Homework Assignments for Collider Physics

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A partial list of assignments accompanying the lectures.

Lecture I: Colliders and Detectors

`Exercise 1.1:` For a π^0, μ^-, or a τ^- respectively, calculate its decay length if the particle has an energy $E = 10$ GeV.

`Exercise 1.2:` An event was identified to have a μ^+ and a μ^- along with some missing energy. What can you say about the kinematics of the system of the missing particles? Consider for both an e^+e^- and a hadron collider.

`Exercise 1.3:` Electron and muon measurements: Estimate the relative errors of energy-momentum measurements for an electron by an electromagnetic calorimetry ($\Delta E/E$) and for a muon by tracking ($\Delta p/p$) at energies of $E = 50$ GeV and 500 GeV, respectively.

`Exercise 1.4:` A 120 GeV Higgs boson will have a production cross section of 20 pb at the LHC. How many events per year do you expect to produce for the Higgs boson with a designed LHC luminosity $10^{33}/\text{cm}^2/\text{s}$? Do you expect it to be easy to observe and why?

Lecture II: Basic Techniques and Tools

`Exercise 2.1:` Assume that $m_a = m_1$ and $m_b = m_2$. Show that

$$\begin{aligned} t &= -2p_{cm}^2(1-\cos\theta_{a1}^*), \\ u &= -2p_{cm}^2(1+\cos\theta_{a1}^*) + \frac{(m_1^2-m_2^2)^2}{s}, \end{aligned}$$

$p_{cm} = \lambda^{1/2}(s, m_1^2, m_2^2)/2\sqrt{s}$ is the momentum magnitude in the c.m. frame.
Note: t is negative dfinite; $t \to 0$ in the collinear limit.

`Exercise 2.2:` A particle of mass M decays to two particles isotropically in its rest frame. What does the momentum distribution look like in a frame in which the particle is moving with a speed β_z? Compare the result with your expectation for the shape change for a basket ball.

`Exercise 2.3:` A particle of mass M decays to 3 particles $M \to abc$. Show that the phase space element can be expressed as

$$dPS_3 = \frac{1}{2^7 \pi^3}\ M^2 dx_a dx_b.$$
$$x_i = \frac{2E_i}{M},\ (i = a, b, c,\ \sum_i x_i = 2).$$

where the integration limits for $m_a = m_b = m_c = 0$ are

$$0 \le x_a \le 1, \quad 1 - x_a \le x_b \le 1.$$

`Exercise 2.4:` Consider a three-body decay of a top quark, $t \to bW^* \to b\ e\nu$. Making use of the phase space recursion relation and the narrow width approximation for the intermediate W boson, show that the partial decay width of the top quark can be expressed as

$$\Gamma(t \to bW^* \to b\ e\nu) \approx \Gamma(t \to bW) \cdot BR(W \to e\nu).$$

`Exercise 2.5:` Appreciate the properties (a) partial wave unitarity and (b) kinematical threshold behavior by explicitly calculating the helicity amplitudes for

$$e^-_L e^+_R \to \gamma^* \to H^- H^+, \quad e^-_L e^+_{L,R} \to \gamma^* \to \mu^-_L \mu^+_R, \quad H^- H^+ \to G^* \to H^- H^+.$$

Lecture III(a): Linear Collider

`Exercise 3.1:` For a resonant production $e^+e^- \to V^*$ with a mass M_V and total width Γ_V, derive the Breit-Wigner formula

$$\sigma(e^+e^- \to V \to X) = \frac{4\pi(2j+1)\Gamma(V \to e^+e^-)\Gamma(V \to X)}{(s - M_V^2)^2 + \Gamma_V^2 M_V^2}\ \frac{s}{M_V^2},$$

Consider a beam energy spread Δ in Gaussian distribution

$$\frac{dL}{d\sqrt{\hat{s}}} = \frac{1}{\sqrt{2\pi}\ \Delta} \exp[\frac{-(\sqrt{\hat{s}} - \sqrt{s})^2}{2\Delta^2}],$$

obtain the appropriate cross section formulas for (a) $\Delta \ll \Gamma_V$ (resonance line-shape) and (b) $\Delta \gg \Gamma_V$ (narrow-width approximation).

`Exercise 3.3:` Derive the Weizsäcker-Williams spectrum for a photon with an energy xE off an electron with an energy E

$$P_{\gamma/e}(x) \approx \frac{\alpha}{2\pi} \frac{1 + (1-x)^2}{x} \ln \frac{E^2}{m_e^2}$$

Lecture III(b): Hadron Collider

`Exercise 4.1:` For a four-momentum $p \equiv p^\mu = (E, \vec{p})$, define

$$
\begin{aligned}
E_T &= \sqrt{p_T^2 + m^2}, \quad p_T^2 = p_x^2 + p_y^2, \quad y = \frac{1}{2}\ln\frac{E+p_z}{E-p_z}, \\
\text{then show } p^\mu &= (E_T \cosh y,\ p_T\cos\phi,\ p_T\sin\phi,\ E_T\sinh y), \\
\text{and, } \frac{d^3\vec{p}}{E} &= p_T dp_T d\phi\ dy = E_T dE_T d\phi\ dy.
\end{aligned}
$$

Due to the random boost between the Lab-frame (O) and the c.m. frame (O') for every event,

$$
y' = \frac{1}{2}\ln\frac{E'+p_z'}{E'-p_z'} = \frac{1}{2}\ln\frac{(1-\beta_{cm})(E+p_z)}{(1+\beta_{cm})(E-p_z)} = y - y_{cm},
$$

where β_{cm} and y_{cm} are the speed and rapidity of the c.m. frame w.r.t. the lab frame.

In the massless limit, the rapidity y defines the pseudo-rapidity:

$$
y \to \eta = \frac{1}{2}\ln\frac{1+\cos\theta}{1-\cos\theta} = \ln\cot\frac{\theta}{2}.
$$

`Exercise 4.3:` Let the pp c.m. energy be S, and the two partons 1 and 2 have energy momentum fractions x_1 and x_2, respectively. Then

$$
s \equiv \tau S, \quad \tau = x_1 x_2 = \frac{s}{S}. \quad y_{cm} = \frac{1}{2}\ln\frac{x_1}{x_2}.
$$

The parton energy fractions are thus given by

$$
x_{1,2} = \sqrt{\tau}\ e^{\pm y_{cm}}.
$$

The integration over the energy fractions can be rewritten in terms of the other two variables as

$$
\int_{\tau_0}^{1} dx_1 \int_{\tau_0/x_1}^{1} dx_2 = \int_{\tau_0}^{1} d\tau \int_{\frac{1}{2}\ln\tau}^{-\frac{1}{2}\ln\tau} dy_{cm}.
$$

$\tau_0 = m_{res}^2/S$ and m_{res} the threshold for the parton final state.

Lecture IV: Kinematics

`Exercise 5.1:` For a two-body massless final state with an invariant mass squared s, show that

$$
\frac{d\hat{\sigma}}{dp_T} = \frac{4p_T}{s\sqrt{1-4p_T^2/s}}\ \frac{d\hat{\sigma}}{d\cos\theta^*}.
$$

where $p_T = p \sin\theta^*$ is the transverse momentum and θ^* is the polar angle in the c.m. frame. Comment on the apparent singularity at $p_T^2 = s/4$.

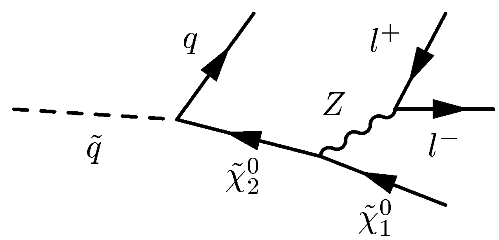
`Exercise 5.2:` Show that for the decay $W^- \to e^- \bar{\nu}_e$, the transverse mass has end points

$$0 \le m_{e\nu\ T}^2 \equiv (E_{eT} + E_{\nu T})^2 - (\vec{p}_{eT} + \vec{p}_{\nu T})^2 \le m_{e\nu}^2.$$

`Exercise 5.3:` Show that if W/Z has some transverse motion, δP_V, then, $p'_{eT} \sim p_{eT}\ [1 + \delta P_V/E_V]$, $m'^2_{e\nu\ T} \sim m^2_{e\nu\ T}\ [1 + (\delta P_V/E_V)^2]$, $m'^2_{ee} = m^2_{ee}$.

`Exercise 5.4:` Consider a squark cascade decay to on-shell neutralinos:

$$\tilde{q} \to q\ \tilde{\chi}_2^0 \to q\ \ell^+\ell^-\ \tilde{\chi}_1^0.$$



Show the existence of the kinematical end-points in invariant mass distributions

$$1^{\rm st}\ \text{edge}:\ \ M^{max}(\ell\ell) = M_{\chi_2^0} - M_{\chi_1^0};$$
$$2^{\rm nd}\ \text{edge}:\ \ M^{max}(\ell\ell j) = M_{\tilde{q}} - M_{\chi_1^0}.$$