Baryons at finite temperature from the lattice

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Mesons in a medium

mesons in a medium very well studied

- hadronic phase: thermal broadening, mass shift
- QGP: deconfinement/dissolution/melting
- quarkonia survival as thermometer
- transport: conductivity/dileptons from vector current
- chiral symmetry restoration

relatively easy on the lattice

- high-precision correlators

what about baryons?

Baryons in a medium

lattice studies of baryons at finite temperature very limited

- screening masses De Tar and Kogut 1987
- ... with a small chemical potential QCD-TARO: Pushkina, de Forcrand, Kim, Nakamura, Stamatescu et al 2005
- temporal correlators Datta, Gupta, Mathur et al 2013

not much more ...

- effective models, mostly at $T \sim 0$ and nuclear density $\Rightarrow$ parity doubling models De Tar & Kunihiro 89 Mukherjee, Schramm, Steinheimer & Dexheimer, Sasaki 17

but understanding highly relevant for e.g. hadron resonance gas (HRG) descriptions in confined phase (e.g. talk by Alba)

Outline

baryons across the deconfinement transition:

- baryon correlators
- FASTSUM collaboration
- in-medium effects below T_c
- parity doubling above T_c
- spectral functions

FASTSUM: PRD 92 (2015) 014503 [arXiv:1502.03603 [hep-lat]]

+ JHEP 06 (2017) 034 [arXiv:1703.09246 [hep-lat]]

+ in preparation

Baryons

- correlators $$G^{\alpha\alpha'}(x) = \langle O^\alpha(x)\,\overline{O}^{\alpha'}(0)\rangle$$
- examples: N, Δ, Ω baryons

$$O_N^\alpha(x) = \epsilon_{abc}\, u_a^\alpha(x) \left(d_b^T(x) C\gamma_5 u_c(x)\right)$$

$$O_{\Delta,i}^\alpha(x) = \epsilon_{abc} \left[2u_a^\alpha(x) \left(d_b^T(x) C\gamma_i u_c(x)\right) + d_a^\alpha(x) \left(u_b^T(x) C\gamma_i u_c(x)\right)\right]$$

$$O_{\Omega,i}^\alpha(x) = \epsilon_{abc}\, s_a^\alpha(x) \left(s_b^T(x) C\gamma_i s_c(x)\right)$$

- essential difference with mesons: role of parity

$$\mathcal{P} O(\tau, \mathbf{x}) \mathcal{P}^{-1} = \gamma_4 O(\tau, -\mathbf{x})$$

- positive/negative parity operators

$$O_\pm(x) = P_\pm O(x) \qquad P_\pm = \frac{1}{2}(1 \pm \gamma_4)$$

Baryons

- positive/negative parity operators

$$O_\pm(x) = P_\pm O(x) \qquad\qquad P_\pm = \frac{1}{2}(1 \pm \gamma_4)$$

- no parity doubling in Nature: nucleon ground state

positive parity: $m_+ = m_N \;\; = 0.939$ GeV

negative parity: $m_- = m_{N^*} = 1.535$ GeV

- thread: what happens as temperature increases?

how are pos/neg parity states encoded in correlators?

$$G_\pm(x-x') = \langle \mathrm{tr} P_\pm O(x)\overline{O}(x')\rangle \qquad \rho_\pm(x-x') = \langle \mathrm{tr} P_\pm \{O(x), \overline{O}(x')\}\rangle$$

Charge conjugation

charge conjugation symmetry (at vanishing density):

$$G_{\pm}(\tau, \mathbf{p}) = -G_{\mp}(1/T - \tau, \mathbf{p}) \qquad \rho_{\pm}(-\omega, \mathbf{p}) = -\rho_{\mp}(\omega, \mathbf{p})$$

- relates pos/neg parity channels

using $G_{+}(\tau, \mathbf{p})$ and $\rho_{+}(\omega, \mathbf{p})$

- positive- (negative-) parity states propagate forward (backward) in euclidean time
- negative part of spectrum of $\rho_{+} \leftrightarrow$ positive part of ρ_{-}

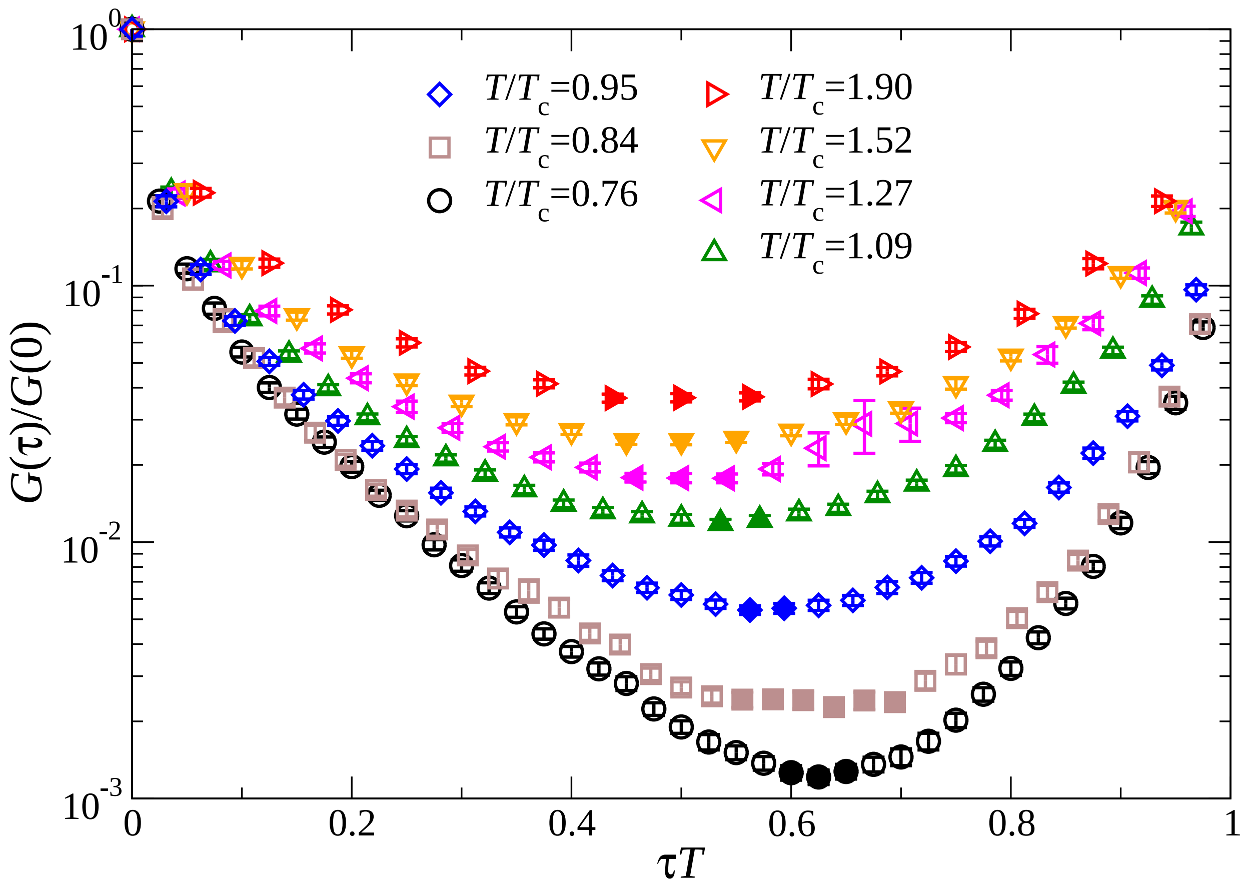
example: single state

$$G_{+}(\tau) = A_{+}e^{-m_{+}\tau} + A_{-}e^{-m_{-}(1/T-\tau)}$$

$$\rho_{+}(\omega)/(2\pi) = A_{+}\delta(\omega - m_{+}) + A_{-}\delta(\omega + m_{-})$$

Nucleon correlators

- euclidean correlator $G_+(\tau)$



- not symmetric around $\tau = 1/2T$ below T_c
- more symmetric as temperature increases

Chiral symmetry

- propagator

$$G(x) = \sum_{\mu} \gamma_\mu G_\mu(x) + \mathbb{1} G_m(x)$$

- chiral symmetry $\qquad \{\gamma_5, G\} = 0 \quad \Rightarrow \quad G_m = 0$
- hence

$$G_+(\tau, \mathbf{p}) = -G_-(\tau, \mathbf{p}) = G_+(1/T - \tau, \mathbf{p}) = 2G_4(\tau, \mathbf{p})$$

degeneracy of $\pm$ parity channels

$$\rho_+(p) = -\rho_-(p) = \rho_+(-p) = 2\rho_4(p)$$

- parity doubling
- in Nature at $T = 0$: no chiral symmetry/parity doubling

Parity and chiral symmetry

however, if chiral symmetry is unbroken ($m_q = 0$ and no SSB)

- degeneracy between pos/neg parity channels already at the level of the correlators

what happens at the confinement/deconfinement transition?

- SU(2)$_\mathrm{A}$ chiral symmetry restored
- expect degeneracies to emerge
- how does this affect mass spectrum?
- role of $m_s > m_{u,d}$?

FASTSUM

- anisotropic $N_f = 2 + 1$ Wilson-clover ensembles

FASTSUM collaboration

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This work

GA, Chris Allton, Simon Hands, Kristi Praki, Jonivar Skullerud

Davide de Boni, Benjamin Jäger

PRD 92 (2015) 014503, arXiv:1502.03603 [hep-lat]

JHEP 06 (2017) 034, arXiv:1703.09246 [hep-lat]

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FASTSUM ensembles

- $N_f = 2 + 1$ dynamical quark flavours, Wilson-clover
- many temperatures, below and above T_c
- anisotropic lattice, $a_s/a_\tau = 3.5$, many time slices
- strange quark: physical value
- two light flavours: somewhat heavy $m_\pi = 384(4)$ MeV

N_s	24	24	24	24	24	24	24	24
N_τ	128	40	36	32	28	24	20	16
T/T_c	0.24	0.76	0.84	0.95	1.09	1.27	1.52	1.90
$N_{\rm cfg}$	140	500	500	1000	1000	1000	1000	1000
$N_{\rm src}$	16	4	4	2	2	2	2	2

- tuning and $N_\tau = 128$ data from HadSpec collaboration

Baryon correlators

computed all octet and decuplet baryon correlators

$S = 0$:	N	Δ	
$S = -1$:	Λ	Σ	Σ^*
$S = -2$:	Ξ	Ξ^*	
$S = -3$:	Ω		

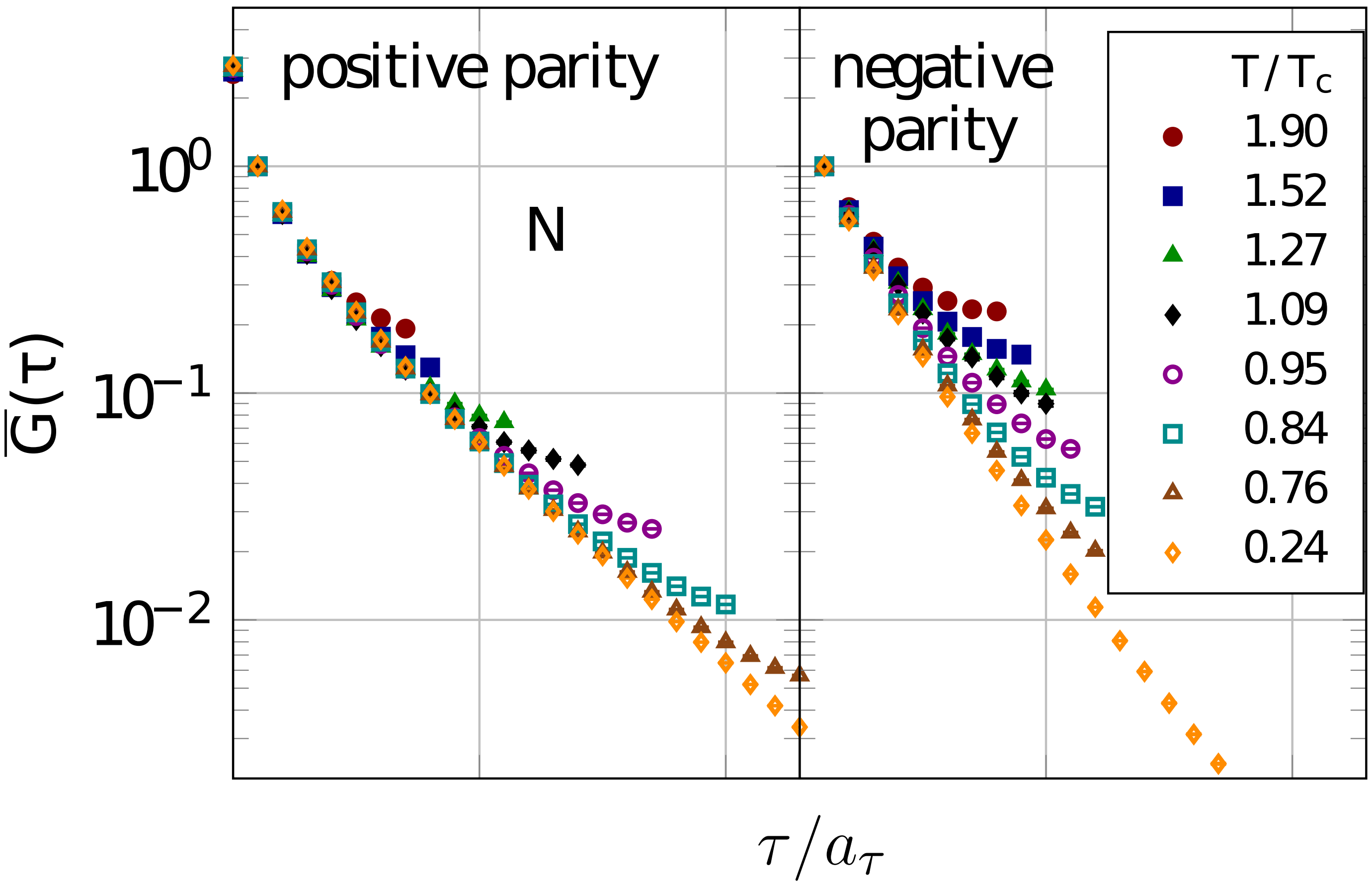
for each baryon: positive and negative parity channels

technical remarks

- studied various interpolation operators
- Gaussian smearing for multiple sources and sinks
- same smearing parameters at all temperatures

Lattice correlators

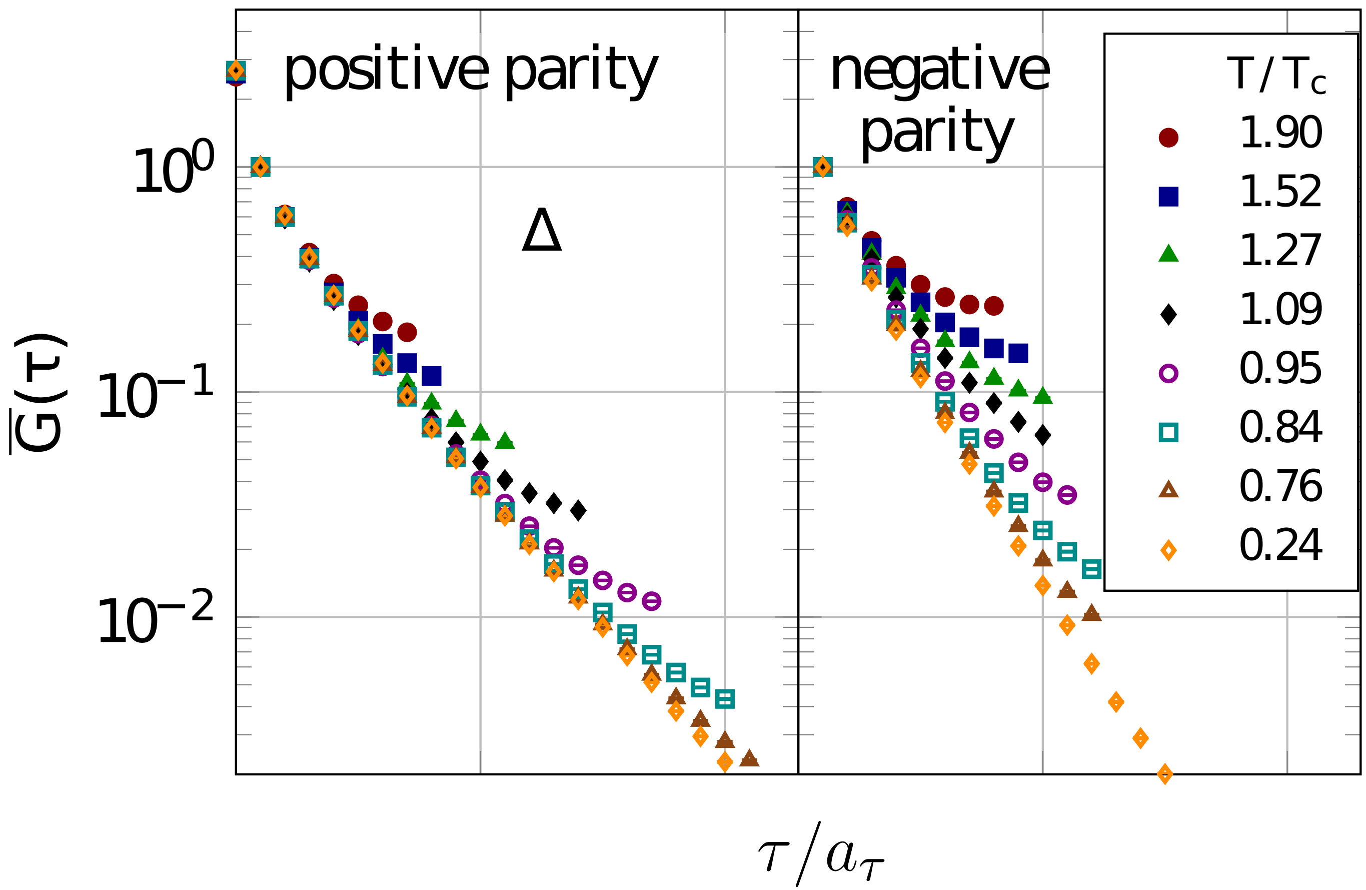
- nucleon



- pos/neg parity channels nondegenerate
- more T dependence in negative-parity channel

Lattice correlators

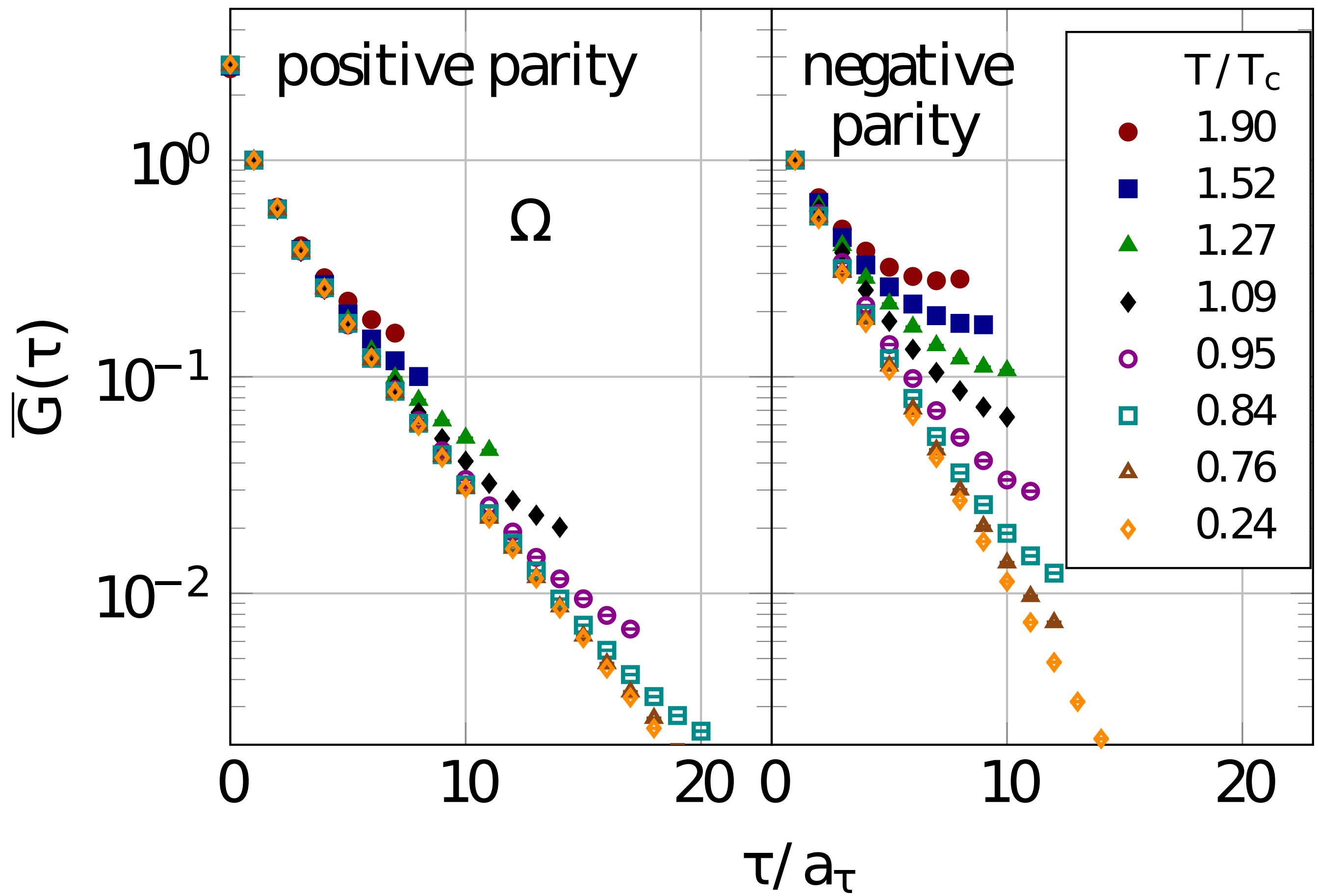
- Δ



- at low T pos/neg parity channels nondegenerate
- more T dependence in negative-parity channel

Lattice correlators

- Ω



- at low T pos/neg parity channels nondegenerate
- more T dependence in negative-parity channel

Baryons in the hadronic phase

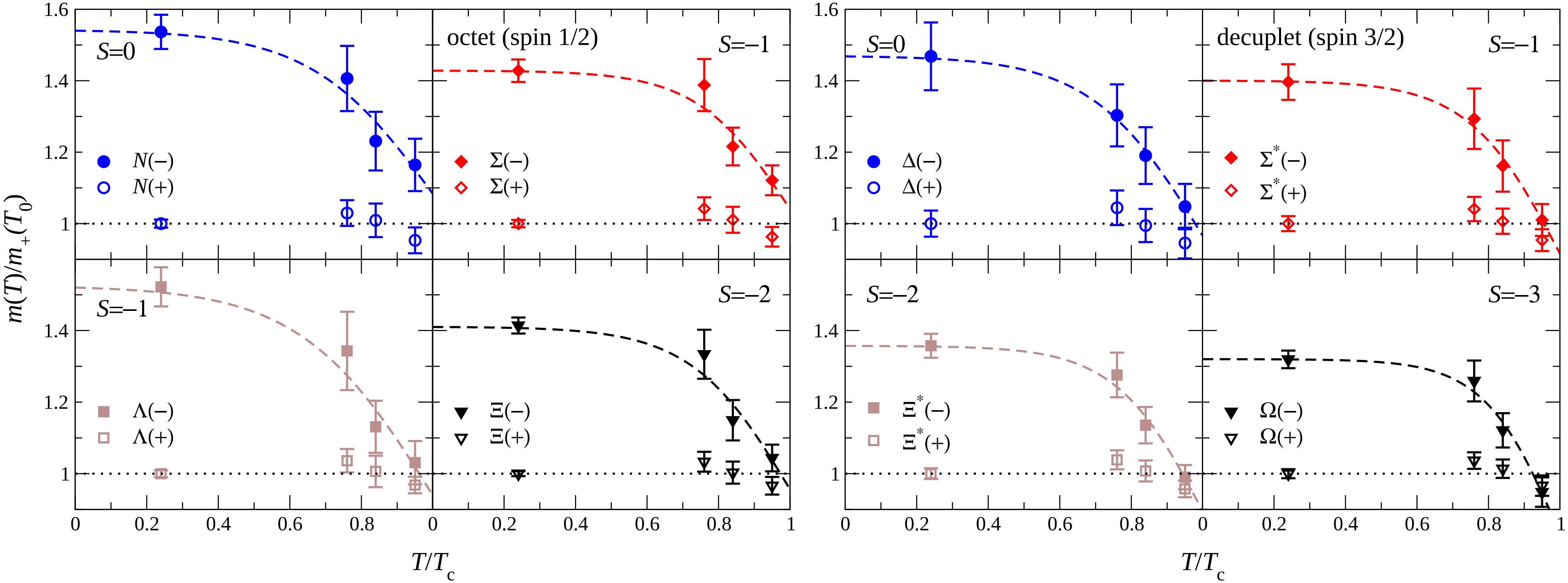
- determine masses of pos/neg parity groundstates
- in-medium effects

Masses of pos/neg parity groundstates (in MeV)

S	T/T_c	0.24	0.76	0.84	0.95	PDG ($T=0$)
0	m_+^N	1158(13)	1192(39)	1169(53)	1104(40)	939
	m_-^N	1779(52)	1628(104)	1425(94)	1348(83)	1535
	m_+^Δ	1456(53)	1521(43)	1449(42)	1377(37)	1232
	m_-^Δ	2138(114)	1898(106)	1734(97)	1526(74)	1700
-1	m_+^Σ	1277(13)	1330(38)	1290(44)	1230(33)	1193
	m_-^Σ	1823(35)	1772(91)	1552(65)	1431(51)	1750
	m_+^Λ	1248(12)	1293(39)	1256(54)	1208(26)	1116
	m_-^Λ	1899(66)	1676(136)	1411(90)	1286(75)	1405–1670
	$m_+^{\Sigma^*}$	1526(32)	1588(40)	1536(43)	1455(35)	1385
	$m_-^{\Sigma^*}$	2131(62)	1974(122)	1772(103)	1542(60)	1670–1940
-2	m_+^Ξ	1355(9)	1401(36)	1359(41)	1310(32)	1318
	m_-^Ξ	1917(27)	1808(92)	1558(76)	1415(50)	1690–1950
	$m_+^{\Xi^*}$	1594(24)	1656(35)	1606(40)	1526(29)	1530
	$m_-^{\Xi^*}$	2164(42)	2034(95)	1810(77)	1578(48)	1820
-3	m_+^Ω	1661(21)	1723(32)	1685(37)	1606(43)	1672
	m_-^Ω	2193(30)	2092(91)	1863(76)	1576(66)	2250

Baryons in the hadronic phase

masses $m_{\pm}(T)$, normalised with m_+ at lowest temperature



in each channel:

- emerging degeneracy around T_c
- negative-parity masses reduced as T increases
- positive-parity masses nearly T independent

Baryons in the hadronic phase

findings

- positive-parity masses nearly T independent
- negative-parity masses reduced as T increases
- characteristic behaviour

$$m_-(T) = w(T,\gamma)m_-(0) + [1 - w(T,\gamma)]m_-(T_c)$$

with one-parameter transition function

$$w(T,\gamma) = \tanh[(1 - T/T_c)/\gamma]/\tanh(1/\gamma)$$

- small (large) $\gamma \Leftrightarrow$ narrow (broad) transition region

fits in each channel

- $0.22 \lesssim \gamma \lesssim 0.35$, mean $\gamma = 0.27(1)$
- $0.85 \lesssim m_-(T_c)/m_+(0) \lesssim 1.1$

Baryons and parity partners

- distinct temperature dependence in hadronic phase
- understand further using
  - effective parity doublet models?
  - holography?
- relevant for heavy-ion phenomenology?

Baryons and parity partners

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  - holography?
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application to HRG

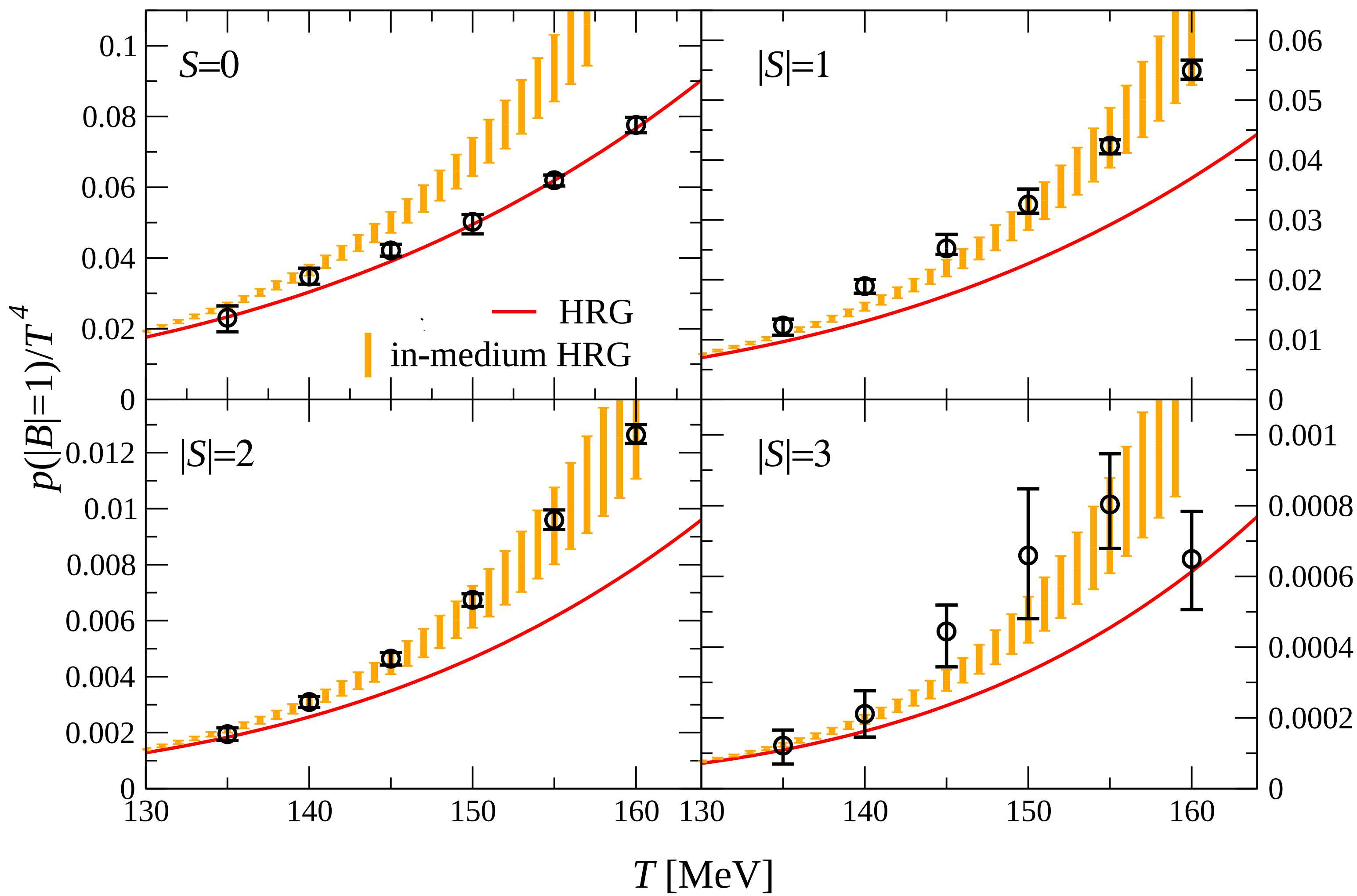
- use states in PDG (not QM)
- T-dependent groundstates in neg parity channels

$$m_-(T) = w(T,\gamma)m_-(0) + [1 - w(T,\gamma)]m_-(T_c)$$

with $\gamma = 0.3$ and $1 < m_-(T_c)/m_+(0) < 1.1$

In-medium HRG

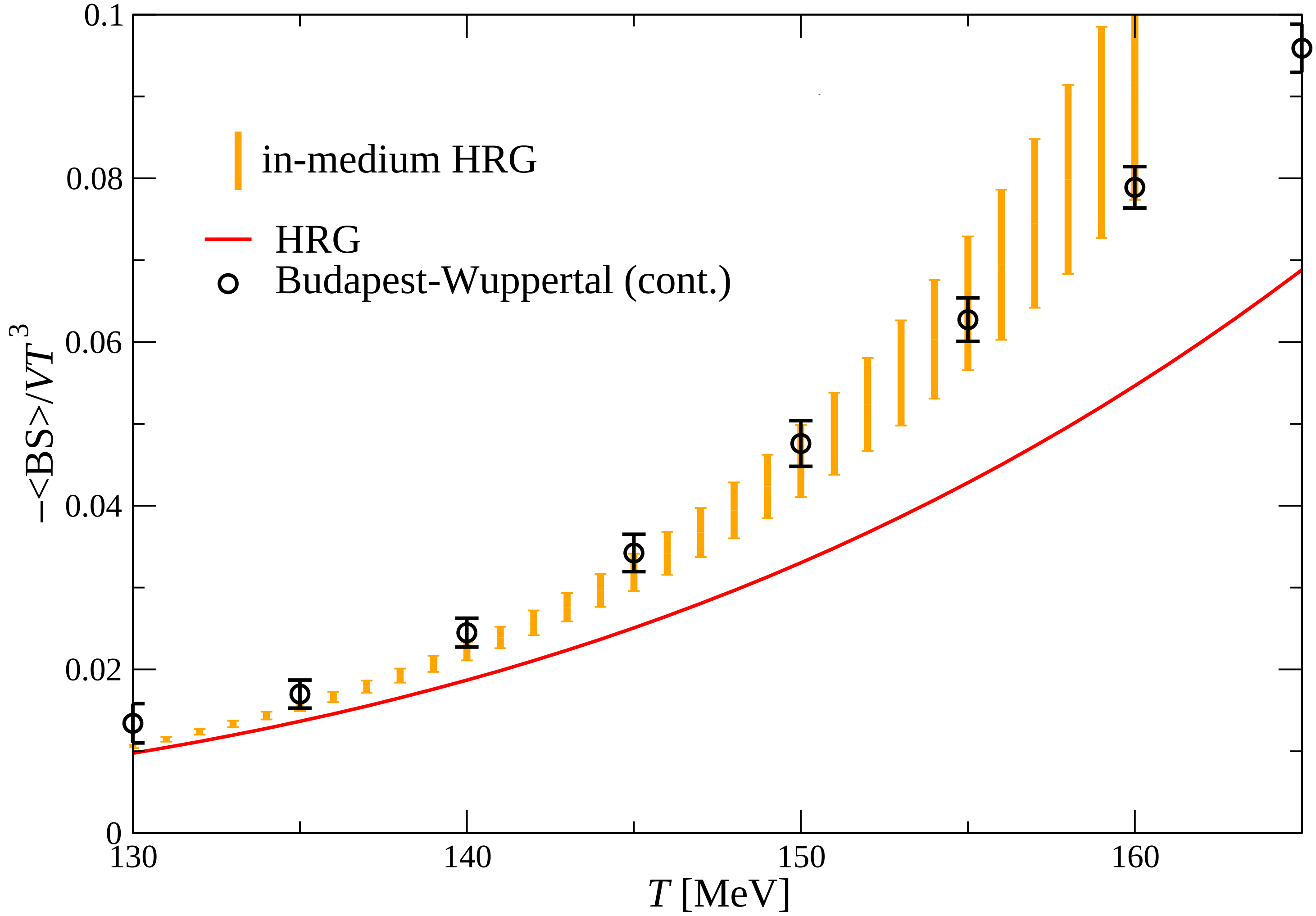
contributions to pressure from baryons with strangeness



compare with lattice data from Alba, Ratti et al, 1702.01113

In-medium HRG

fluctuations of strange baryons $\langle BS \rangle$



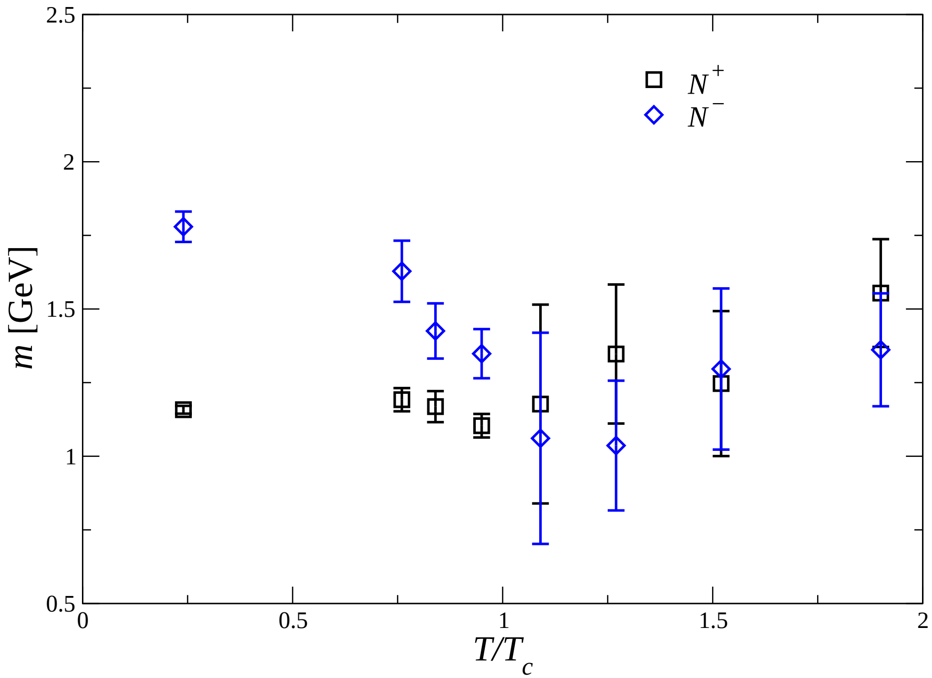
compare with lattice data from Budapest-Wuppertal

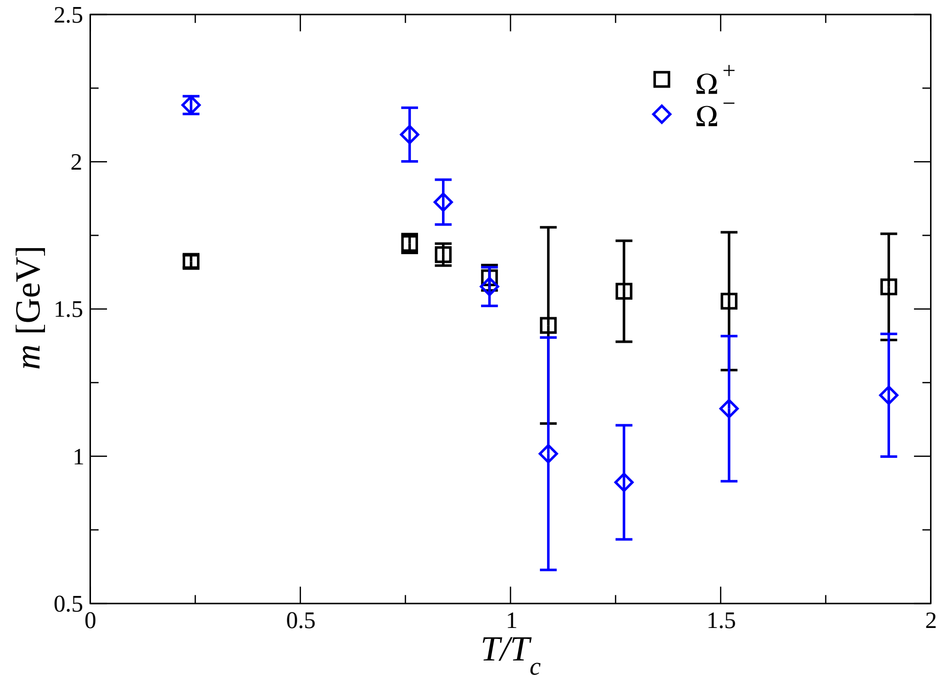
QGP: fate of light baryons

consider now the quark-gluon plasma

- no clearly identifiable groundstates: baryons dissolved

example: use conventional exponential fits





no clearly defined groundstates above T_c

QGP: fate of light baryons

- no clearly identifiable groundstates: baryons dissolved
- chiral symmetry restoration $\Leftrightarrow$ parity doubling
- study correlator ratio Datta, Gupta, Mathur et al 2013

$$R(\tau) = \frac{G_+(\tau) - G_-(\tau)}{G_+(\tau) + G_-(\tau)}$$

- no parity doubling and $m_- \gg m_+$: $R(\tau) = 1$
- parity doubling: $R(\tau) = 0$

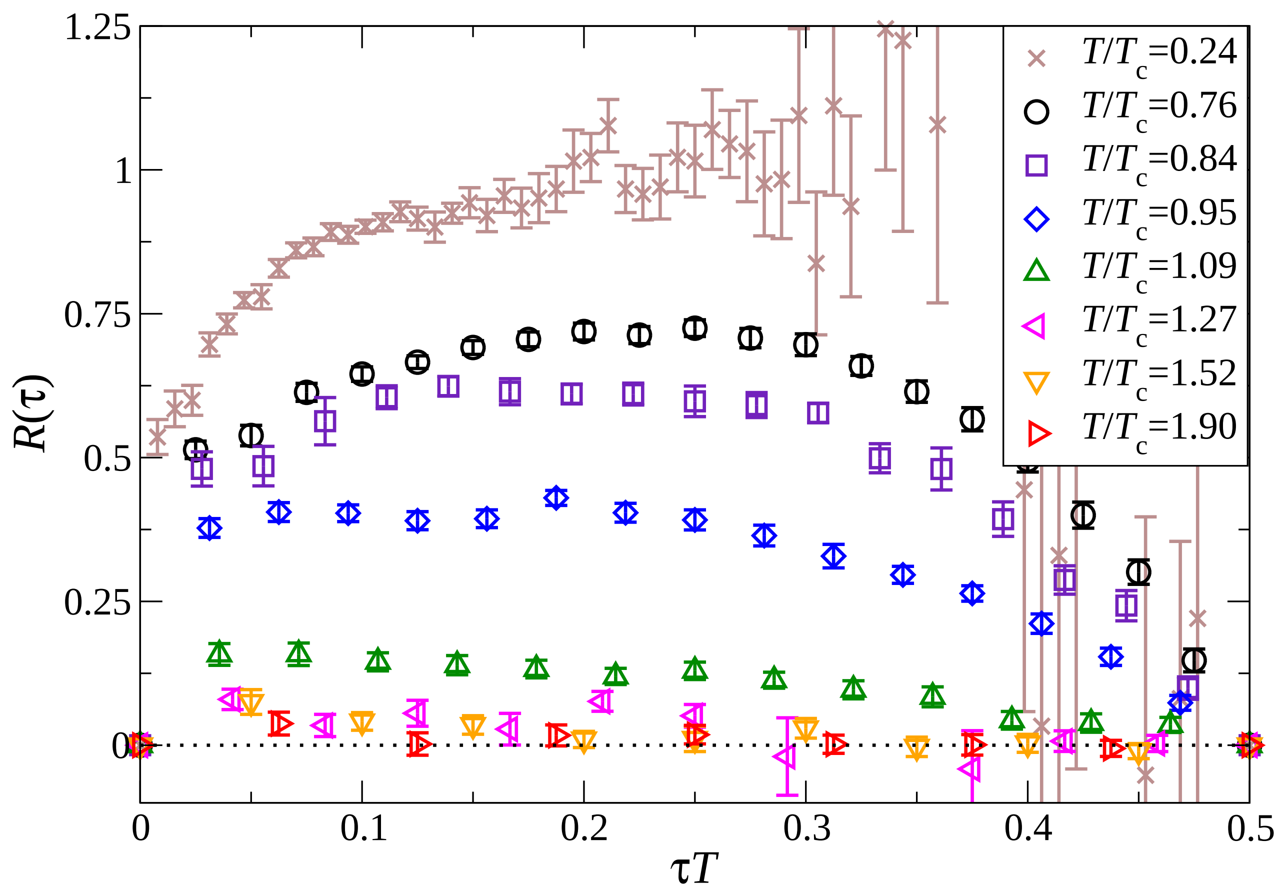
by construction: $R(1/T - \tau) = -R(\tau)$ and $R(1/2T) = 0$

- integrated ratio

$\Rightarrow$ quasi-order parameter

$$R = \frac{\sum_n R(\tau_n)/\sigma^2(\tau_n)}{\sum_n 1/\sigma^2(\tau_n)}$$

Nucleon channel

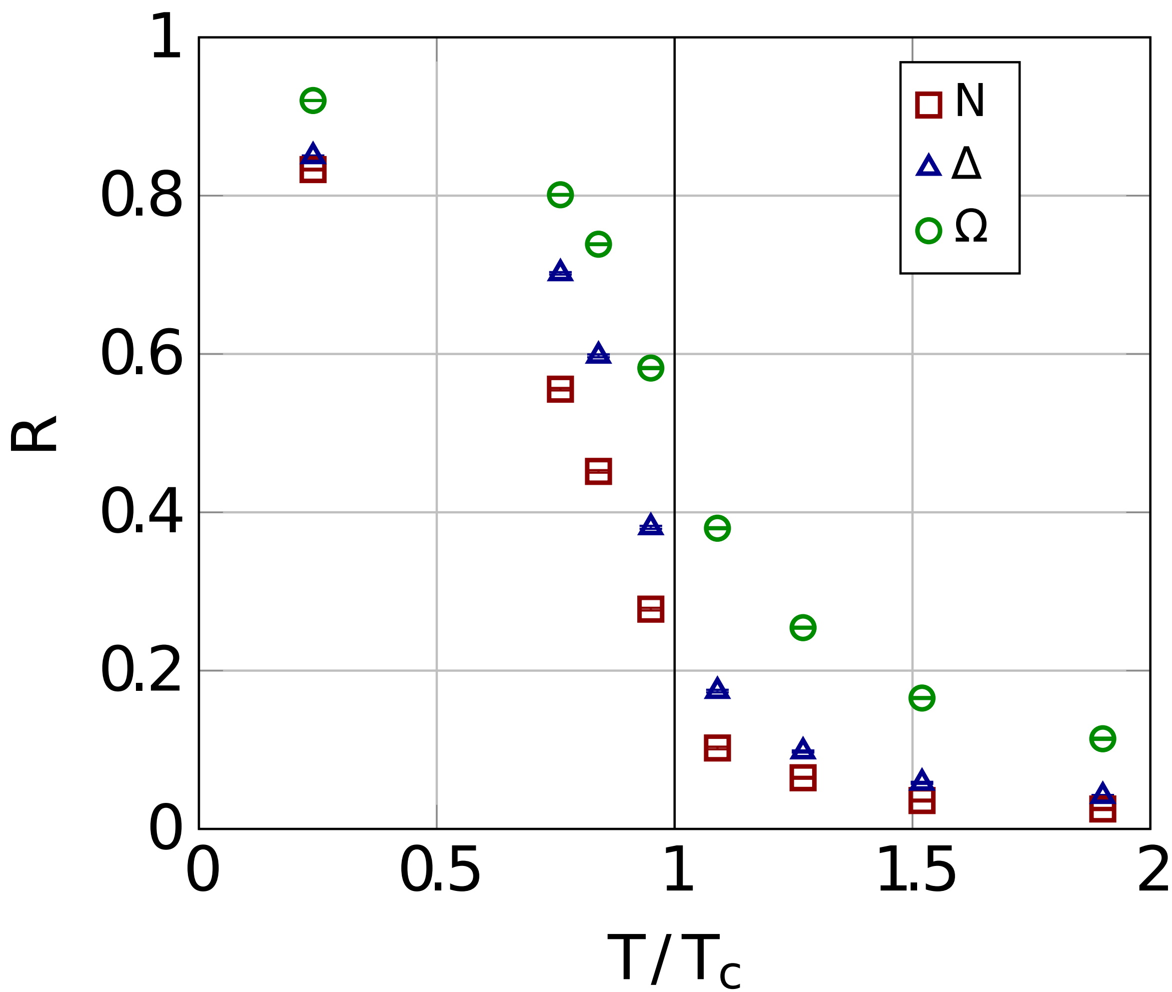


- ratio close to 1 below T_c, decreasing uniformly
- ratio close to 0 above T_c, parity doubling

Quasi-order parameter

- integrated ratio

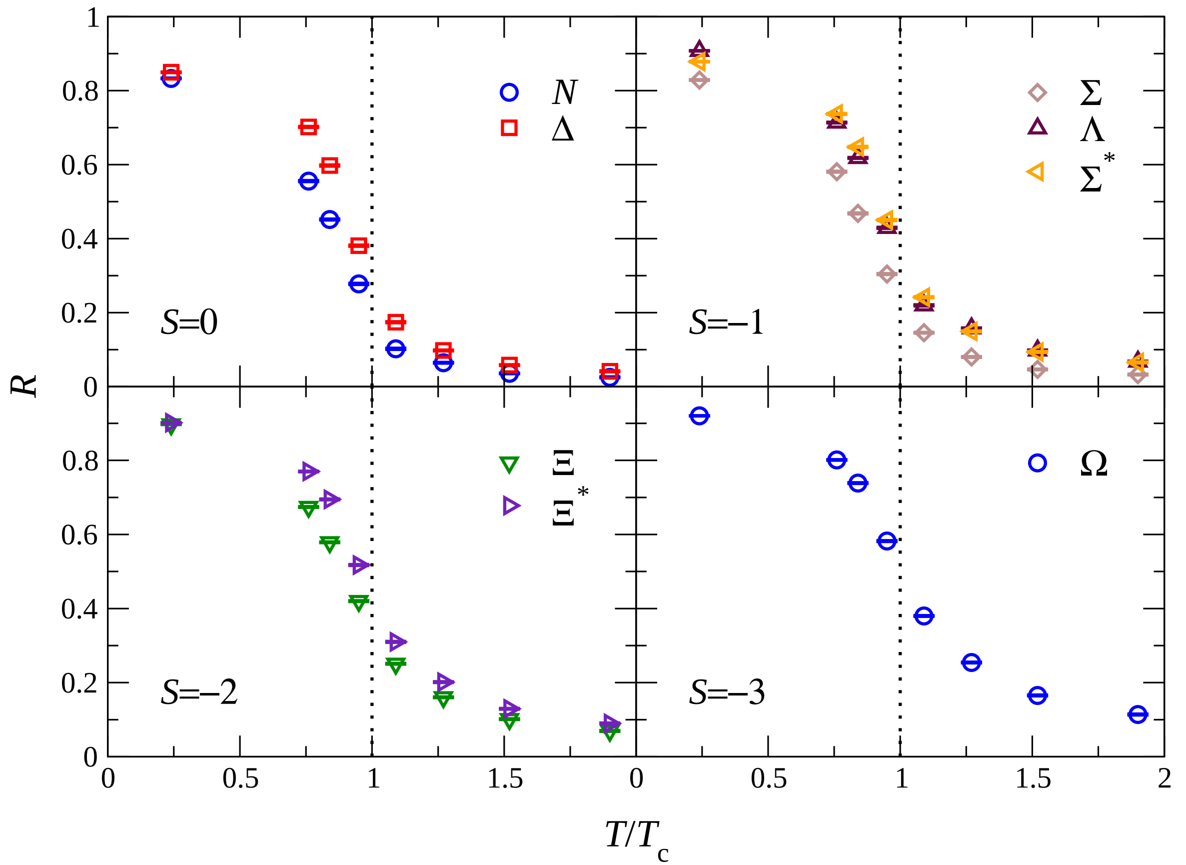
$$R = \frac{\sum_n R(\tau_n)/\sigma^2(\tau_n)}{\sum_n 1/\sigma^2(\tau_n)}$$



- crossover behaviour, tied with deconfinement transition and hence chiral transition – note: $m_q \neq 0$
- effect of heavier s quark visible

Quasi-order parameter

parity doubling in the QGP: $R \sim 1 \to 0$



- crossover behaviour, tied with deconfinement transition and hence chiral transition – note: $m_q \neq 0$
- effect of heavier s quark visible

Parity doubling

- clear signal for parity doubling
  even with finite quark masses
- crossover behaviour, coinciding with transition to QGP
- visible effect of heavier s quark

spectral functions

Spectral properties: fermions

$$G^{\alpha\alpha'}(\tau, \mathbf{p}) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} K(\tau, \omega) \rho^{\alpha\alpha'}(\omega, \mathbf{p})$$

- fermionic Matsubara frequencies

$$K(\tau, \omega) = T \sum_n \frac{e^{-i\omega_n \tau}}{\omega - i\omega_n} = \frac{e^{-\omega\tau}}{1 + e^{-\omega/T}} = e^{-\omega\tau} \left[1 - n_F(\omega)\right]$$

- kernel not symmetric, instead

$$K(1/T - \tau, \omega) = K(\tau, -\omega)$$

- positivity: $\rho_4(p), \pm\rho_\pm(p) \geq 0$ for all ω
- $\rho_m(p) = [\rho_+(p) + \rho_-(p)]/4$ not sign definite

Spectral functions

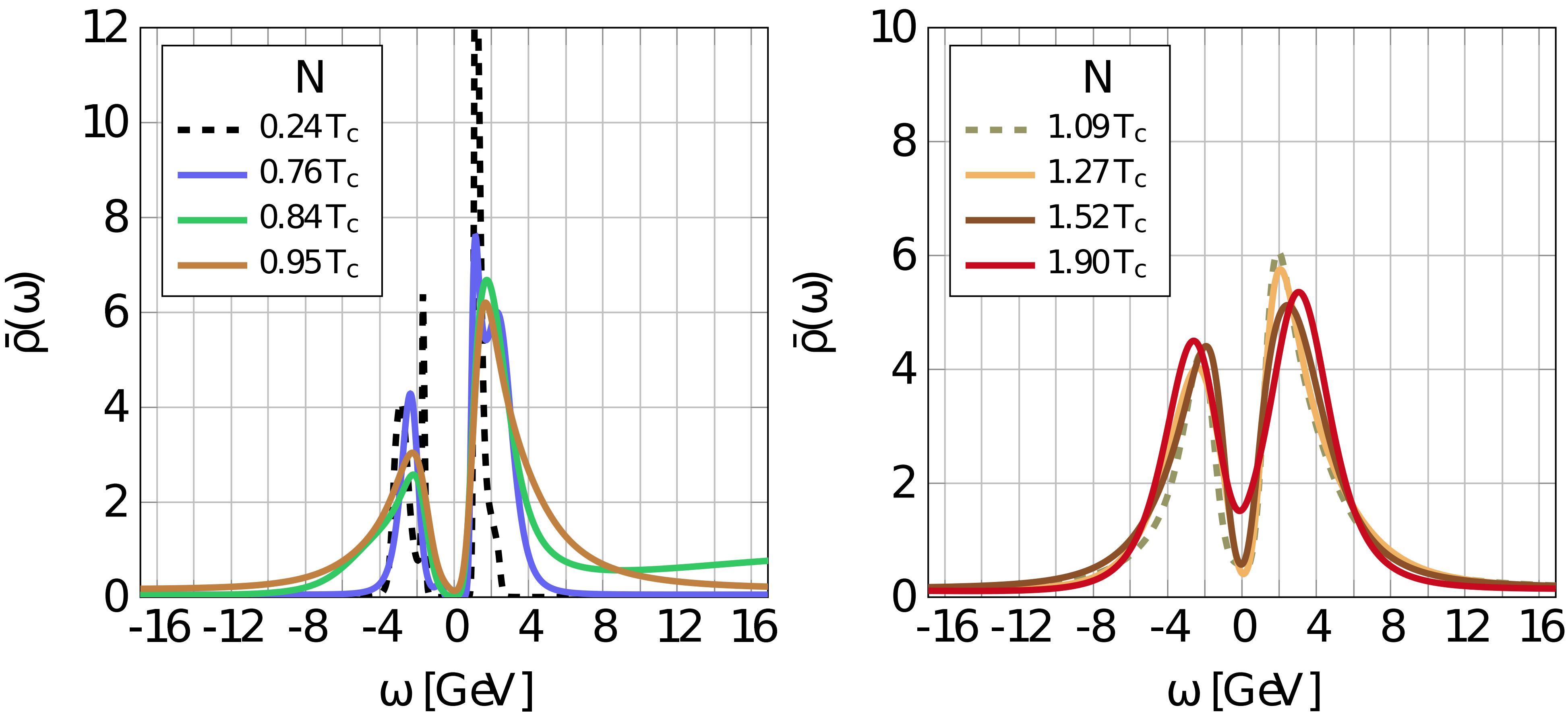
extract same information from spectral functions

$$G_{\pm}(\tau) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} K(\tau,\omega)\rho_{\pm}(\omega) \qquad K(\tau,\omega) = \frac{e^{-\omega\tau}}{1 + e^{-\omega/T}}$$

- *ill-posed* inversion problem
- use Maximum Entropy Method (MEM)
- featureless default model
- construct $\rho_+(\omega) \geq 0$ for all ω
- $\rho_-(\omega) = -\rho_+(-\omega)$

Baryon spectral functions

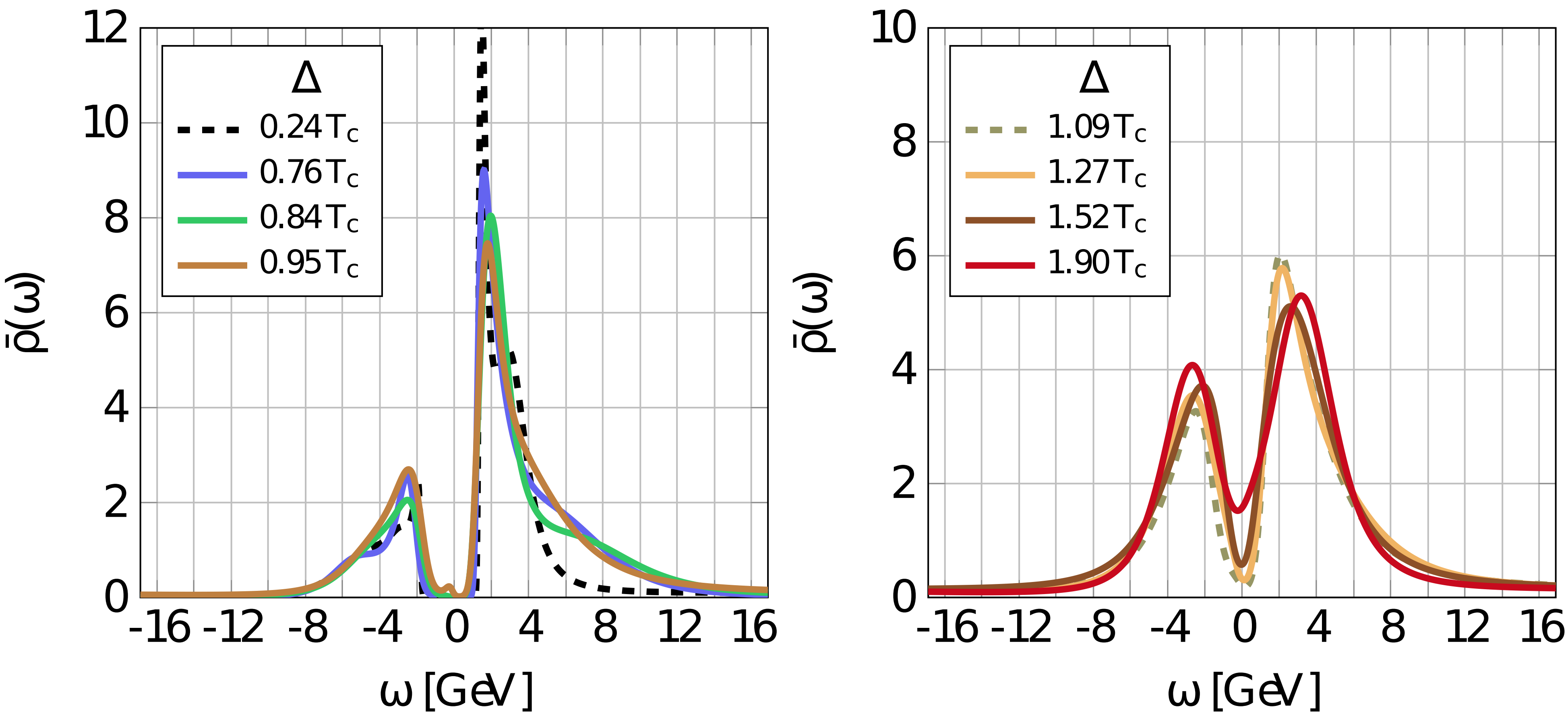
- nucleon



- groundstates below T_c
- degeneracy emerging above T_c

Baryon spectral functions

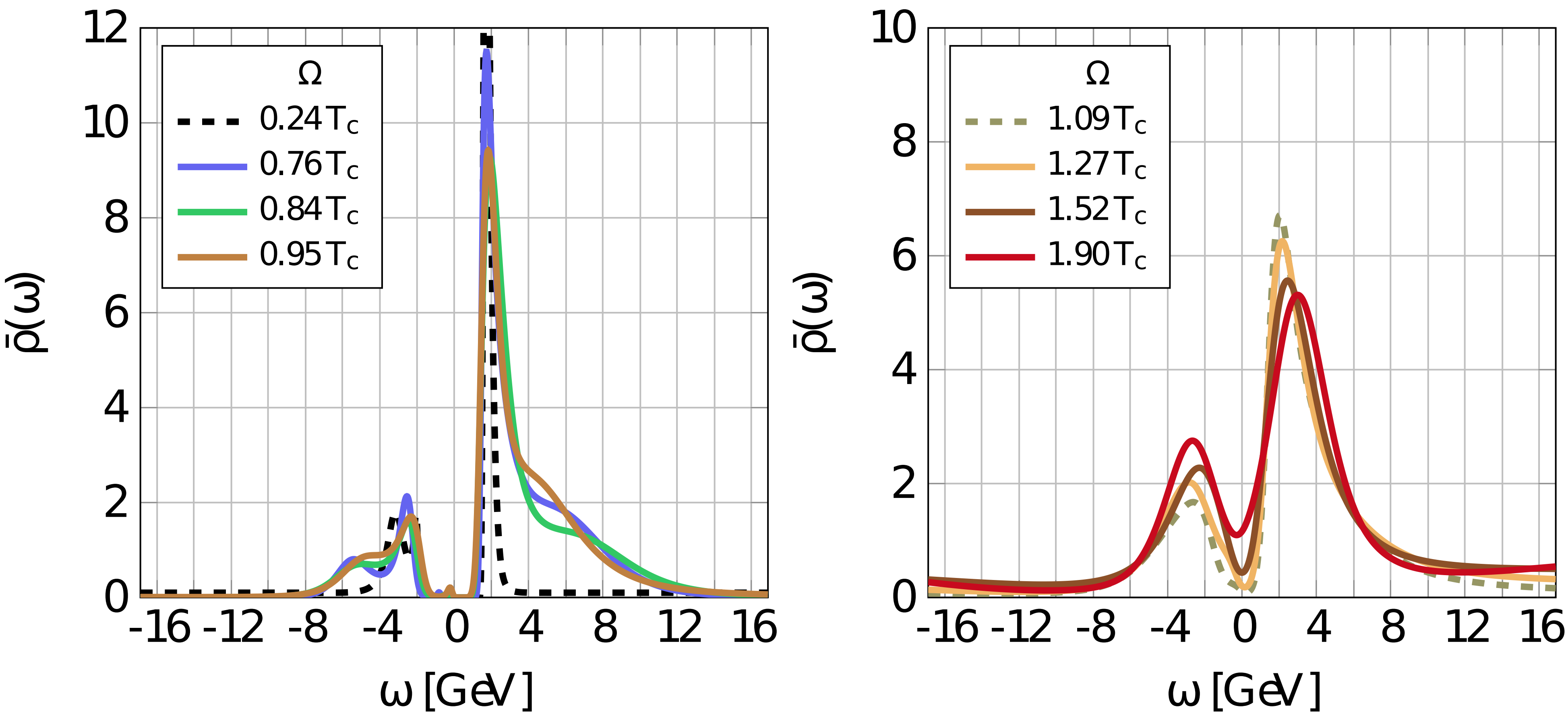
- Δ



- groundstates below T_c
- degeneracy emerging above T_c

Baryon spectral functions

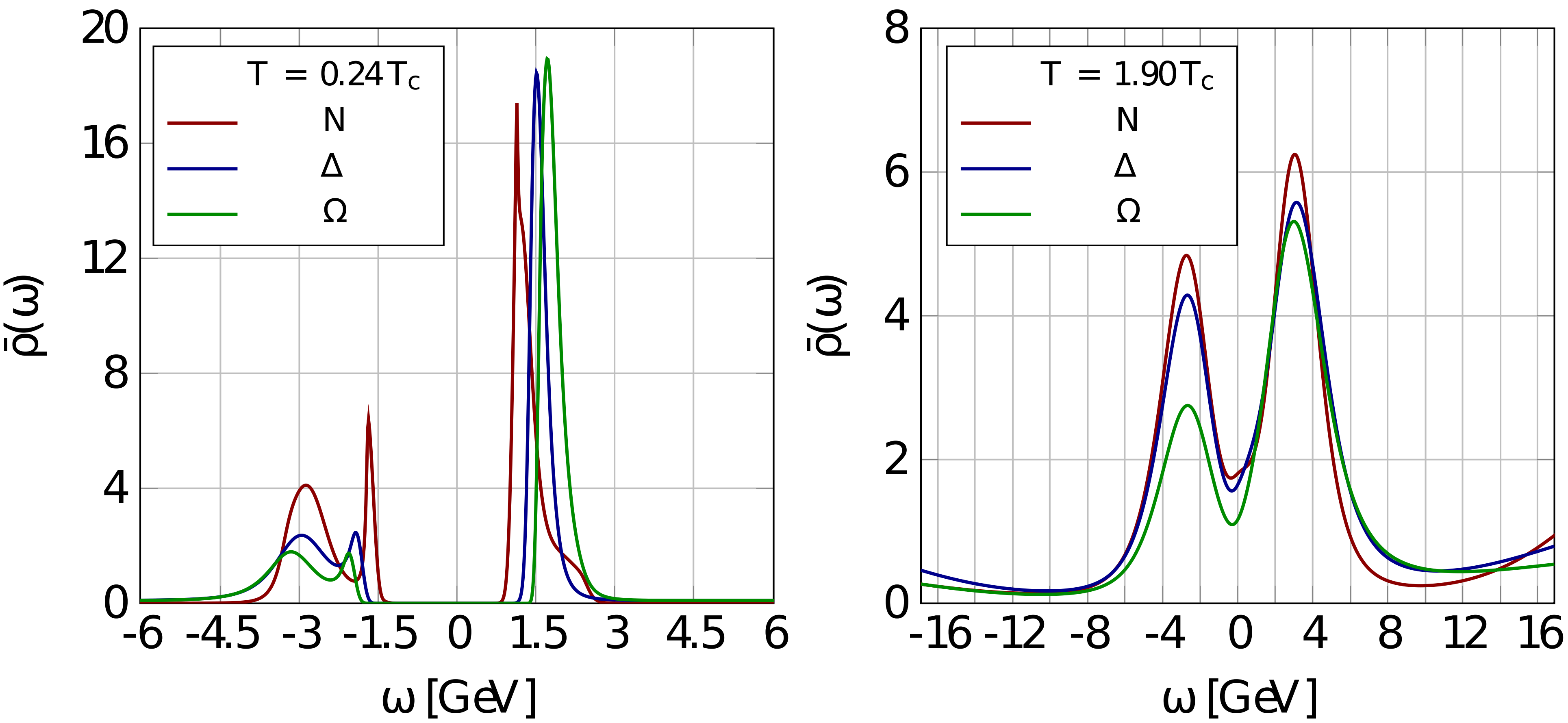
- Ω



- groundstates below T_c
- degeneracy emerging above T_c, finite m_s

Baryon spectral functions

- all channels: low and high temperature



- groundstates below T_c
- degeneracy emerging above T_c

Baryon spectral functions

- results consistent with correlator analysis
- latter is on firmer ground, due to inversion uncertainties
- effect of heavier s quark visible

Summary: baryons in medium

in hadronic phase

- pos-parity groundstates mostly T independent
- stronger T dependence in neg-parity groundstates
  reduction in mass, near degeneracy close to T_c
- relevant for heavy-ion phenomenology?

in quark-gluon plasma

- pos/neg parity channels degenerate: parity doubling
- linked to deconfinement transition and chiral symmetry restoration
- correlator and spectral function analysis consistent
- effect of heavier s quark noticeable