Thermodynamics of Parity Doublers in Effective Theory

Chihiro Sasaki
Institute of Theoretical Physics
University of Wroclaw
Outlines

I. Hadrons in a hot/dense medium
   - Parity doublers: mesons and baryons
   - Survival masses vs. trace anomaly

II. Fluctuations and correlations
   - In-medium Hadron Resonance Gas
   - S-matrix approach
   - Chiral-confinement interplay

III. Summary and conclusions
I. Hadrons in a hot/dense medium
Spectra in a chirally broken world

- Lowest pseudo-scalar mesons as NG bosons
- Mass splitting between positive and negative parity hadrons
Spectra in a chirally restored world

- Lowest scalar meson $\rightarrow$ O(4) vector with pion
- Parity partners degenerate $\rightarrow$ chiral partners

$[m_q \approx 0$: helicity eigenstates $\approx$ parity eigenstates]
Lattice QCD tells us ...

- Screening masses of mesons and nucleons
  [DeTar-Kogut, 1987]
Lattice QCD tells us ...

- Temporal correlations in baryonic channels
  [Arts et al. 2015-17: \( \text{mpi} \approx 400 \text{ MeV}, \text{mk} \approx 500 \text{ MeV}, Tc_h = 185 \text{ MeV} \)]
Hadron masses vs. CSB

- Gell-Mann—Levy model (1960)

\[ \mathcal{L}_{GL} = i\bar{N} \not{\partial} N - g\bar{N} (\sigma + i\gamma_5 \vec{\tau} \cdot \vec{\pi}) N + \mathcal{L}_{\text{meson}} \]

\[ \psi_L \rightarrow L \psi_L, \quad \psi_R \rightarrow R \psi_R \quad m_N = g \langle \sigma \rangle \]

- When CS restored \( \rightarrow \) massless nucleon

- Mesons: massless (P), massive (S) by CSB

- Vector mesons (V,A) in LSM can stay massive.

\[ R^\mu \rightarrow U_R R^\mu U_R^\dagger, \quad L^\mu \rightarrow U_L L^\mu U_L^\dagger \quad \frac{m_1^2}{2} \text{Tr} \left[ (L^\mu)^2 + (R^\mu)^2 \right] \]
Non-SCB mass of nucleons

SU(2) chiral transformation of 2 nucleons

→ how to assign 2 indep. rotation to them?

\[ \psi_{1L} \to g_l \psi_{1L}, \quad \psi_{1R} \to g_r \psi_{1R} \sim \psi_{1L} : (1/2, 0) \quad \psi_{1R} : (0, 1/2) \]

\[ \psi_{2L} \to g_r \psi_{2L}, \quad \psi_{2R} \to g_l \psi_{2R} \sim \psi_{2L} : (0, 1/2) \quad \psi_{2R} : (1/2, 0) \]

\[ \mathcal{L}_m = m_0 \left( \bar{\psi}_2 \gamma_5 \psi_1 - \bar{\psi}_1 \gamma_5 \psi_2 \right) \Rightarrow m_{N\pm} = \frac{1}{2} \left[ \sqrt{c_1 \sigma^2 + 4m_0^2} + c_2 \sigma \right] \]

[DeTar-Kunihiro, 1989]
Origin of the survival masses?

- Emergence of a scale in QCD $\rightarrow$ trace anomaly

$$\partial_\mu J^\mu = T^\mu_\mu \propto \langle H | G^2 | H \rangle$$

- In hot matter [Miller, 2007: lattice QCD EoS]

$$\langle G^2 \rangle_{T_{\text{chiral}}} \simeq \frac{1}{2} \langle G^2 \rangle_{T=0}$$

- In nuclear matter [Cohen et al., 1995: Feynman-Hellmann theorem & low-density approx.]

$$\left\langle \frac{\alpha_s}{\pi} G^a_{\mu\nu} G^{a\mu\nu} \right\rangle_{\rho_N} - \left\langle \frac{\alpha_s}{\pi} G^a_{\mu\nu} G^{a\mu\nu} \right\rangle_{\text{vac}}$$

$$= -\frac{8}{9} (M_N - \sigma_N - S) \rho_N + \cdots \quad 5\% \text{ smaller}$$
How large is $m_0$?

- At Tch: $\langle G^2 \rangle_T \rightarrow m_0 = 210$ MeV [CS et al.]
- Vacuum: $m_0 = 270 - 460$ MeV [DeTar-Kunihiro, Nemoto et al., Gallas et al.]
- Nuclear matter
  - Ground state: binding energy, saturation point
  - Preferred $m_0 \approx 500$-800 MeV (w/ and w/o 4Q)
    [Zschiesche et al. 2007, Gallas et al. 2011]
- Swansea LQCD: near Tch, zero chem.pot.
  $m_0(\text{octet,decuplet}) \leq m^+(T=0)$
Let’s take $m_0 \approx 900$ MeV!
Parity doubling of baryons

- Baryon octet and decuplet with finite $m_0$
- Consistent with established phenomenology:
  - Gell-Mann-Okubo mass formula
  \[ \frac{3}{4}m_\Lambda + \frac{1}{4}m_\Sigma - \frac{1}{2}(m_N + m_\Xi) = 0 \]
  - Gell-Mann’s equal spacing rule
  \[ m_\Sigma^* - m_\Delta = m_\Xi^* - m_\Sigma^* = m_\Omega - m_\Xi^* \]
Chiral condensates

- Quark condensates from a model vs. LQCD
- Pion mass dependence: $\pi_\text{m} = 140, 400$ MeV
Mass splitting: octet

\[ \text{M}_{\text{pi}} = 140 \text{ MeV} \]

\[ \text{400 MeV} \]
Mass splitting: decuplet
Remarks

- Nucleons and deltas: $\delta m$ drops substantially.
- Much milder trend in hyperons: s-quark effect
- Heavier mpi $\rightarrow$ mk: $\delta m(u,d) \approx \delta m(s)$
  - more explicit breaking but exact SU(3)
  - Swansea’s setup: SU(3) rather than SU(2+1)
  - Still, Omega-baryon mass needs to be understood.
  - Missing piece(s)? --- the onset of deconfinement
- Need simulations with physical mpi!
II. Fluctuations and correlations
Signal of chiral symmetry restoration

- Lattice QCD shows clearly $\langle qq\bar{q}\rangle$ dropping!
- More deviation from HRG in higher-order fluctuations $\rightarrow$ Missing states? Interactions? and/or in-medium effects?
In-medium HRG

- T-dep. motivated by Lattice findings [Aarts et al.]

\[ M^{-}(T) = M^{-}(T = 0) \omega(T, b) + M^{-}(T_c)(1 - \omega(T, b)) \]
\[ \omega(T, b) = \tanh[(1 - T/T_c)/b]/\tanh(1/b) \]

[Morita, Redlich and Sasaki, to appear]
Fluctuations of net-baryon number

\[ \chi_{ijk}^{BQS} = \frac{\delta^{i+j+k}}{\partial^{i}(\mu_B/T)\partial^{j}(\mu_Q/T)\partial^{k}(\mu_S/T)} p(T,\mu_B,\mu_Q,\mu_S)/T^4 \]

[Morita, Redlich and Sasaki, to appear]
What is missing? --- finite width

- Thermodynamics of broad resonances
  - S matrix approach [Dashen, Ma and Bernstein, 1969]
    - Grand canonical potential
      
      \[
      \Omega = \Omega_0 + \Omega_{\text{int}}
      \]

      \[
      \Delta \ln Z = \int dE e^{-\beta E} \frac{1}{4\pi i} \text{tr} \left[ S^{-1} \frac{\partial}{\partial E} S \right]_c
      \]

    - Leading contribution: 2-body [Beth-Uhlenbech, 1937]
      
      \[
      \Delta \ln Z = \int dE e^{-\beta E} \times \frac{1}{\pi} \frac{\partial}{\partial E} \text{tr} (\delta E)
      \]
What is missing? --- finite width

- $K0^*/\kappa$ (800) meson: chiral partner of kaon

NOTE: omitted from PDG summary table

- $S$ matrix approach [Friman et al. 2015]

✓ Empirical $\pi$-K phase shift from experiment
Pi-Nucleon system

[Lo et al., to appear]
Chiral-confinement interplay
A missing piece: (de)confinement

- Modeling conf. of QCD thermodynamics
  - Confined phase = $Z(N_c)$ unbroken phase in YM
    \[ \hat{L}(\vec{x}) = \mathcal{P} \exp \left[ i \int_0^{1/T} d\tau A_4(\vec{x}, \tau) \right] \langle \Phi(\vec{x}) \rangle \sim e^{-F_q(\vec{x})/T} \]
  - Full QCD: PNJL/PQM $\approx$ GL theory of $\sigma$ and $\Phi$
  - At finite density? --- $Z(N_c)$ strongly violated!

- How to suppress quarks at low density?
  \[ \rightarrow \text{IR/UV cutoff in fermion dist. functions} \]
  \[ T=mu=0 \quad [\text{Ebert et al. 1996; NJL, SD, AdS/QCD}] \]
  \[ n_q = \theta(|\vec{p}| - b) f_q, \quad n_{N\pm} = \theta(b - |\vec{p}|) f_{N\pm} \]

\[ \text{IR} \quad \text{nucleons} \quad \text{quarks} \quad \text{UV} \]
Polyakov loop vs. IR cutoff

- Hadron size $\approx 1/b \rightarrow$ depends on $T$ and $\mu$!
  - Otherwise SB limit not achieved. [Benic et al. 2015]
    \[ \Omega = \sum_{X=N_\pm,q} \Omega_X + V_\sigma + V_\omega + V_b, \quad V_b = -\frac{k_b^2}{2}b^2 + \frac{\lambda_b}{4}b^4 \]
  - Const. $b \rightarrow$ condensation of a scalar field $b$: $\langle b \rangle$

- $T \neq 0, \mu = 0$ thermodynamics vs. PQM

\[ T_c^{\text{bQPM}} \approx \left( \frac{12\lambda_x\chi_0^2}{\gamma_q g^2} + \frac{3}{\pi^2}b^2 \right)^{1/2} \]
\[ T_c^{\text{PQM}} = \left( \frac{12\lambda_x\chi_0^2}{\gamma_q g^2} + \frac{2}{\pi^2}\Phi^2 \right)^{1/2} \]
\[ \langle \Phi \rangle \approx 1 - \langle b \rangle / \langle b_0 \rangle \]
Onset of different fermions

- Fractions of particle num. density

[Marczenko-Sasaki, to appear]
Chiral-confinement interplay

- Dirac zero mode = non-vanishing $<\text{qqbar}>$
  - when CS is unbroken, then no conf.?
    - String tension unchanged!
    - Chiral restoration $\neq$ deconf.
      [Gattringer; Bruckmann et al.; Gongyo et al.]
  
- Anomaly matching: e.g. $\pi^0 \rightarrow \gamma\gamma$
  - No WZW w/ unbroken chiral
  - Anomaly can match only if deconf.
  - New collective modes?

- The best we can do: systematic case study
III. Summary and conclusions
Emergent parity-doubling structure as a manifestation of restored chiral symmetry

- Lattice QCD, effective theories
- In dense QCD: rich phase structure
- Interplay between CSB and confinement

- Naive “in-medium HRG” does not work.
- Effect of resonance widths – beyond HRG
- Toward more realistic EoS