UNIVERSIDADE DE COIMBRA

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The QCD phase diagram in the presence of external magnetic fields

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Outline

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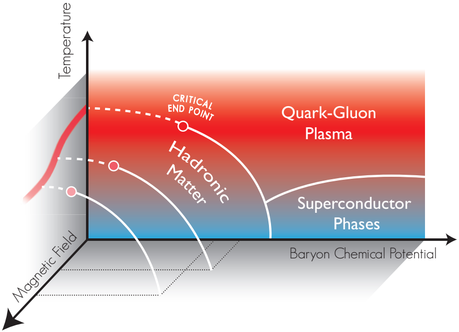
2. PNJL model in the presence of an external magnetic field
3. Chiral transition and the Critical End Point (CEP) at $B = 0$
4. Net baryon fluctuations at $B = 0$
5. How does an external magnetic field influence the QCD phase diagram?
6. Net baryon fluctuations in a strong magnetic field
7. Sound speed around the CEP at finite B
8. Conclusions

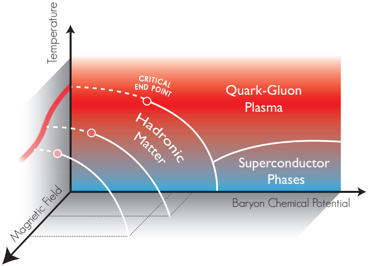
Motivation

Analyze the effects of an external magnetic field
on the QCD phase diagram



- What are the effects of B on chiral symmetry breaking and confinement?
- What happens to the CEP?

QCD phase Diagram in the presence of a magnetic field



Important for:

- **Measurements in Heavy Ion Collisions**[1]:
  - RHIC energy scale: $eB_{max} \approx 5 \times 10^{18}$ G $(5 \times m_\pi^2)$
  - LHC energy scale: $eB_{max} \approx 5 \times 10^{19}$ G $(15 \times m_\pi^2)$
- **Magnetized neutron stars:** low T and high μ_B region ($B \sim 10^{18-20}$ G in the interior?)[2]
- **First phases of the Universe:** high T and low μ_B region

1− [V. V. Skokov, et al., IJMPA 24 (2009) 5925]; 2− [E. J. Ferrer, et al., Phys.Rev. C82 (2010) 065802]

Framework: the PNJL model

Polyakov loop extended Nambu-Jona–Lasinio model

$$\mathcal{L} = \bar{q}\left[i\gamma_\mu D^\mu - \hat{m}_c\right]q + \mathcal{L}_{\mathsf{sym}} + \mathcal{L}_{\mathsf{det}} + \mathcal{U}\left(\Phi, \bar{\Phi}; T\right) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

where

$$\mathcal{L}_{\mathsf{sym}} = G_s \sum_{a=0}^{8}\left[(\bar{q}\lambda_a q)^2 + (\bar{q}i\gamma_5\lambda_a q)^2\right]$$

$$\mathcal{L}_{\mathsf{det}} = -K\left\{\det\left[\bar{q}(1+\gamma_5)q\right] + \det\left[\bar{q}(1-\gamma_5)q\right]\right\}$$

- Covariant derivative: $D^\mu = \partial^\mu - iq_f A^\mu_{EM} - iA^\mu$
- Constant B field in the z direction: $A^{EM}_\mu = \delta_{\mu 2} x_1 B$
- For the Polyakov loop potential we use

$$\frac{\mathcal{U}\left(\Phi, \bar{\Phi}; T\right)}{T^4} = -\frac{a\left(T\right)}{2}\bar{\Phi}\Phi + b(T)\ln\left[1 - 6\bar{\Phi}\Phi + 4(\bar{\Phi}^3 + \Phi^3) - 3(\bar{\Phi}\Phi)^2\right]$$

Framework: the PNJL model

- Regularization: 3-momentum cutoff Λ
- NJL parametrization[3]:

$$m_u = m_d = 5.5 \text{ MeV}, \quad m_s = 140.7 \text{ MeV}$$
$$G_s\Lambda^2 = 3.67, \quad K\Lambda^5 = -12.36, \quad \Lambda = 602.3 \text{ MeV}$$

$\Rightarrow$ Fixed to reproduce several physical vacuum properties (f_π, M_π, M_K, and $M_{\eta'}$)

- $\mathcal{U}\left(\Phi, \bar{\Phi}; T\right)$ parametrization[4]:

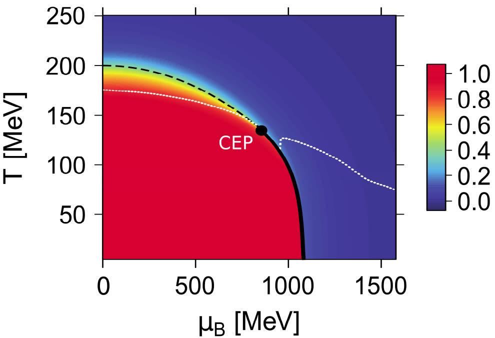
$$a_0 = 3.51, \quad a_1 = -2.47, \quad a_2 = 15.2, \quad b_3 = -1.75$$
$$T_0 = 210 \text{ MeV}$$

$\Rightarrow$ Chosen to reproduce lattice results ($T^{dec} = 171$ MeV)

3– [P. Rehberg, et al. PRC53, 410]; 4– [S. Roessner, et al. PRD75, 034007]

Phase diagram for $B = 0$: Up/Down quark

- Isospin symmetry: $\langle \bar{u}u \rangle = \left\langle \bar{d}d \right\rangle$
- Vacuum normalized **up-quark condensate**: $\langle \bar{u}u \rangle (T, \mu_B) / \langle \bar{u}u \rangle (0, 0)$

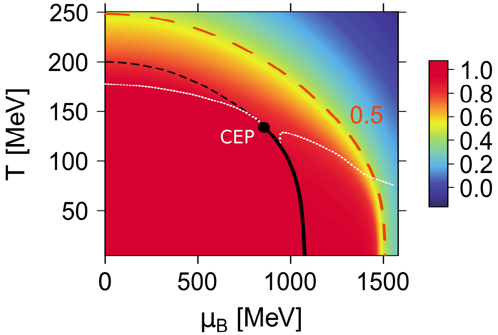


- $T^{\chi}(\mu_B = 0) = 200$ MeV
- $T^{\Phi}(\mu_B = 0) = 171$ MeV
- **CEP** at $\begin{cases} T^{\mathtt{CEP}} = 133\,\text{MeV} \\ \mu_B^{\mathtt{CEP}} = 862\,\text{MeV} \end{cases}$

- Crossover transition for $T > 133$ MeV
- Second-order phase transition at $T = 133$ MeV
- First-order phase transition for $T < 133$ MeV

Phase diagram for $B = 0$: Strange quark

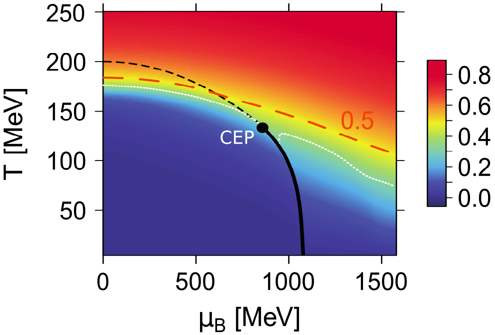
- Vacuum normalized **s-quark condensate**: $\langle \bar{s}s \rangle (T, \mu_B) / \langle \bar{s}s \rangle (0,0)$



- Small variation at the chiral transition
- Crossover transition for every T

Phase diagram for $B = 0$: Polyakov loop

- **Polyakov loop** value: $\Phi(T, \mu_B)$
  - $\Phi \to 0$: *confined* phase (low temperatures)
  - $\Phi \to 1$: *deconfined* phase (high temperatures)



- $T^{\Phi}(\mu_B = 0) = 171$ MeV
- Small variation at the chiral transition
- Crossover transition for every T

Net-baryon fluctuations

- The nth-order **net-baryon fluctuations** (susceptibility):

$$\chi_B^n(T,\mu_B) = \frac{\partial^n \left(P(T,\mu_B)/T^4\right)}{\partial(\mu_B/T)^n}$$

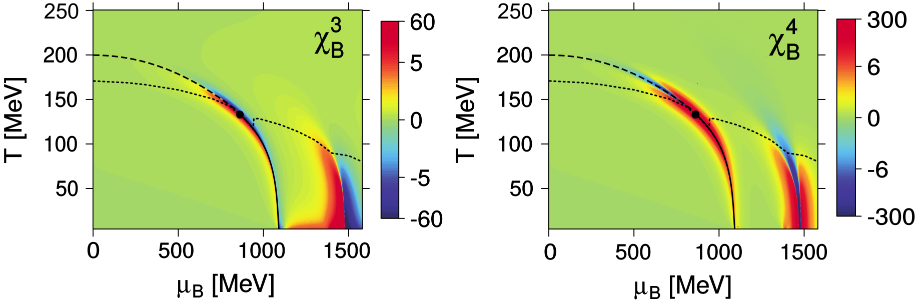
- Susceptibilities ratios have no volume dependence:

$$\chi_B^4/\chi_B^2 = \kappa\sigma^2 \qquad \chi_B^3/\chi_B^1 = S\sigma^3/M$$

- They measure the kurtosis and skewness of the net-baryon distribution.

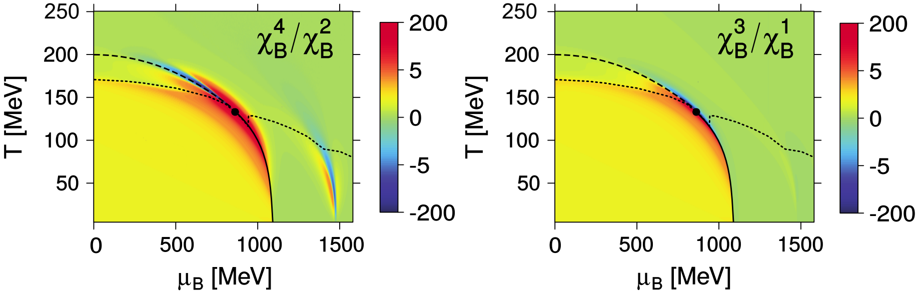
- Fluctuations provide important information on critical phenomena:
    - possible experimental signatures for the presence of a CEP and the onset of deconfinement

χ_B^3 and χ_B^4 fluctuations ($B=0$)



- Non-monotonic dependencies around the CEP
- Positive fluctuations of χ_B^3 on the chiral restored phase
- χ_B^4 fluctuations are symmetric with respect to the chiral transition
- A similar non-monotonic dependence occurs at high μ_B (strange quark transition) but **no CEP is present**
  - A stronger G_s would give rise to a first-order transition and a CEP
  - An external magnetic has this effect

χ_B^4/χ_B^2 and χ_B^3/χ_B^1 fluctuations ratios ($B = 0$)

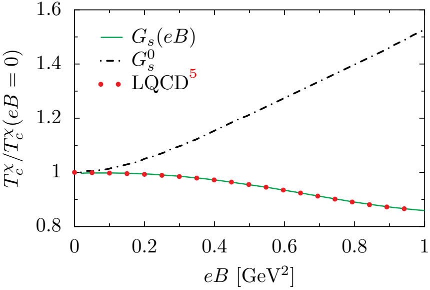


- Clear distinction between the broken/restored chiral symmetry region
- There is a pronounced variation around deconfinement transition
- The non-monotonic dependence at higher μ_B is still visible

Can the non-monotonic (critical) region still persist in the absence of a CEP?

Two models at finite B: different scalar couplings

- Constant coupling: $G_s = G_s^0 = 3.67/\Lambda^2$
- Magnetic field dependent coupling: $G_s = G_s(eB)$



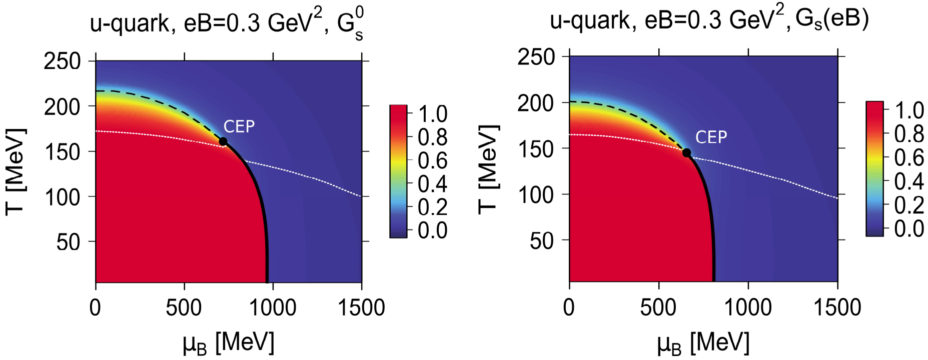
[M. Ferreira, et al., Phys. Rev. D89 (2014) 116011-9]

- Same vacuum properties for both models:
  - $G_s(eB) \to G_s^0$ as $B \to 0$

5– [G. Bali, et al., JHEP 1202 (2012) 044]

Phase diagram at finite B: Up quark

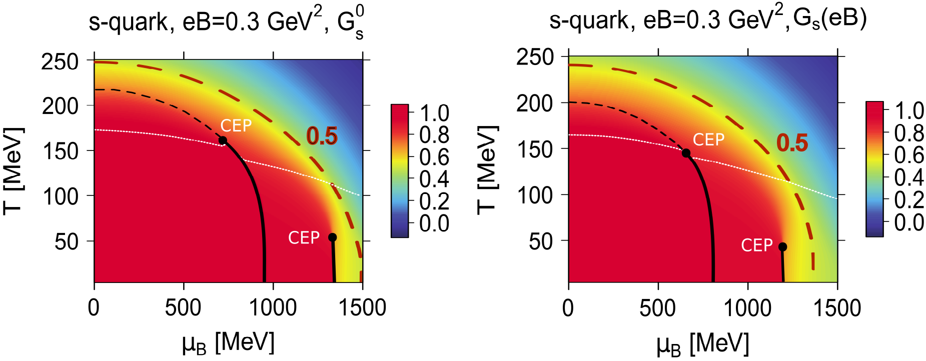
- External magnetic field: $eB = 0.3$ GeV^2



- $G_s(eB)$ model shows **Inverse Magnetic Catalysis**
  - Both T^χ and T^Φ decrease with B
- $\mu_B^{\text{crit}}(T = 0)$ also decreases with B for the $G_s(eB)$ model
- $G_s(eB)$ model $\Longrightarrow$ smaller region for the chiral broken phase

Phase diagram at finite B: Strange quark

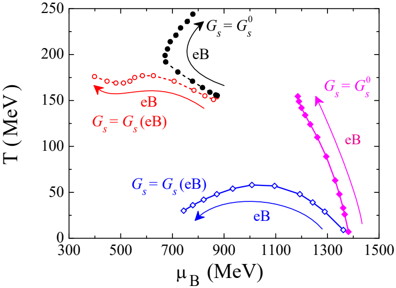
- External magnetic field: $eB = 0.3$ GeV^2



- A new first-order phase transition happens at higher μ_B
- A CEP related with the strange quark appears
- The strange CEP also occurs at lower μ_B for the $G_s(eB)$ model

Location of the CEP in the presence of a magnetic field

Distinct behaviors between models at large B:



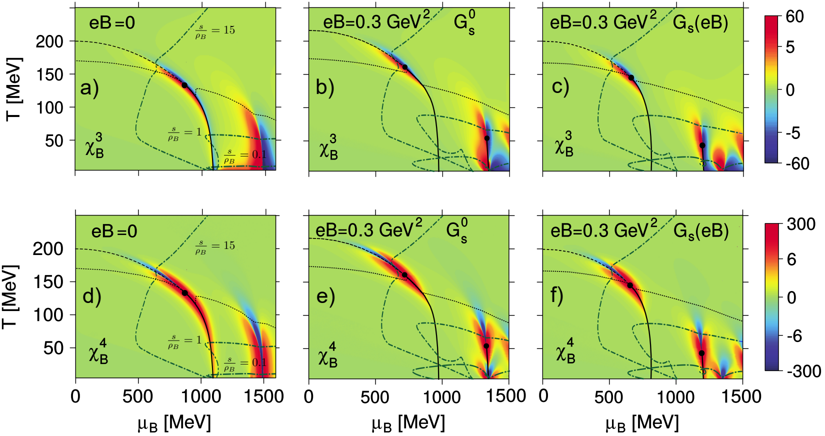
Case G_s^0:

- increasing magnetic fields shift both CEPs to higher temperatures

Case $G_s(eB)$:

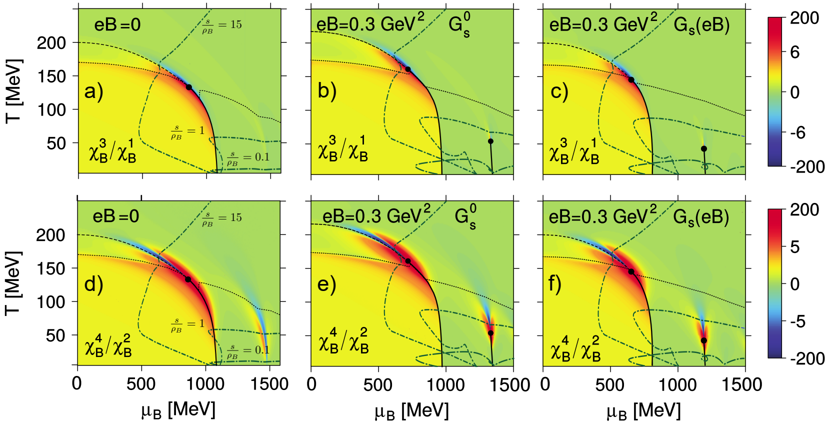
- light CEP: it occurs at smaller μ_B and at a practically unchanged temperatures
- strange CEP: it occurs at smaller μ_B and, at high values of B, also at smaller temperatures

χ_B^3 and χ_B^4 fluctuations in a strong B



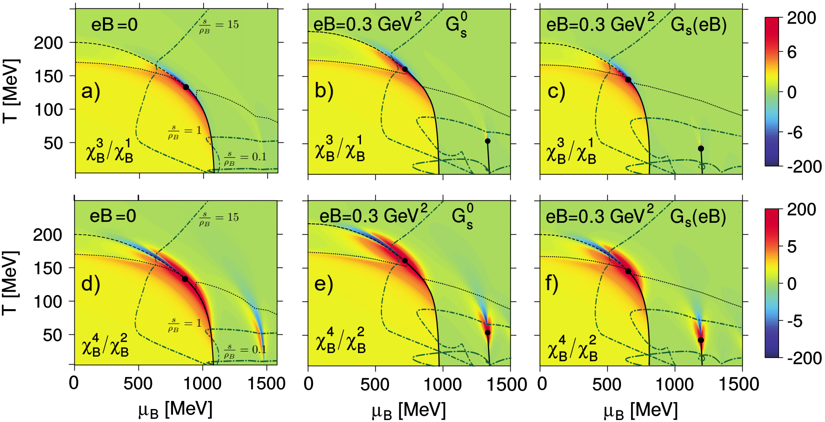
- Three CEP like structures at high μ_B:
  - 1^{st}: s-quark first-order phase transition
  - 2^{nd}: population of a new LL for the d-quark
  - 3^{rd}: s-quark first-order phase transition at higher μ_B

χ_B^4/χ_B^2 and χ_B^3/χ_B^1 fluctuations ratios in a strong B



- Only the non-monotonic behavior around the CEPs remains
- B concentrates the high fluctuation region around the CEP
- The behavior of isentropic trajectories shows the presence of several CEPs: slight focusing effect towards the strange CEP

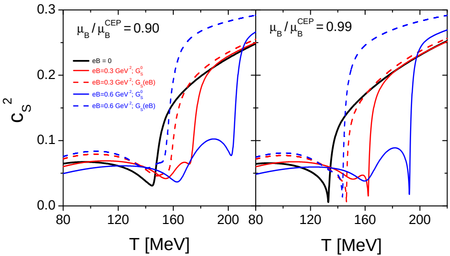
χ_B^4/χ_B^2 and χ_B^3/χ_B^1 fluctuations ratios in a strong B



Case $G_s(eB)$:

- μ_B^{CEP} decreases with $B \Longrightarrow$ the chiral crossover at $\mu_B = 0$ possibly turns into a 1^{st}-order phase transition for high enough B
- Increasing of $\chi_B^{3(4)}/\chi_B^{1(2)}$ at small μ_B due to the dragging of the critical region by the CEP $\Longrightarrow$ sign the vicinity of a CEP at low μ_B?

Sound speed around the (ligh) CEP at finite B



$B = 0$:

- local minimum that tends to zero as the CEP gets closer
- very close to the CEP (crossover region) both transitions coincide

$B \neq 0$:

- two minima related with both transitions (chiral and deconfinement)
- 2^{nd} minimum (related with chiral transition) tends to zero as the CEP gets closer
- near the CEP c_s^2 reflects the changes in the QCD phase diagram due to $B \Longrightarrow$ it is sensitive to both the chiral symmetry restoration and deconfinement transitions

Conclusions

- The presence of an external magnetic field causes a complex pattern of multiple phase transitions

- B induces multiple phase transitions, and thus the emergence of several CEPs also in the strange sector

- The CEP's location strongly depends on whether the IMC is taken into account:
  - The G_s^0 predicts that μ_B^{CEP} increases with B ($eB > 0.3$ GeV^2)
    - Suppression of fluctuations at low μ_B
  - The $G_s(eB)$ predicts that μ_B^{CEP} decreases with B
    - Enhancement of fluctuations at low μ_B

- Fluctuations do not necessarily indicate the existence of a CEP

- The speed of sound shows a much richer structure in quark magnetized matter and allows to identify both chiral and deconfinement transitions

Acknowledgments

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- P. Costa is supported by FCT under Contract No. SFRH/BPD/102273/2014

(Backup slides) PNJL model in an external magnetic field

Thermodynamic potential:

$$
\begin{aligned}
\Omega(T,\mu_i) \quad &= \quad \mathcal{U}(\Phi,\bar{\Phi},T) - N_c \sum_{i=u,d,s} \frac{|q_i|eB}{2\pi} \sum_{n=0}^{\infty} \alpha_n \int_{-\infty}^{\infty} \frac{dp_z}{2\pi} \Bigg(E_i \\
&+ \quad \frac{T}{3} \ln \left\{ 1 + 3\bar{\Phi} \mathrm{e}^{-(E_i-\mu_i)/T} + 3\Phi \mathrm{e}^{-2(E_i-\mu_i)/T} + \mathrm{e}^{-3(E_i-\mu_i)/T} \right\} \\
&+ \quad \frac{T}{3} \ln \left\{ 1 + 3\Phi \mathrm{e}^{-(E_i+\mu_i)/T} + 3\bar{\Phi} \mathrm{e}^{-2(E_i+\mu_i)/T} + \mathrm{e}^{-3(E_i+\mu_i)/T} \right\} \Bigg) \\
&+ \quad G_s \sum_{\{i=u,d,s\}} \langle \bar{q_i} q_i \rangle^2 + 4K \langle \bar{q_u} q_u \rangle \langle \bar{q_d} q_d \rangle \langle \bar{q_s} q_s \rangle
\end{aligned}
$$

- In the presence of an external magnetic field $B = B\hat{z}$:

$$
E_i = \sqrt{2n|q_i|eB + p_z^2 + M_i^2}
$$

  - $n = 0, 1, 2, \ldots$ is the Landau level.

- Dimensional reduction: $D \to D-2 \longmapsto k_x, k_y, k_z \to k_z$.
- The model is modified in the following:

$$
2\int \frac{d^3p}{(2\pi)^3} f(E_i) \to \frac{|qB|}{2\pi} \sum_{n=0}^{\infty} \alpha_n \int \frac{dp_z}{2\pi} f\left(\sqrt{p_z^2 + 2|qB|n + M_i^2}\right)
$$

with $\alpha_0 = 1$ and $\alpha_{n \neq 0} = 2$

(Backup slides) Parameters and results

Physical quantities	Parameter set and constituent quark masses
$f_\pi = 92.4$ MeV	$m_u = m_d = 5.5$ MeV
$M_\pi = 135.0$ MeV	$m_s = 140.7$ MeV
$M_K = 497.7$ MeV	$\Lambda = 602.3$ MeV
$M_{\eta'} = 960.8$ MeV	$G_s^0\Lambda^2 = 3.67$
$M_\eta = 514.8$ MeV*	$K\Lambda^5 = 12.36$
$f_K = 97.7$ MeV*	$M_u = M_d = 367.7$ MeV*
$M_\sigma = 728.8$ MeV*	$M_s = 549.5$ MeV*
$M_{a_0} = 873.3$ MeV*	
$M_\kappa = 1045.4$ MeV*	
$M_{f_0} = 1194.3$ MeV*	
$\theta_P = -5.8^{o*}$; $\theta_S = 16^{o*}$	

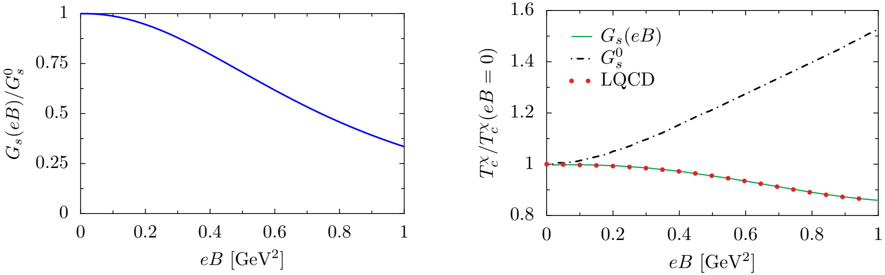
– [S. P. Klevansky, et al., Phys. Rev. C53 (1996) 410]

– [P. Costa, et al., Phys. Rev. D71 (2005) 116002]

(Backup slides) The magnetic field dependence of G_s

- The $T_c^\chi(B)/T_c^\chi(eB=0)$ (given by LQCD[6]) is obtained by the following $G_s(eB)$ dependence

$$G_s(\zeta) = G_s^0 \left(\frac{1 + a\,\zeta^2 + b\,\zeta^3}{1 + c\,\zeta^2 + d\,\zeta^4} \right), \quad \text{where} \quad \zeta = eB/\Lambda_{QCD}^2$$

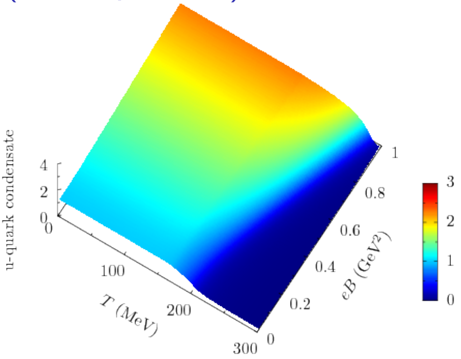


[M. Ferreira, et al., Phys. Rev. D89 (2014) 116011-9]

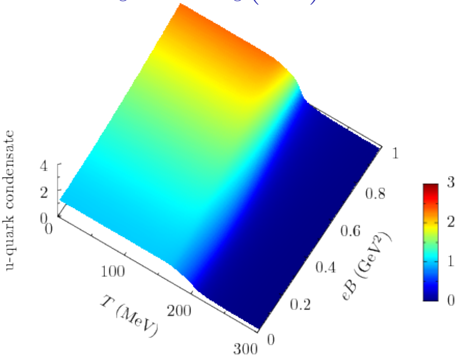
- The pseudocritical temperature decrease ratio is possible via the $G_s(eB)$
- Furthermore, the crossover nature of the transitions is preserved

6 – [G. Bali, et al., JHEP 1202 (2012) 044]

(Backup slides) Quark condensate: G_s vs. $G_s(eB)$



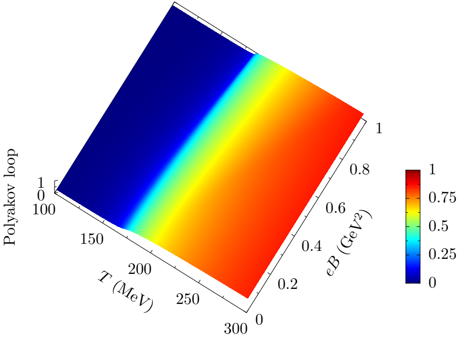
$G_s = G_s^0$



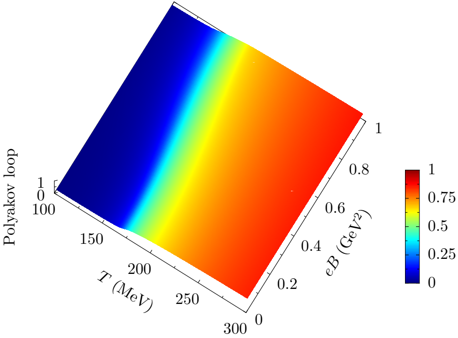
$G_s(eB)$

- $G_s(eB)$ still leads to MC at low temperatures
  - B enhances the quark condensate
- $G_s(eB)$ generates IMC on the transition temperature region
  - B weakens the quark condensate

(Backup slides) Polyakov loop: G_s vs. $G_s(eB)$



$G_s = G_s^0$



$G_s(eB)$

- The Polyakov loop shows the following trends (as in LQCD):
  - for a given temperature, it increases with B and changes strongly on the transition region
  - The inflection point moves to smaller temperatures with increasing B