Studies of large-N reduction with adjoint fermions

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"Non-perturbative volume-reduction of large-N QCD with adjoint fermions"
BB and S.R.Sharpe, 0906.3538

"Large-N volume reduction of lattice QCD with adjoint Wilson fermions at weak coupling"
BB, JHEP, 0905.2406

Main motivation

Study a large-N limit of QCD where quarks are back-reacting on gauge fields

→ Natural to put quarks in two-antisymmetric :

QCD(AS)
Corrigan-Ramond,
Armoni-Shifman-Veneziano,
Sannino et al.

Armoni-Veneziano-Shifman:

QCD(AS)
infinite volume

$\xleftrightarrow{\text{Orientifold}}$

QCD(Adj)
infinite volume

Kovtun-Unsal-Yaffe:

QCD(Adj)
infinite volume

$\xleftrightarrow{\text{Orbifold}}$

QCD(Adj)
"zero" volume

"Eguchi-Kawai"
volume reduction
works if have light
adjoint quarks around

→ **Study QCD(Adj) on a single-site gives physics of QCD(AS) at infinite volume**

More motivations

QCD + adjoints is an interesting theory:

- N_f=1/2 is softly broken $\mathcal{N}$=1 SUSY
- N_f=2 is in (or close by to) the conformal window.
- Any value of N_f=2 and heavy enough quarks is YM.

Can study large-N limit of all these with method I will describe.

Does it really provide a workable Eguchi-Kawai ? (finally, after 27 years...)

- Original Eguchi-Kawai. Jan `82. But Bhanot-Heller-Neuberger Feb `82.
- Quenched Eguchi-Kawai. Bhanot Heller Neuberger, Feb `82 But BB-Sharpe, 2008.
- Twisted Eguchi-Kawai. Gonzalez-Arroyo Okawa, July `82 But, Teper-Vairinhos, Hanada-et-al, Bietenholtz et al. 2006.

- Adjoint Eguchi-Kawai. Kovtun-Unsal-Yaffe, 2007 This talk.

No "But"'s ?

- Deformed Eguchi-Kawai. Unsal-Yaffe, 2008 BB, Vairinhos, 2008

I. What is the Eguchi-Kawai equivalence.

Given SU(N) gauge theory on an $L_1 x L_2 x L_3 x L_4$ lattice, defined by $g^2 N, am, a\mu, \ldots$

Then if:

- Translation symmetry is intact.
- Z_N center symmetry is intact.
- large-N factorization holds.

at N=oo →

Wilson loops, Hadron spectra, condensates, etc. are independent of $L_{1,2,3,4}$.

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This talk: fate of Z_N. First at weak coupling, then non-perturbatively

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Not "academic" requirements:

breakdown of EK equivalence by formation of a baryon crystal

BB, PRD,0811.4141, 0901.4035

QEK model

BB+Sharpe 2008

at N=oo

Wilson loops, Hadron spectra, condensates, etc. are independent of $L_{1,2,3,4}$.

This talk: fate of Z_N. First at weak coupling, then non-perturbatively

II. Weak coupling analysis: $L_{2,3,4} = \infty$, **$L_1=1$.** BB, JHEP, 0905.2406

Fields:

$$U_{\mu=0}(\vec{x}, x_0 = 1) \equiv \Omega_{\vec{x}}$$

$$U_{\mu=1,2,3}(\vec{x}, x_0 = 1) \equiv U_{\mu=1,2,3}(\vec{x})$$

$$\psi(\vec{x}, x_0 = 1) \equiv \psi_{\vec{x}}$$

Action:

Gauge: Wilson, $b = 1/g^2 N$

Fermions: Wilson, $\kappa = \dfrac{1}{8 + 2am_0}$

kappa=1/8 : chiral quarks
kappa=0 : infinitely massive quarks

Weak coupling expansion:

$g^2 N \to 0$:

$$\Omega^{ab}_{\vec{x}} = \delta^{ab}\, e^{i\theta^a} + \delta\Omega^{ab}(\vec{x}) + \ldots$$

$$U_{\vec{x},\mu} = 1 + iA_\mu(\vec{x}) + \ldots$$

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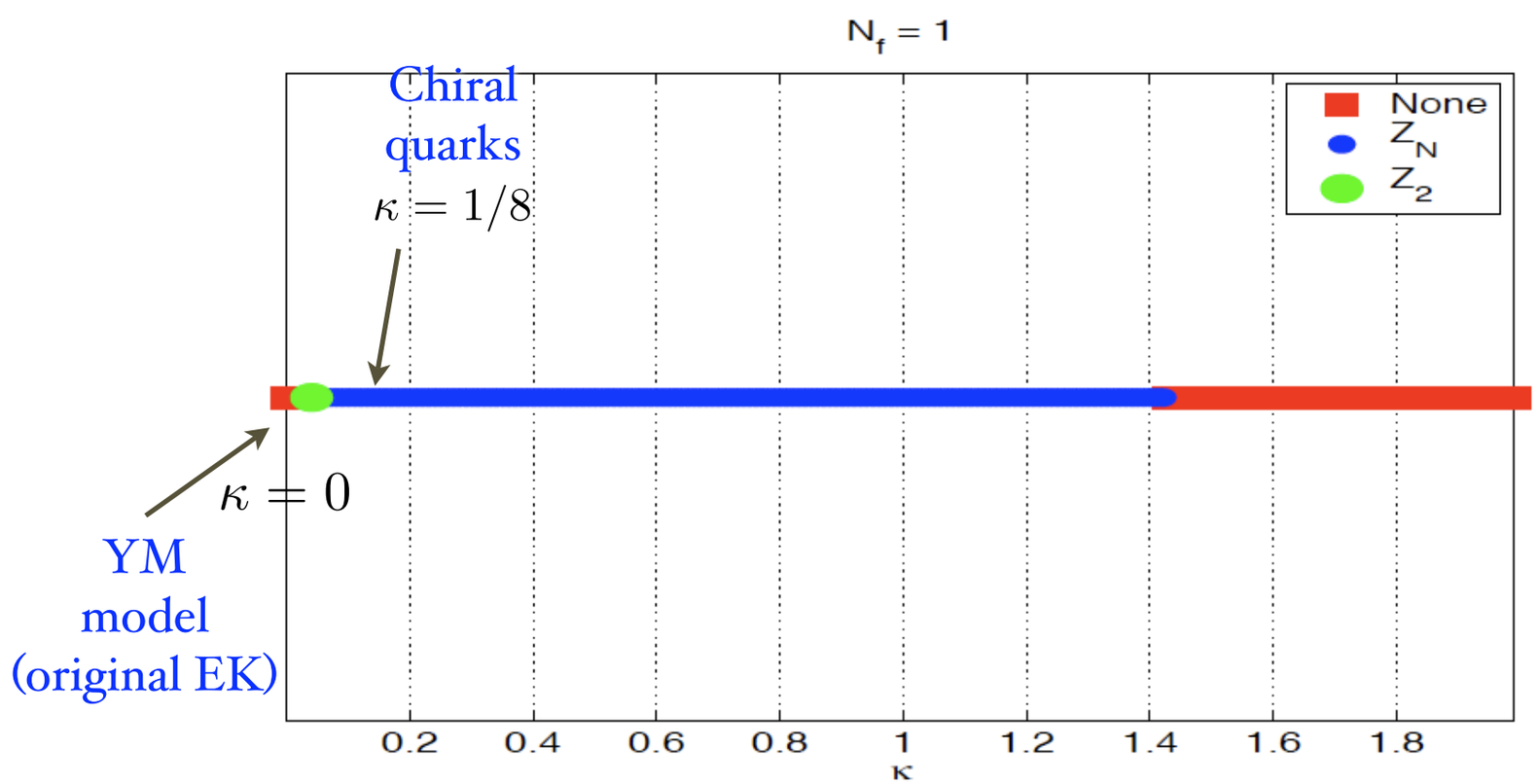
Get familiar form

$$V(\theta) = \sum_{a\neq b} \int \left(\frac{dp}{2\pi}\right)^3 \left\{ \log\left[\hat{p}^2 + 4\sin^2\left(\frac{\theta^a-\theta^b}{2}\right)\right] - 2N_f \log\left[\hat{\hat{p}}^2 + \sin^2\left(\theta^a-\theta^b\right) + m_W^2(\theta,p)\right]\right\}$$

$$\hat{p}^2 = 4\sum_{i=1}^{3}\sin^2 p_i/2 \xrightarrow{a\to 0} \vec{p}^2 \qquad \hat{\hat{p}}^2 = \sum_{i=1}^{3}\sin^2 p_i \xrightarrow{a\to 0} \vec{p}^2 \qquad m_W = am_0 + 2\left[\sin^2(p/2) + \sin^2((\theta^a-\theta^b)/2)\right]$$

Calculate $V(\theta)$ potential for different θ^a corresponding to Z_N, $Z_N \to \emptyset$, $Z_N \to Z_2$.



These are good news:

Reduction works !

II. Weak coupling analysis: $L_{2,3,4} = \infty$, **$L_1=1$.** BB, JHEP, 0905.2406

Aside : Comparing some weak coupling calculations.

Kovtun et al. '07	Lattice calculation	Bedaque et al. '08
4D YM + adjoints continuum	4D YM+Wilson adjoints and $L_1=1$	Think about $L_1=1$ as 3D theory defined on spatial continuum
Z_N	Z_N	$Z_N \to Z_2$

Useful to understand the reason for the difference

II. Weak coupling analysis: $L_{2,3,4} = \infty$, $L_1=1$. BB, JHEP, 0905.2406

Bedaque et al. (focus on gauge dynamics first)

$$S_{\text{gauge}} = \frac{2N}{\lambda}\,\text{Re}\sum_{\substack{x\\ i<j}}\text{Tr}\left(U_{x,i}U_{x+i,j}U^\dagger_{x+j,i}U^\dagger_{x,j}\right)$$

$$+\frac{2N}{\lambda}\,\text{Re}\sum_{\substack{x\\ i}}\text{Tr}\left(U_{x,i}\Omega_{x+i}U^\dagger_{x,i}\Omega^\dagger_{x}\right)$$

$$\xrightarrow{\text{“}a_s \to 0\text{”}}$$

$$S^{\text{one-site}} = \int d^3x\left(\frac{1}{g^2}\text{Tr}\sum_{i<j\in[1,3]}F^2_{ij} + f^2\,\text{Tr}\sum_i |D_i\,\Omega|^2\right)$$

$$D_i\Omega(x) = \partial_i\Omega(x) + i[A_i,\Omega]$$

$$\Omega \in SU(N)$$

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$$= \Lambda^3 + \Lambda\sum_{a\neq b}\sin^2\left(\frac{\theta^a-\theta^b}{2}\right) + \ldots$$

$$= \Lambda^3 + \Lambda\,|\text{tr}\,\Omega_{\text{classical}}|^2 + \ldots,\ \Omega^{ab}_{\text{classical}} = e^{i\theta^a}$$

$\longrightarrow$ $S^{\text{one-site}}$ is non-renormalizable ... need counter-terms...

Bernard&Appelquist `80, Longhitano `80, Banks&Ukawa `84, Gasser-Leutwyler ‘84, Arkani-Hamed-Cohen-Georgi `01, Pisarski `06,

II. Weak coupling analysis: $L_{2,3,4} = \infty$, $L_1=1$. BB, JHEP, 0905.2406

What does this teach us?

- Continuum limit in space with $L_1=1$ (or for any $D > 2L_1$).
  - need counter-terms $\longrightarrow$ new **L**ow **E**nergy **C**onstants (**LEC**).
  - means treating theory as an **E**ffective **F**ield **T**heory (**EFT**), and at one-loop:

$$V(\theta) \longrightarrow V(\theta) + b_1 \left|\mathrm{tr}\,\Omega_{\mathrm{classical}}\right|^2 + b_2 \left|\mathrm{tr}\,\Omega^2_{\mathrm{classical}}\right|^2$$

Amusing: these are (Mithat and Larry)'s terms

Dim-reg hides this and implicitly sets $b_1=b_2=0 \longrightarrow b_{1,2} > 0$ fixes $Z_N \longrightarrow Z_2$ breaking.

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EFT point of view is not necessary

In any case, large-N reduction defined at fix lattice cutoff:

But Z_N-realization may depend on lattice action ...

Results I got cannot be anticipated in advance (from Kovtun et al.)

In any case, we saw that :
Wilson fermions at weak coupling obey reduction

But really need a non-perturbative lattice study

- Really interested in $L_{1,2,3,4}=1$, but IR div's.
- What happens at $g^2 N \simeq 1 - 3$?
- Non-perturbative effect (e.g. QEK and TEK model).

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- Wilson gauge + $N_f=1$ Wilson fermions, Metropolis. BB+S.Sharpe, 0906.3538
- SU(N) with N=8,10,11,13,15.
- A variety of of spacings and masses.

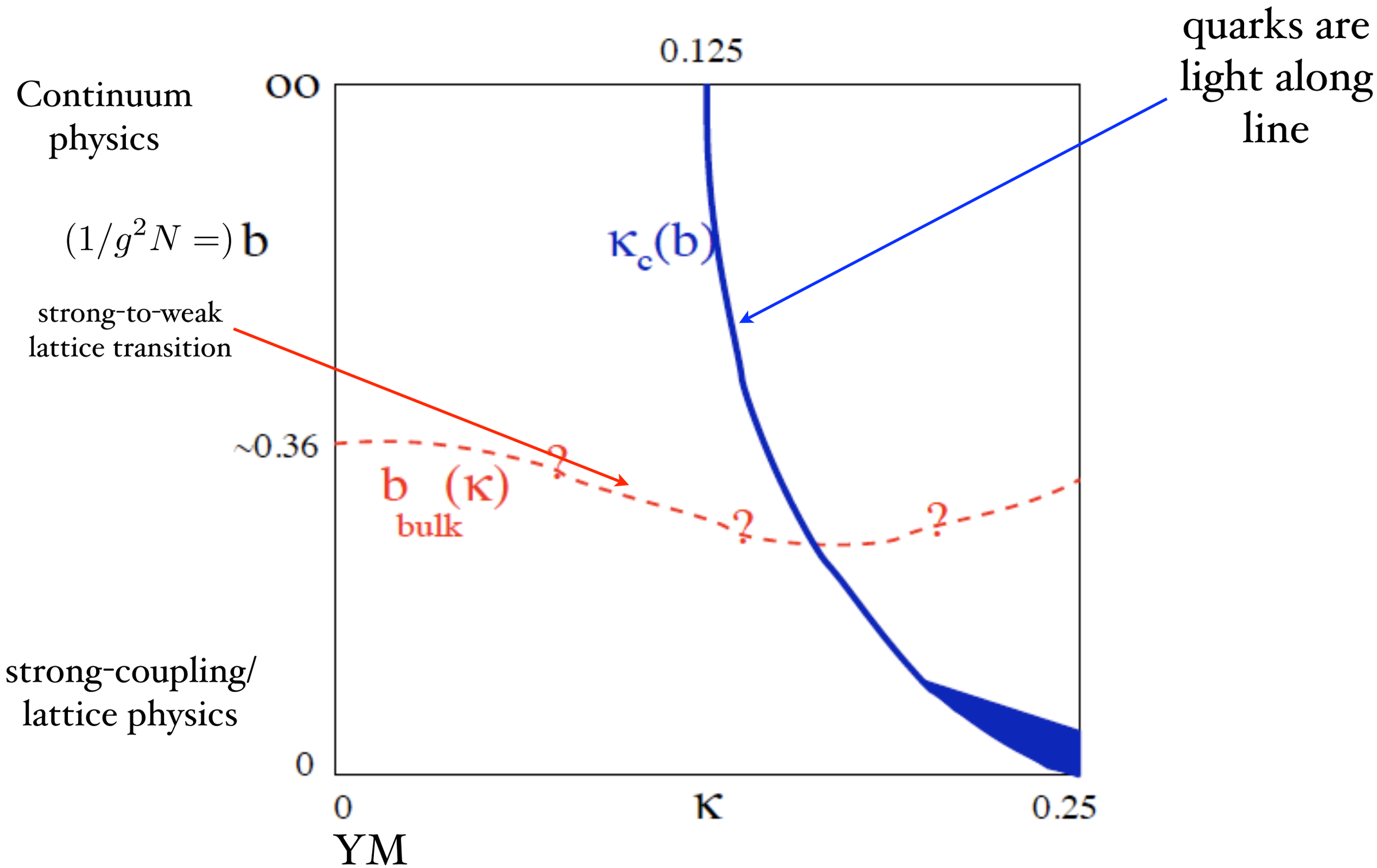
Goal : Map single-site theory in κ and $g^2 N$

Look for intact ZN.

III. Results of non-perturbative MC lattice simulations

BB+S.Sharpe, 0906.3538

What should we expect? $L_{1,2,3,4}=\infty$:



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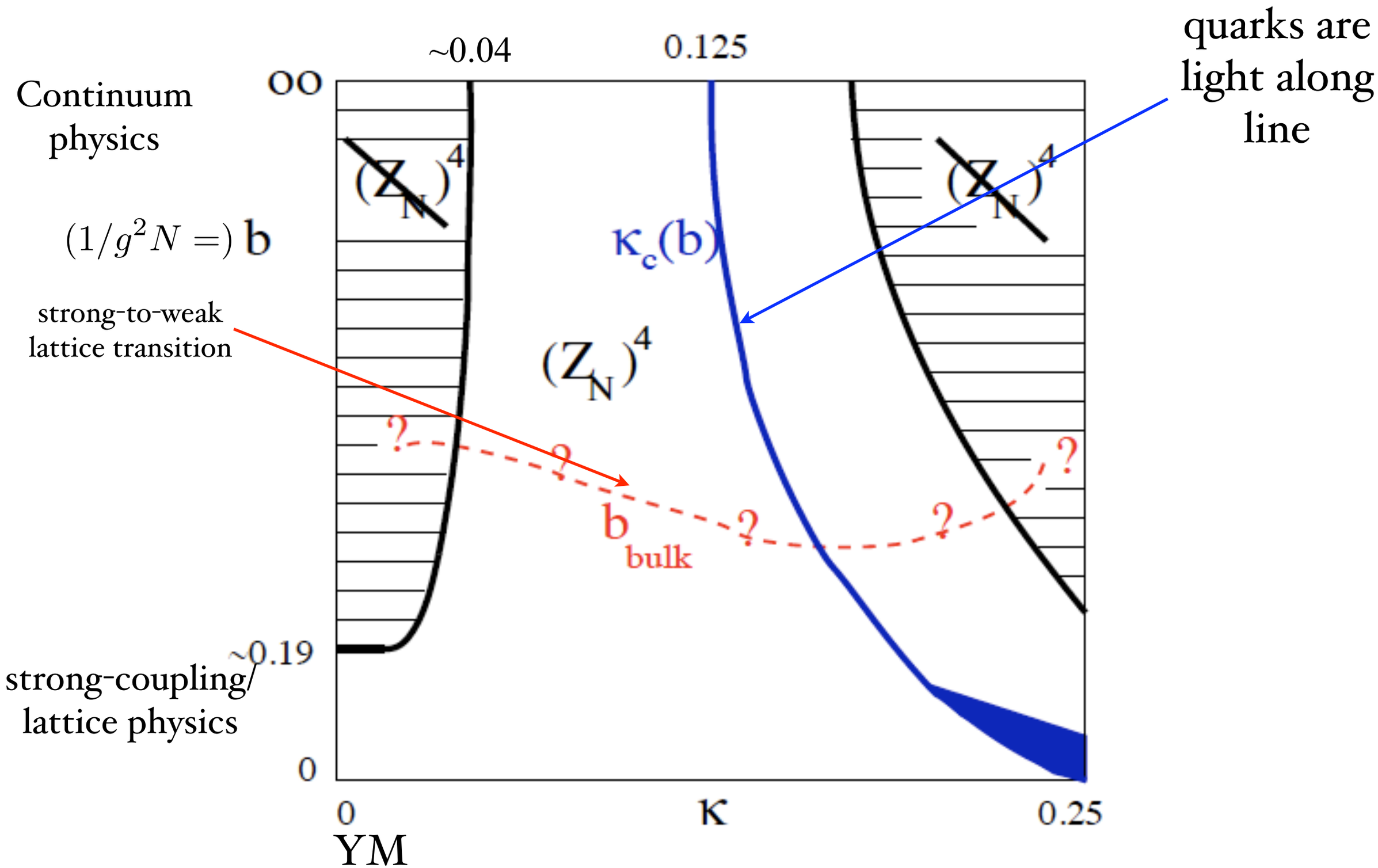

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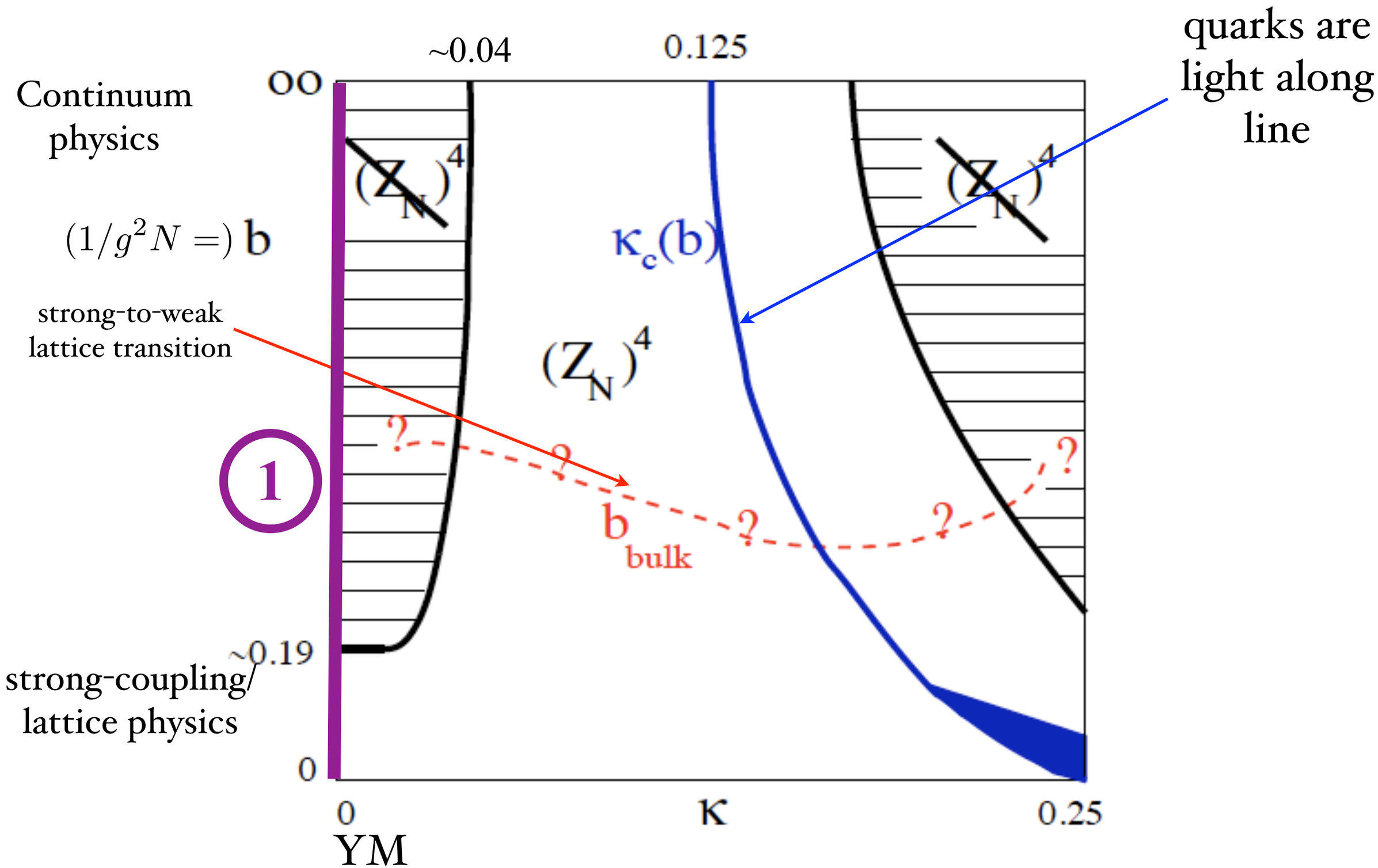

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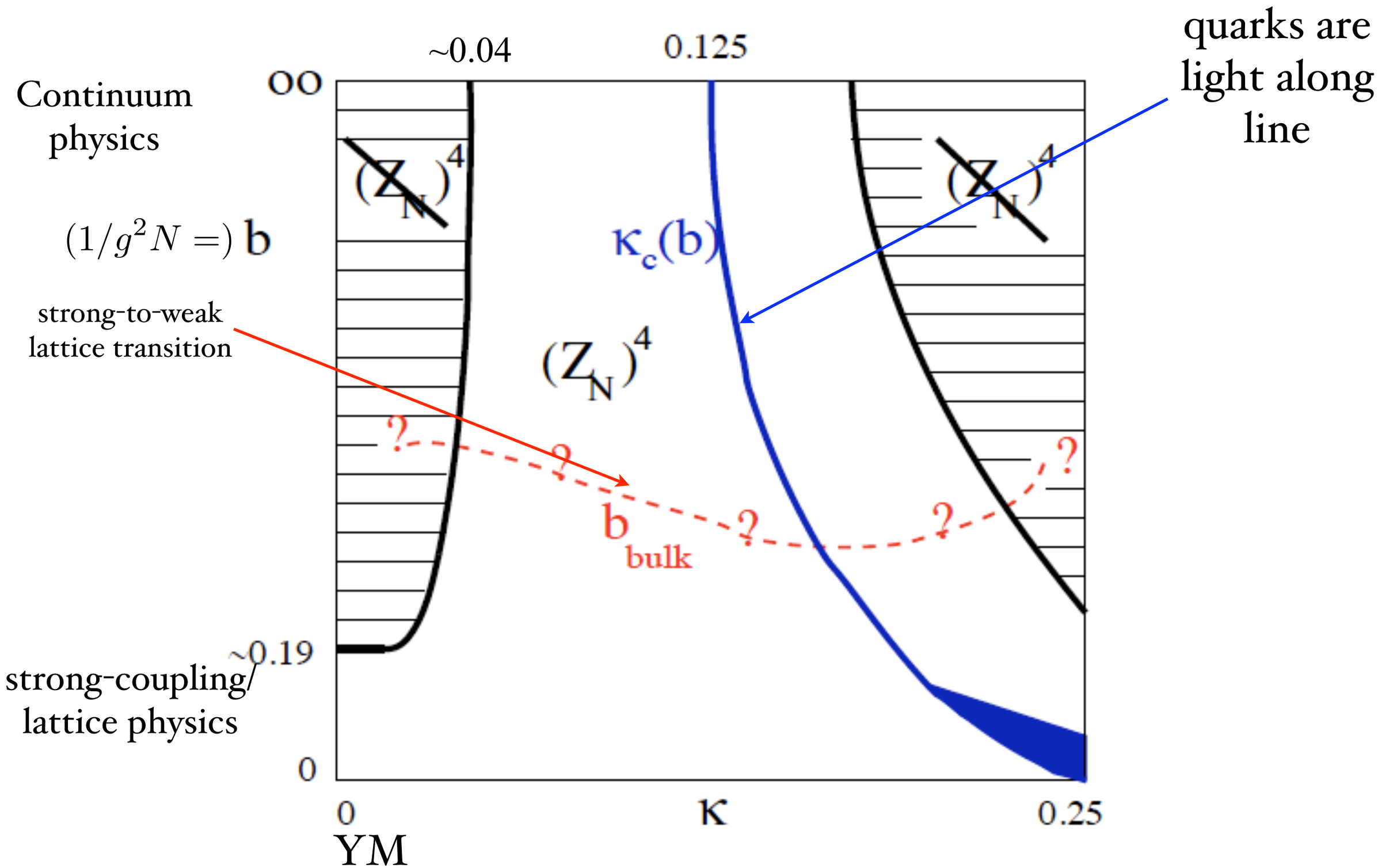

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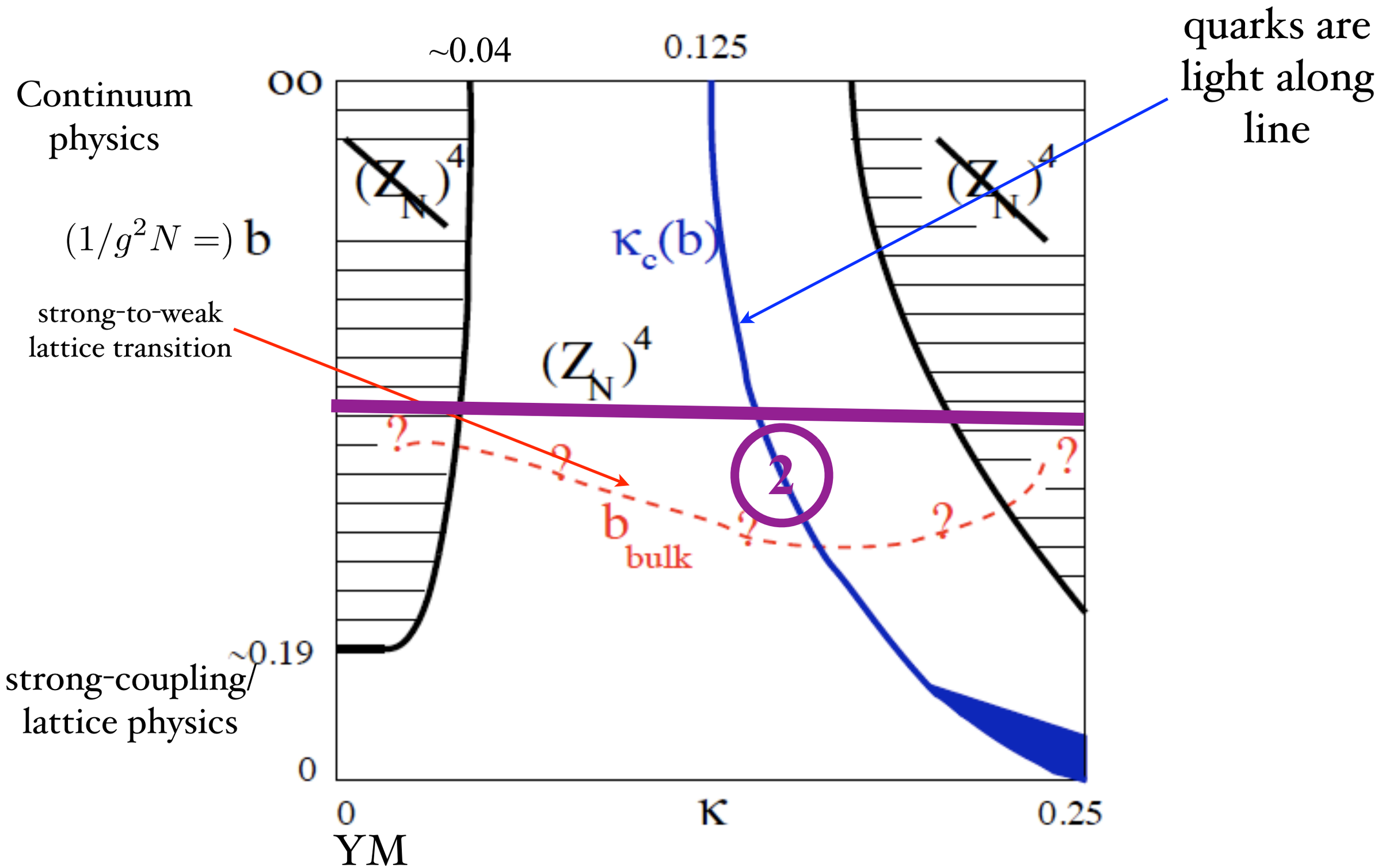

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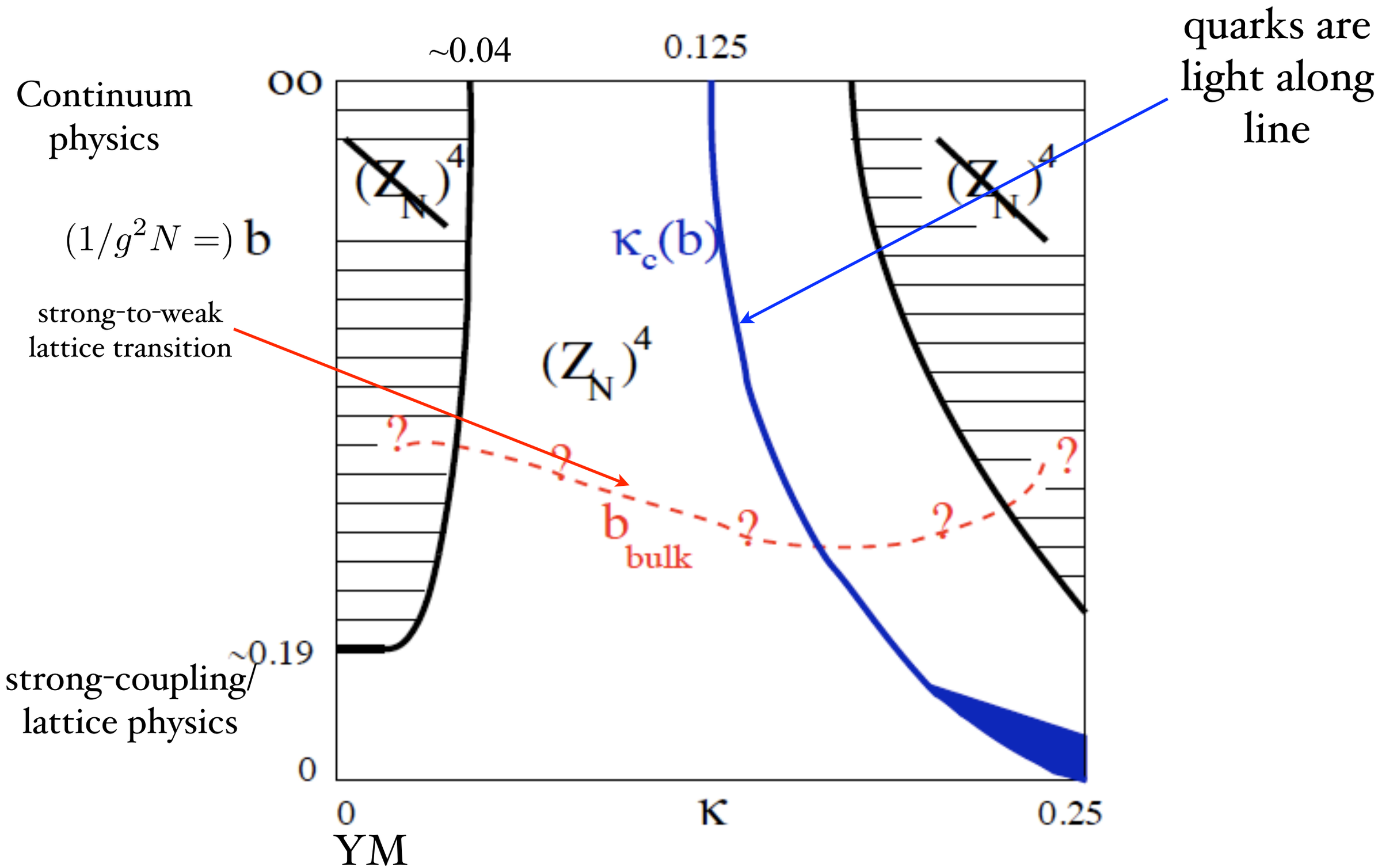

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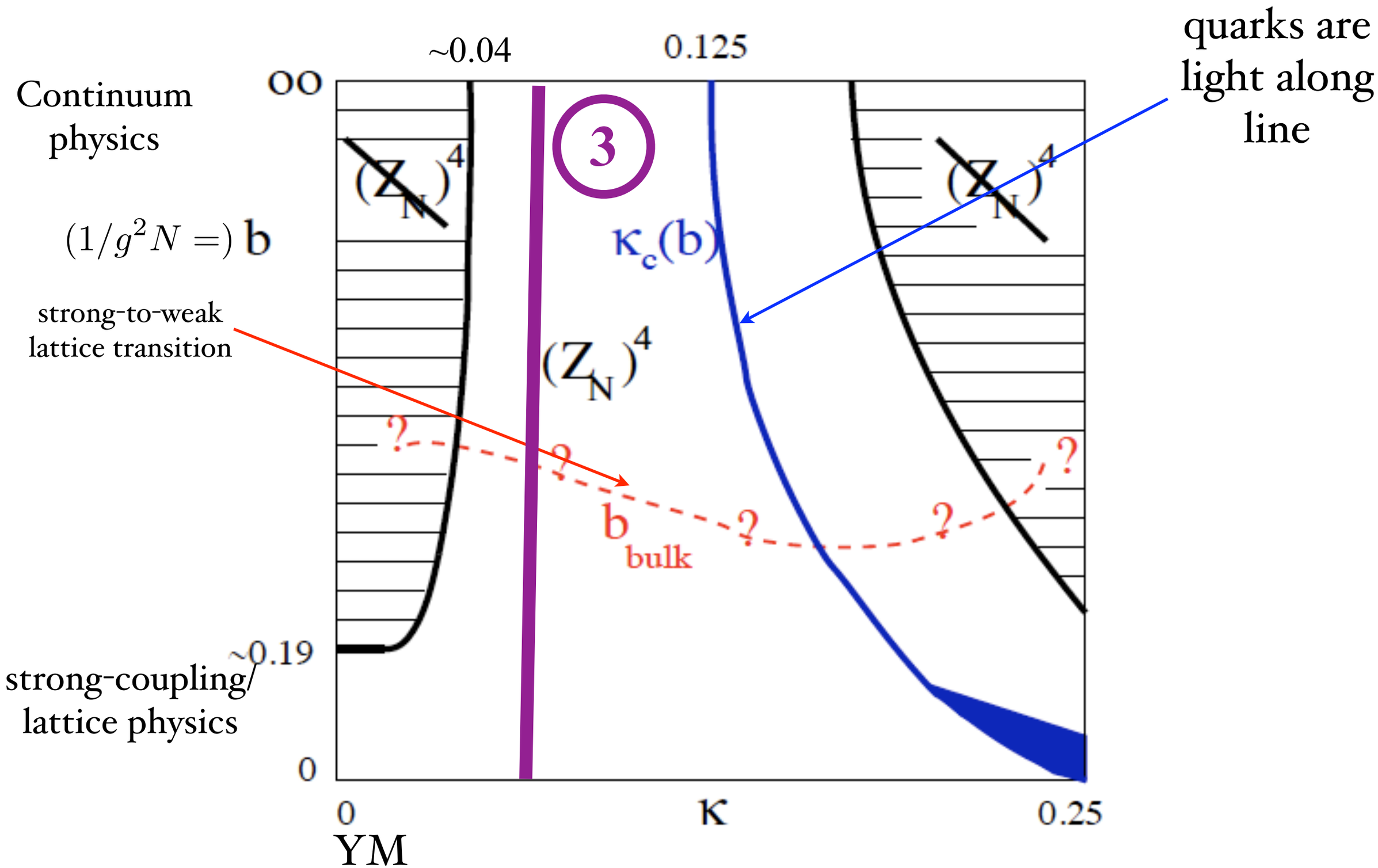

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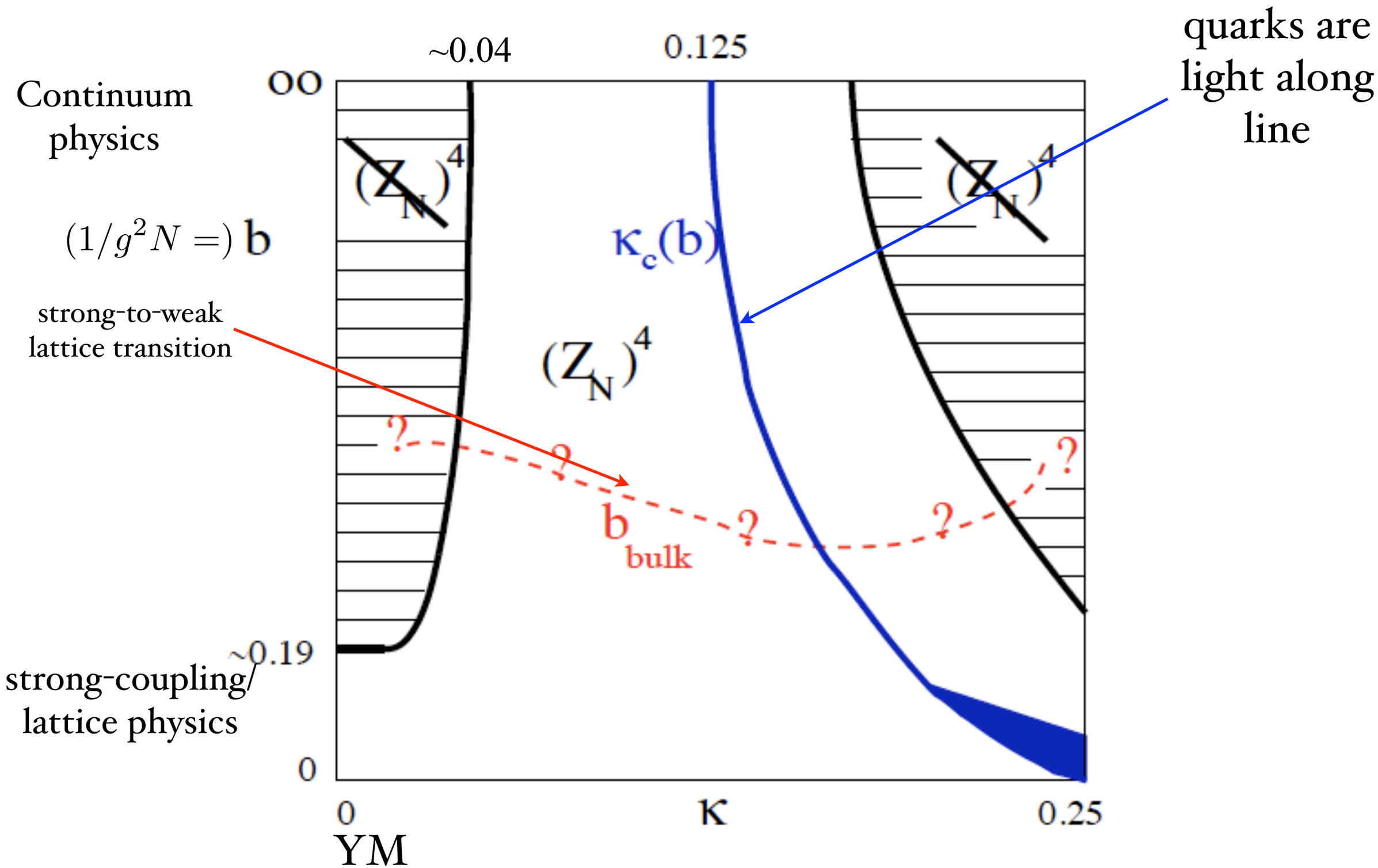

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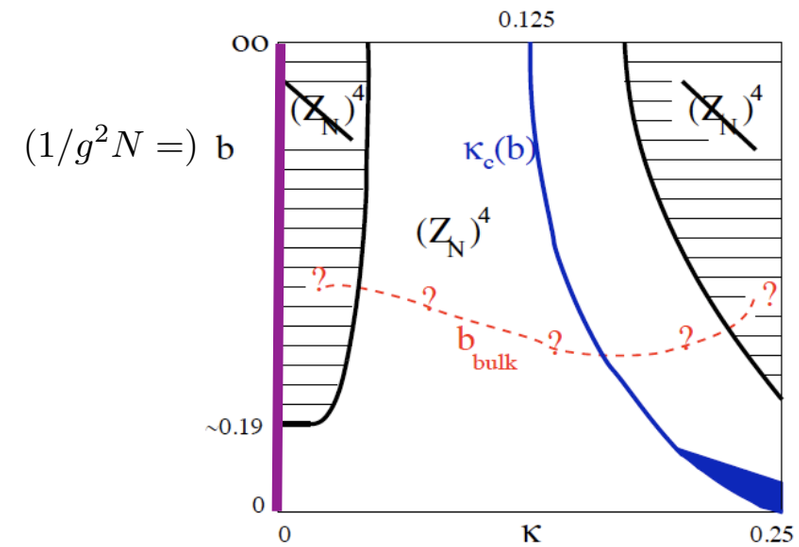

Scan no. 1 : infinitely massive quarks

X-axis: Real(Polyakov)
Y-axis: Imag(Polyakov)

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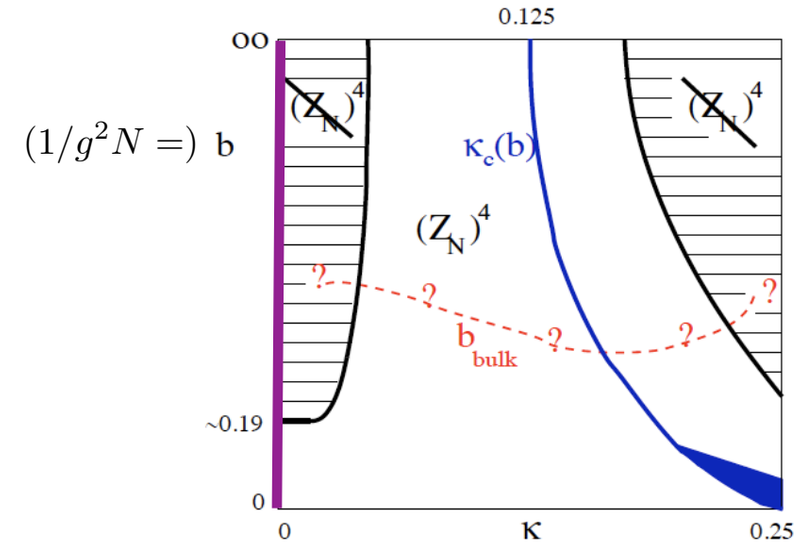

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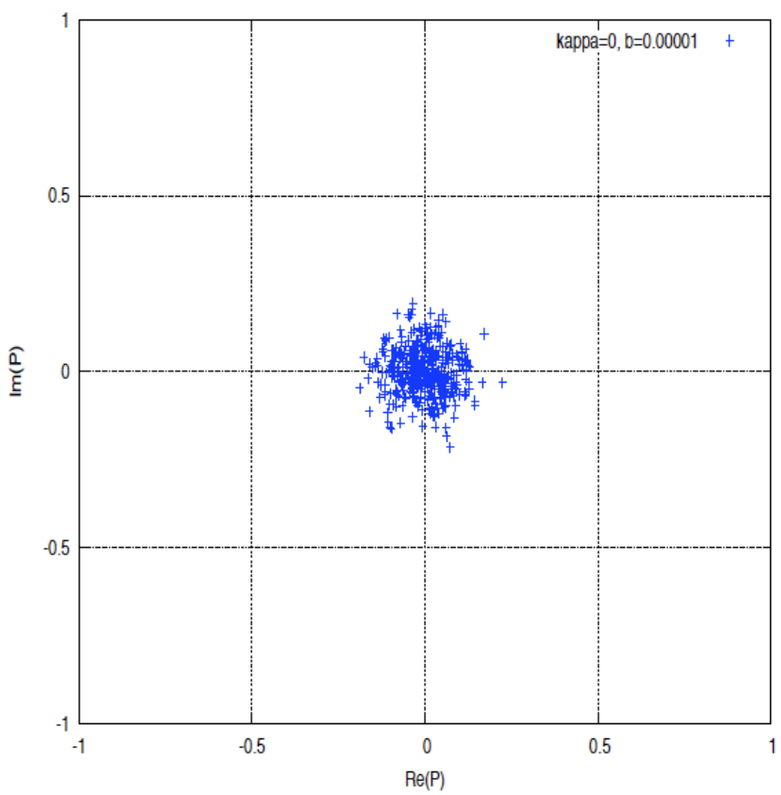

$b = 0$

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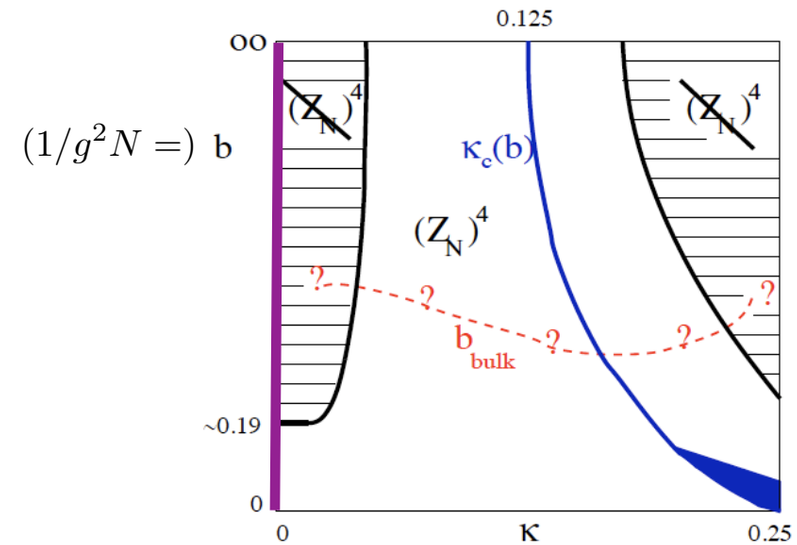

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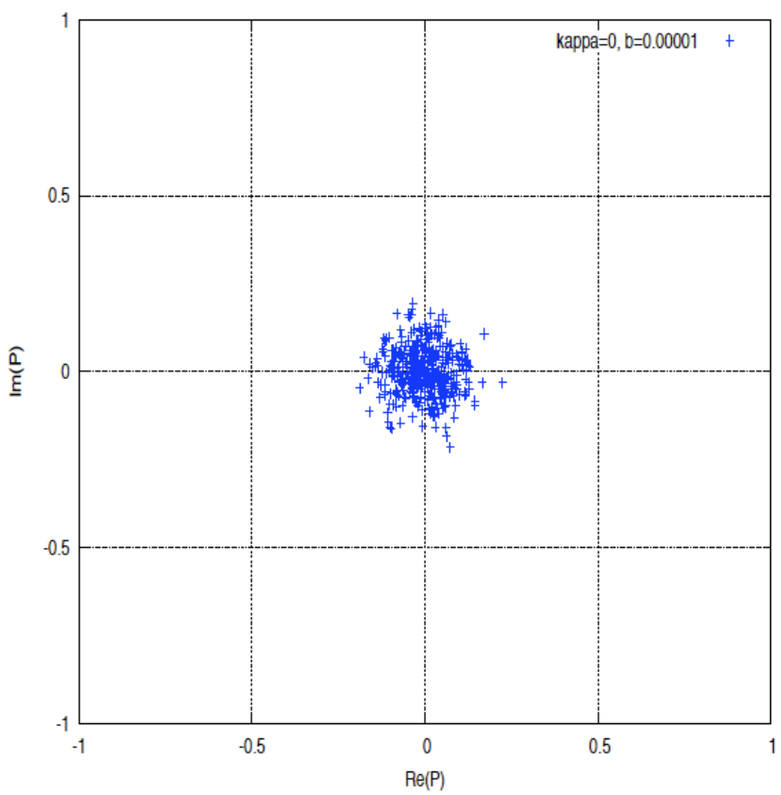

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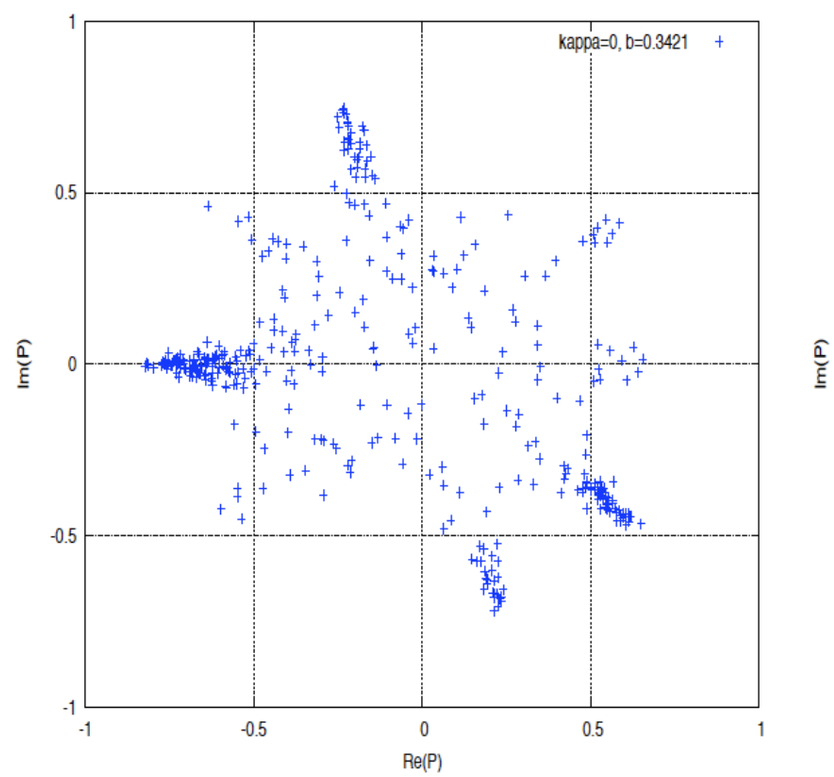

$b \approx 0.3$

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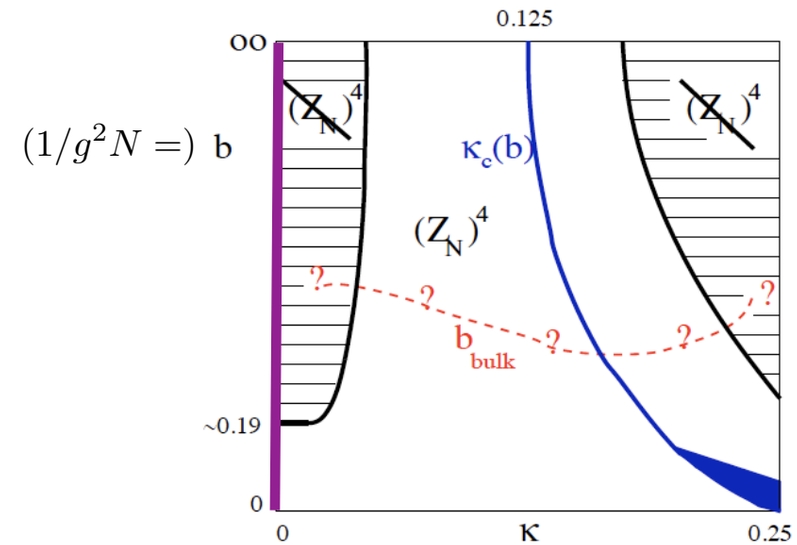

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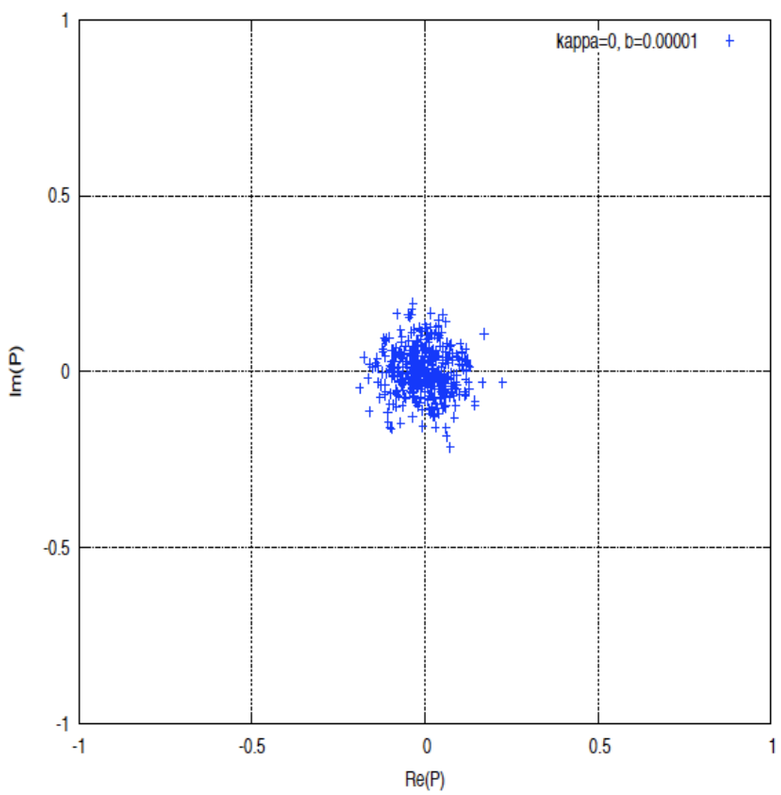

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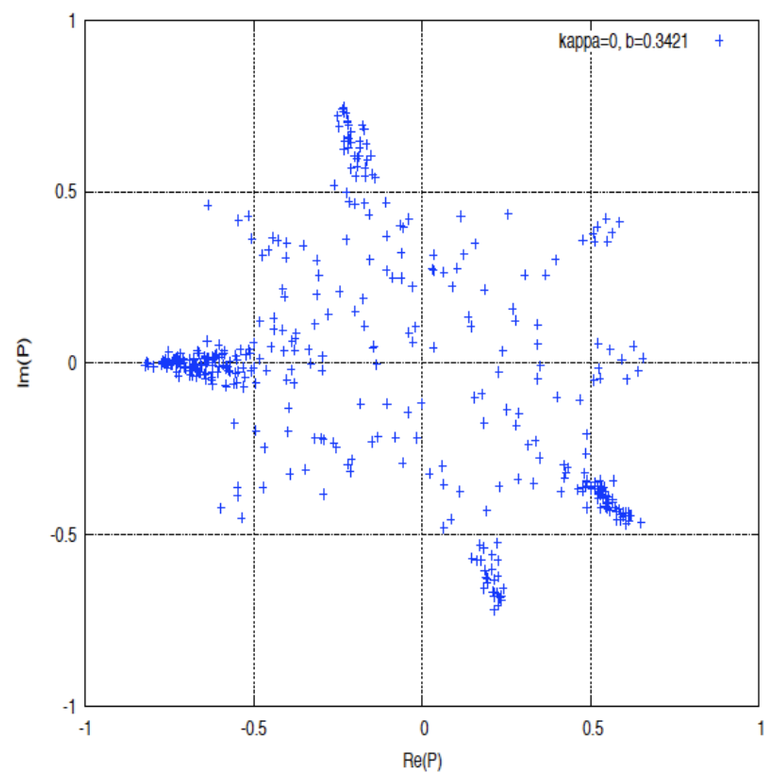

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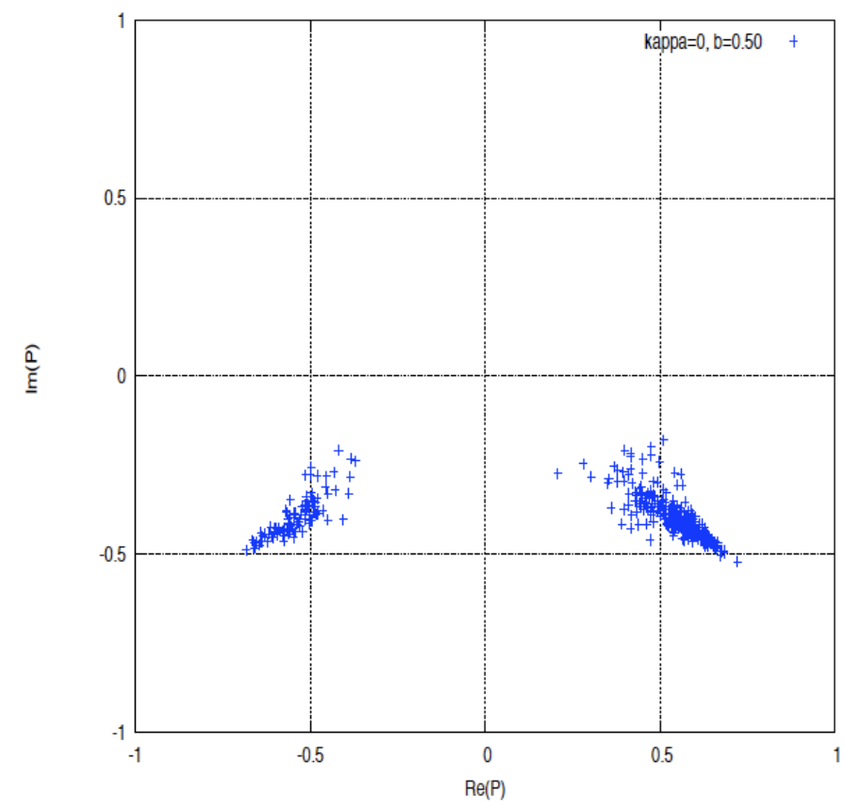

$b = 0.5$

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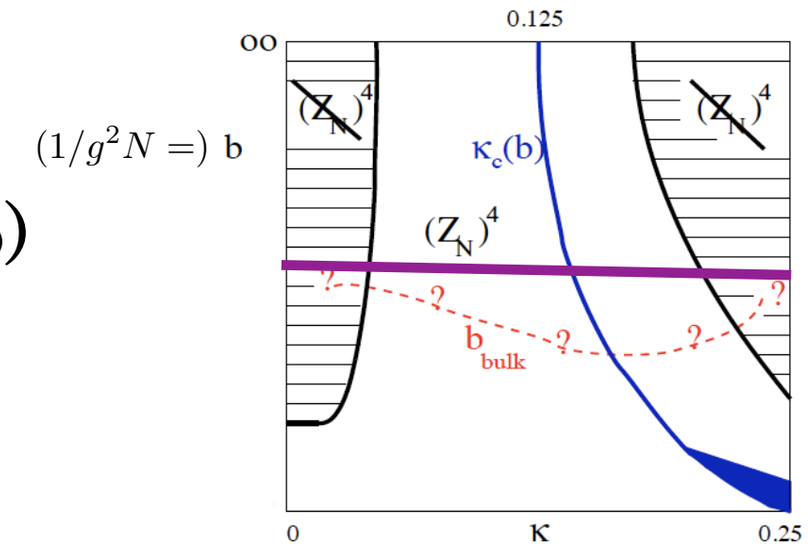
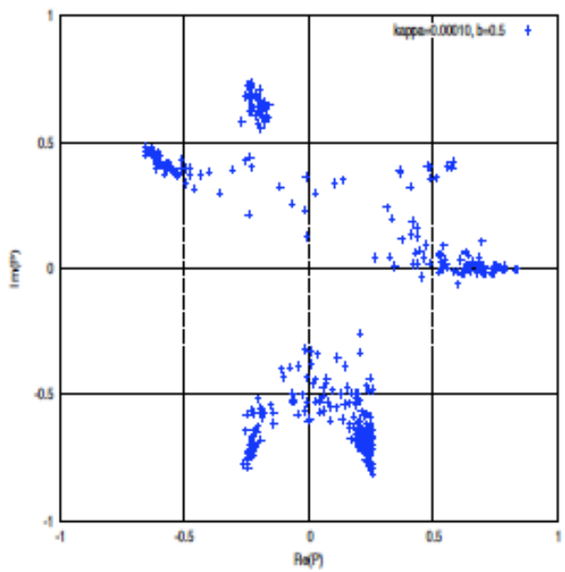


Scan no. 2 : decreasing the quark mass b=0.5 ,SU(10)

$\kappa \approx 0$



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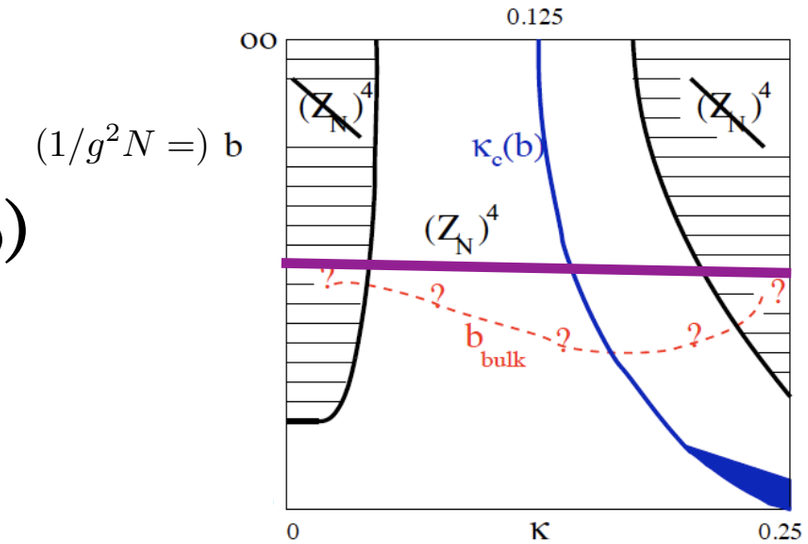

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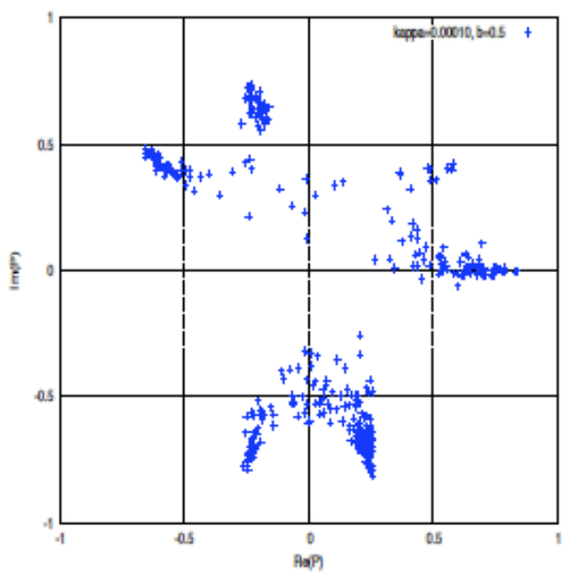
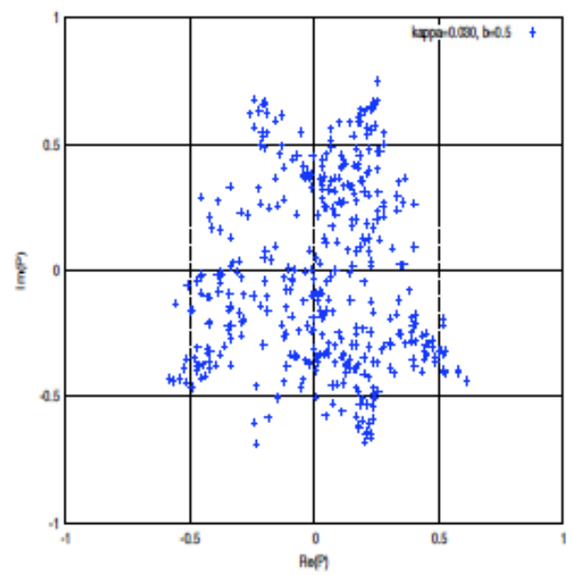


$\kappa = 0.03$



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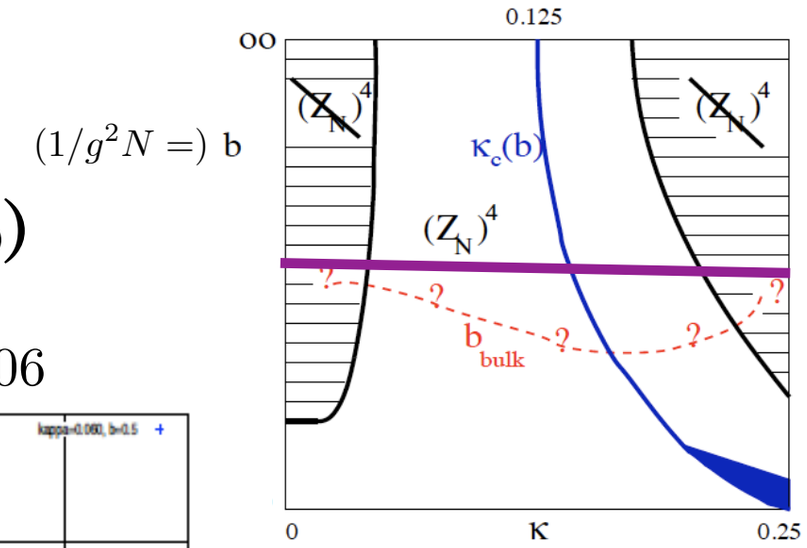
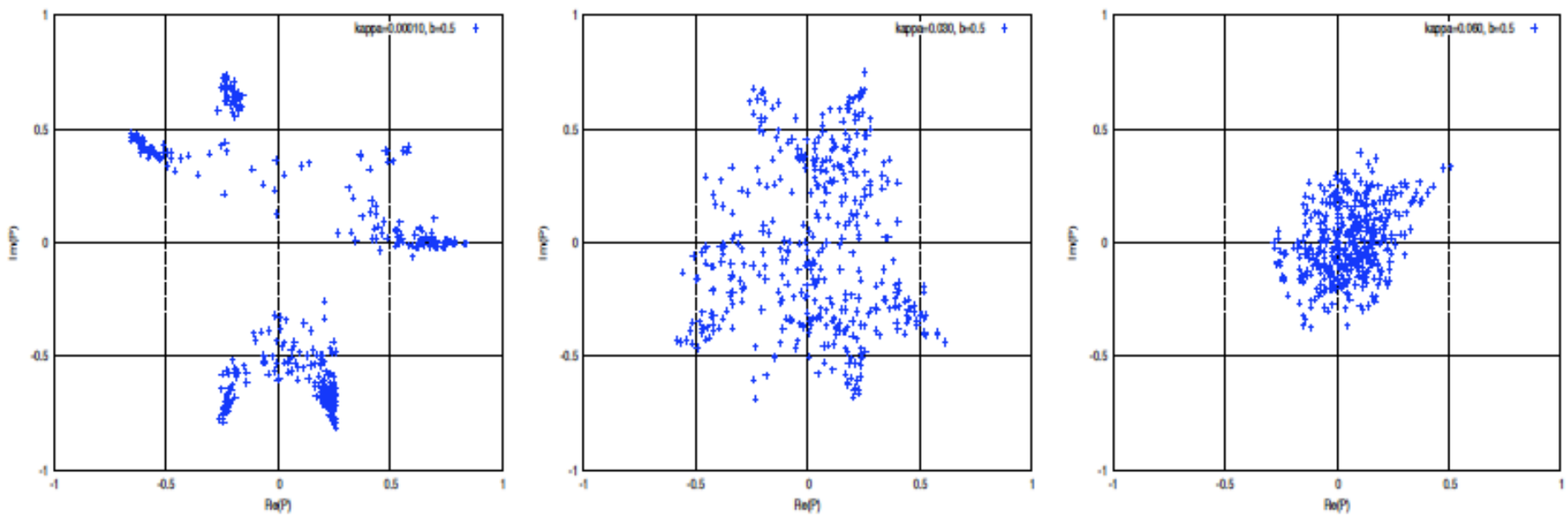


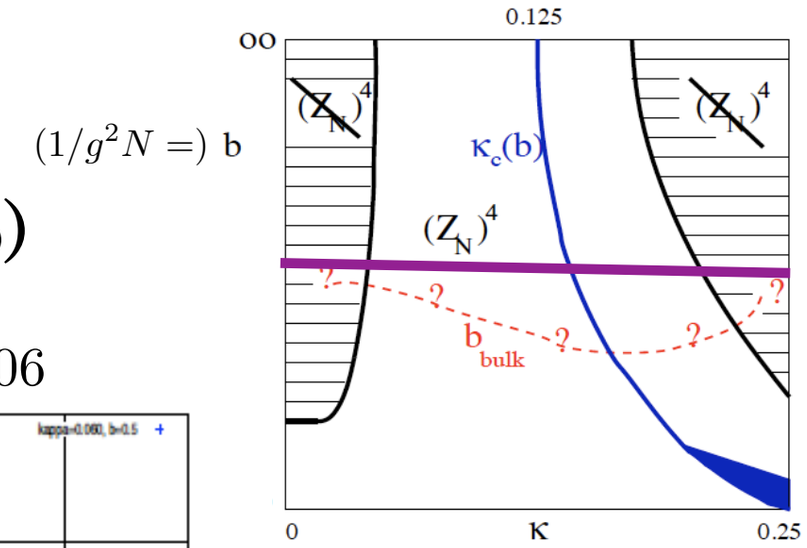
Scan no. 2 : decreasing the quark mass b=0.5 ,SU(10)

$\kappa \approx 0$ $\qquad$ $\kappa = 0.03$ $\qquad$ $\kappa = 0.06$



III. Results of non-perturbative MC lattice simulations

BB+S.Sharpe, 0906.3538



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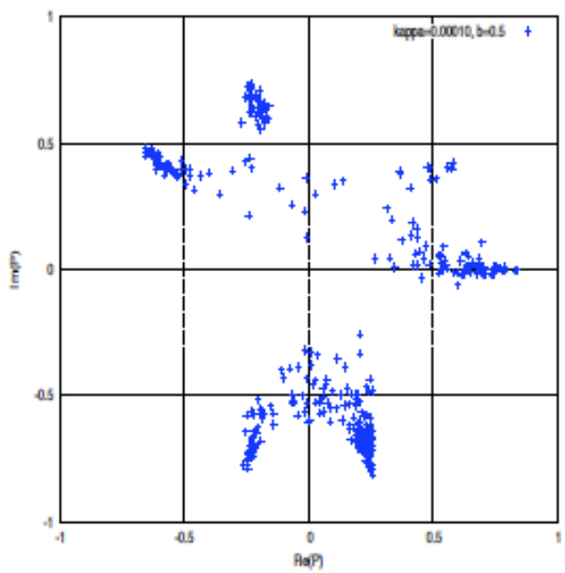

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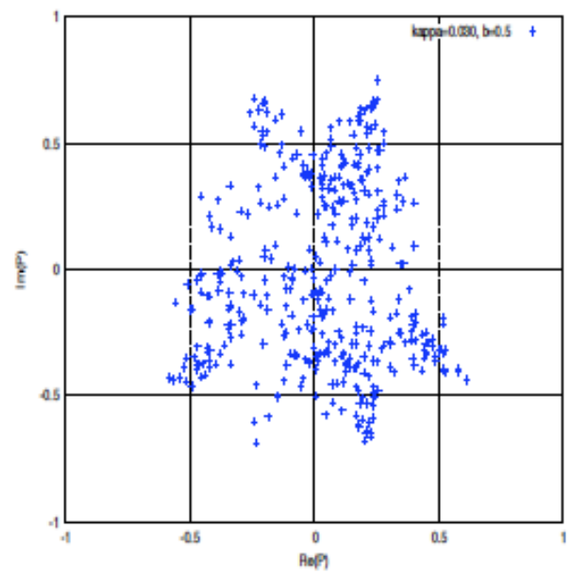
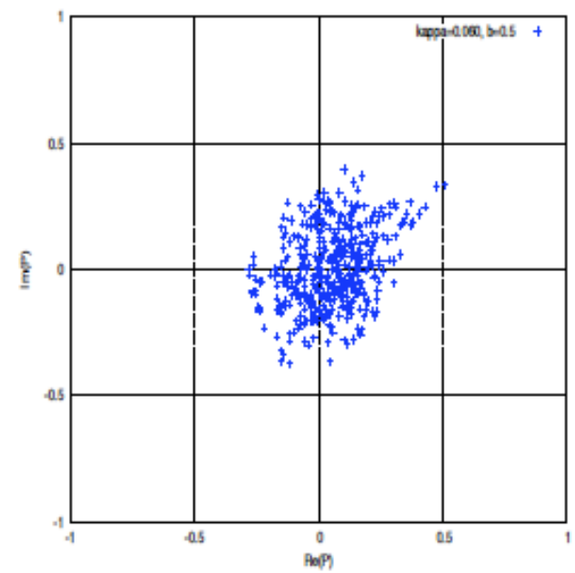
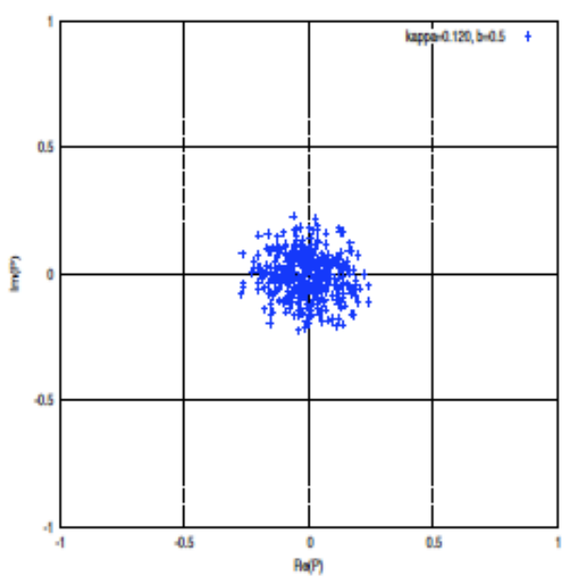


$\kappa = 0.06$

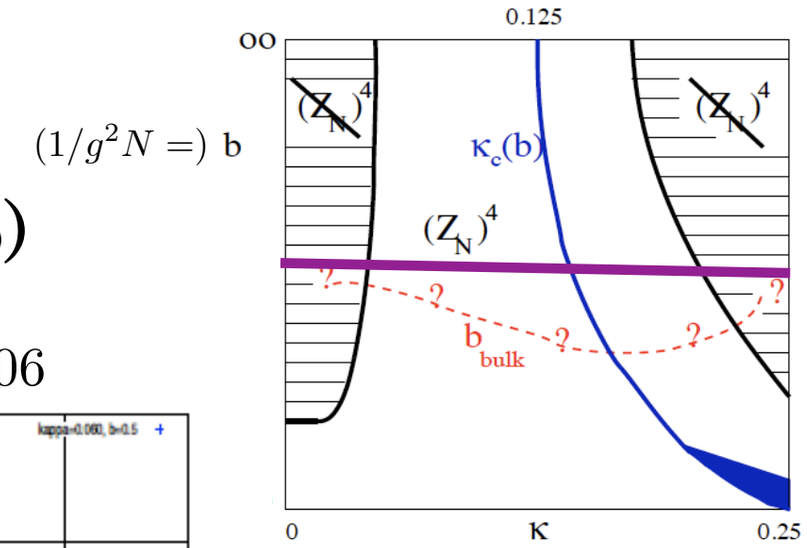




$\kappa = 0.12$

III. Results of non-perturbative MC lattice simulations

BB+S.Sharpe, 0906.3538



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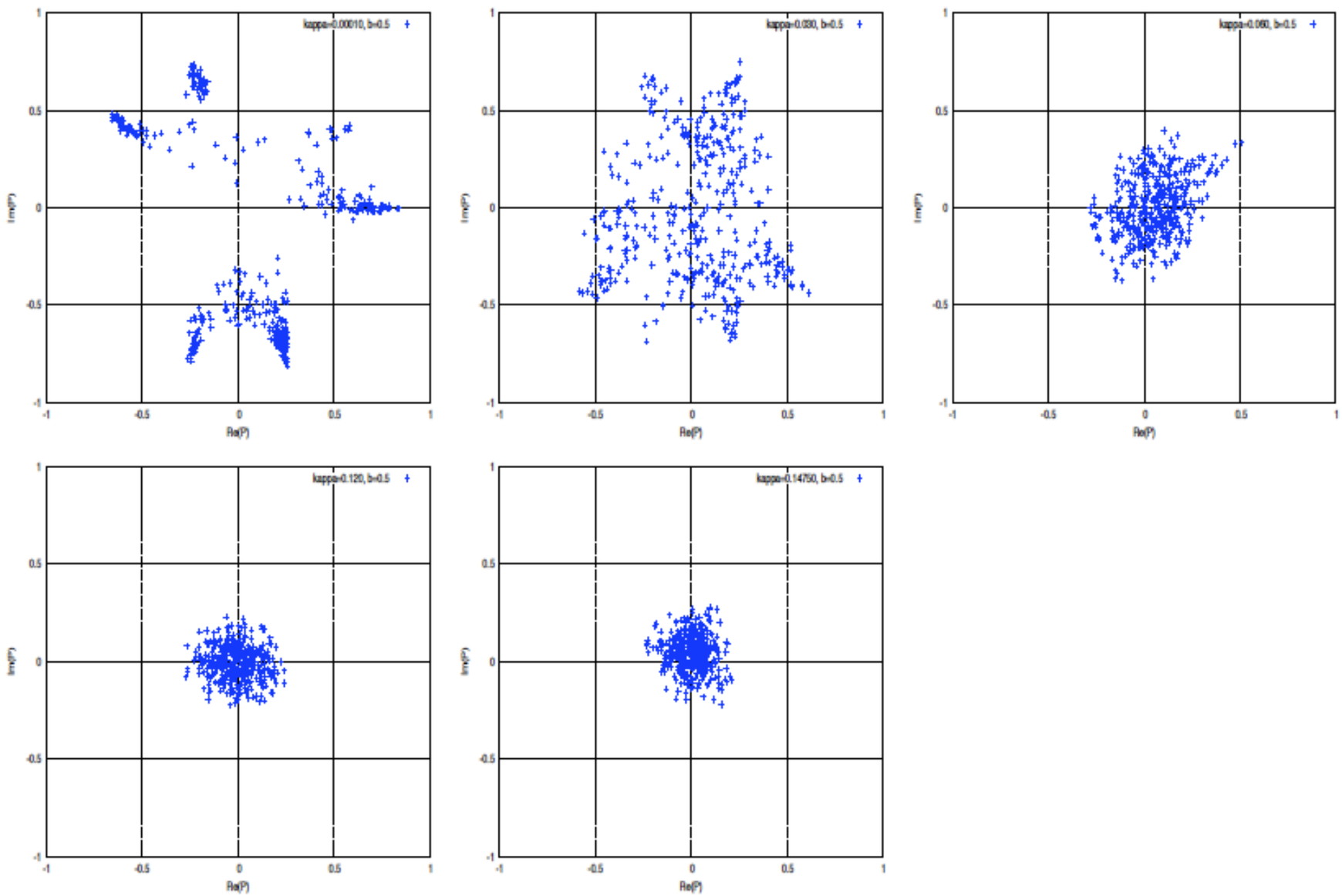

$\kappa = 0.12$ $\kappa = 0.1475$

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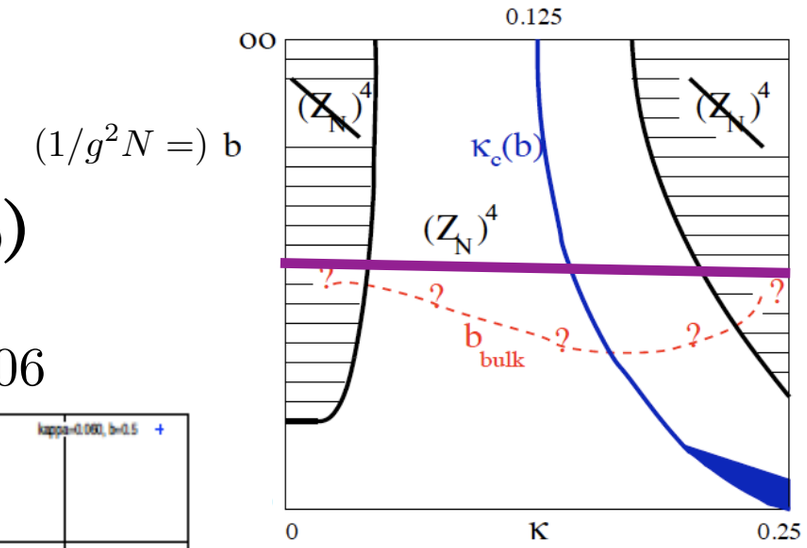

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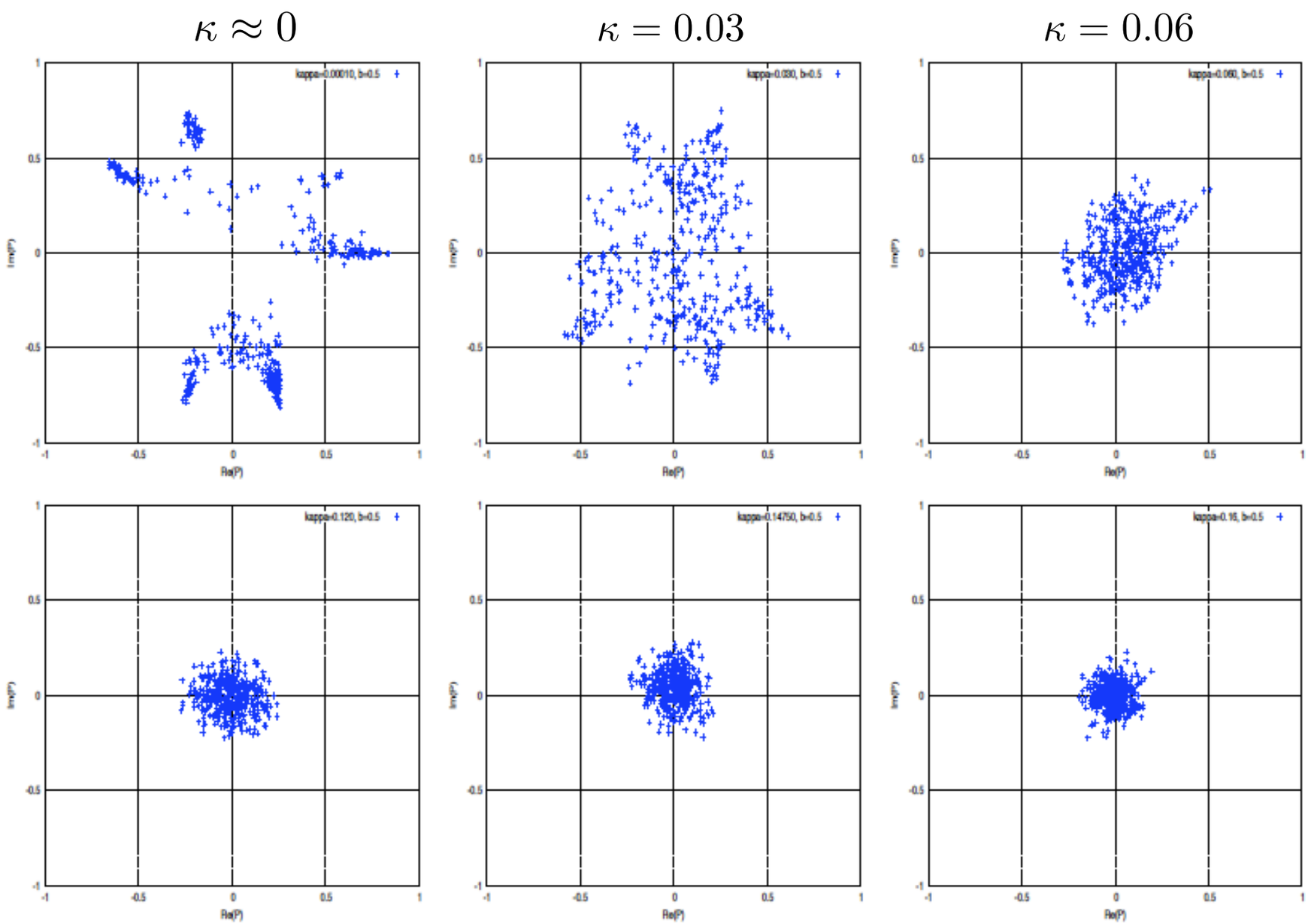

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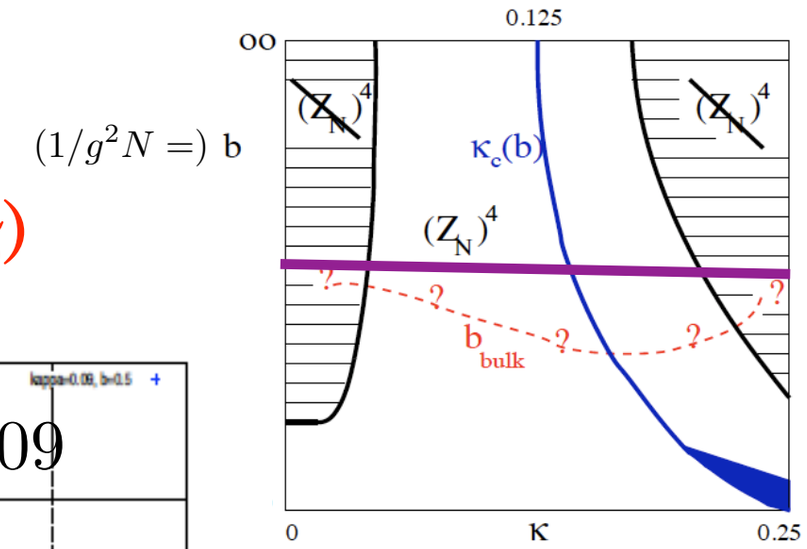
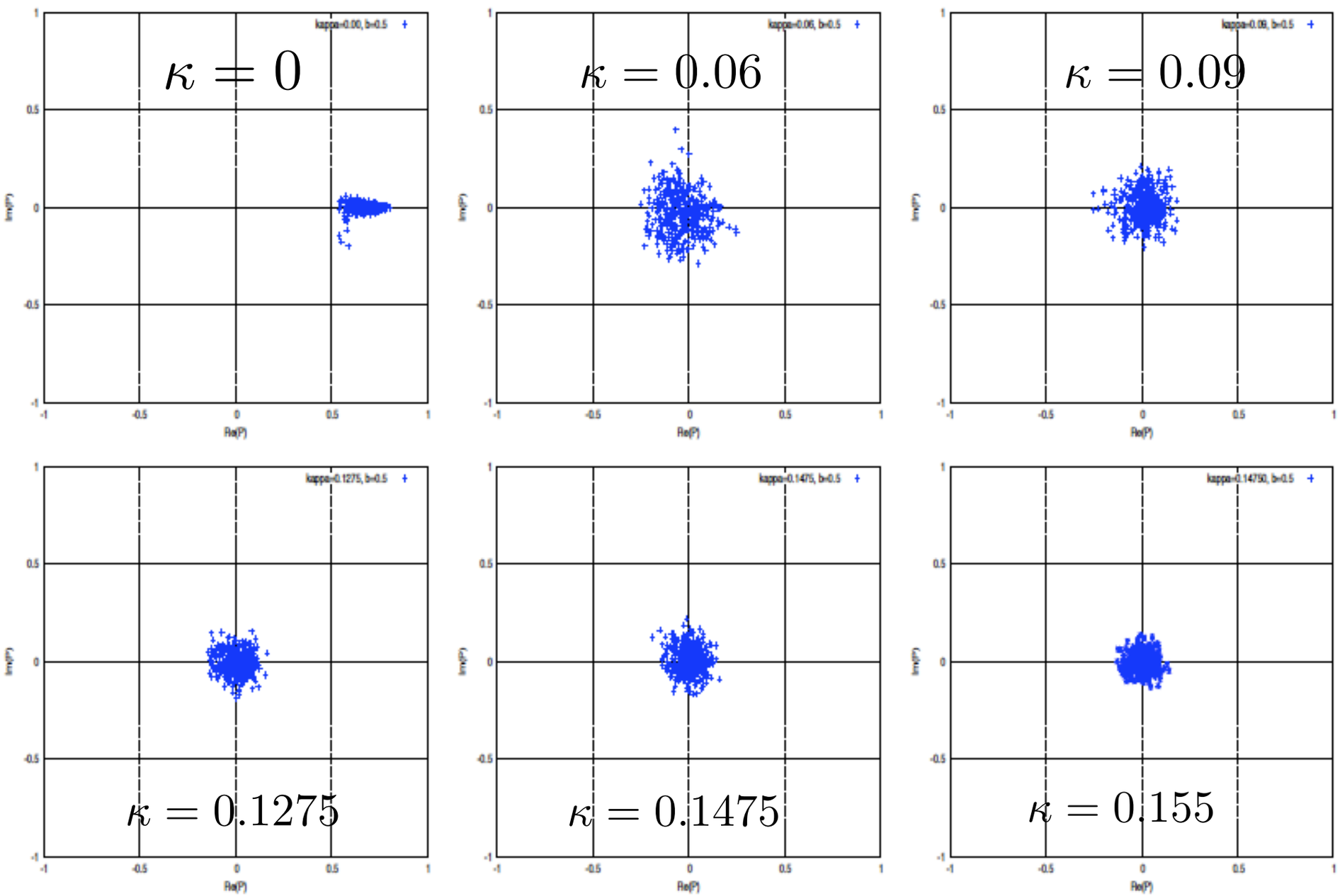


Scan no. 2 : decreasing the quark mass b=0.5 ,SU(15)



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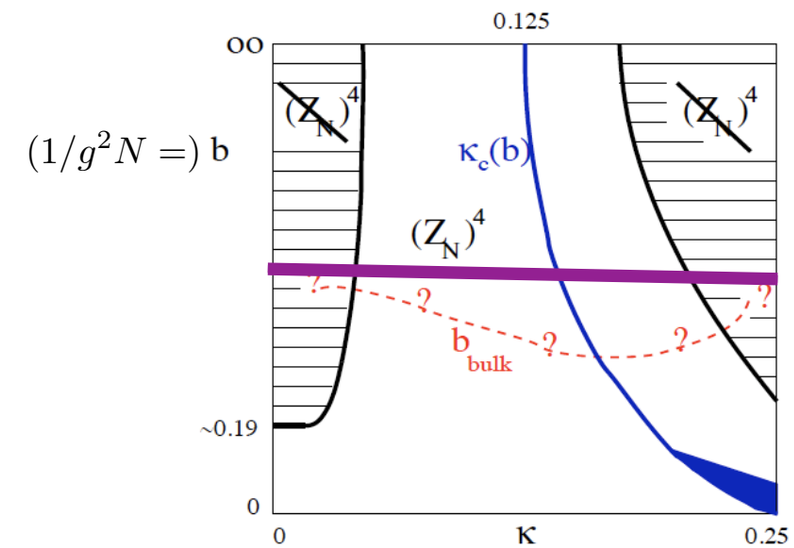
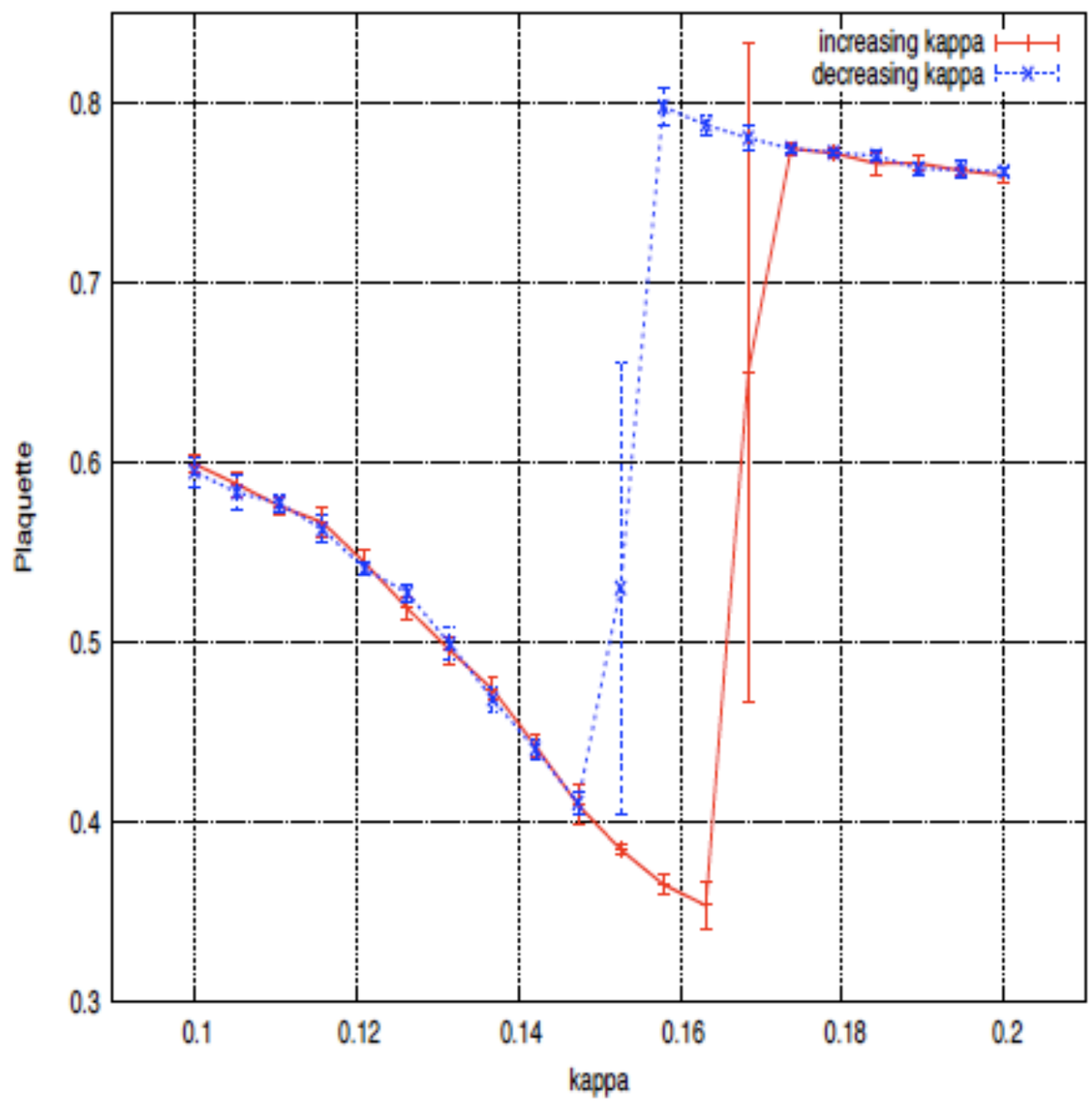
BB+S.Sharpe, 0906.3538



Scan no. 2 : looking for the "critical" line



b=0.35:
1st transition
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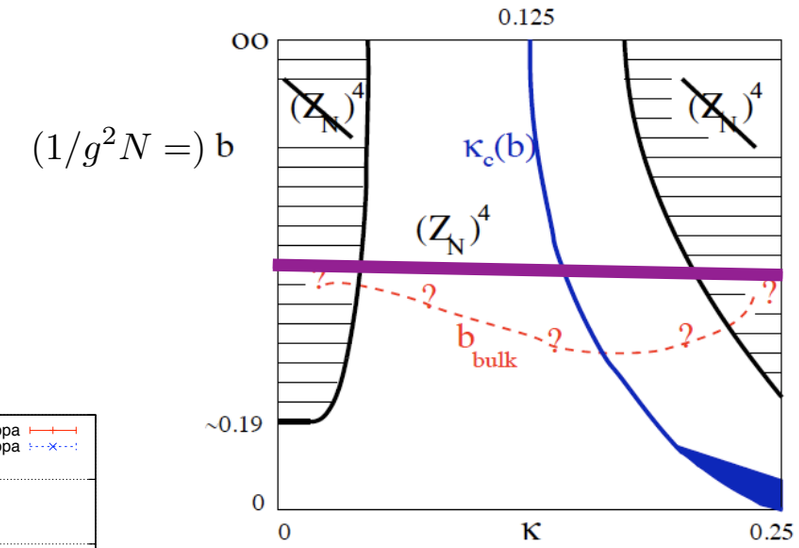

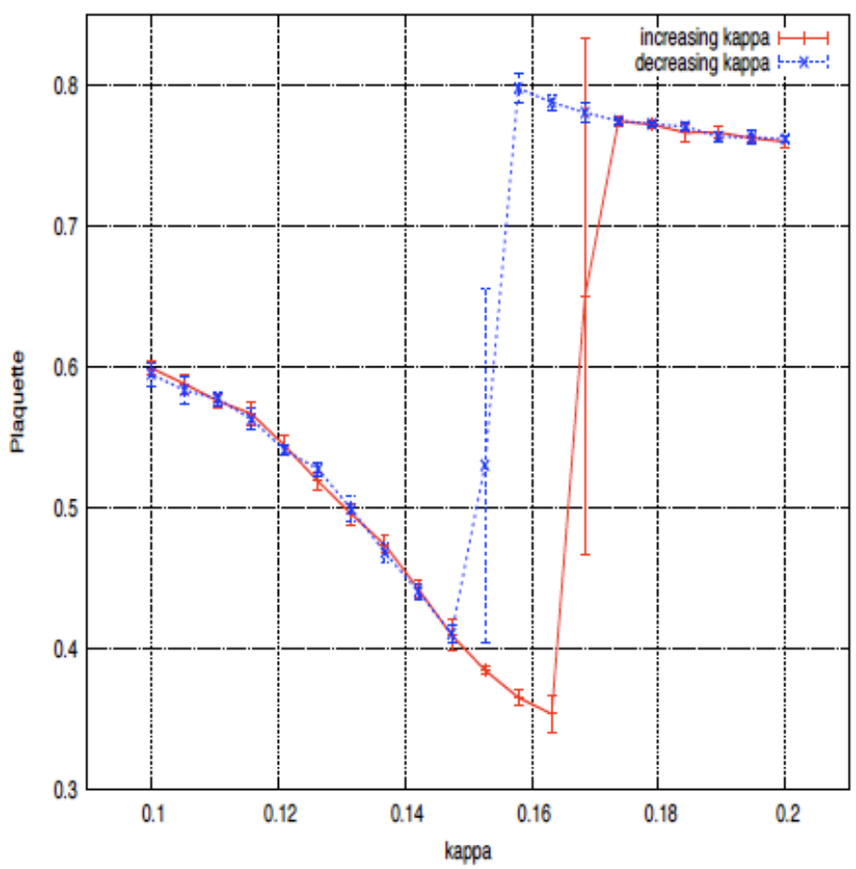

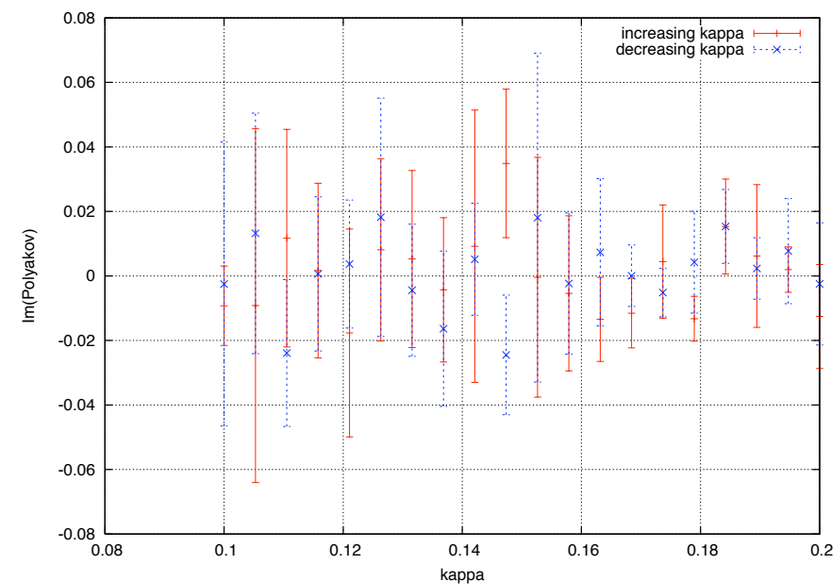

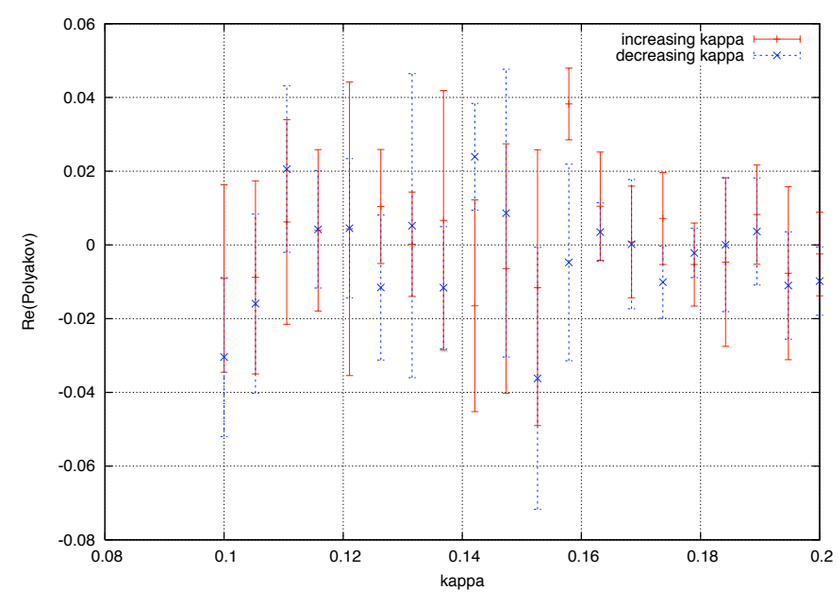

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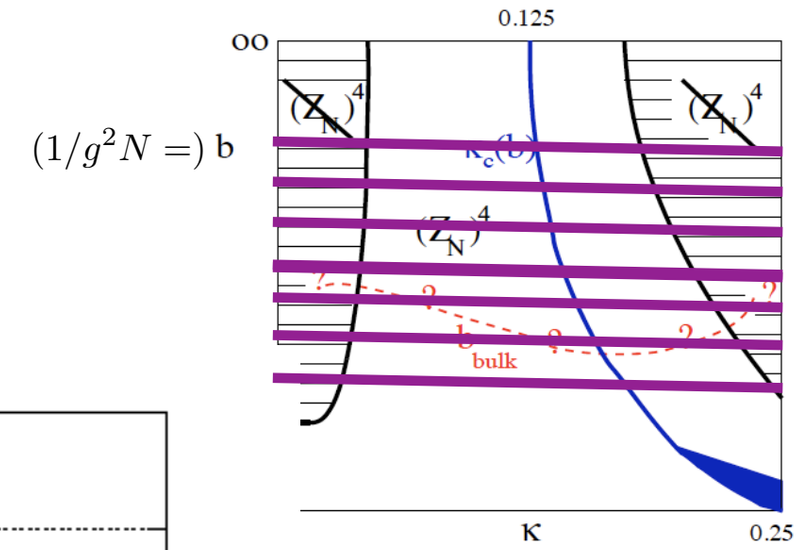

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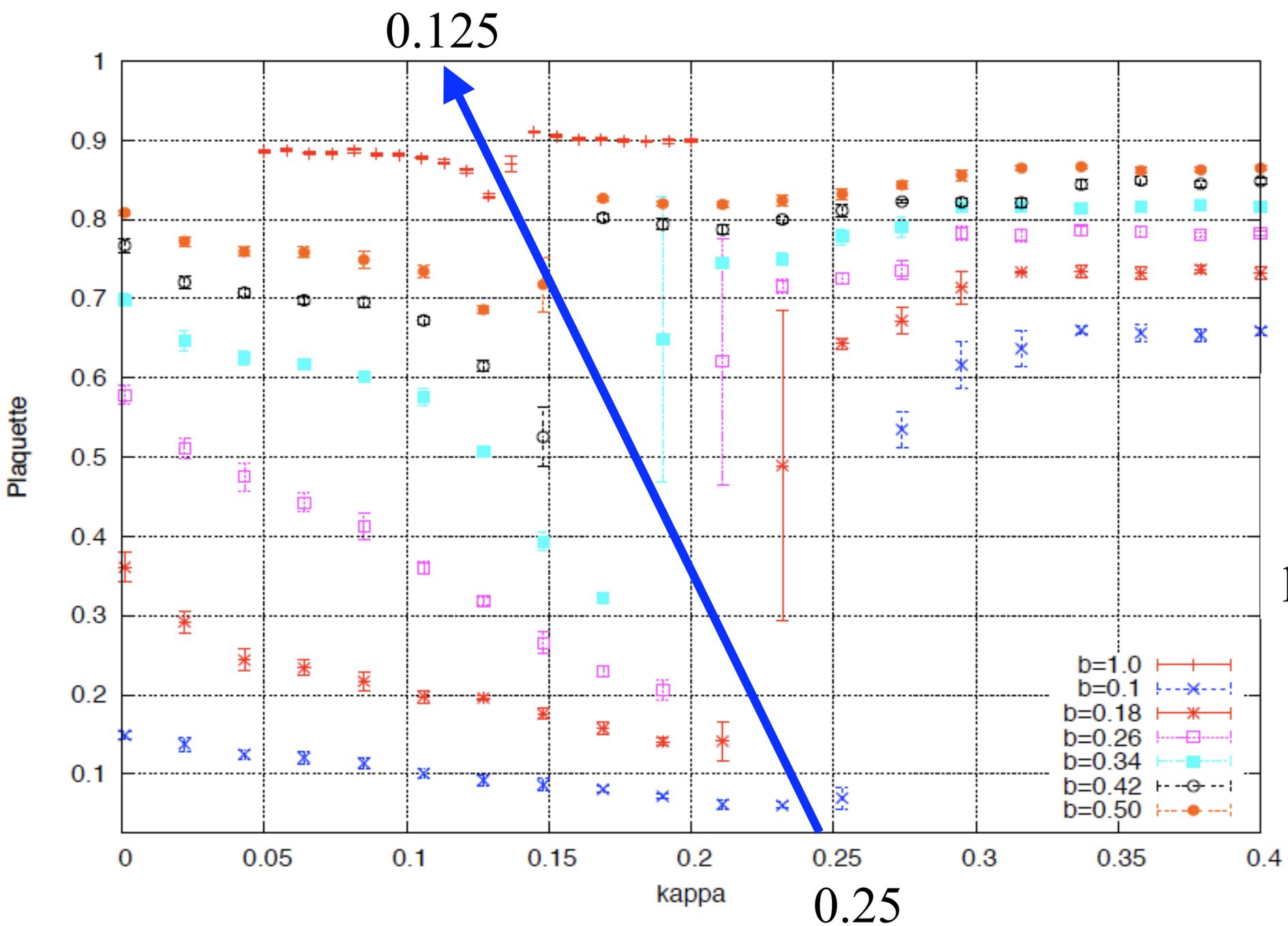
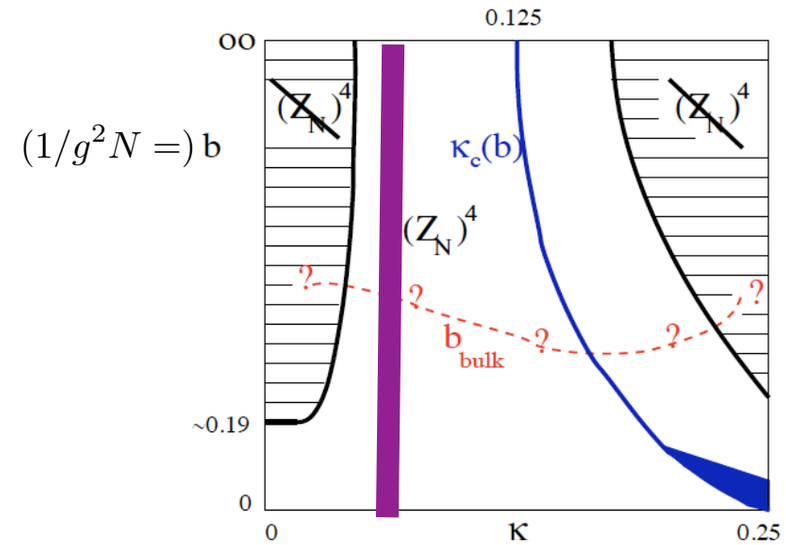


1st transition structure at all b. Extending from kappa=0.25 to 0.125

III. Results of non-perturbative MC lattice simulations

BB+S.Sharpe, 0906.3538

All results consistent with phase diagram and validity of reduction.



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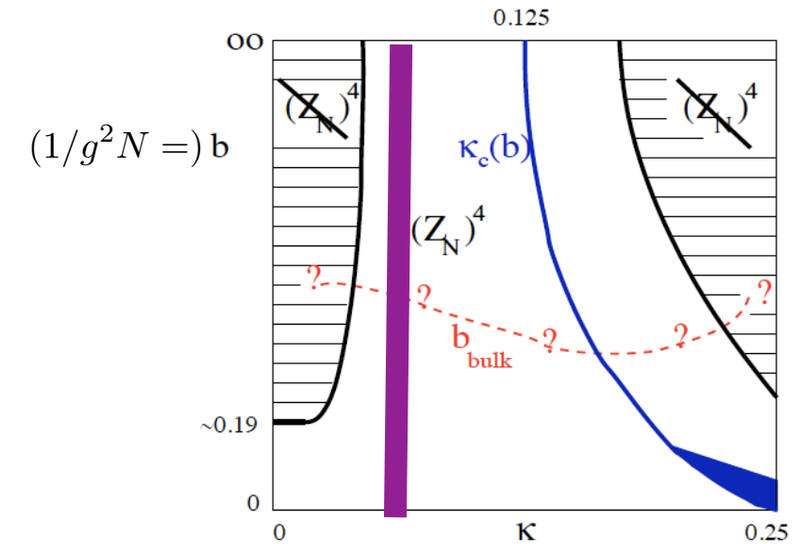

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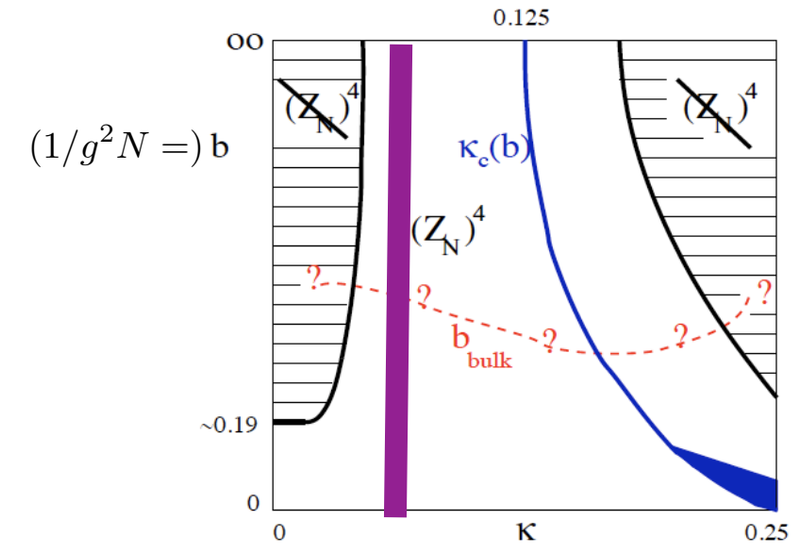
- More order parameters for nontrivial breaking of Z_N.

III. Results of non-perturbative MC lattice simulations

BB+S.Sharpe, 0906.3538



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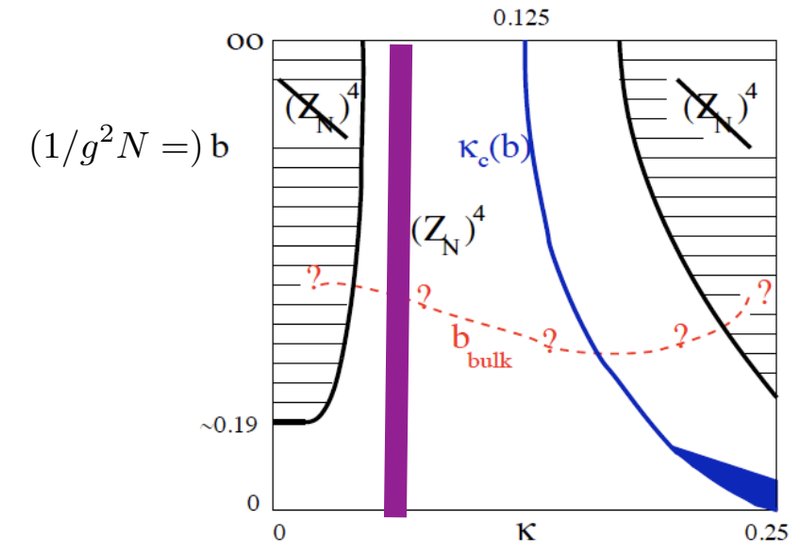
→ Perform long runs and measure order parameters of the form

$$\mathrm{tr}\,[P_1^{n_1}\,P_2^{n_2}\,P_3^{n_3}\,P_4^{n_4}\,] \quad \text{with} \quad n_i \in [-5,5]$$ → 14641 order parameters !

III. Results of non-perturbative MC lattice simulations

BB+S.Sharpe, 0906.3538



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Still no sign of breakdown of Z_N, up to b=0.5-1.0

IV. Conclusions and future prospects.

Weak coupling with $L_{2,3,4}=\infty$, $L_1=1$

- **Large-N volume reduction works for YM+Wilson adjoints fermions.**

Non-perturbative lattice Monte-Carlo of $N_f=1$ case.

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This is exciting :
Eguchi-Kawai finally works
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- Comparisons with RMT.
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- Open to suggestions ...

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