Studies of large-N reduction with adjoint fermions

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“Non-perturbative volume-reduction of large-N QCD with adjoint fermions”
BB and S.R. Sharpe, 0906.3538

“Large-N volume reduction of lattice QCD with adjoint Wilson fermions at weak coupling”
BB, JHEP, 0905.2406
Main motivation

Study a large-N limit of QCD where quarks are back-reacting on gauge fields

Armoni-Veneziano-Shifman:

QCD(AS)
infinite volume

Orientifold

QCD(Adj)
infinite volume

Kovtun-Unsal-Yaffe:

QCD(Adj)
infinite volume

Orbifold

QCD(Adj)
“zero” volume

“Eguchi-Kawai”
volume reduction
works if have light
adjoint quarks around

Study QCD(Adj) on a single-site gives physics of QCD(AS) at infinite volume
More motivations

QCD + adjoints is an interesting theory:

- \( N_f=1/2 \) is softly broken \( \mathcal{N}=1 \) SUSY
- \( N_f=2 \) is in (or close by to) the conformal window.
- Any value of \( N_f=2 \) and heavy enough quarks is YM.

Does it really provide a workable Eguchi-Kawai? (finally, after 27 years...)

- Original Eguchi-Kawai. Jan `82.
- Quenched Eguchi-Kawai. Bhanot, Heller, Neuberger, Feb `82
- Twisted Eguchi-Kawai. Gonzalez-Arroyo, Okawa, July `82

Can study large-N limit of all these with method I will describe.

But Bhanot-Heller-Neuberger Feb `82.
But BB-Sharpe, 2008.
But, Teper-Vairinhos, Hanada-et-al, Bietenholtz et al. 2006.

This talk. No “But”’s?
I. What is the Eguchi-Kawai equivalence.

Given SU(N) gauge theory on an $L_1 \times L_2 \times L_3 \times L_4$ lattice, defined by $g^2N, am, a\mu, \ldots$

Then if:

- Translation symmetry is intact.
- $Z_N$ center symmetry is intact.
- large-N factorization holds.

At $N=\infty$, Wilson loops, Hadron spectra, condensates, etc. are independent of $L_{1,2,3,4}$. 
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This talk: fate of $Z_N$. First at weak coupling, then non-perturbatively
I. What is the Eguchi-Kawai equivalence.

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Wilson loops, Hadron spectra, condensates, etc.
are independent of $L_{1,2,3,4}$.

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Not “academic” requirements:

Breakdown of EK equivalence by formation of a baryon crystal

BB, PRD, 0811.4141, 0901.4035

QEK model
BB+Sharpe 2008

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at $N=\infty$
II. Weak coupling analysis: $L_{2,3,4} = \infty$, $L_1 = 1$.  

Fields: 

$$U_{\mu=0}(\vec{x}, x_0 = 1) \equiv \Omega_{\vec{x}}$$

$$U_{\mu=1,2,3}(\vec{x}, x_0 = 1) \equiv U_{\mu=1,2,3}(\vec{x})$$

$$\psi(\vec{x}, x_0 = 1) \equiv \psi_{\vec{x}}$$

Action:

Gauge: Wilson, $b = 1/g^2 N$

Fermions: Wilson, $\kappa = \frac{1}{8 + 2am_0}$

kappa=$1/8$ : chiral quarks

kappa=$0$ : infinitely massive quarks

Weak coupling expansion:

$$\Omega_{\vec{x}}^{ab} = \delta^{ab} e^{i\theta^a} + \delta \Omega^{ab}(\vec{x}) + \ldots$$

$$g^2 N \to 0 :$$

$$U_{\vec{x},\mu} = 1 + iA_\mu(\vec{x}) + \ldots$$
II. Weak coupling analysis: $L_{2,3,4} = \infty$, $L_1 = 1$. BB, JHEP, 0905.2406

Get familiar form

$$V(\theta) = \sum_{a \neq b} \int \left(\frac{dp}{2\pi}\right)^3 \left\{ \log \left[ \hat{p}^2 + 4 \sin^2 \left(\frac{\theta^a - \theta^b}{2}\right) \right] - 2N_f \log \left[ \hat{p}^2 + \sin^2 (\theta^a - \theta^b) + m_W^2 (\theta, p) \right] \right\}$$

$$\hat{p}^2 = 4 \sum_{i=1}^{3} \sin^2 p_i/2 \overset{a \rightarrow b}{\longrightarrow} \hat{p}^2 \quad \hat{p}^2 = \sum_{i=1}^{3} \sin^2 p_i \overset{a \rightarrow b}{\longrightarrow} \hat{p}^2 \quad m_W = a m_0 + 2 \left[ \sin^2(p/2) + \sin^2((\theta^a - \theta^b)/2) \right]$$

Calculate $V(\theta)$ potential for different $\theta^a$ corresponding to $Z_N$, $Z_N \rightarrow \emptyset$, $Z_N \rightarrow Z_2$.

These are good news:
Reduction works!
II. Weak coupling analysis: $L_{2,3,4} = \infty$, $L_1=1$.  

Aside: Comparing some weak coupling calculations.

<table>
<thead>
<tr>
<th>Kovtun et al. ‘07</th>
<th>Lattice calculation</th>
<th>Bedaque et al. ‘08</th>
</tr>
</thead>
<tbody>
<tr>
<td>4D YM + adjoints continuum</td>
<td>4D YM + Wilson adjoints and $L_1=1$</td>
<td>Think about $L_1=1$ as 3D theory defined on spatial continuum</td>
</tr>
<tr>
<td>$Z_N$</td>
<td>$Z_N$</td>
<td>$Z_N \rightarrow Z_2$</td>
</tr>
</tbody>
</table>

Useful to understand the reason for the difference
II. Weak coupling analysis: $L_{2,3,4} = \infty$, $L_1=1$. BB, JHEP, 0905.2406

Bedaque et al. (focus on gauge dynamics first)

\[
S_{\text{gauge}} = \frac{2N}{\lambda} \text{Re} \sum_{x} \text{Tr} \left( U_{x,i} U_{x+1,j} U_{x+1,j}^\dagger U_{x,j}^\dagger \right) + \frac{2N}{\lambda} \text{Re} \sum_{x} \text{Tr} \left( U_{x,i} \Omega_{x,i} U_{x,i}^\dagger \Omega_{x,i}^\dagger \right)
\]

\[
S_{\text{one-site}} = \int d^3 x \left( \frac{1}{g^2} \text{Tr} \sum_{i<j}^3 F_{ij}^2 + f^2 \text{Tr} \sum_i \left| D_i \Omega \right|^2 \right)
\]

\[
D_i \Omega(x) = \partial_i \Omega(x) + i[A_i, \Omega]
\]

$\Omega \in SU(N)$
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\[ \Rightarrow \text{"} a_s \rightarrow 0 \text{"} \]

$$S_{\text{one-site}} = \int d^3x \left( \frac{1}{g^2} \text{Tr} \sum_{i<j \in [1,3]} F_{ij}^2 + f^2 \text{Tr} \sum_i |D_i \Omega|^2 \right)$$

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"$a_s \to 0$"

$$S_{\text{one-site}} = \int d^3x \left( \frac{1}{g^2} \text{Tr} \sum_{i<j\in[1,3]} F_{ij}^2 + f^2 \text{Tr} \sum_i |D_i \Omega|^2 \right)$$

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$$= \Lambda^3 + \Lambda \sum_{a \neq b} \sin^2 \left( \frac{\theta^a - \theta^b}{2} \right) + \ldots$$

$$= \Lambda^3 + \Lambda \left| \text{tr} \Omega_{\text{classical}} \right|^2 + \ldots, \Omega_{\text{classical}}^{ab} = e^{i\theta^a}$$

---

$S_{\text{one-site}}$ is non-renormalizable ... need counter-terms...

Bernard&Appelquist `80, Longhitano `80, Banks&Ukawa `84, Gasser-Leutwyler `84, Arkani-Hamed-Cohen-Georgi `01, Pisarski `06,
II. Weak coupling analysis: $L_{2,3,4} = \infty$, $L_1=1$. BB, JHEP, 0905.2406

What does this teach us?

- Continuum limit in space with $L_1=1$ (or for any $D > 2L_1$).
- Need counter-terms $\rightarrow$ new **Low Energy Constants (LEC)**.
- Means treating theory as an **Effective Field Theory (EFT)**, and at one-loop:

\[ V(\theta) \rightarrow V(\theta) + b_1 |\text{tr} \Omega_{\text{classical}}|^2 + b_2 |\text{tr} \Omega_{\text{classical}}^2|^2 \]

Amusing: these are (Mithat and Larry)'s terms

Dim-reg hides this and **implicitly** sets $b_1=b_2=0 \rightarrow b_{1,2} > 0$ fixes $Z_N \rightarrow Z_2$ breaking.
II. Weak coupling analysis: \( L_{2,3,4} = \infty, \ L_1 = 1 \). \( \text{BB, JHEP, 0905.2406} \)

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EFT point of view is not necessary

In any case, large-\( N \) reduction defined at fix lattice cutoff:

But \( \mathbb{Z}_N \)-realization may depend on lattice action ...

Results I got cannot be anticipated in advance (from Kovtun et al.)
In any case, we saw that: Wilson fermions at weak coupling obey reduction

But really need a non-perturbative lattice study

- Really interested in $L_{1,2,3,4}=1$, but IR div’s.
- What happens at $g^2 N \sim 1 - 3$?
- Non-perturbative effect (e.g. QEK and TEK model).
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Wilson gauge + $N_f=1$ Wilson fermions, Metropolis.  
SU($N$) with $N=8,10,11,13,15$.
A variety of spacings and masses.

Goal: Map single-site theory in $\kappa$ and $g^2 N$  
Look for intact $ZN$.  

BB+S.Sharpe, 0906.3538
III. Results of non-perturbative MC lattice simulations

What should we expect? $L_{1,2,3,4} = oo$:

- Continuum physics
  
  $(1/g^2 N = ) b$

- Strong-to-weak lattice transition

- Strong-coupling/lattice physics

- Quarks are light along line

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Continuum physics

strong-coupling/lattice physics

$\frac{1}{g^2 N} = b$

$0.25$ $0.125$ $0.04$ $0.19$
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Continuum physics

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$\sim 0.04$

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- $\sim 0.04$
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- quarks are light along line
- $\kappa_c(b)$
- $\mathcal{Z}_N^4$
- $0.125$
- $0.25$

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  - At $b\approx 0.19$

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  - At $b\approx 0.04$
  - $\kappa_c(b)$
  - $b_{\text{bulk}}$

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III. Results of non-perturbative MC lattice simulations

Scan no. 1: infinitely massive quarks

X-axis: Real(Polyakov)
Y-axis: Imag(Polyakov)
III. Results of non-perturbative MC lattice simulations

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\[ b = 0 \]
III. Results of non-perturbative MC lattice simulations

Scan no. 1: infinitely massive quarks

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\[ b = 0 \quad \text{and} \quad b \approx 0.3 \]
III. Results of non-perturbative MC lattice simulations

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X-axis: Real(Polyakov)
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\[ b = 0 \quad b \approx 0.3 \quad b = 0.5 \]
III. Results of non-perturbative MC lattice simulations

Scan no. 2: decreasing the quark mass $b=0.5$, SU(10)
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$k \approx 0$

$k = 0.03$
III. Results of non-perturbative MC lattice simulations

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$\kappa = 0.1475$
III. Results of non-perturbative MC lattice simulations

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III. Results of non-perturbative MC lattice simulations

Scan no. 2: decreasing the quark mass $b=0.5$, $\text{SU}(15)$

$\kappa = 0$

$\kappa = 0.06$

$\kappa = 0.09$

$\kappa = 0.1275$

$\kappa = 0.1475$

$\kappa = 0.155$
III. Results of non-perturbative MC lattice simulations

Scan no. 2: looking for the “critical” line

b = 0.35: 1st transition at kappa ~ 0.15

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Scan no. 2: looking for the “critical” line

III. Results of non-perturbative MC lattice simulations

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\[ b = 0.35: \]

1st transition at \( \kappa \sim 0.15 \)
III. Results of non-perturbative MC lattice simulations

Scan no. 2: looking for the "critical" line

1st transition structure at all $b$. Extending from $\kappa=0.25$ to 0.125
III. Results of non-perturbative MC lattice simulations

All results consistent with phase diagram and validity of reduction.

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But:

- See long autocorrelation times there at very very weak coupling.
- More order parameters for nontrivial breaking of $Z_N$. 

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Perform long runs and measure order parameters of the form

$$\text{tr } [P_1^{n_1} P_2^{n_2} P_3^{n_3} P_4^{n_4}] \text{ with } n_i \in [-5, 5]$$

14641 order parameters!
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14641 order parameters!

Still no sign of breakdown of $Z_N$, up to $b=0.5-1.0$
IV. Conclusions and future prospects.

Weak coupling with $L_{2,3,4}=\infty$, $L_1=1$

- Large-N volume reduction works for YM+Wilson adjoints fermions.

Non-perturbative lattice Monte-Carlo of $N_f=1$ case.

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This is exciting:
Eguchi-Kawai finally works
start extracting physics of QCD(Adj) and QCD(AS)

- Mesons, baryons (see Carlos Q. Hoyos’s talk).
- Realization of chiral symmetry (of both adjoint sea-quarks and valence fundamental).
- Comparisons with RMT.
- Static potential, string tensions.
- Open to suggestions ...

A new UNEXPLORED physically relevant and rich theory
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pretty cool... 😎