QCD phase diagram in an extended effective Lagrangian approach

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Outline

Introduction

QCD: the Theory of **Strong Interactions**

- Very successfull pQCD at high energy
- Non-perturbative low energy regime requires the use of other tools for instance:
  - lQCD
  - AdS/QCD
  - Dyson-Schwinger
  - FRG
  - Chiral pertubation theory
  - Effective models
    - **Dynamical**/**Explicit** **Chiral Symmetry Breaking** plays a big role in low energy phenomenology

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Phase diagram for strongly interacting matter

A clear and present challenge [1]:

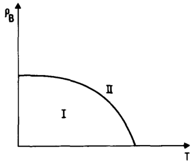

Fig. 1. Schematic phase diagram of hadronic matter. ρ_B is the density of baryonic number. Quarks are confined in phase I and unconfined in phase II.

[1] N. Cabbibo, G. Parisi Phys.Lett. 59B (1975) 67-69; Kenji Fukushima, Tetsuo Hatsuda Rept.Prog.Phys.74:014001,2011; http://nica.jinr.ru/physics.php

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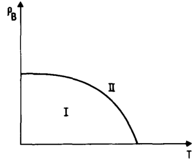

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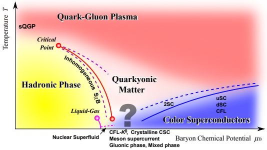

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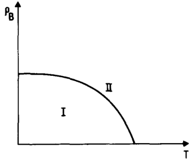

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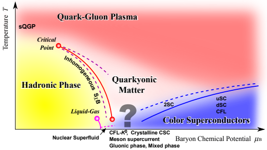

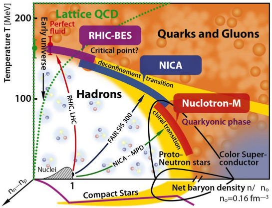

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Nambu–Jona-Lasinio Model

NJL: effective model for the non-perturbative low energy regime of **QCD** with Dynamical Chiral Symmetry Breaking (**DχSB**)

- **NJL** shares the global symmetries with **QCD**
- Dynamical generation of the **constituent mass**
- Light pseudoscalar as (quasi) **Nambu-Goldstone boson**
- **Quark condensates** as order parameter
- No gluons (no confinement/deconfinement)
- Local and non renormalizable

Multi-quark interations (u, d and s) [2]

$$\mathcal{L}_{\text{eff}} = \overline{q} \imath \partial\!\!\!/ q + \mathcal{L}_m$$

[2] $\Sigma = (s_a - \imath p_a) \frac{1}{2} \lambda_a$, $s_a = \bar{q} \lambda_a q$, $p_a = \bar{q} \lambda_a \imath \gamma_5 q$, and $a = 0, 1, \ldots, 8$

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$$\mathcal{L}_{\rm eff} = \bar{q} \imath \partial\!\!\!/ q + \mathcal{L}_m$$

- **Explicit Chiral symmetry breaking**

$$\mathcal{L}_m = \bar{q}\hat{m}q$$

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Multi-quark interations (u, d and s) [2]

$$\mathcal{L}_{\text{eff}} = \overline{q}\imath\partial\!\!\!/ q + \mathcal{L}_m + \mathcal{L}_{NJL}$$

- $\mathcal{L}_m = \overline{q}\hat{m}q$

- **Nambu–Jona-Lasinio** (4 q)

$$\mathcal{L}_{NJL} = G\,\text{tr}\left[\Sigma^\dagger\Sigma\right]$$

[2] $\Sigma = (s_a - \imath p_a)\frac{1}{2}\lambda_a$, $s_a = \bar{q}\lambda_a q$, $p_a = \bar{q}\lambda_a\imath\gamma_5 q$, and $a = 0, 1, \ldots, 8$

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Multi-quark interations (u, d and s) [2]

$$\mathcal{L}_{\text{eff}} = \overline{q}\imath\partial\!\!\!/ q + \mathcal{L}_m + \mathcal{L}_{NJL} + \mathcal{L}_H$$

- $\mathcal{L}_m = \overline{q}\hat{m}q$
- $\mathcal{L}_{NJL} = G\,\text{tr}\left[\Sigma^\dagger\Sigma\right]$
- **'t Hooft determinant** (6 q)

$$\mathcal{L}_H = \kappa\left(\det\left[\Sigma\right] + \det\left[\Sigma^\dagger\right]\right)$$

[2] $\Sigma = (s_a - \imath p_a)\frac{1}{2}\lambda_a$, $s_a = \bar{q}\lambda_a q$, $p_a = \bar{q}\lambda_a \imath\gamma_5 q$, and $a = 0, 1, \ldots, 8$

Multi-quark interations (u, d and s) [2]

$$\mathcal{L}_{\rm eff} = \bar{q}\imath\partial\!\!\!/ q + \mathcal{L}_m + \mathcal{L}_{NJL} + \mathcal{L}_H + \mathcal{L}_{8q}$$

- $\mathcal{L}_m = \bar{q}\hat{m}q$
- $\mathcal{L}_{NJL} = G\,\mathrm{tr}\left[\Sigma^\dagger\Sigma\right]$
- $\mathcal{L}_H = \kappa\left(\det\left[\Sigma\right] + \det\left[\Sigma^\dagger\right]\right)$
- **Eight quark interaction term**

$$\mathcal{L}_{8q} = \mathcal{L}_{8q}^{(1)} + \mathcal{L}_{8q}^{(2)}, \quad \mathcal{L}_{8q}^{(1)} = g_1\left(\mathrm{tr}\left[\Sigma^\dagger\Sigma\right]\right)^2, \quad \mathcal{L}_{8q}^{(2)} = g_2\mathrm{tr}\left[\Sigma^\dagger\Sigma\Sigma^\dagger\Sigma\right]$$

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- $\mathcal{L}_m = \bar{q}\hat{m}q$

- $\mathcal{L}_{NJL} = G\,\mathrm{tr}\left[\Sigma^\dagger\Sigma\right]$
- $\mathcal{L}_H = \kappa\left(\det\left[\Sigma\right] + \det\left[\Sigma^\dagger\right]\right)$
- $\mathcal{L}_{8q} = \mathcal{L}_{8q}^{(1)} + \mathcal{L}_{8q}^{(2)}, \quad \mathcal{L}_{8q}^{(1)} = g_1\left(\mathrm{tr}\left[\Sigma^\dagger\Sigma\right]\right)^2, \quad \mathcal{L}_{8q}^{(2)} = g_2\mathrm{tr}\left[\Sigma^\dagger\Sigma\Sigma^\dagger\Sigma\right]$

OZI violation in $\mathcal{L}_H$ and $\mathcal{L}_{8q}^{(1)}$.

[2] $\Sigma = (s_a - \imath p_a)\frac{1}{2}\lambda_a$, $s_a = \bar{q}\lambda_a q$, $p_a = \bar{q}\lambda_a\imath\gamma_5 q$, and $a = 0, 1, \ldots, 8$

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Multi-quark interations (u, d and s) [2]

$$\mathcal{L}_{\text{eff}} = \bar{q}\imath\partial\!\!\!/ q + \mathcal{L}_m + \mathcal{L}_{NJL} + \mathcal{L}_H + \mathcal{L}_{8q} + \mathcal{L}_\chi$$

- $\mathcal{L}_m = \bar{q}\hat{m}q$
- **Extended Explicit Chiral symmetry breaking $\mathcal{L}_\chi$**
- $\mathcal{L}_{NJL} = G\,\mathrm{tr}\left[\Sigma^\dagger\Sigma\right]$
- $\mathcal{L}_H = \kappa\left(\det\left[\Sigma\right] + \det\left[\Sigma^\dagger\right]\right)$
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Non canonical explicit chiral symmetry breaking terms

[2] $\Sigma = (s_a - \imath p_a)\frac{1}{2}\lambda_a$, $s_a = \bar{q}\lambda_a q$, $p_a = \bar{q}\lambda_a\imath\gamma_5 q$, and $a = 0, 1, \ldots, 8$

Inclusion of explicit chiral symmetry breaking terms

$$\mathcal{L}_\chi = \sum_{i=1}^{10} \mathcal{L}_\chi^i,$$

$$\mathcal{L}_\chi^1 = -\kappa_1 \; e_{ijk} e_{mnl} \Sigma_{im} \chi_{jn} \chi_{kl} + h.c., \qquad \mathcal{L}_\chi^2 = \kappa_2 \; e_{ijk} e_{mnl} \chi_{im} \Sigma_{jn} \Sigma_{kl} + h.c.,$$

$$\mathcal{L}_\chi^3 = g_3 \,\mathrm{tr}\left[\Sigma^\dagger \Sigma \Sigma^\dagger \chi\right] + h.c., \qquad \mathcal{L}_\chi^4 = g_4 \,\mathrm{tr}\left[\Sigma^\dagger \Sigma\right] \mathrm{tr}\left[\Sigma^\dagger \chi\right] + h.c.,$$

$$\mathcal{L}_\chi^5 = g_5 \,\mathrm{tr}\left[\Sigma^\dagger \chi \Sigma^\dagger \chi\right] + h.c. \qquad \mathcal{L}_\chi^6 = g_6 \,\mathrm{tr}\left[\Sigma \Sigma^\dagger \chi \chi^\dagger + \Sigma^\dagger \Sigma \chi^\dagger \chi\right],$$

$$\mathcal{L}_\chi^7 = g_7 \left(\mathrm{tr}\left[\Sigma^\dagger \chi\right] + h.c.\right)^2 \qquad \mathcal{L}_\chi^8 = g_8 \left(\mathrm{tr}\left[\Sigma^\dagger \chi\right] - h.c.\right)^2,$$

$$\mathcal{L}_\chi^9 = -g_9 \,\mathrm{tr}\left[\Sigma^\dagger \chi \chi^\dagger \chi\right] + h.c. \qquad \mathcal{L}_\chi^{10} = -g_{10} \,\mathrm{tr}\left[\chi^\dagger \chi\right] \mathrm{tr}\left[\chi^\dagger \Sigma\right] + h.c.$$

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- $\kappa_1,\ g_9,\ g_{10} \to 0$ without loss of generality

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- $\kappa_1, \; g_9, \; g_{10} \to 0$ without loss of generality
- $\chi \to \frac{1}{2}\hat{m}$

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- $\kappa_1,\ g_9,\ g_{10} \to 0$ without loss of generality
- $\chi \to \frac{1}{2}\hat{m}$
- $\kappa_1, \kappa_2, g_4, g_7, g_8, g_{10}$ OZI violating

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Thermodynamic potential [3]

$$\Omega = \mathcal{V}_{st}\left[h_i\right] + \sum_i \frac{N_c}{8\pi^2}\left(J_{-1}\left[M_i\left[h_i\right], T, \mu_i\right] + C\left[T, \mu_i\right]\right)$$

$$\begin{aligned}
\mathcal{V}_{st}\left[h_i\right] =& \frac{1}{16}\Bigg(4G\left(h_i^2\right) + 3g_1\left(h_i^2\right)^2 + 3g_2\left(h_i^4\right) + 4g_3\left(h_i^3 m_i\right) \\
& + 4g_4\left(h_i^2\right)\left(h_j m_j\right) + 2g_5\left(h_i^2 m_i^2\right) + 2g_6\left(h_i^2 m_i^2\right) + 4g_7\left(h_i m_i\right)^2 \\
& + 8\kappa h_u h_d h_s + 8\kappa_2\left(m_u h_d h_s + h_u m_d h_s + h_u h_d m_s\right)\Bigg)\Bigg|_0^{M_i} \\
\Delta_f =& M_f - m_f \\
=& -G h_f - \frac{g_1}{2} h_f (h_i^2) - \frac{g_2}{2}(h_f^3) - \frac{3g_3}{4} h_f^2 m_f - \frac{g_4}{4}\left(m_f\left(h_i^2\right) + 2h_f(m_i h_i)\right) \\
& - \frac{g_5 + g_6}{2} h_f m_f^2 - g_7 m_f (h_i m_i) - \frac{\kappa}{4} t_{fij} h_i h_j - \kappa_2 t_{fij} h_i m_j
\end{aligned}$$

[3] For details see: J. Moreira, J. Morais, B. Hiller, A. A. Osipov, and A. H. Blin, Phys. Rev. D 91, 116003 (2015), arXiv:1409.0336 [hep-ph]

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&+4g_4\left(h_i^2\right)\left(h_j m_j\right) + 2g_5\left(h_i^2 m_i^2\right) + 2g_6\left(h_i^2 m_i^2\right) + 4g_7\left(h_i m_i\right)^2 \\
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Thermodynamic potential: the fermionic integrals [4]

$$\Omega = \mathcal{V}_{st}[h_i] + \sum_i \frac{N_c}{8\pi^2}\left(J_{-1}[M_i[h_i], T, \mu_i] + C[T, \mu_i]\right)$$

$$J_{-1}^{vac}[M, \Lambda] = -16\pi^2 \int \frac{\mathrm{d}^4 p}{(2\pi)^4} \int_{E_0^2}^{E_M^2} \mathrm{d}E^2 \hat{\rho} \frac{1}{E^2 + p_4^2}$$

$$J_{-1}[M, \Lambda, \mu, T] = -16\pi^2 \int \frac{\mathrm{d}^3 \vec{p}}{(2\pi)^3} \int_{E_0^2}^{E_M^2} \mathrm{d}E^2\, T \sum_{n=-\infty}^{+\infty} \hat{\rho} \frac{1}{E^2 + (\pi(2n+1)T - i\mu)^2}$$

$$= -\int \frac{\mathrm{d}^3 \vec{p}}{(2\pi)^3} \int_{E_0^2}^{E_M^2} \mathrm{d}E^2 \hat{\rho} \frac{8\pi^2}{E}\left(1 - n_q[E, \mu, T] - n_{\overline{q}}[E, \mu, T]\right)$$

$$\hat{\rho}_{PV}^{E} = 1 - \left(1 - \Lambda^2 \frac{\partial}{\partial E^2}\right) \mathrm{e}^{-\Lambda^2 \frac{\partial}{\partial E^2}}$$

$$C(T, \mu) = \int \frac{\mathrm{d}^3 p}{(2\pi)^3} 16\pi^2 T \log\left(\left(1 + \mathrm{e}^{-\frac{|\boldsymbol{p}| - \mu}{T}}\right)\left(1 + \mathrm{e}^{-\frac{|\boldsymbol{p}| + \mu}{T}}\right)\right)$$

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$$= -\int \frac{\mathrm{d}^3 \overrightarrow{p}}{(2\pi)^3} \int_{E_0^2}^{E_M^2} \mathrm{d}E^2 \hat{\rho} \frac{8\pi^2}{E}\left(1 - n_q\left[E, \mu, T\right] - n_{\overline{q}}\left[E, \mu, T\right]\right)$$

$$\hat{\rho}_{PV}^{E} = 1 - \left(1 - \Lambda^2 \frac{\partial}{\partial E^2}\right) \mathrm{e}^{-\Lambda^2 \frac{\partial}{\partial E^2}}$$

$$C(T, \mu) = \int \frac{\mathrm{d}^3 p}{(2\pi)^3} 16\pi^2 T \log\left(\left(1 + \mathrm{e}^{-\frac{|\boldsymbol{p}| - \mu}{T}}\right)\left(1 + \mathrm{e}^{-\frac{|\boldsymbol{p}| + \mu}{T}}\right)\right)$$

[4] For details see: J. Moreira, J. Morais, B. Hiller, A. A. Osipov, and A. H. Blin, Phys. Rev. D 91, 116003 (2015), arXiv:1409.0336 [hep-ph]

CFisUC

Thermodynamic potential: the fermionic integrals [4]

$$\Omega = \mathcal{V}_{st}[h_i] + \sum_i \frac{N_c}{8\pi^2}\left(J_{-1}[M_i[h_i], T, \mu_i] + C[T, \mu_i]\right)$$

$$J_{-1}^{vac}[M,\Lambda] = -16\pi^2 \int \frac{\mathrm{d}^4 p}{(2\pi)^4} \int_{E_0^2}^{E_M^2} \mathrm{d}E^2 \hat{\rho} \frac{1}{E^2 + p_4^2}$$

$$J_{-1}[M,\Lambda,\mu,T] = -16\pi^2 \int \frac{\mathrm{d}^3 \vec{p}}{(2\pi)^3} \int_{E_0^2}^{E_M^2} \mathrm{d}E^2\, T \sum_{n=-\infty}^{+\infty} \hat{\rho} \frac{1}{E^2 + (\pi(2n+1)T - i\mu)^2}$$

$$= -\int \frac{\mathrm{d}^3 \vec{p}}{(2\pi)^3} \int_{E_0^2}^{E_M^2} \mathrm{d}E^2 \hat{\rho} \frac{8\pi^2}{E}\left(1 - n_q[E,\mu,T] - n_{\overline{q}}[E,\mu,T]\right)$$

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Inclusion of the Polyakov loop.

Introduce homogeneous background A_4 gluonic field

$$\partial^\mu \rightarrow D^\mu = \partial^\mu + \imath A^\mu, \quad A^\mu = \delta_0^\mu g A_a^0 \frac{\lambda^a}{2}, \quad L = \mathcal{P} e^{\int_0^\beta \mathrm{d}x_4 \imath A_4}, \quad \phi = \frac{1}{N_c}\mathrm{Tr} L, \quad \overline{\phi} = \frac{1}{N_c}\mathrm{Tr} L^\dagger$$

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- ∼order parameter (exact in the quenched limit) for (de)confinement ($\phi = 0 \leftrightarrow$ confined)
- enters the action as an imaginary μ

$$n_q(M,p,\mu,T) = \left(1 + e^{\left(\sqrt{M^2+p^2}-\mu\right)/T}\right)^{-1}$$

$$n_{\overline{q}}(M,p,\mu,T) = \left(1 + e^{\left(\sqrt{M^2+p^2}+\mu\right)/T}\right)^{-1}$$

$$\tilde{n}_q(M,p,\mu,T,\phi,\overline{\phi}) \equiv \frac{1}{N_c}\sum_{i=1}^{N_c} n_q(\sqrt{M^2+p^2}, \mu + \imath\,(A_4)_{ii}\,, T)$$

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- $\Omega\left[M_i, T, \mu, \phi, \overline{\phi}\right] = \mathcal{V}_{st}\left[h_i\right] + \frac{N_c}{8\pi^2}\sum_{f=u,d,s}\left(J_{-1}\left[M_f, T, \mu, \phi, \overline{\phi}\right] + C(T,\mu)\right) + \mathcal{U}\left[\phi, \overline{\phi}, T\right]$

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Polyakov potentials

- **Logarithmic form** [5]
  ($a_0 = 3.51,\ a_1 = -2.47,\ a_2 = 15.2,\ b_3 = -1.75,\ T_0 = 200\,\mathrm{MeV}$) :

$$\frac{\mathcal{U}_I}{T^4} = -\frac{1}{2} a[T]\,\bar{\phi}\phi + b[T]\ln\left[1 - 6\bar{\phi}\phi + 4\left(\bar{\phi}^3 + \phi^3\right) - 3\left(\bar{\phi}\phi\right)^2\right]$$

$$a[T] = a_0 + a_1\frac{T_0}{T} + a_2\left(\frac{T_0}{T}\right)^2; \quad b[T] = b_3\left(\frac{T_0}{T}\right)^3$$

- **Exponential K-Log form** [6]
  ($a_0 = 6.75,\ a_1 = -9.8,\ a_2 = 0.26,\ b_3 = 0.805,\ b_4 = 7.555,\ K = 0.1,\ T_0 = 175\,\mathrm{MeV}$):

$$\frac{\mathcal{U}_{II}}{T^4} = -\frac{1}{2} a[T]\,\bar{\phi}\phi - \frac{b_3}{6}\left(\bar{\phi}^3 + \phi^3\right) + \frac{b_4}{4}\left(\bar{\phi}\phi\right)^2 + K\ln\left[\frac{27}{24\pi^2}\left(1 - 6\bar{\phi}\phi + 4\left(\bar{\phi}^3 + \phi^3\right) - 3\left(\bar{\phi}\phi\right)^2\right)\right]$$

$$a[T] = a_0 + a_1\left(\frac{T_0}{T}\right)e^{-a_2\frac{T_0}{T}}$$

[5] S. RöSSner, C. Ratti, and W. Weise, Phys. Rev. D 75, 034007 (2007)

[6] A. Bhattacharyya, S. K. Ghosh, S. Maity, S. Raha, R. Ray, K. Saha, and S. Upadhaya, Phys. Rev. D 95, 054005 (2017)

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NJL: C_s^2 and $\Theta^{\mu}{}_{\mu}\,|_{\mu=0}$ *vs* lQCD [7]

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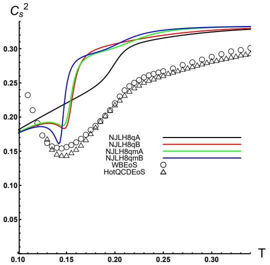

[7] WB: S. Borsanyi, Z. Fodor, C. Hoelbling, S. D. Katz, S. Krieg, and K. K. Szabo, Phys. Lett. B730, 99 (2014), arXiv:1309.5258 [hep-lat]
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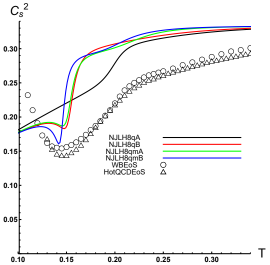

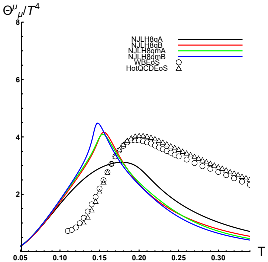

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NJL: $\chi_2^B, \chi_2^B, \chi_2^S\,|_{\mu=0}$ *vs* lQCD [8]

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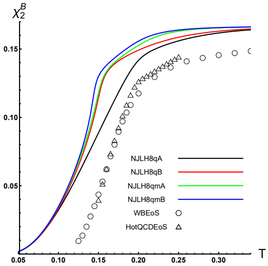

$\chi_2^B = \frac{1}{9}(\chi_2^u + \chi_2^d + \chi_2^s + 2\chi_{11}^{us} + 2\chi_{11}^{ds} + 2\chi_{11}^{ud})$

[8] WB: S. Borsanyi, Z. Fodor, S. D. Katz, S. Krieg, C. Ratti, and K. Szabo, JHEP 01, 138 (2012), arXiv:1112.4416 [hep-lat].
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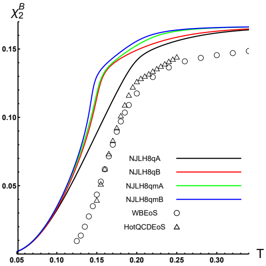

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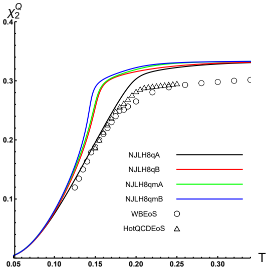

$\chi_2^Q = \frac{1}{9}(4\chi_2^u + \chi_2^d + \chi_2^s - 4\chi_{11}^{us} + 2\chi_{11}^{ds} - 4\chi_{11}^{ud})$

[8] WB: S. Borsanyi, Z. Fodor, S. D. Katz, S. Krieg, C. Ratti, and K. Szabo, JHEP 01, 138 (2012), arXiv:1112.4416 [hep-lat].
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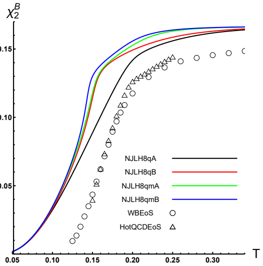
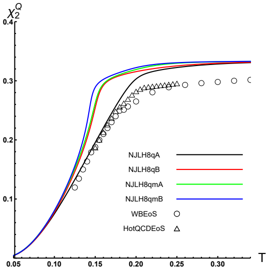


$\chi_2^B = \frac{1}{9}(\chi_2^u + \chi_2^d + \chi_2^s + 2\chi_{11}^{us} + 2\chi_{11}^{ds} + 2\chi_{11}^{ud})$



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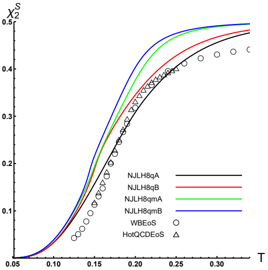

$\chi_2^S = \chi_2^s$

[8] WB: S. Borsanyi, Z. Fodor, S. D. Katz, S. Krieg, C. Ratti, and K. Szabo, JHEP 01, 138 (2012), arXiv:1112.4416 [hep-lat].
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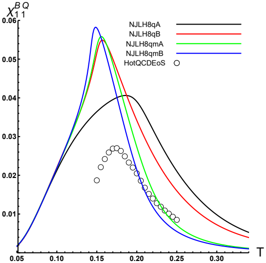

$\chi_{11}^{BQ} = \frac{1}{9}(2\chi_2^u - \chi_2^d - \chi_2^s + \chi_{11}^{ud} + \chi_{11}^{us} - 2\chi_{11}^{ds})$

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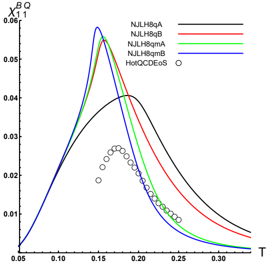

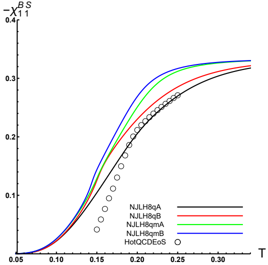

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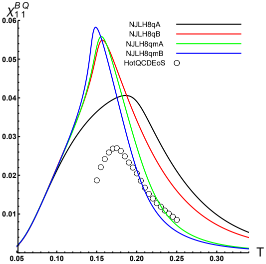

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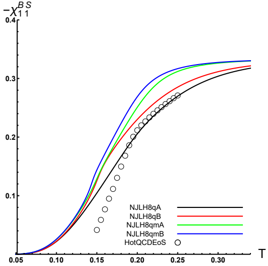

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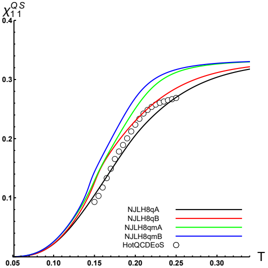

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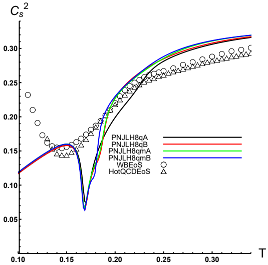

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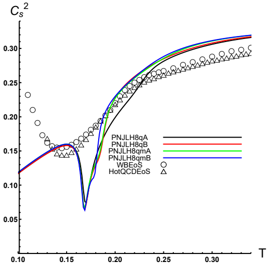

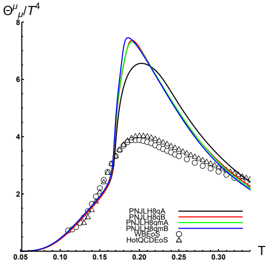

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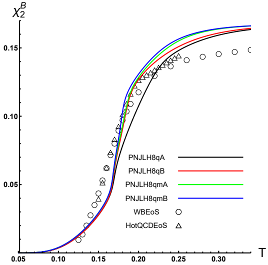

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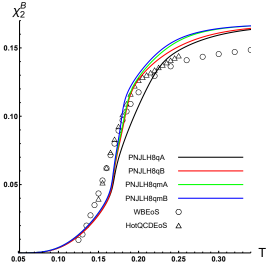

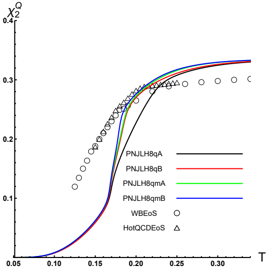

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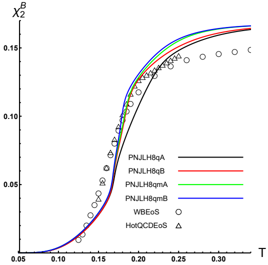
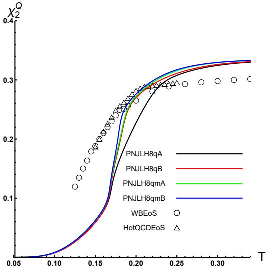
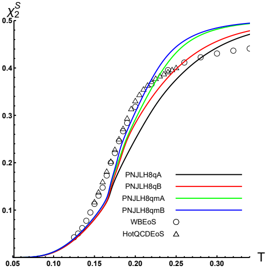
PNJL (Log): χ_2^B, χ_2^B, $\chi_2^S \,|_{\mu=0}$ *vs* lQCD [11]

- $\chi_2^B = \frac{1}{2} \frac{\partial^2 \Omega / T^4}{\partial \left(\frac{\mu_B}{T} \right)^2}$
- $\chi_2^Q = \frac{1}{2} \frac{\partial^2 \Omega / T^4}{\partial \left(\frac{\mu_Q}{T} \right)^2}$
- $\chi_2^S = \frac{1}{2} \frac{\partial^2 \Omega / T^4}{\partial \left(\frac{\mu_S}{T} \right)^2}$







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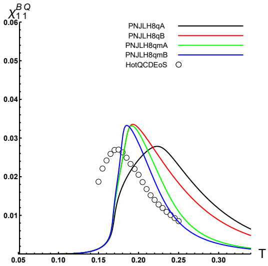
PNJL (Log): $\chi_{11}^{BQ}, \chi_{11}^{BS}, \chi_{11}^{QS}|_{\mu=0}$ *vs* lQCD [12]

- $$\chi_{11}^{BQ} = \frac{\partial^2 \Omega / T^4}{\partial(\frac{\mu_B}{T})\partial(\frac{\mu_Q}{T})}$$



[12] HotQCD: A. Bazavov et al. (HotQCD), Phys. Rev. D86, 034509 (2012), arXiv:1203.0784 [hep-lat].

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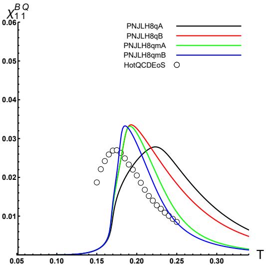
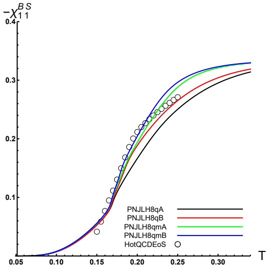
- $\chi_{11}^{BS} = \frac{\partial^2 \Omega / T^4}{\partial(\frac{\mu_B}{T})\partial(\frac{\mu_S}{T})}$





[12] HotQCD: A. Bazavov et al. (HotQCD), Phys. Rev. D86, 034509 (2012), arXiv:1203.0784 [hep-lat].

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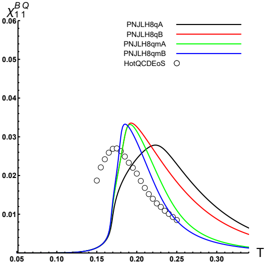
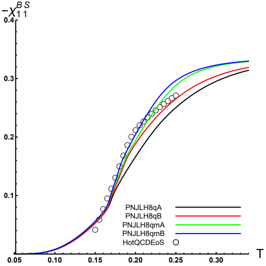
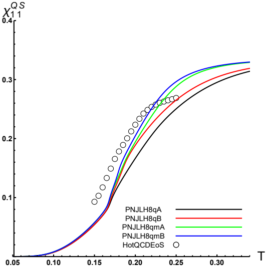
- $\chi_{11}^{QS} = \dfrac{\partial^2\Omega/T^4}{\partial(\frac{\mu_Q}{T})\partial(\frac{\mu_S}{T})}$



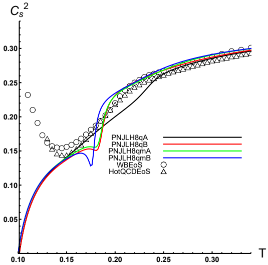




[12] HotQCD: A. Bazavov et al. (HotQCD), Phys. Rev. D86, 034509 (2012), arXiv:1203.0784 [hep-lat].

PNJL (Exp K-Log): C_s^2 and $\Theta^{\mu}{}_{\mu}\,|_{\mu=0}$ *vs* lQCD [13]

- $C_s^2 \equiv \frac{\partial P}{\partial \epsilon} = \frac{-\frac{\partial \Omega}{\partial T}}{T\frac{\partial^2 \Omega}{\partial T^2}}$



[13] WB: S. Borsanyi, Z. Fodor, C. Hoelbling, S. D. Katz, S. Krieg, and K. K. Szabo, Phys. Lett. B730, 99 (2014), arXiv:1309.5258 [hep-lat]
HotQCD: A. Bazavov et al. (HotQCD), Phys. Rev. D90, 094503 (2014), arXiv:1407.6387 [hep-lat]

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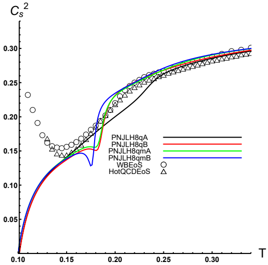
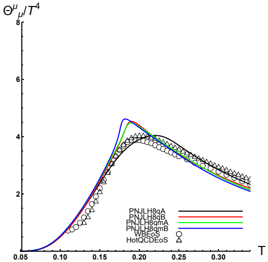
- $\Theta^\mu{}_\mu = \epsilon - 3P$





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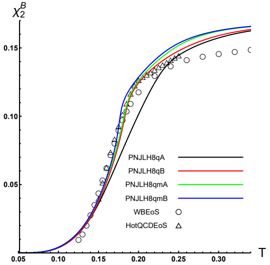
PNJL (Exp K-Log): $\chi_2^B, \chi_2^B, \chi_2^S\,|_{\mu=0}$ *vs* lQCD [14]

- $\chi_2^B = \frac{1}{2}\frac{\partial^2 \Omega/T^4}{\partial\left(\frac{\mu_B}{T}\right)^2}$



[14] WB: S. Borsanyi, Z. Fodor, S. D. Katz, S. Krieg, C. Ratti, and K. Szabo, JHEP 01, 138 (2012), arXiv:1112.4416 [hep-lat].
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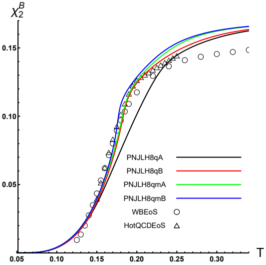

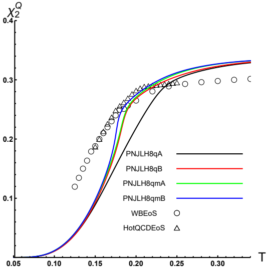

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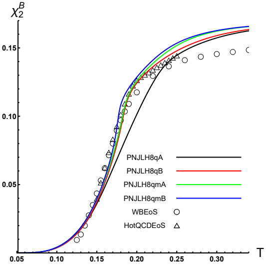
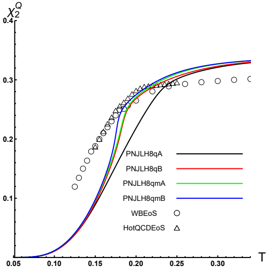
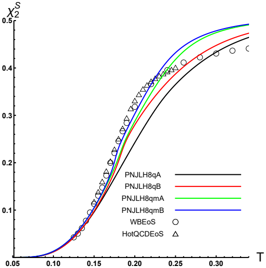
- $\chi_2^S = \frac{1}{2}\frac{\partial^2 \Omega/T^4}{\partial\left(\frac{\mu_S}{T}\right)^2}$







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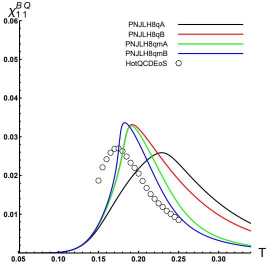
HotQCD: A. Bazavov et al. (HotQCD), Phys. Rev. D86, 034509 (2012), arXiv:1203.0784 [hep-lat].

PNJL (Exp K-Log): $\chi^{BQ}_{11}, \chi^{BS}_{11}, \chi^{QS}_{11}|_{\mu=0}$ vs lQCD [15]

- $\chi^{BQ}_{11} = \frac{\partial^2 \Omega / T^4}{\partial(\frac{\mu_B}{T})\partial(\frac{\mu_Q}{T})}$



[15] HotQCD: A. Bazavov et al. (HotQCD), Phys. Rev. D86, 034509 (2012), arXiv:1203.0784 [hep-lat].

PNJL (Exp K-Log): $\chi_{11}^{BQ}, \chi_{11}^{BS}, \chi_{11}^{QS}|_{\mu=0}$ vs lQCD [15]

- $\chi_{11}^{BQ} = \frac{\partial^2 \Omega / T^4}{\partial(\frac{\mu_B}{T})\partial(\frac{\mu_Q}{T})}$
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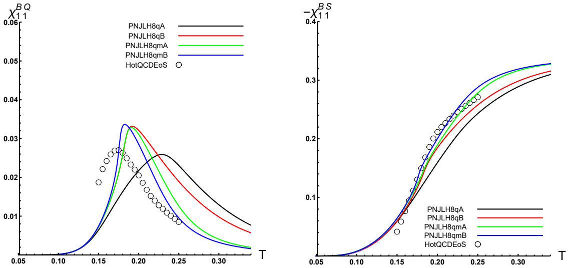
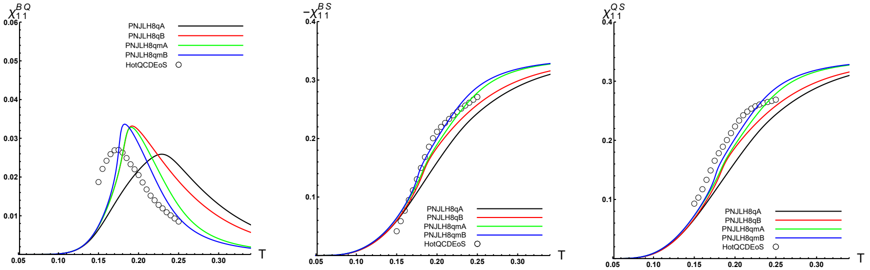


[15] HotQCD: A. Bazavov et al. (HotQCD), Phys. Rev. D86, 034509 (2012), arXiv:1203.0784 [hep-lat].

PNJL (Exp K-Log): $\chi_{1\,1}^{B\,Q}, \chi_{1\,1}^{B\,S}, \chi_{1\,1}^{Q\,S}|_{\mu=0}$ *vs* lQCD [15]

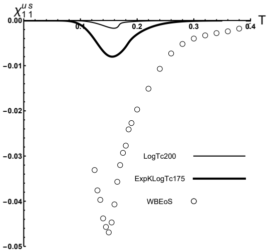
- $\chi_{11}^{BQ} = \frac{\partial^2\Omega/T^4}{\partial(\frac{\mu_B}{T})\partial(\frac{\mu_Q}{T})}$
- $\chi_{11}^{BS} = \frac{\partial^2\Omega/T^4}{\partial(\frac{\mu_B}{T})\partial(\frac{\mu_S}{T})}$
- $\chi_{11}^{QS} = \frac{\partial^2\Omega/T^4}{\partial(\frac{\mu_Q}{T})\partial(\frac{\mu_S}{T})}$



[15] HotQCD: A. Bazavov et al. (HotQCD), Phys. Rev. D86, 034509 (2012), arXiv:1203.0784 [hep-lat].

PNJL: $\chi_{11}^{us}|_{\mu=0}$ *vs* lQCD [16] : gluonic signature?

- $$\chi_{1\,1}^{u\,s} = \frac{\partial^2 \Omega / T^4}{\partial\left(\frac{\mu_u}{T}\right)\partial\left(\frac{\mu_s}{T}\right)}$$



[16] WB: S. Borsanyi, Z. Fodor, S. D. Katz, S. Krieg, C. Ratti, and K. Szabo, JHEP 01, 138 (2012), arXiv:1112.4416 [hep-lat].

Conclusions

- Multiquark interaction and full pattern of explicit chiral symmetry breaking play a key role in the reproduction of several key lQCD results
- Perfect fit across the board is not achieved with this Polykov potential but very promising results
- PNJL can however shift several results in temperature towards lQCD data
- Correlations in the *uds* base dissapear without Polyakov loop

Additional information:

- "Thermodynamical properties of strongly interacting matter in a model with explicit chiral symmetry breaking interactions" J. Moreira, J. Morais, B. Hiller, A.A. Osipov, A.H. Blin, e-Print: arXiv:1806.00327

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  - FCT through grant SFRH/BPD/110421/2015.
  - Networking support by the COST Action CA15213 THOR.
  - AFIF-Associação de Física de Interacções Fortes.

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