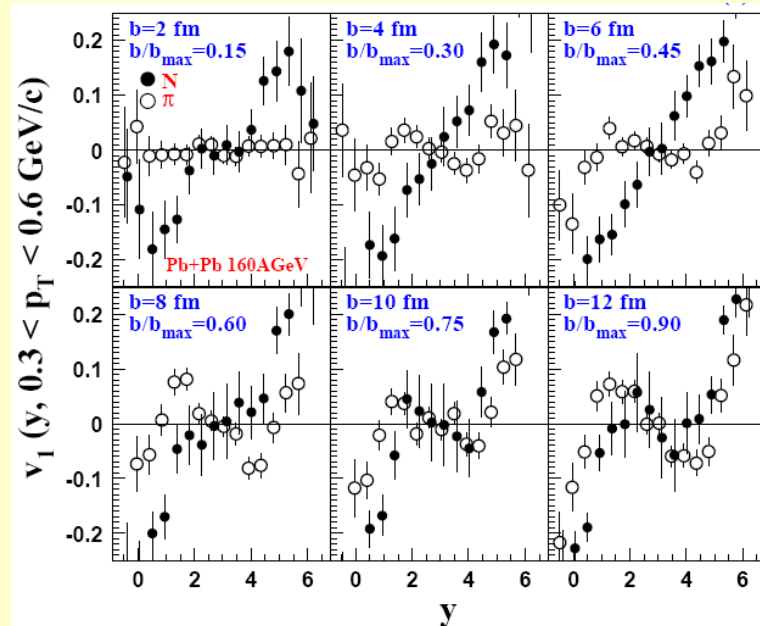
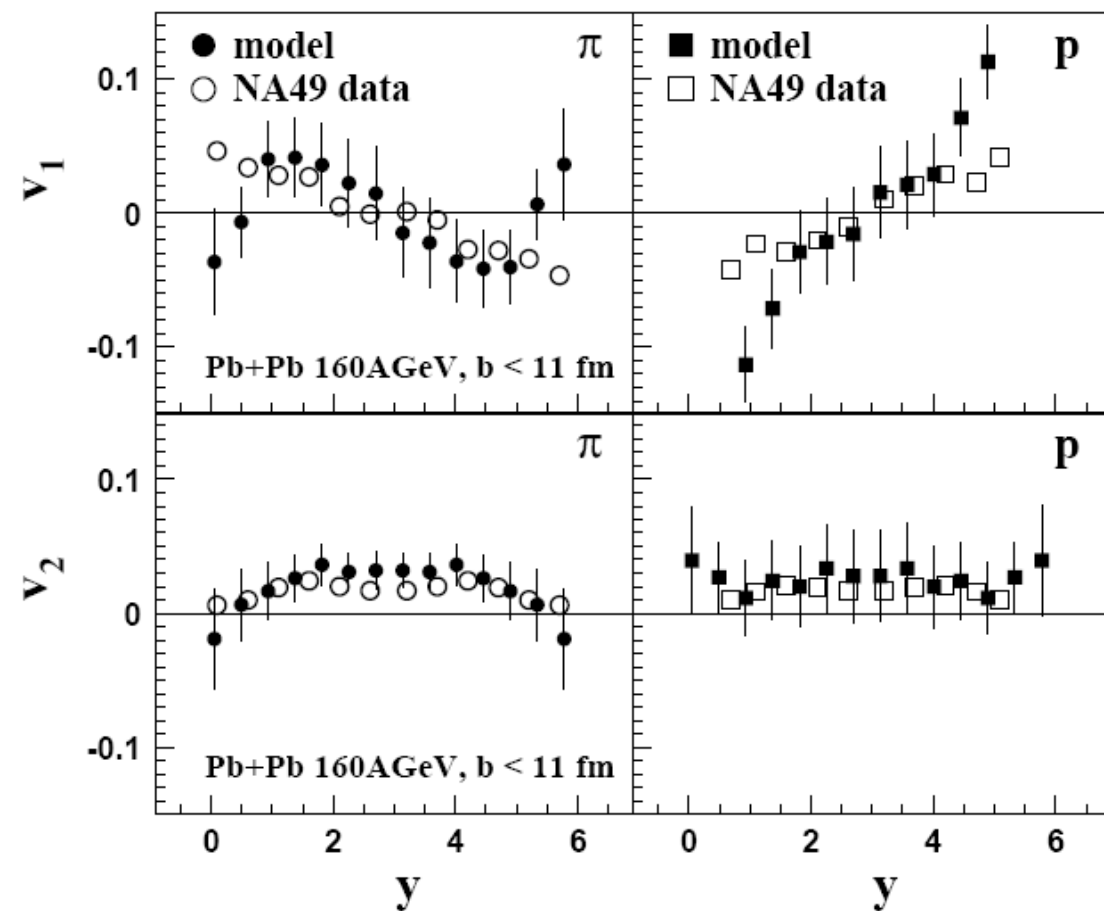


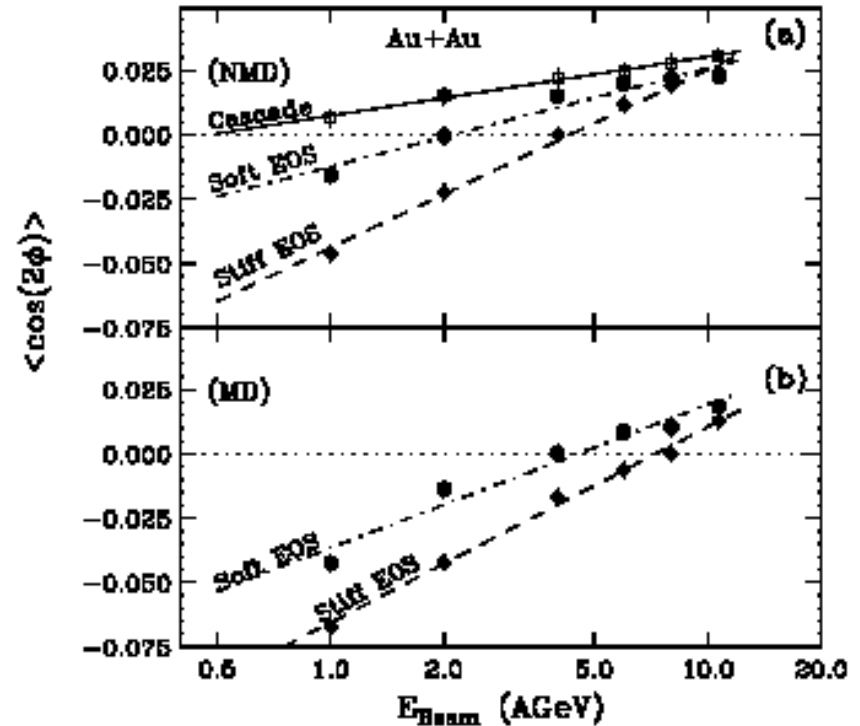
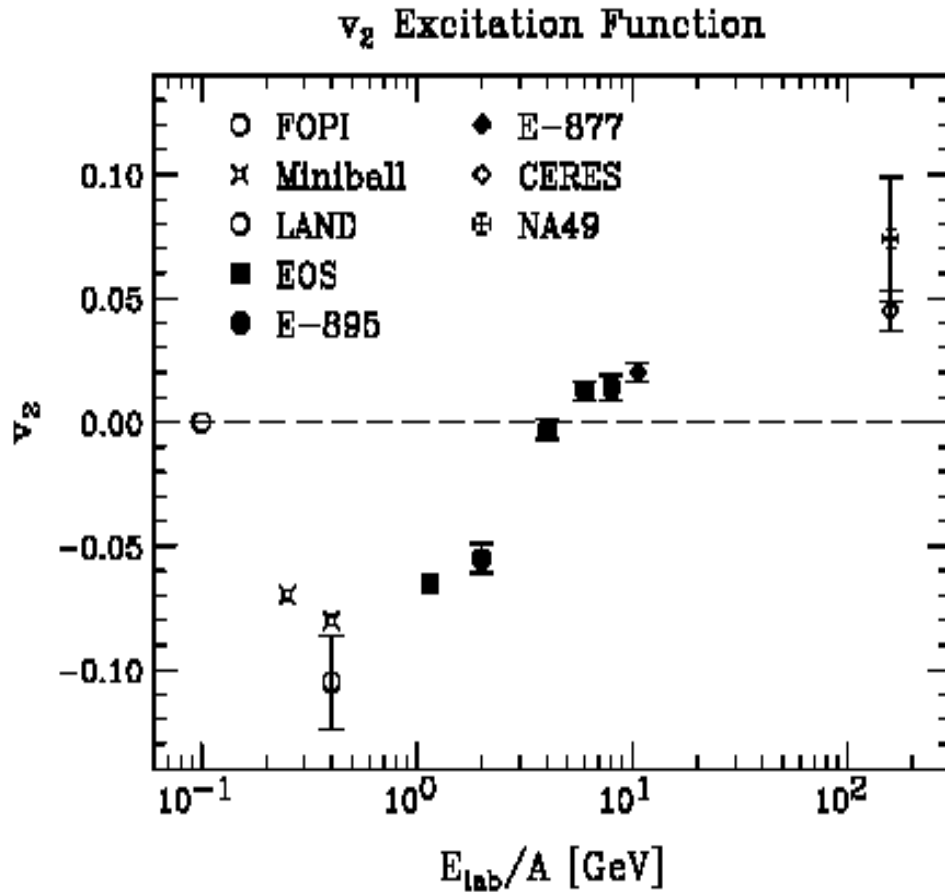
Figure 6. Same as in Fig. 5 but for $E_{lab} = 30$ A GeV.

Comparison with QGSM calculations



E. Zabrodin et al., PRC 63 (2003) 034902;
L. Bravina et al., PRC 61 (2000) 064802

Features of elliptic flow



Elliptic flow **changes** from **out-of-plane** to **in-plane** with rising bombarding energy. The harder the EOS, the more out-of-plane flow gets

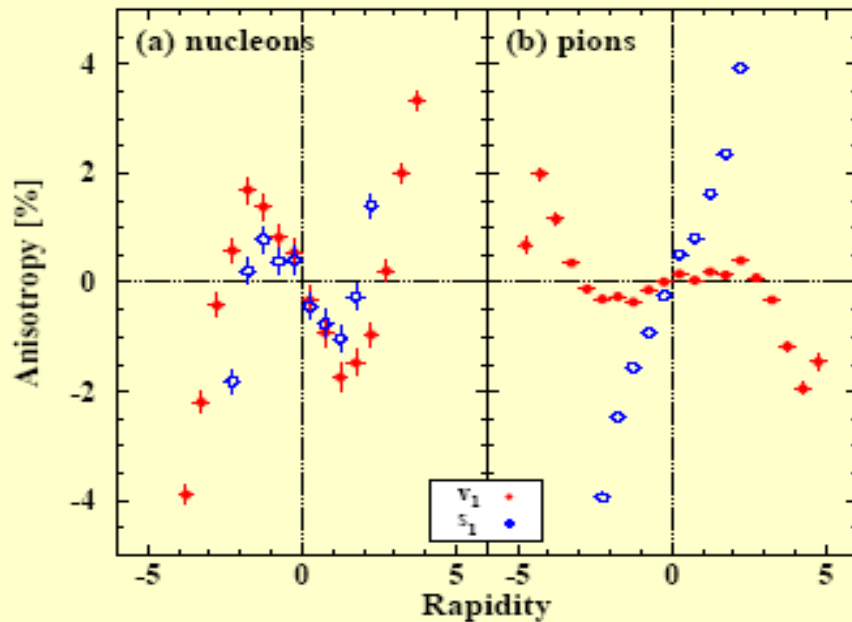
Transverse flow at RHIC

Directed flow in microscopic simulations

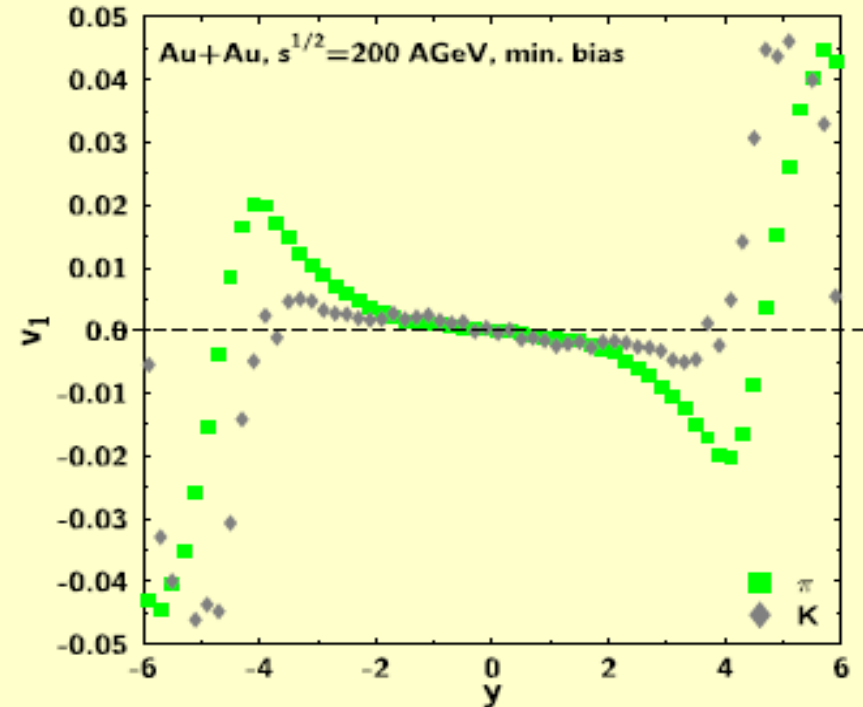
R.J.M. Snellings *et al.*, PRL 84 (2000) 2803

M. Bleicher and H. Stöcker, PLB 526 (2002) 309

Rapidity dependence of the v_1 of π , N



Rapidity dependence of the v_1 of π , K



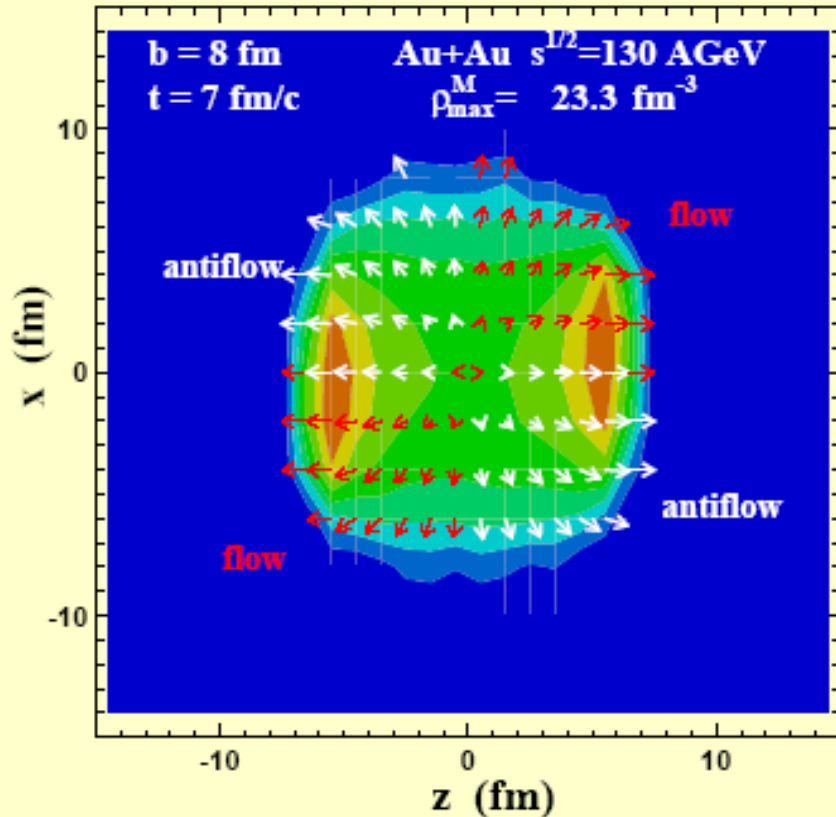
RQMD calculations for Au+Au at $\sqrt{s} = 200$ AGeV ($5 fm \leq b \leq 10 fm$)

UrQMD calculations for Au+Au at $\sqrt{s} = 200$ AGeV (minimum bias events)

Antiflow / vanishing of flow at midrapidity

Directed flow in microscopic simulations

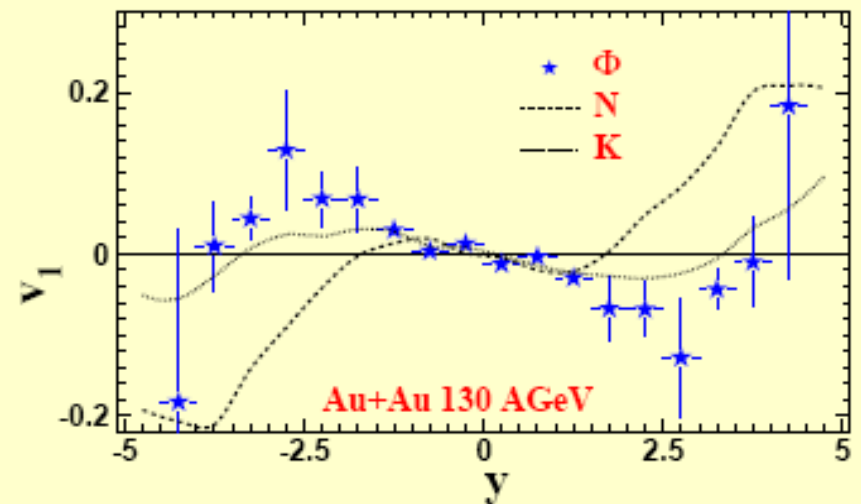
Resulting flow = normal flow - antiflow



Although the normal flow component is always slightly larger than the antiflow one, in central rapidity window the anti-flow can overshadow its normal counterpart

L. Bravina *et al.*, NPA 715 (2003) 665c

Directed flow $v_1(y, \text{all } p_t)$ of ϕ, N, K in minimum bias Au+Au events at $\sqrt{s} = 130$ AGeV



Directed flow of ϕ mesons $v_1(y)$ has negative slope (antiflow) at $|y| \leq 2$. This distribution is similar to those of other hadrons at $|y| \leq 2$ in Au + Au at $\sqrt{s} = 130$ AGeV because of similarities of their production and dynamics

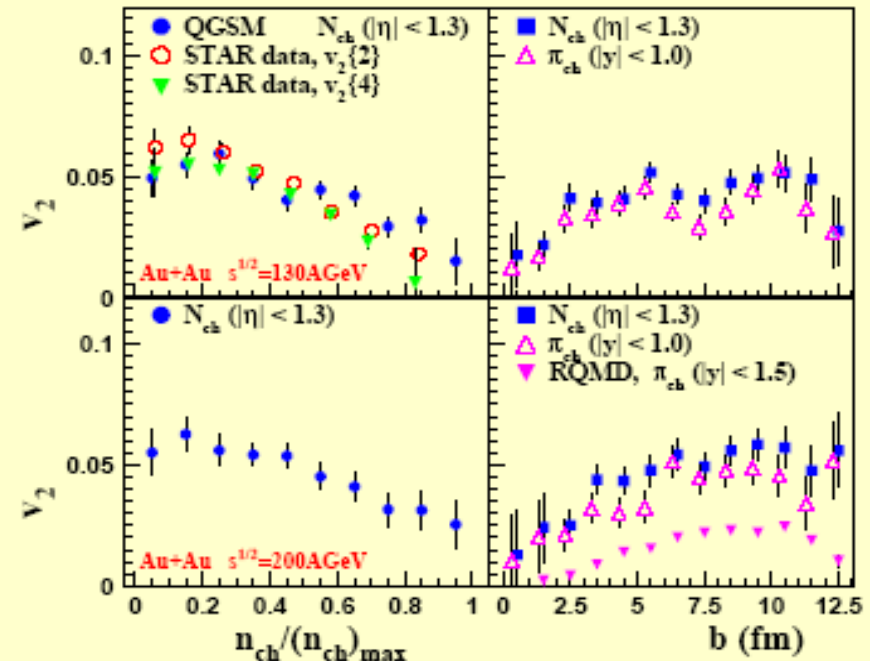
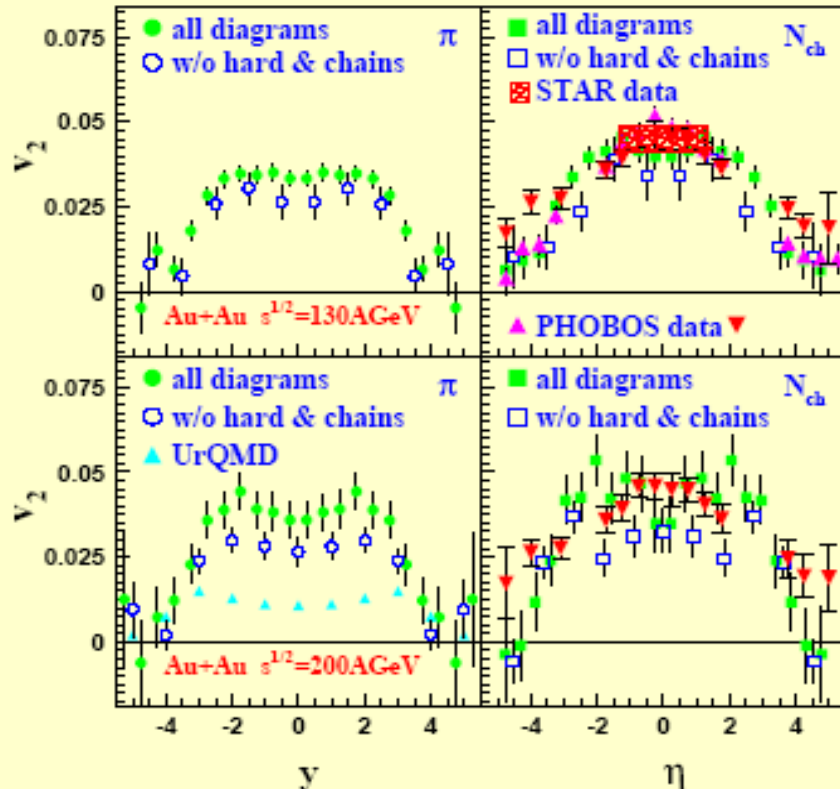
Elliptic flow in microscopic simulations

E. Z., L. Bravina, A. Faessler, C. Fuchs, PLB 508 (2001) 184

PPNP 53 (2004) 183

M. Bleicher and H. Stöcker, PLB 526 (2002) 309

S. Manly *et al.* (PHOBOS Collab.), NPA 715 (2003) 614c



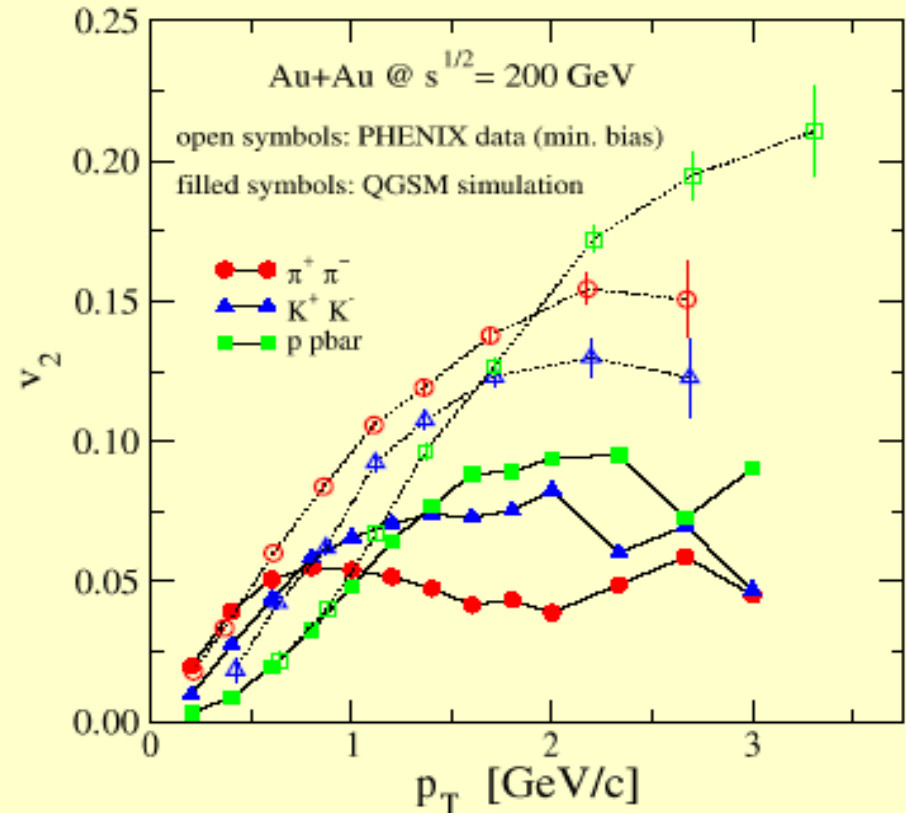
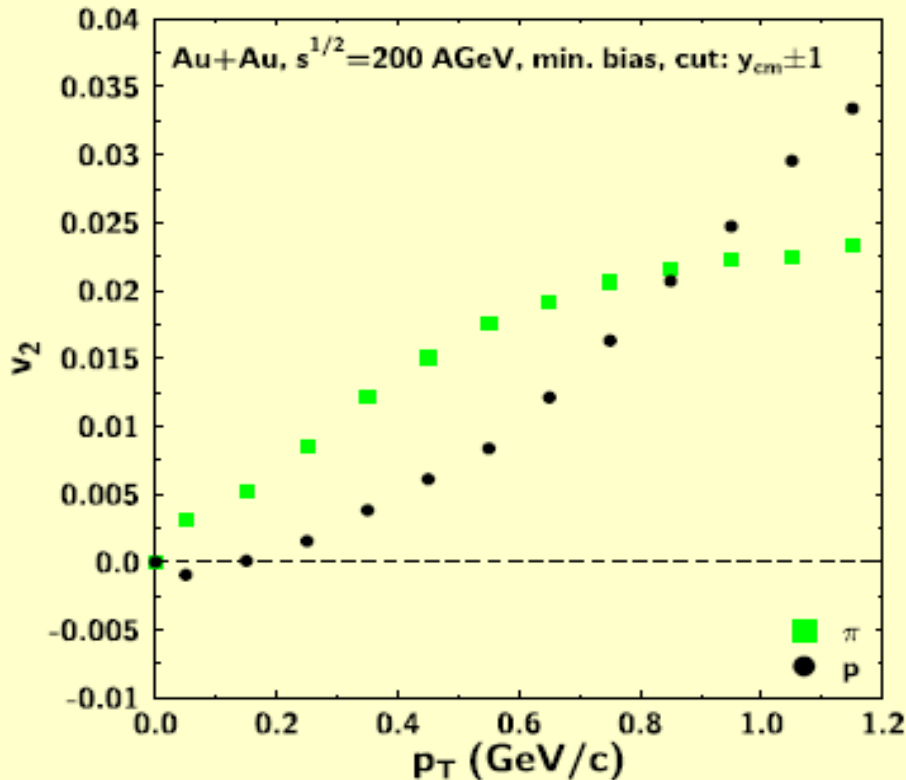
(Pseudo)rapidity dependencies of the elliptic flow of charged particles in the whole η range at both energies were obtained *before* the experimental data became available

Elliptic flow in microscopic simulations

Transverse momentum dependence of v_2

M. Bleicher and H. Stöcker, PLB 526 (2002) 309

G. Burau et al., nucl-th/0411117

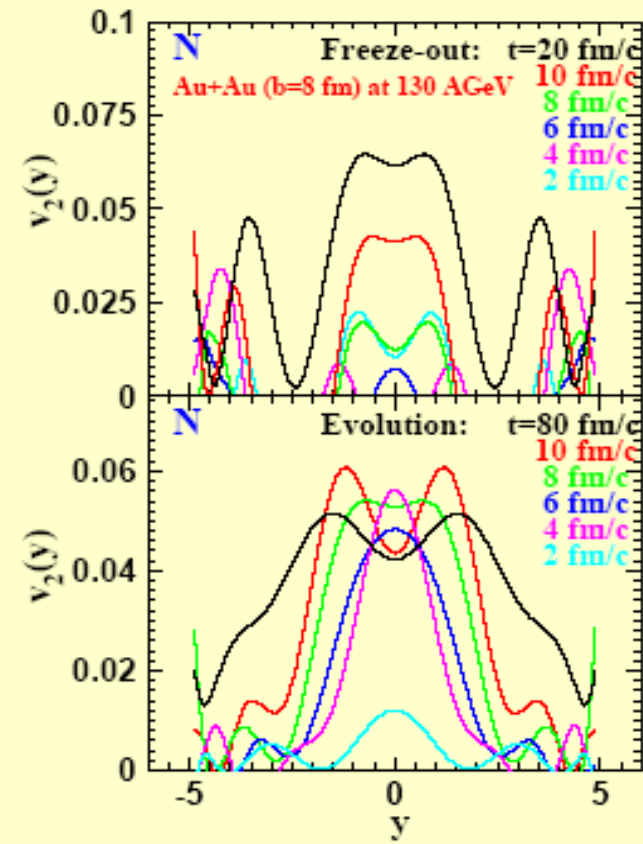
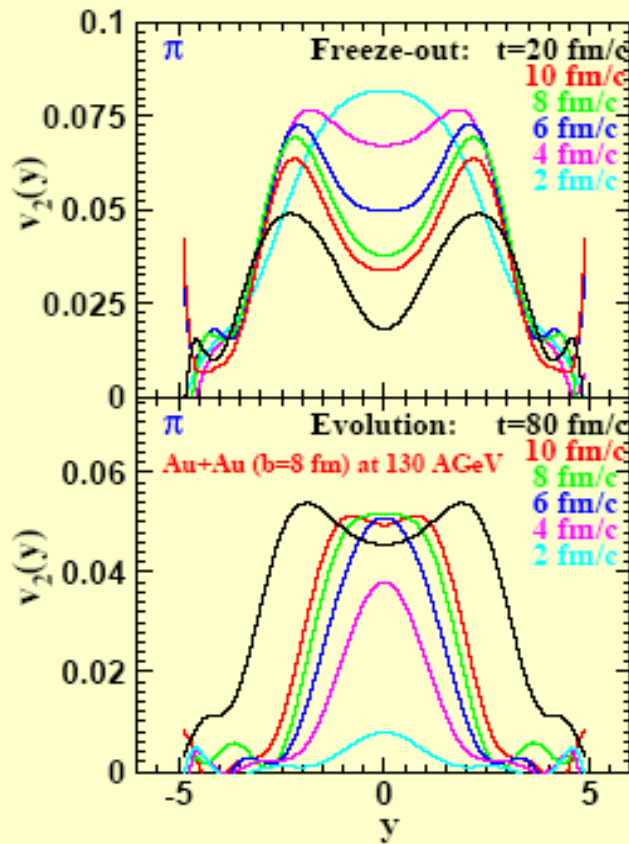


Both **UrQMD** and **QGSM** show crossing of the elliptic flow for mesons and baryons. This agrees with the experimental data

Time evolution of elliptic flow

PIONS (Au+Au, 130 AGeV, b=8 fm)

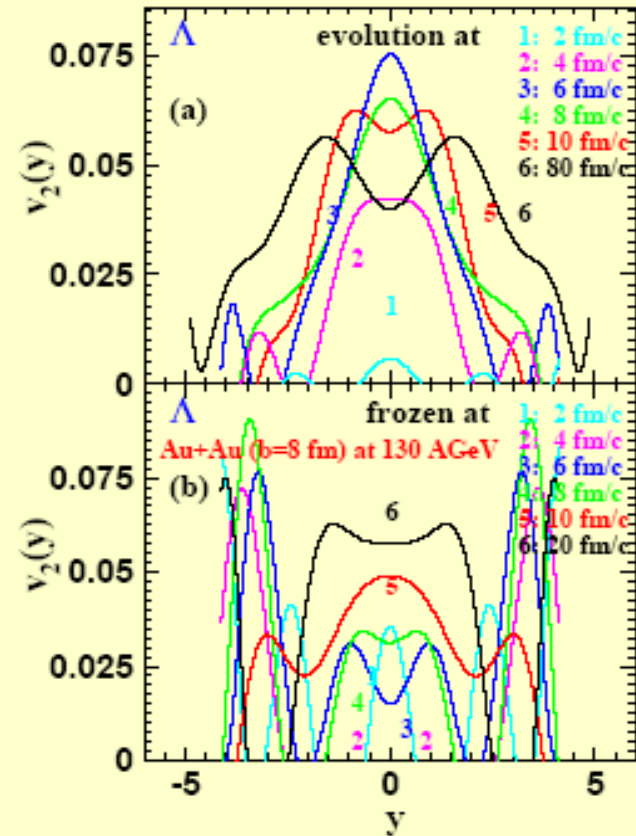
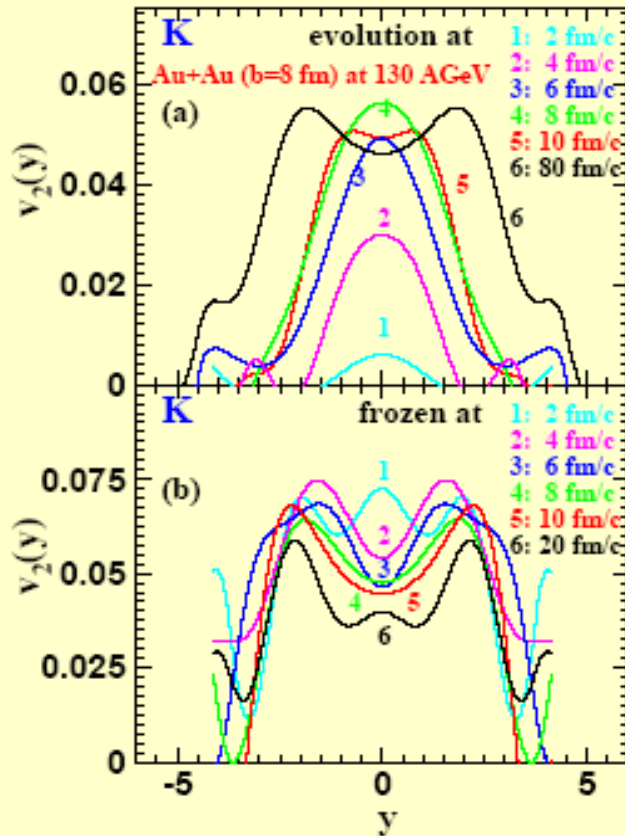
NUCLEONS



- (1) The **earlier** the freeze-out of **pions**, the stronger their elliptic flow
- (2) The **later** the freeze-out of **nucleons**, the stronger their elliptic flow
- (3) The flow formation is not over e.g. at $t = 6 \text{ fm}/c$ due to continuous freeze-out of particles

Time evolution of elliptic flow

KAONS (Au+Au, 130 AGeV, b=8 fm) LAMBIDAS

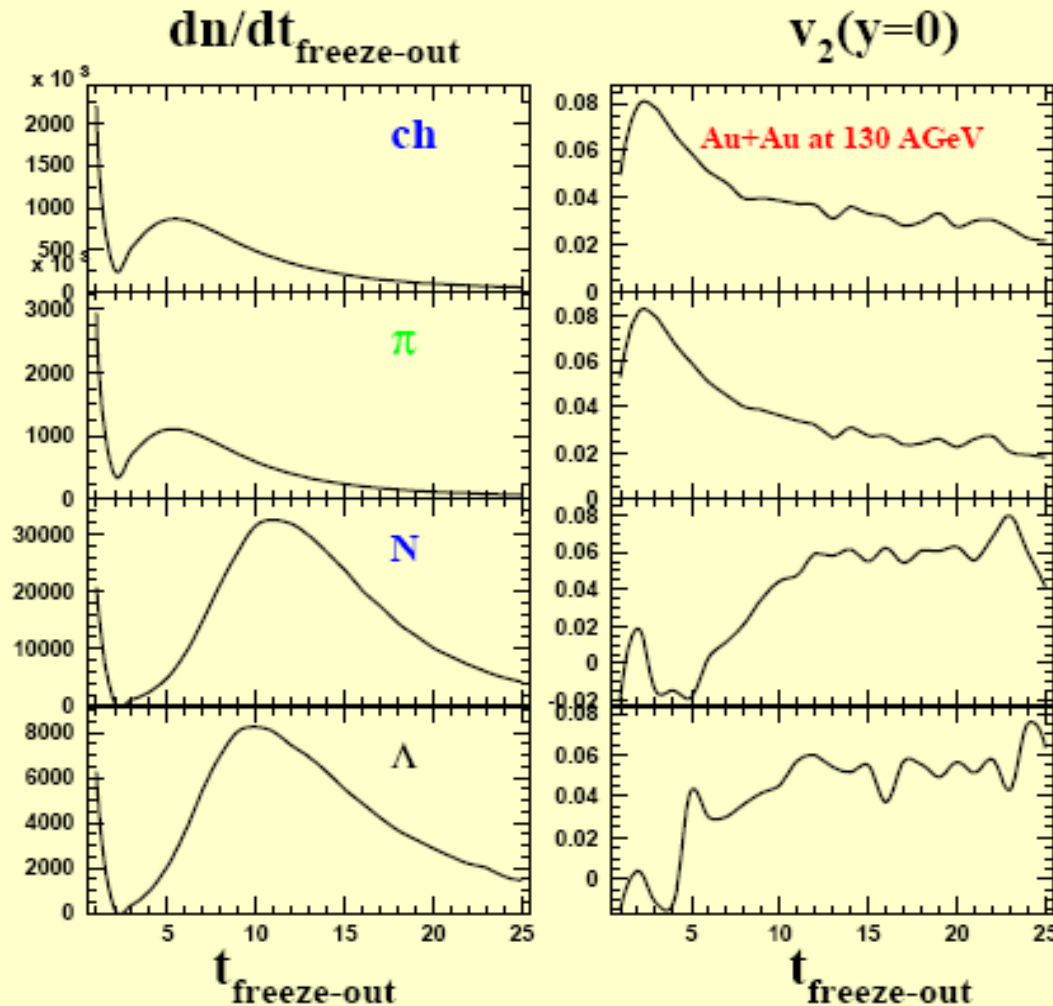


- (1) The **earlier** the freeze-out of **kaons**, the stronger their elliptic flow
- (2) The **later** the freeze-out of **lambdas**, the stronger their elliptic flow

This is the main difference in the formation of the elliptic flow of mesons and baryons

Elliptic flow and freeze-out

Au+Au (b=8 fm) at $\sqrt{s} = 130$ AGeV



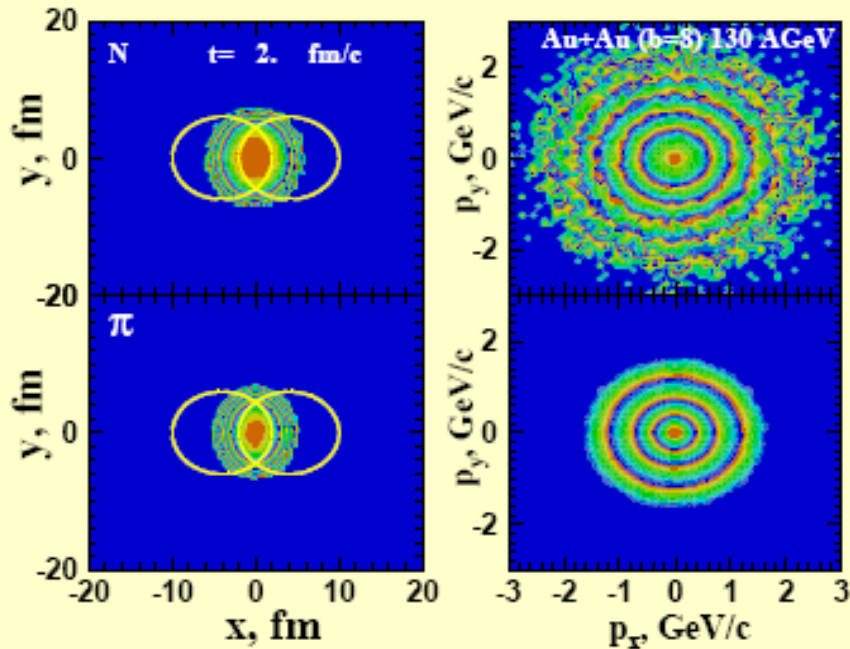
(1) Substantial part of hadrons leaves the system immediately after their production within the first two fm/c.

(2) Baryons and mesons are completely different: pions emitted within the first few fm/c carry the strongest flow. In contrast to pions, the baryon fraction acquires stronger elliptic flow during the subsequent rescatterings, developing the hydro-like flow.

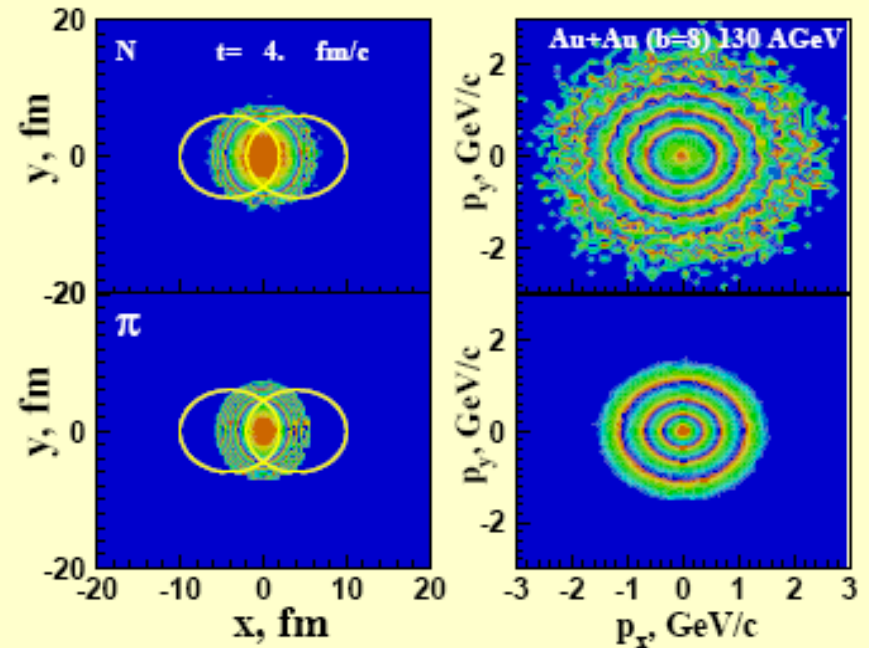
Elliptic flow and freeze-out

Anisotropy in coordinate space and elliptic flow of nucleons and pions in **Au+Au** collisions at $\sqrt{s} = 130$ **AGeV** with the impact parameter $b = 8$ fm.

$t = 2$ fm/c



$t = 4$ fm/c

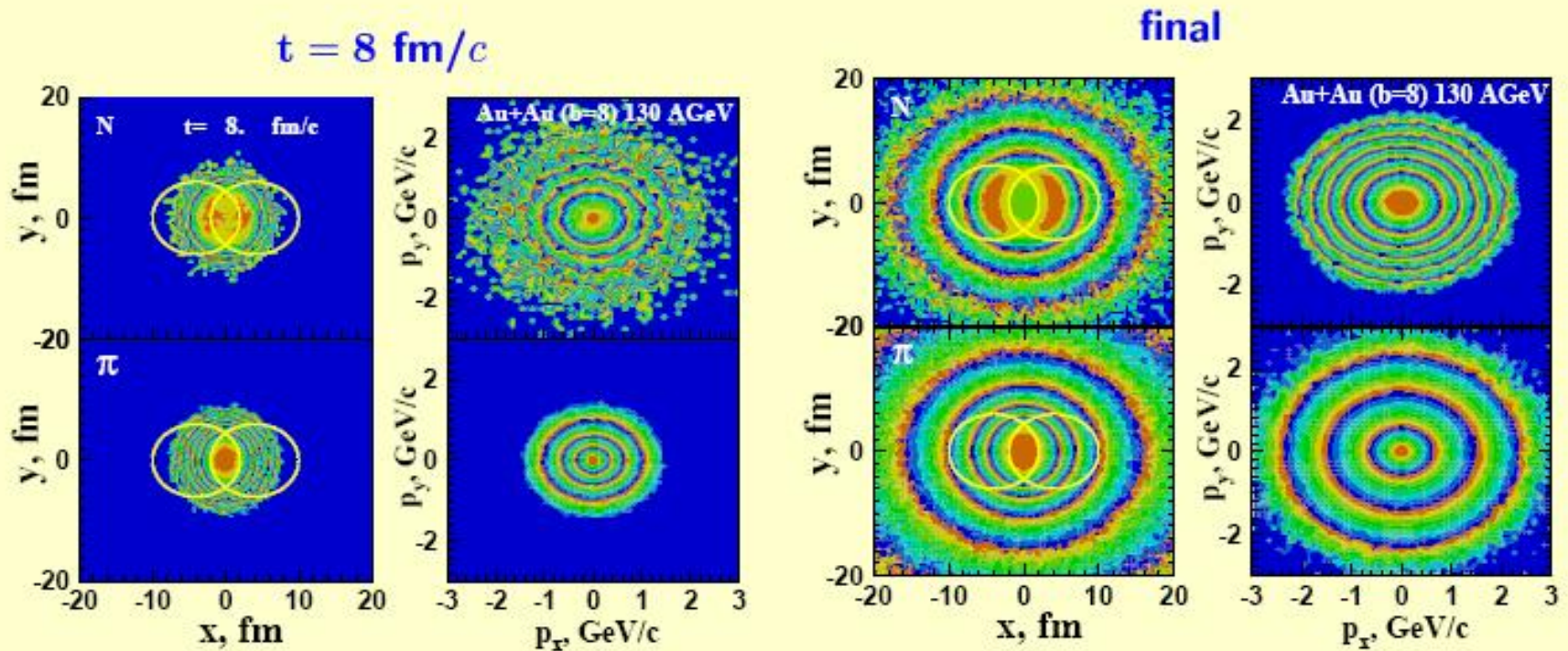


Strong anisotropy in coordinate space,
but weak anisotropy in the momentum
space

Anisotropy starts to develop in the
momentum space for low momenta

Elliptic flow and freeze-out

Anisotropy in coordinate space and elliptic flow of nucleons and pions in **Au+Au** collisions at $\sqrt{s} = 130$ **AGeV** with the impact parameter $b = 8$ fm.

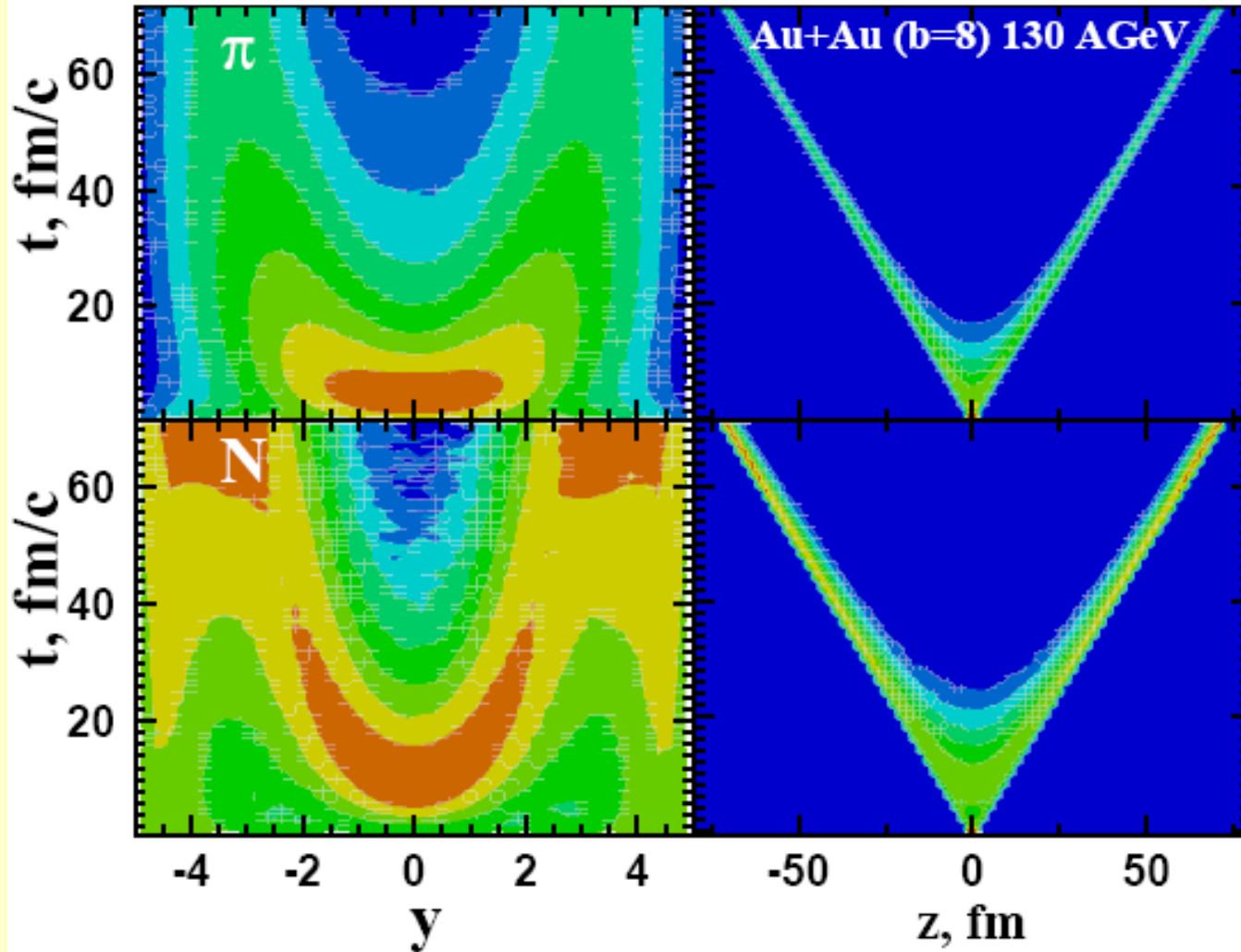


Nucleons leave the overlapping region; at the end of the reaction we see the pronounced maxima at centers of nuclei. Most of the **pions** is staying within the overlapping area till the end of reaction

The anisotropy in coordinate space (almond-shaped region) is transformed into the anisotropy in the momentum space.

Elliptic flow and freeze-out

L. Bravina et al., PLB 631 (2005) 109



Pions and nucleons are coming from **different** areas

CONCLUSIONS

◆ **Directed flow** = **Normal Flow** – **Antiflow**

Normal Flow \geq **Antiflow** (except of the midrapidity range)

◆ The softening of the flow can be misinterpreted as the softening of EOS due to formation of the QGP, but:

QGP \rightarrow the effect is stronger for semi-central collisions

Cascade \rightarrow the effect is stronger for semi-peripheral and peripheral ones

◆ At energies about few GeV: normal directed flow of protons at midrapidity in central collisions and antiflow in peripheral ones. Mesons – antiflow for all centralities.

◆ At RHIC/LHC: the directed flow of both mesons and baryons is almost zero or antiflow at midrapidity for all centralities

◆ The directed flow of high- P_T is elongated in normal direction

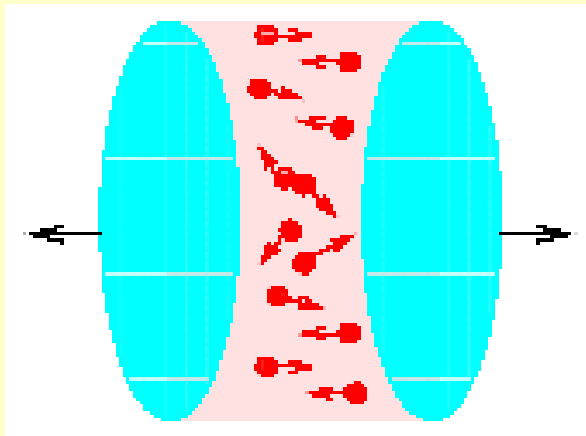
◆ Development of both directed and elliptic flow of hadrons at midrapidity in transport models takes about 6-8 fm/c (or even longer)

◆ Collective phenomena, such as anisotropic flow, should be studied together with the freeze-out conditions

Back-up Slides

Quark-Gluon String Model

Nuclear Collisions

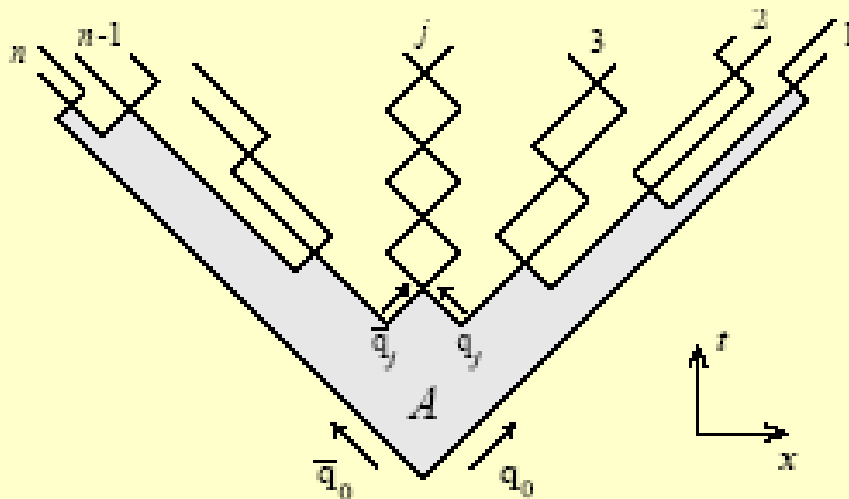


- ✦ A nucleus-nucleus collision is treated as incoherent superposition of elementary collisions at the parton, string, and hadron levels
- ✦ The newly produced particles can interact after a certain formation time τ_0 . It comes from the uncertainty principle

$$\tau_0 \geq \frac{\hbar}{m_T}$$

However, for composite particles (hadrons) this is an open and model dependent issue.

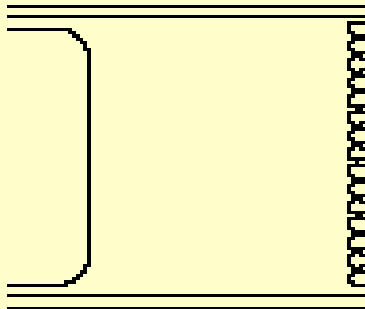
- ✦ For heavy-ion collisions QGSM includes the secondary interactions of the produced hadrons with primary target or projectile nucleons and with secondary hadrons



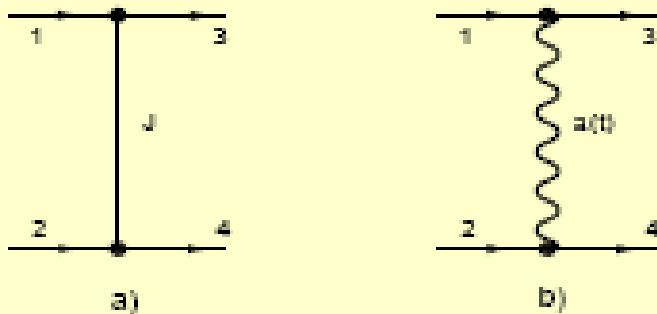
1/N-topological expansion

Planar digram

In hh interactions a one-string mechanism (with quark-antiquark annihilation) is possible in $p\bar{p}$ collisions but not in pp

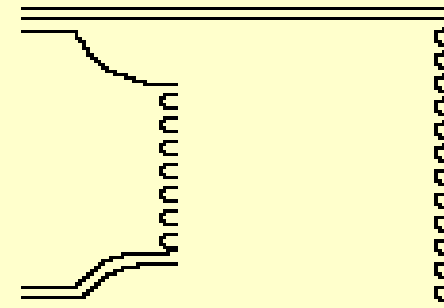


Such a diagram corresponds to the **Reggeon exchange** in the GRT (weight $\propto 1/N$; contribution $\propto s^{-1/2}$)

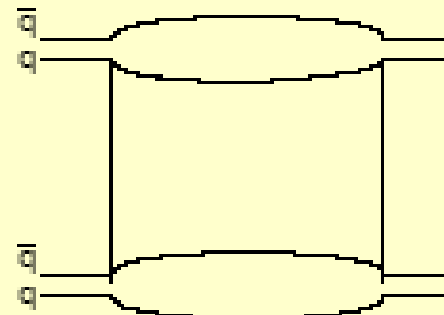


Cylinder digram

The simplest topology which contribution does not vanish at $s \rightarrow \infty$ is a two-string diagram



Such a diagram corresponds to the **Pomeron exchange** in the GRT (weight $\propto 1/N^2$). Its square has the topology of a cylinder



Inelastic cross section

The inelastic hh cross section $\sigma_{\text{in}}(s)$ can be calculated via the real part of the eikonal $u(s, b)$

$$\sigma_{\text{in}}(s) = 2\pi \int_0^{\infty} \{1 - \exp[-2u^{\text{R}}(s, b)]\} b db$$

The eikonal can be presented as a sum of three terms corresponding to soft and hard Pomeron exchange, and triple Pomeron exchange, which is responsible for the single diffraction process,

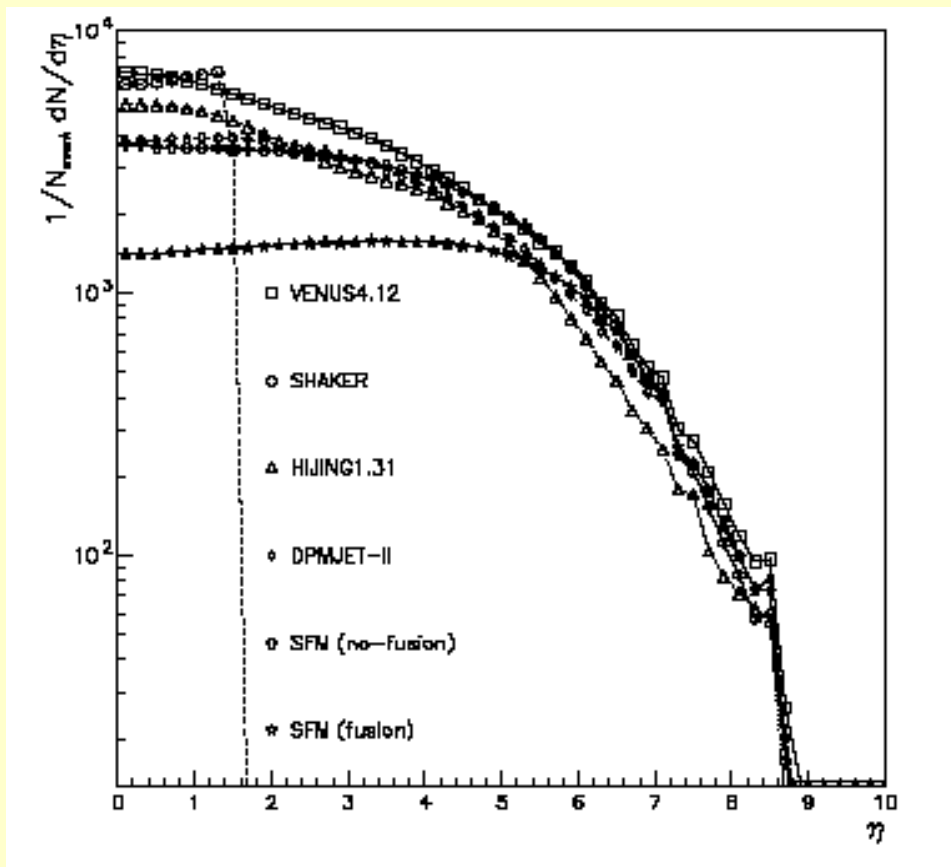
$$u^{\text{R}}(s, b) = u_{\text{soft}}^{\text{R}}(s, b) + u_{\text{hard}}^{\text{R}}(s, b) + u_{\text{triple}}^{\text{R}}(s, b)$$

Using the Abramovskii-Gribov-Kancheli (AGK) cutting rules we get

$$\begin{aligned} \sigma_{\text{in}}(s) &= \sum_{i,j,k=0; i+j+k \geq 1} \sigma_{ijk}(s), \\ \sigma_{ijk}(s) &= 2\pi \int_0^{\infty} b db \exp[-2u^{\text{R}}(s, b)] \\ &\quad \times \frac{[2u_{\text{soft}}^{\text{R}}(s, b)]^i}{i!} \frac{[2u_{\text{hard}}^{\text{R}}(s, b)]^j}{j!} \frac{[2u_{\text{triple}}^{\text{R}}(s, b)]^k}{k!}. \end{aligned}$$

The last equation enables one to determine the number of strings and hard jets.

Predictions for Pb+Pb at 5500 GeV

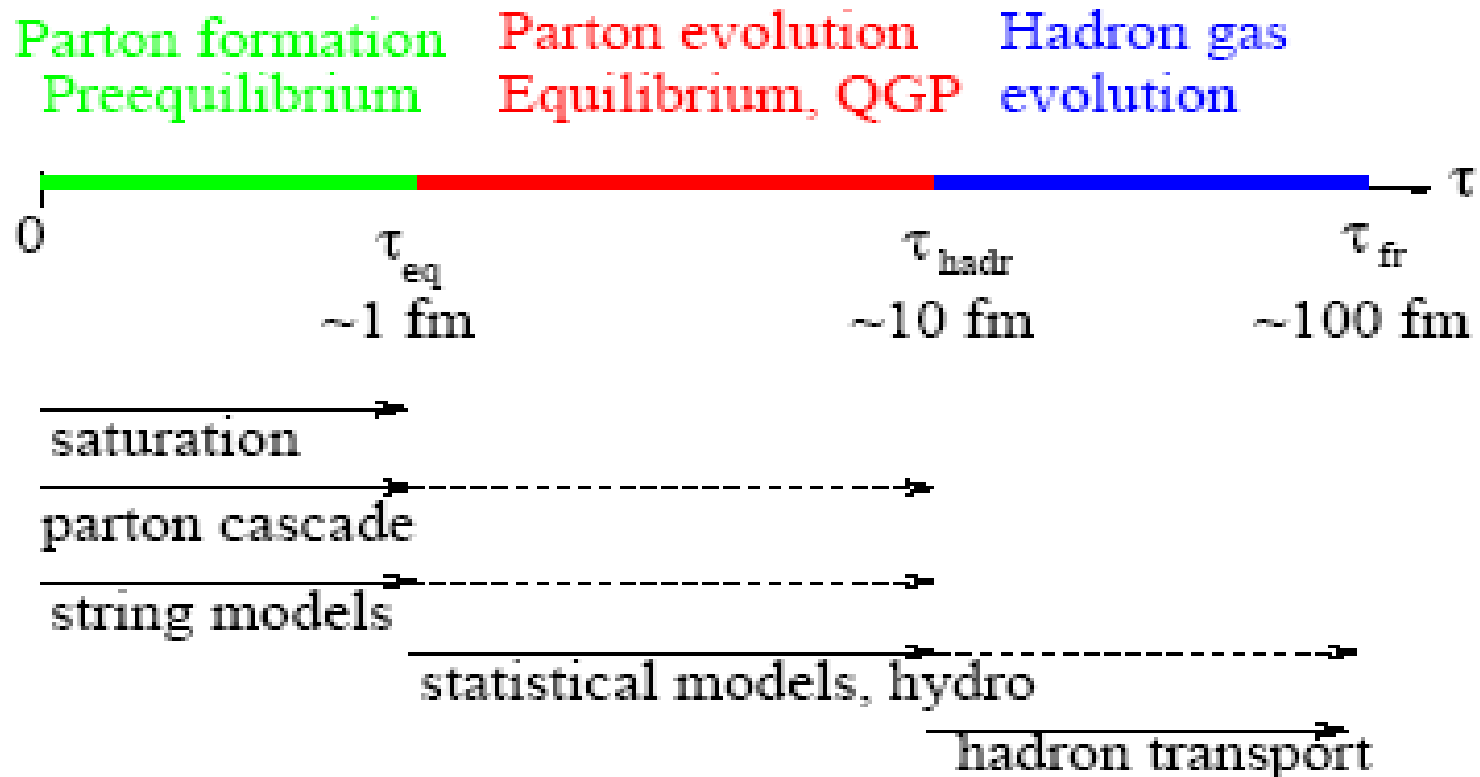


Different Monte Carlo model predictions:

*Alice technical proposal
CERN/LHCC 95-71*

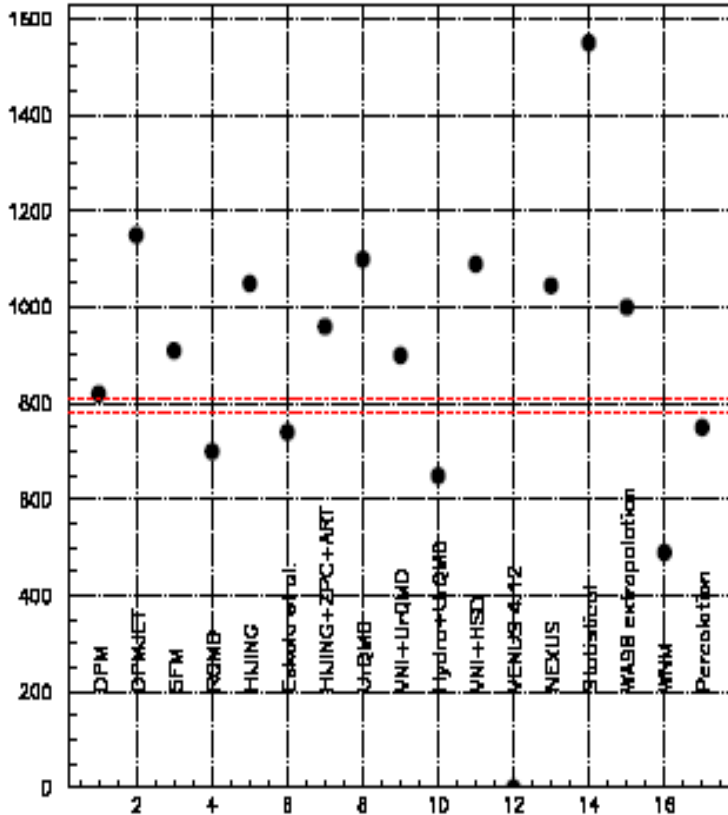
Venus
Shaker
HIJING
DPMJET
SFM w/o SF
SFM with SF

Applicability of different models

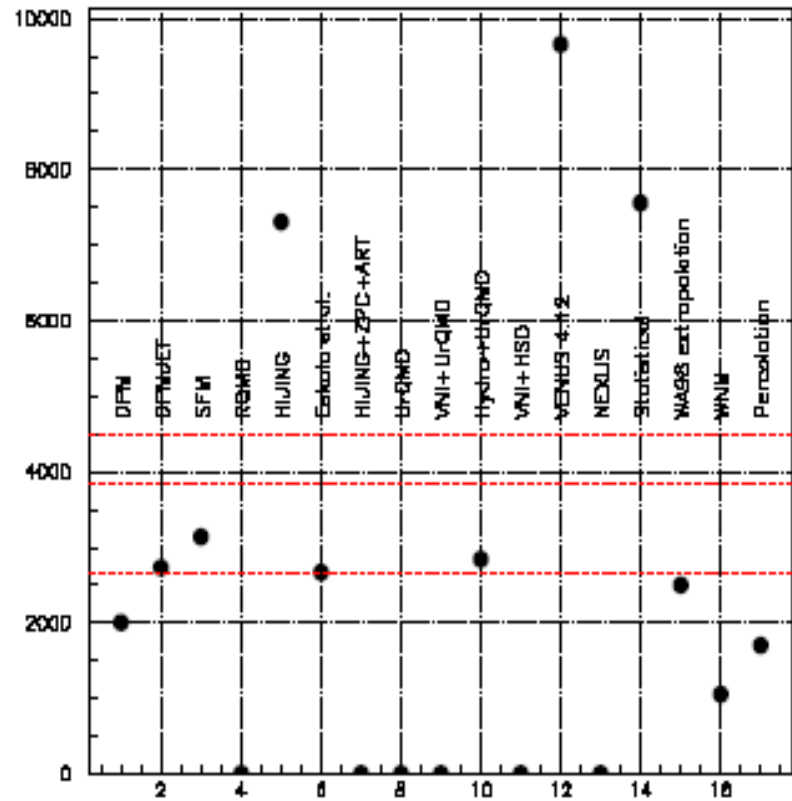


Predictions for A+A at 200 and 5500 GEV

dN/dy at $y=0$ for charged particles at RHIC, 1/20 centrality

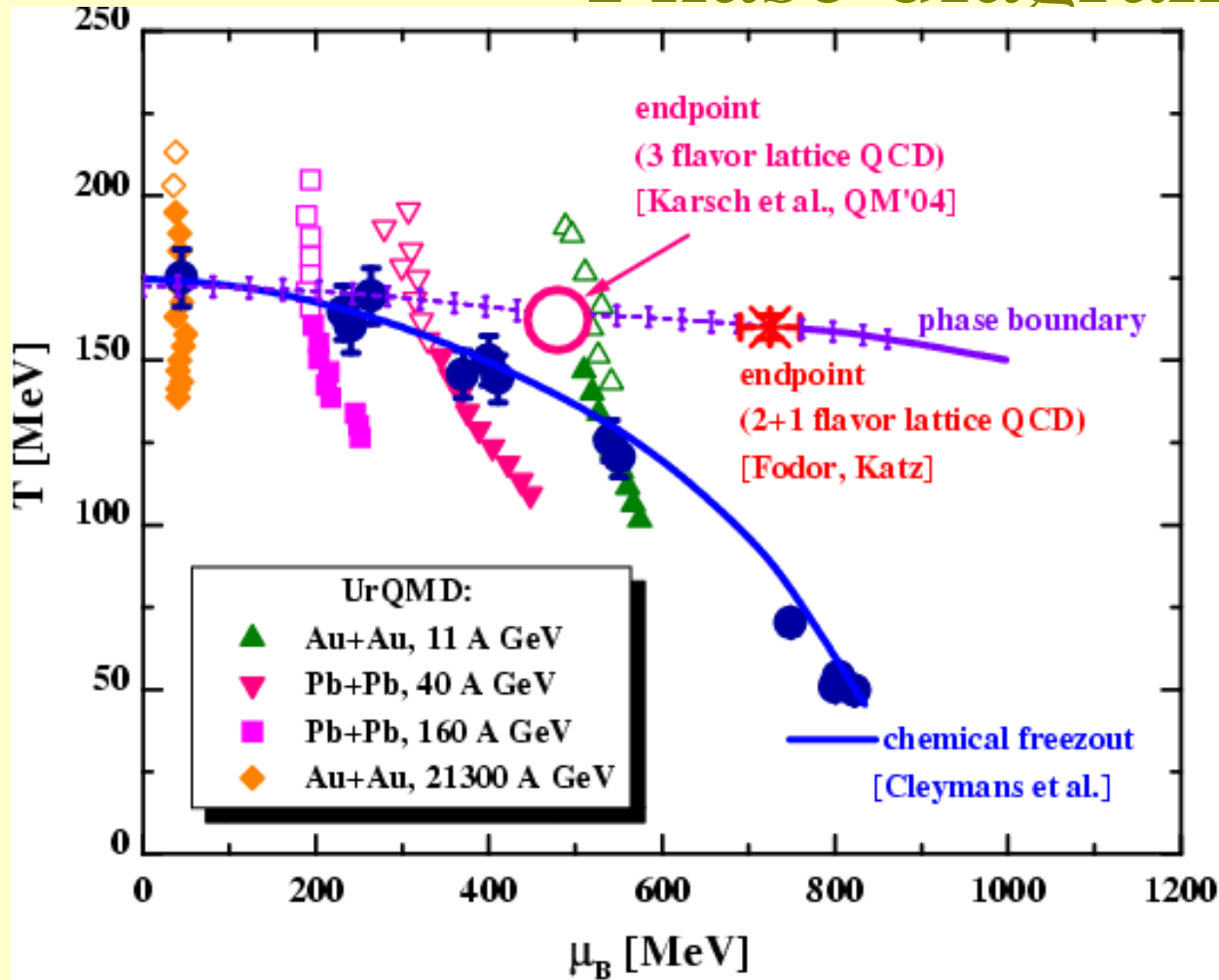


dN/dy at $y=0$ for charged particles at LHC, 1/20 centrality





Phase diagram



- QGP might be reached already at low SPS energy
- Tricritical point around 10-40 GeV
- No phase transition at RHIC
- Necessary to explore 10-30 AGeV energy region to study the phase transition