Non-Supersymmetric Brane Configurations, Seiberg Duality, and Dynamical Symmetry Breaking

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Introduction

Seiberg duality (Seiberg 1994) is a highly non-trivial statement about strongly coupled $\mathcal{N} = 1$ SQCD. It provides insights about the IR degrees of freedom of the strongly coupled gauge theory.

The statement is that

An 'electric' $SU(N_c)$ theory with N_f quarks

and

A 'magnetic' $SU(N_f - N_c)$ theory with N_f 'dual' quarks and a 'meson', interacting via a superpotential $W = Mq\tilde{q}$,

become equivalent in the IR.

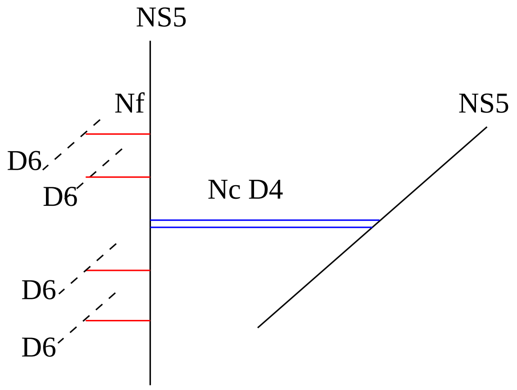
In particular, when $N_c + 2 < N_f < \frac{3}{2}N_c$, the magnetic theory is IR free and contains massless "colored" degrees of freedom.

The result is surprising since in ordinary QCD we do not expect a description of the IR physics in terms of a dual gauge theory, but by a sigma model of mesons (a "chiral Lagrangian").

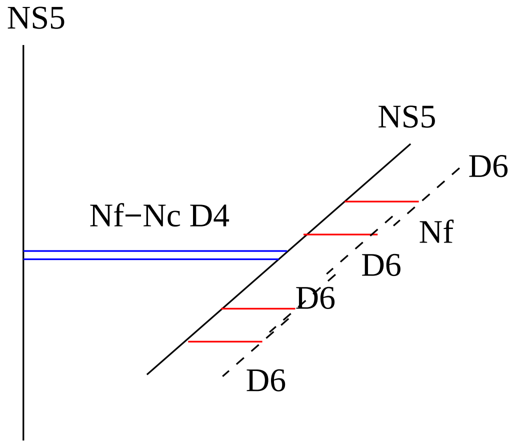
Introduction 2

The duality is supported by the matching of moduli spaces, 't Hooft global anomaly matching and by string theory.

The realization of the 'electric' $U(N_c)$ theory in type IIA is via the following Hanany-Witten brane configuration



By swapping the NS5 branes we arrive at the $U(N_f - N_c)$ magnetic theory (Elitzur, Giveon, Kutasov, 1997)



Motivation

In this talk I will discuss examples of *non-supersymmetric* Seiberg dualities.

The evidence for the duality is based on string theory, 't Hooft anomaly matching conditions and the large-N limit.

We will see how phenomena in string theory are related to phenomena in gauge theories. In particular a potential between branes and an orientifold plane is interpreted as a Coleman-Weinberg potential.

The minimum of the potential leads to a dynamical symmetry breaking pattern which is consistent with both the Coleman-Witten analysis and the Vafa-Witten theorem.

The electric theory

Electric Theory			
	$Sp(2N_c)$	$SU(2N_f)$	$U(1)_R$
A_μ	$\square\square$ $N_c(2N_c+1)$	$\bullet$	0
λ	$\begin{array}{c}\square\\[-4pt]\square\end{array}$ $N_c(2N_c-1)$	$\bullet$	1
Φ	$\square$ $2N_c$	$\square$ $2N_f$	$\frac{N_f-N_c+1}{N_f}$
Ψ	$\square$ $2N_c$	$\square$ $2N_f$	$\frac{-N_c+1}{N_f}$

Table 1: *The matter content of the electric theory.*

The magnetic theory

Magnetic Theory			
	$Sp(2\tilde{N}_c)$	$SU(2N_f)$	$U(1)_R$
a_μ	$\square\!\square$ $\tilde{N}_c(2\tilde{N}_c+1)$	$\bullet$	0
l	$\begin{smallmatrix}\square\\ \square\end{smallmatrix}$ $\tilde{N}_c(2\tilde{N}_c-1)$	$\bullet$	1
ϕ	$\square$ $2\tilde{N}_c$	$\bar{\square}$ $2\bar{N}_f$	$\frac{N_c-1}{N_f}$
ψ	$\square$ $2\tilde{N}_c$	$\bar{\square}$ $2\bar{N}_f$	$\frac{N_c-N_f-1}{N_f}$
M	$\bullet$	$\begin{smallmatrix}\square\\ \square\end{smallmatrix}$ $N_f(2N_f-1)$	$\frac{2N_f-2N_c+2}{N_f}$
χ	$\bullet$	$\square\!\square$ $N_f(2N_f+1)$	$\frac{N_f-2N_c+2}{N_f}$

Table 2: The magnetic theory. $\tilde{N}_c = N_f - N_c + 2$.

Anomaly matching

The following set of global anomalies match, as in $SO(2N)$ SQCD

$$SU(2N_f)^3 \quad 2N_c d^3(\Box)$$

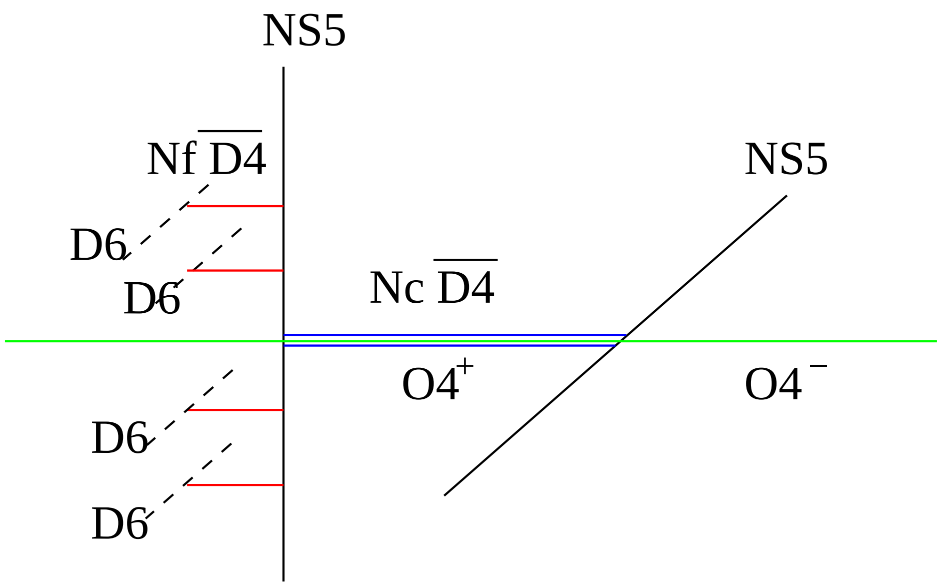
$$SU(2N_f)^2 U(1)_R \quad \frac{-2N_c^2 + 2N_c}{N_f} d^2(\Box)$$

$$U(1)_R \quad -2N_c^2 + 3N_c$$

$$U(1)_R^3 \quad N_c \left(2N_c - 1 - 4\frac{(N_c-1)^3}{N_f^2} \right)$$

String theory setup

We consider a brane configuration similar to the configuration that gives rise to $Sp(2N)$ SQCD, apart from replacing the D4 branes by *anti* D4 branes



The presence of the *anti* D4 branes and the orientifold planes break supersymmetry and lead to the electric theory, except that on the branes the global symmetry is $SO(2N_f)$.

There are no open or closed string tachyons in the spectrum. The above brane configuration is stable.

Evidence for Seiberg Duality from String Theory

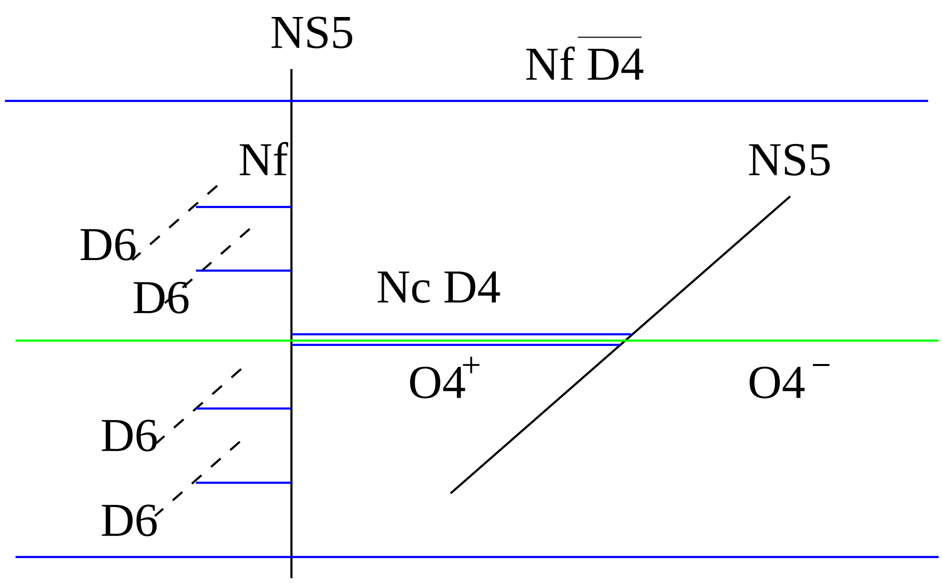
Starting from the brane configuration that includes *anti-branes*, we can swap the NS5 branes to arrive at the proposed magnetic theory. When we swap the NS5 branes two anti-branes are created see Evans et.al..

In the SUSY case it leads to a dual theory based on $Sp(2\tilde{N}_c)$ gauge group, with $\tilde{N}_c = N_f - N_c - 2$. In the present case, the two anti-branes gives rise to a dual theory based on $Sp(2\tilde{N}_c)$, but with $\tilde{N}_c = N_f - N_c + 2$, similar to the supersymmetric SO case.

Another Argument based on String Theory

Another argument, due to Shigeki Sugimoto (unpublished) is as follows:

Consider the supersymmetric brane configuration. Now add to the configuration a set of N_f anti D4 branes. After brane annihilation, the electric theory becomes the non-susy $Sp(2(N_f - N_c))$ *magnetic* theory, while the magnetic theory becomes the non-susy $Sp(2(N_c + 2))$ *electric theory*.

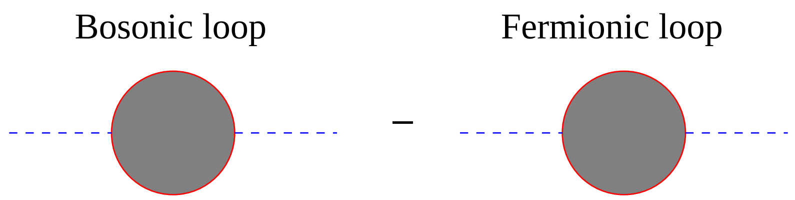


According to this argument, Seiberg duality for the non-supersymmetric theory follows from Seiberg duality for SQCD.

Dynamics of the Electric Theory

The electric theory contains a scalar. At infinite-N_c the electric theory is supersymmetric and the scalar is massless. Seiberg duality is trivial at infinite N_c.

When N_c is finite the electric "squark" acquires a mass due to its interaction with the "gluon" and the "gluino"



$$M_\Phi^2 \sim g_e^2 \Lambda^2 = \frac{\lambda_e}{N_c}\Lambda^2$$

As a result the "squark" decouples and we end up with a QCD-like theory.

We anticipate confinement and a formation of a quark condensate that will break the global symmetry

$$SU(2N_f) \rightarrow Sp(2N_f)$$

Dynamics of the Magnetic Theory - The Squark

Similarly to the electric theory, the magnetic theory contains a scalar (a "squark").

The analysis of the one-loop Coleman-Weinberg potential yields

$$M_\phi^2 \sim (g_m^2 - y^2)\Lambda^2\,,$$

with y - the Yukawa coupling of the squark to the meson and the mesino.

A two-loop analysis of the beta functions for g_m and y, carried out by Oehme, shows that the two couplings reduce to one coupling if

$$\frac{g_m^2}{y^2} = 3\frac{N_f}{N_c} - 1\,.$$

Hence $g_m^2 > y^2$, $M_\phi^2 > 0$ and the magnetic squark decouples.

Dynamics of the Magnetic Theory - The Meson

We now turn to the dynamics of the meson field. The meson interacts with a massive squark and a massless quark, hence it acquires a potential. It is easy to see that the meson acquires a *tachyonic* mass, and therefore the naive vacuum with $\langle M_{[ij]}\rangle = 0$ is *unstable.*

The Coleman-Weinberg potential for the vev $\langle M_{[ij]}\rangle \equiv m_{[ij]}$ takes the form

$$V(\{m_{[ij]}\}) =$$
$$\tilde{N}_c \left(\mathrm{tr} \int \frac{d^4p}{(2\pi)^4} \log(p^2 + y^2(mm^\dagger) + g_m^2\Lambda^2) \right.$$
$$\left. - \mathrm{tr} \int \frac{d^4p}{(2\pi)^4} \log(p^2 + y^2(mm^\dagger)) \right) .$$

We arrive at

$$\hat{V}(\{\hat{m}_{[ij]}\}) =$$
$$\frac{1}{4\pi^2} \mathrm{tr} \left\{ (\hat{m}\hat{m}^\dagger + g_m^2)^2 \log(\hat{m}\hat{m}^\dagger + g_m^2) \right.$$
$$\left. - (\hat{m}\hat{m}^\dagger)^2 \log(\hat{m}\hat{m}^\dagger) \right\}$$

where $\hat{V} = V/(\tilde{N}_c\Lambda^4)$ and $\hat{m} = ym/\Lambda$.

The Meson Potential

The function $\hat{V}$ is plotted below

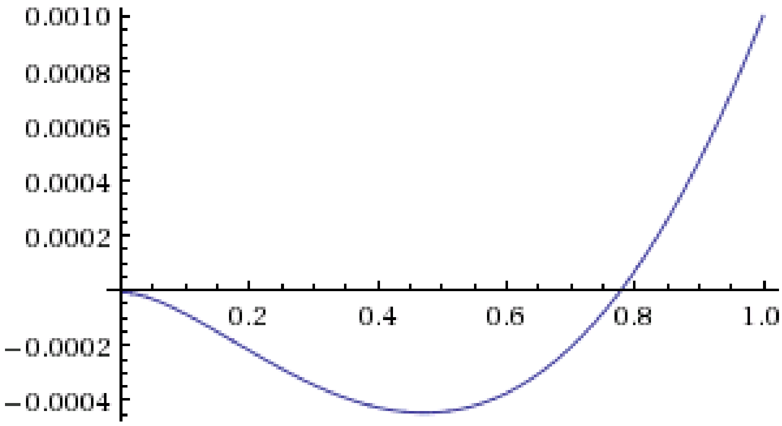


Figure 1: The potential $4\pi^2\hat{V}$ for the meson field. In this figure we used $g_m^2 = 0.001$. The minimum is at $\hat{m}\hat{m}^\dagger = \exp(-3/2)$.

In the vacuum $SU(2N_f)$ is dynamically broken to $Sp(2N_f)$, resulting in $(2N_f)^2 - 1 - \frac{1}{2}2N_f(2N_f+1)$ massless Nambu-Goldstone mesons that belong to the coset $SU(2N_f)/Sp(2N_f)$.

In addition, the $U(1)_R$ symmetry also gets broken, resulting in an additional Nambu-Goldstone boson.

The Quark Condensate

What is the corresponding condensate in the electric theory ? Let us use the dictionary of the supersymmetric theory

$$M_{[ij]} = \Phi^a_{[i}\Phi^a_{j]} \, ,$$

where Φ is the electric squark. The equations of motion for the massive field Φ^a_i relate it to $\lambda\Psi$ as follow

$$\Phi^a_i \sim \lambda^a_b \Psi^b_i \, ,$$

hence the meson condensate can be identified with the four fermion condensate

$$\langle M_{[ij]} \rangle \sim \langle \lambda\Psi_{[i}\lambda\Psi_{j]} \rangle$$

A consistency check of the above identification is that both the meson operator and the four fermion electric operator have the same $U(1)_R$ charge, with $R = \frac{2(N_f - N_c + 1)}{N_f}$. Dynamical symmetry breaking is thus understood as due to quark condensation, similarly to the chiral quark condensate formation in QCD.

The Spectrum of the Magnetic Theory

Let us comment on the fate of the $Sp(2\tilde{N}_c)$ gauge theory. As the meson condenses, both the squark and the quark acquire a mass due to the superpotential $W = \frac{1}{\mu} Mqq$. The color and flavor theories will decouple. The $Sp(2\tilde{N}_c)$ theory is expected to confine and to exhibit a mass gap, similar to pure $\mathcal{N} = 1$ Super Yang-Mills theory. The glueballs of the color theory are massive and hence decouple from the IR theory. Therefore the only massless fields of the magnetic theory are the Nambu-Goldstone bosons associated with the breaking of the $SU(2N_f)$ flavor symmetry and an additional Nambu-Goldstone boson associated with the breaking of the $U(1)_R$ symmetry.

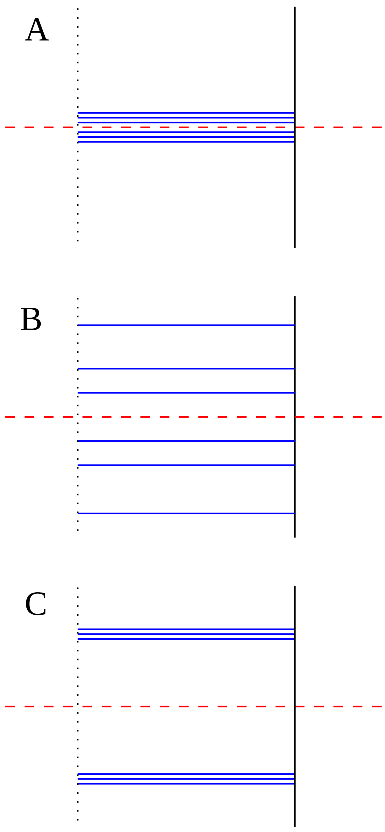
Back to the Brane Configuration

The tachyonic nature of the meson field means that the magnetic brane configuration is unstable.

The Coleman-Weinberg potential for the meson has a simple interpretation in terms of the brane configuration. It represents the effective potential between the orientifold plane and the flavor branes.

A priori, given that the theory is non-supersymmetric, we could have imagined several scenarios for the vacuum brane configuration.

The Vacuum of the Magnetic Theory



String theory chooses the third configuration where the branes attract each other and get repelled away from the orientifold plane. This configuration where the eigenvalues (branes) coincide, but differ from zero, corresponds to $SO(2N_f) \to U(N_f)$.

This is a manifestation of the Vafa-Witten theorem in string theory.

$SU(N)$ Theories

We wish to consider a theory which is closer to QCD: A theory based on the gauge group $SU(N_c)$ with a chiral $SU(N_f) \times SU(N_f)$ global symmetry.

A natural setup is type 0' string theory - a non-supersymmetric string theory that was constructed by Sagnotti.

By using either non-critical string theory or a type 0A brane configuration, it was suggested (A.A., Israel, Moraitis, Niarchos, 2008) that the following electric $U(N_c)$ and magnetic $U(N_f - N_c + 4)$ theories admit a Seiberg duality.

The electric theory

Type 0 Electric Theory				
	$\mathrm{U}(N_c)$	$\mathrm{SU}(N_f)$	$\mathrm{SU}(N_f)$	$\mathrm{U}(1)_R$
A_μ	adjoint	$\bullet$	$\bullet$	0
λ	$\begin{array}{c}\Box\\\Box\end{array}$	$\bullet$	$\bullet$	1
$\tilde{\lambda}$	$\overline{\begin{array}{c}\Box\\\Box\end{array}}$	$\bullet$	$\bullet$	1
Φ	$\overline{\Box}$	$\Box$	$\bullet$	$\frac{N_f - N_c + 2}{N_f}$
Ψ	$\Box$	$\Box$	$\bullet$	$\frac{-N_c + 2}{N_f}$
$\tilde{\Phi}$	$\Box$	$\bullet$	$\overline{\Box}$	$\frac{N_f - N_c + 2}{N_f}$
$\tilde{\Psi}$	$\overline{\Box}$	$\bullet$	$\overline{\Box}$	$\frac{-N_c + 2}{N_f}$

Table 3: *The non-susy electric theory.*

By swapping the NS5 branes we arrive at the magnetic theory.

The magnetic theory

Type 0 Magnetic Theory ($\tilde{N}_c = N_f - N_c + 4$)				
	$\mathrm{U}(\tilde{N}_c)$	$\mathrm{SU}(N_f)$	$\mathrm{SU}(N_f)$	$\mathrm{U}(1)_R$
A_μ	adjoint	$\bullet$	$\bullet$	0
λ	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	$\bullet$	$\bullet$	1
$\tilde{\lambda}$	$\overline{\begin{smallmatrix}\square\\\square\end{smallmatrix}}$	$\bullet$	$\bullet$	1
ϕ	$\square$	$\overline{\square}$	$\bullet$	$\frac{N_c-2}{N_f}$
ψ	$\overline{\square}$	$\overline{\square}$	$\bullet$	$\frac{N_c-N_f-2}{N_f}$
$\tilde{\phi}$	$\overline{\square}$	$\bullet$	$\square$	$\frac{N_c-2}{N_f}$
$\tilde{\psi}$	$\square$	$\bullet$	$\square$	$\frac{N_c-N_f-2}{N_f}$
M	$\bullet$	$\square$	$\overline{\square}$	$\frac{2N_f-2N_c+4}{N_f}$
χ	$\bullet$	$\square\square$	$\bullet$	$\frac{N_f-2N_c+4}{N_f}$
$\tilde{\chi}$	$\bullet$	$\bullet$	$\overline{\square\square}$	$\frac{N_f-2N_c+4}{N_f}$

Table 4: *The matter content of the proposed magnetic theory with number of colours* $\tilde{N}_c = N_f - N_c + 4$.

Arguments in favor of Seiberg duality

The technical details are similar to the previous case. In the Veneziano large N_c limit the electric and magnetic theories are equivalent to $U(N_c)$ and $U(N_f - N_c)$ electric and magnetic SQCD.

Moreover, the 't Hooft anomalies match.

And finally, the duality is supported by string theory (the brane swapping argument).

What are the implications of the duality in this case ?

Seiberg duality - Implications

The one-loop Coleman-Weinberg potential for the meson admits a unique minimum where $SU(N_f) \times SU(N_f) \to SU(N_f)$. As a result there will be $N_f^2 - 1$ massless Nambu-Goto mesons. In addition, the $U(1)_R$ is also broken - resulting in an additional massless meson.

There meson condensate is interpreted as a four fermion condensate of the form

$$\langle M_{[ij]} \rangle \sim \langle \lambda \Psi_{[i} \lambda \Psi_{j]} \rangle$$

similar to the QCD chiral condensate.

Other degrees of freedom acquire a mass and decouple.

Conclusions

By using insights from string theory we proposed a non-supersymmetric Seiberg duality.

The 'electric' and 'magnetic' theories admit a description in terms of a non-supersymmetric brane configuration which consists of an orientifold plane.

In the first case ($Sp(2N_c)$) susy is broken because branes are replaced by anti-branes.

In the second case ($U(N_c)$) susy is broken because type 0 string theory is a non-supersymmetric theory (it can be viewed as an orbifold the type II string).

Both field theories are very similar to each other. In both cases the meson admits a potential with properties predicted by Coleman-Witten almost 40 years ago!

Could we learn more about strongly coupled gauge theories by studying non-supersymmetric brane configurations?