Particle Physics and Cosmology: Experiment 3

Relic Neutrinos As Dark Matter

Introduction

If we denote the number density of a species X by n_X and its number density relative to the number of photons by r_X, we have the following equation for the evolution of r_X with time:

$$\frac{dr_X}{dt} = -\beta_X n_\gamma (r_X^2 - r_{X_{eqbm}}^2)$$

where $r_{X_{eqbm}}$ is the relative number density if the X's are in equilibrium, n_γ is the number density of photons and $\beta_X n_X^2$ is the annihilation rate of X's.

In the standard model of particle physics neutrinos are massless, but experimentally there are only lower bounds on their masses. If we introduce masses for the neutrinos they become dark matter candidates.

The reaction we are interested in is

$$\nu + \bar{\nu} \to Z^0 \to y\bar{y}$$

where y is any fermion lighter than the massive neutrino. For neutrinos of mass $m >> MeV$ we have

$$\beta_\nu \sim G_F^2 m^2$$

where

$$G_F = \frac{\sqrt{2}e^2}{8\sin^2\theta_W M_W^2} = \frac{\sqrt{2}e^2}{8\sin^2\theta_W \cos^2\theta_W M_Z^2} = (290\text{GeV})^{-2}$$

Solving the number density evolution equation allows us to calculate the *relic density* of neutrinos. If the relic density for a given mass is too high, we can rule out neutrinos of that mass. The aim of the experiment is to find what range of neutrino masses is ruled out.

1: Equilibrium Density

Recall that the equilibrium number density of X's is given by

$$n_X = \frac{g_X}{(2\pi)^3}\int f(\mathbf{k})d^3k$$

where the energy E is given by $E^2 = \mathbf{k}.\mathbf{k} + m_X^2$ and

$$f(\mathbf{k}) = \frac{1}{\mathrm{e}^{(E-\mu_X)/T} \pm 1}$$

where μ_X is the chemical potential.

Using a computer languauge of your choice, calculate $r_{\nu_{eqbm}}$ as a function of T. For a massive neutrino $g_\nu = 4$ and μ_ν can be assumed to be small.

2: Time Temperature Relation

The dynamics of the universe is governed by the Friedmann equation,

$$\frac{\dot{R}^2}{R^2} + \frac{k}{R^2} = \frac{8}{3}\pi G\rho.$$

We will be interested in times less than a few minutes after the big bang, so the universe will be radiation dominated. Assuming the universe to be flat, solve the Friedmann equation, analytically to find the temperature as a function of time. You will need to put back all factors of c and $\hbar$ by reconciling dimensions.

3: Solving For r_ν

You can consider the evolution in time or $1/T$. Reconcile the dimensions in the evolution equation and select a suitable unit to use for the independent variable. Numerically solve the evolution equation for a range of neutrino masses, calculate the relic number density and energy density. Compare the energy density with the critical density and deduce whether or not the neutrinos close the universe.