Evolution of multiplicity fluctuations

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COST THOR WG1+WG2 meeting
Lisbon

13.6.2018

Motivation

- Determination of chemical freeze-out T_{ch} and μ_B
  - from particle multiplicities or their ratios (first moments of multiplicity distribution)
  - from higher moments of multiplicity distribution – directly put in connection with simulations of lattice QCD
- disagreement in T extracted by these two methods – T from fluctuations seems to be lower
- Can we have an ensemble of systems with ongoing chemical reactions (inelastic collisions) where different moments of multiplicity distribution evolve differently?
- (Such a situation could arise in cooling fireball.)

The tool: master equation

- describes the evolution of $P_n(t)$ – the probability to have n pieces of the studied species
- applied in this study: process of the type $a_1 + a_2 \leftrightarrow b_1 + b_2$
  - b_1, b_2 conserve $U(1)$ charge
  - e.g. strangeness production $\pi + N \leftrightarrow K + \Lambda$
  - n is the number of b_1, b_2 pairs
- the master equation

$$\frac{dP_n}{dt} = \frac{G}{V}\langle N_{a_1}\rangle\langle N_{a_2}\rangle\left[P_{n-1} - P_n\right] - \frac{L}{V}\left[n^2 P_n - (n+1)^2 P_{n+1}\right]$$

$$\{\text{creation of } b\text{-pair}\} \quad - \quad \{\text{annihilation of } b\text{-pair}\}$$

- $G = \langle\sigma_G v_{rel}\rangle$, $L = \langle\sigma_L v_{rel}\rangle$
- assumes heatbath of species a_1, a_2
- V is the proper volume

[C.M. Ko, *et al.*, Phys. Rev. Lett. **86** (2001) 5438]

Rescaling the time variable

Introduce

$$t_0 = \frac{V}{L} \qquad \tau = \frac{t}{t_0} \qquad \varepsilon = \frac{G}{L}\langle N_{a_1}\rangle\langle N_{a_2}\rangle$$

Master equation in dimensionless time

$$\frac{dP_n}{d\tau} = \varepsilon\left[P_{n-1} - P_n\right] - \left[n^2 P_n - (n+1)^2 P_{n+1}\right]$$

- depends only on ε
- describes common behaviour for any set of reactants

Best suitable for studying relaxation and equilibrium values.

[S. Jeon, et al., Nucl. Phys. A **697** (2002) 546]

Generating function

Introduce

$$g(x, \tau) = \sum_{n=0}^{\infty} x^n P_n(\tau)$$

Master equation is rewritten into the form

$$\frac{\partial g(x, \tau)}{\partial \tau} = \frac{L}{V}(1 - x)(xg'' + g' - \varepsilon g)$$

where $g' = \frac{\partial g}{\partial x}$

Equilibrium solution must obey:

$$xg_{eq}''(x) + g_{eq}'(x) - \varepsilon g_{eq}(x) = 0$$

The equilibrium solution:

$$g_{eq}(x) = \frac{I_0(2\sqrt{\varepsilon x})}{I_0(2\sqrt{\varepsilon})}$$

[C.M. Ko, *et al.*, Phys. Rev. Lett. **86** (2001) 5438]

Equilibrium values of factorial moments

Factorial moments are calculated from the generating function as

$$\langle n(n-1)\ldots(n-m+1)\rangle(\tau) = \frac{\partial^m g(x,\tau)}{\partial x^m}\bigg|_{x=1}$$

Derived equilibrium values

$$\langle n\rangle_{eq} = \frac{\partial g_{eq}(x)}{\partial x}\bigg|_{x=1} = \sqrt{\varepsilon}\frac{I_1(2\sqrt{\varepsilon})}{I_0(2\sqrt{\varepsilon})}$$

$$\langle n(n-1)\rangle_{eq} = \frac{\partial^2 g_{eq}(x)}{\partial x^2}\bigg|_{x=1} = -\frac{1}{2}\sqrt{\varepsilon}\frac{I_1(2\sqrt{\varepsilon})}{I_0(2\sqrt{\varepsilon})} + \frac{1}{2}\varepsilon\frac{I_2(2\sqrt{\varepsilon}) + I_0(2\sqrt{\varepsilon})}{I_0(2\sqrt{\varepsilon})}$$

[S. Jeon, *et al.*, Nucl. Phys. A **697** (2002) 546]

Relaxation of scaled factorial moments

Observe scaled factorial moments

$$
\begin{aligned}
F_2(\tau) &= \langle n(n-1) \rangle / \langle n \rangle^2 \\
F_3(\tau) &= \langle n(n-1)(n-2) \rangle / \langle n \rangle^3 \\
F_4(\tau) &= \langle n(n-1)(n-2)(n-3) \rangle / \langle n \rangle^4
\end{aligned}
$$

Binomial initial conditions

- at most one b-pair per event
- average number of b pairs: N_0

$$
\begin{aligned}
P_0(\tau = 0) &= 1 - N_0 \\
P_1(\tau = 0) &= N_0 \\
P_m(\tau = 0) &= 0 \qquad m > 1
\end{aligned}
$$

Relaxation of scaled factorial moments: results

parameters of calculation: $\varepsilon = 0.1$ and $N_0 = 0.005$
Plot: scaled factorial moments divided by their equilibrium values
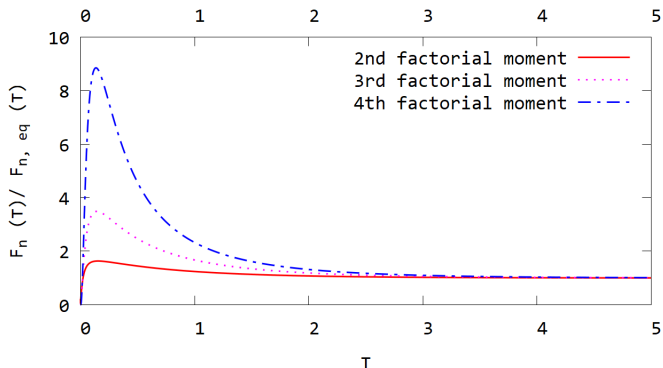
All moments relax at the same time.
Higher moments fluctuate further away from the equilibrium value.

# An expanding and cooling fireball

- assume simple 1D Bjorken expansion

$$T^3(t) = T_0^3 \frac{t_0}{t} \qquad V(t) = V_0 \frac{t}{t_0}$$

- assume kinetic freeze-out (end of evolution) at
  - $T = 100$ MeV
  - $t = 10$ fm/c
- intial values: chemical freeze-out
  - $T_0 = 165$ MeV
  - $t_0 = 2.2$ fm/c (calculated from Bjorken model)
  - $V_0 = 125$ fm^3
- time scale introduced $\Rightarrow$ need to specify the reaction and cross-section

The chemical reaction investigated

$\pi + N \leftrightarrow K + \Lambda$
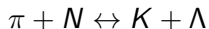
- vacuum cross section assumed (*plan to use in-medium values in the future*)

- high reaction threshold at these temperatures

- relaxation time much longer than expansion time scale

- want to study this system also as proxy for other chemical reactions

- $\Rightarrow$ a quick solution: scale the $\langle \sigma v_{rel} \rangle$ term
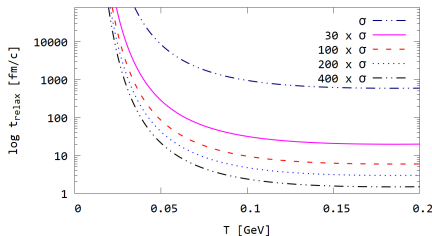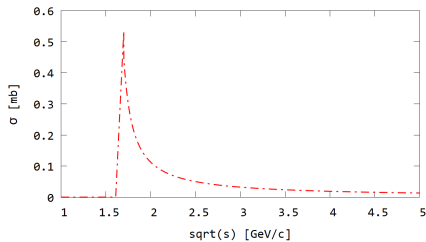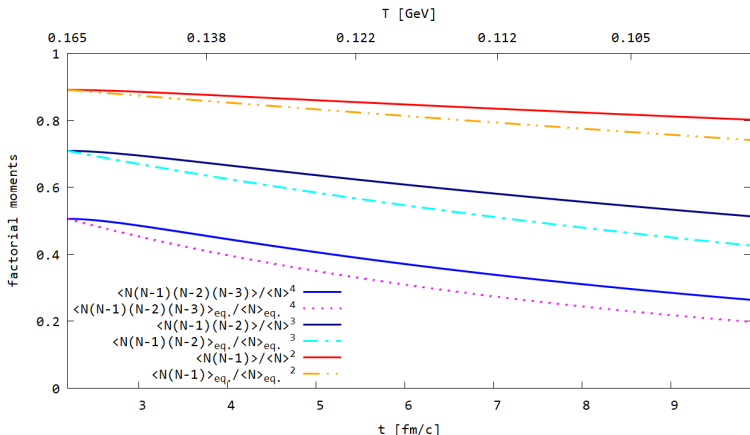
Time evolution of F_2 with different scales of σ
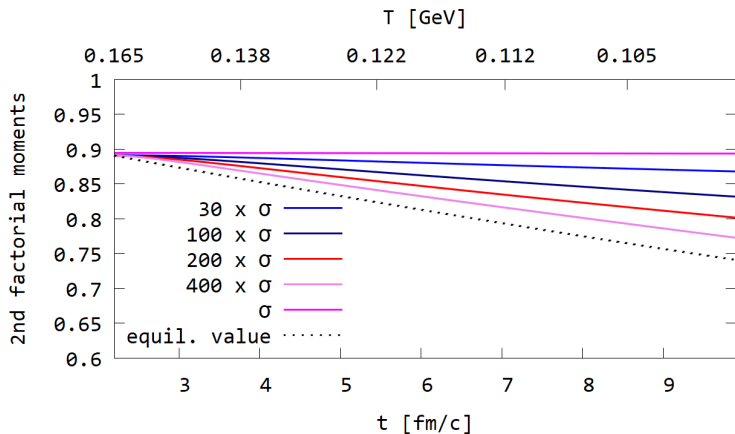
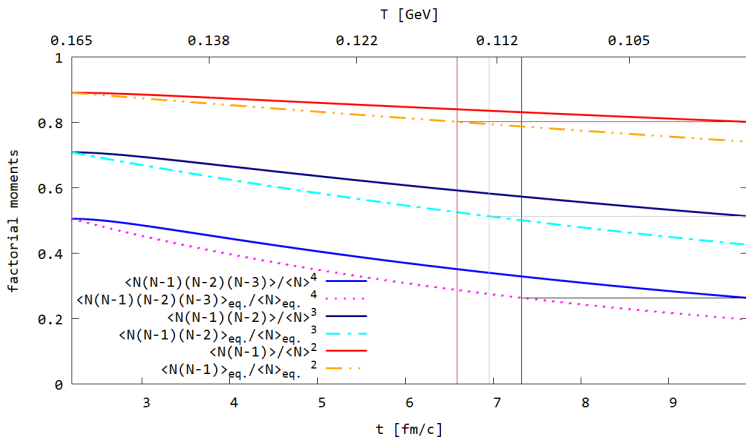# Time evolution of different orders of factorial moments

$\langle \sigma v_{rel} \rangle$ scaled 200x, $\langle N_\pi \rangle = 15$, $\langle N_N \rangle = 10$

The relative deviation from equilibrium value is higher for higher moments.

The seeming freeze-out temperature

What happens if a particular observed value of a factorial moment is interpreted as coming from thermalised system?
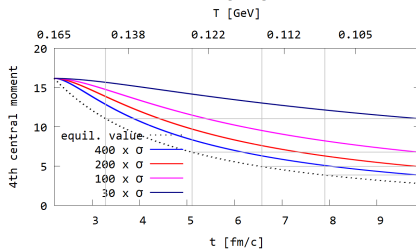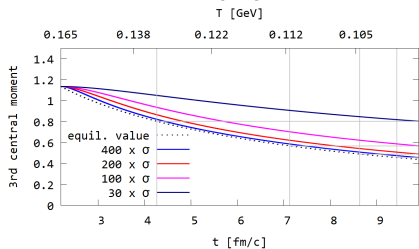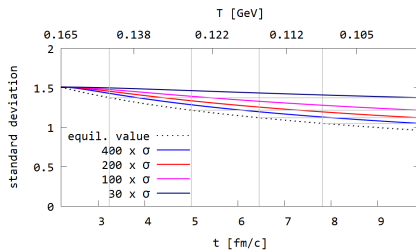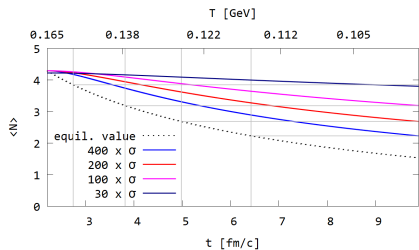

Each individual moment leads to a different (seeming) FO temperature!

Central moments and volume-independent ratios

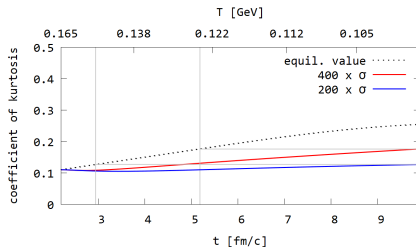Measured experimentally and determined in lattice QCD.

- $\mu_2 = \langle N^2 \rangle - \langle N \rangle^2$.
- $\mu_3 = \langle (N - \langle N \rangle)^3 \rangle$.
- $\mu_4 = \langle (N - \langle N \rangle)^4 \rangle$.

- Coefficient of skewness $S = \dfrac{\mu_3}{\sigma^3} = \dfrac{\langle (N - \langle N \rangle)^3 \rangle}{\langle (N - \langle N \rangle)^2 \rangle^{3/2}}$.

- Coefficient of kurtosis $\kappa = \dfrac{\mu_4}{\sigma^4} - 3 = \dfrac{\langle (N - \langle N \rangle)^4 \rangle}{\langle (N - \langle N \rangle)^2 \rangle^2} - 3$.

Time evolution of central moments

Different seeming FO temperatures than from factorial moments!

Seeming FO temperature from skewness and kurtosis
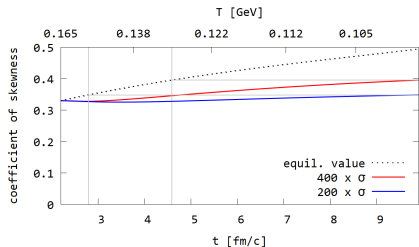
Equilibrium value is increasing with decreasing temperature!

Evolution with smaller cross sections never meets the equilibrium value.

The extracted seeming FO temperature depends on the (combination of) moments used!

Isospin randomisation

- proton number fluctuations are measured as a proxy for baryon number fluctuations
- protons can be turned into neutrons and vice versa
- apply the master equation to the reaction

$$p + \pi^0 \leftrightarrow \Delta^+ \leftrightarrow n + \pi^+$$

- process not appropriate for this master equation, because $I_3 = -1/2$ channel not included
- large cross section and no threshold $\Rightarrow$ fast process
- despite of all this: calculate to get an *educated guess*

Moments of proton number multiplicity in cooling fireball
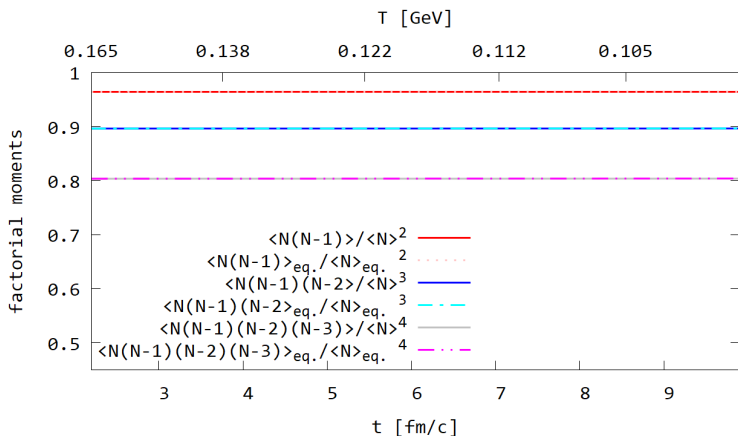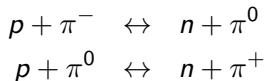

No change of any moments of the multiplicity distribution!

- proton number fluctuations are not affected by isospin randomisation
- the argument survives also when calculation is done with the appropriate master equation

Derivation of the master equations

Consider the isospin randomisation reactions

$$p + \pi^- \quad \leftrightarrow \quad n + \pi^0$$
$$p + \pi^0 \quad \leftrightarrow \quad n + \pi^+$$

5 species: p, n, π^-, π^0, π^+
3 constraints (conserved numbers):

I_3 isospin

n_N number of nucleons

n_π number of all pions

choose 2 independent variables:

a number of protons

b number of π^-

denote reaction rates:

k for $p + \pi^- \to n + \pi^0$

l for $p + \pi^- \leftarrow n + \pi^0$

m for $p + \pi^0 \to n + \pi^+$

n for $p + \pi^0 \leftarrow n + \pi^+$

Follow the evolution of $P(a, b; t)$.
8 entries in the master equation:
creation and annihilation of a state
with (a, b) protons and π^-
along each of the arrows.

The master equation for isospin randomisation

$$
\begin{aligned}
\frac{dP(a, b; t)}{dt} &= k(a+1)(b+1)P(a+1, b+1; t) \\
&+ l(n_N - a + 1)\left(n_\pi - I_3 - \frac{n_N}{2} + a - 2b + 1\right) \\
&\qquad\qquad\qquad\qquad\qquad P(a-1, b-1; t) \\
&+ m(a+1)\left(n_\pi - I_3 - \frac{n_N}{2} + a - 2b + 1\right)P(a+1, b; t) \\
&+ n(n_N - a + 1)\left(n_\pi - I_3 - \frac{n_N}{2} + a - 2b + 1\right)P(a-1, b; t) \\
&- \left[kab + l(n_N - a)\left(n_\pi - I_3 - \frac{n_N}{2} + a - 2b\right)\right. \\
&\quad + ma\left(n_\pi - I_3 - \frac{n_N}{2} + a - 2b\right) \\
&\quad \left. + n(n_N - a)\left(n_\pi - I_3 - \frac{n_N}{2} + a - 2b\right)\right]P(a, b; t)
\end{aligned}
$$

Master equation for simple systems can quickly become complicated.

Summary

- Determining chemical freeze-out parameters from higher moments can be misleading if due to expansion and cooling multiplicity distribution is out of equilibrium.

- Proton number distribution is not altered through isospin randomisation into neutrons.
  also [M. Kitazawa, M. Asakawa PRC **85** (2012) 021901(R) and **86** (2012) 024904]

- Use the formalism for studies of fluctuations of *rare* species with only few ways of being produced and annihilated.

BACKUP SLIDES

Equilibrium values of 3 and 4 order factorial moment

$$
\begin{aligned}
\langle n(n-1)(n-2)\rangle_{eq} = g_{eq}'''(1) \;=\;& \frac{3}{4}\sqrt{\varepsilon}\frac{I_1(2\sqrt{\varepsilon})}{I_0(2\sqrt{\varepsilon})} - \frac{5}{4}\varepsilon\frac{I_2(2\sqrt{\varepsilon})+I_0(2\sqrt{\varepsilon})}{I_0(2\sqrt{\varepsilon})} \\
& + \frac{1}{4}\varepsilon^{3/2}\frac{I_3(2\sqrt{\varepsilon})+3I_1(2\sqrt{\varepsilon})}{I_0(2\sqrt{\varepsilon})}
\end{aligned}
$$

$$
\begin{aligned}
\langle n(n-1)(n-2)(n-3)\rangle_{eq} \;=\;& g_{eq}^{(4)}(1) = \frac{23}{8}\varepsilon\frac{I_2(2\sqrt{\varepsilon})+I_0(2\sqrt{\varepsilon})}{I_0(2\sqrt{\varepsilon})} \\
& -\frac{15}{8}\sqrt{\varepsilon}\frac{I_1(2\sqrt{\varepsilon})}{I_0(2\sqrt{\varepsilon})} \\
& -\varepsilon^{3/2}\frac{4I_3(2\sqrt{\varepsilon})+7I_1(2\sqrt{\varepsilon})}{4I_0(2\sqrt{\varepsilon})} \\
& +\frac{1}{8}\varepsilon^2\frac{I_4(2\sqrt{\varepsilon})+2I_2(2\sqrt{\varepsilon})+I_0(2\sqrt{\varepsilon x})}{I_0(2\sqrt{\varepsilon})}
\end{aligned}
$$