Is QCD a string theory?

[arXiv:0901.0494]

Schrödinger's cat?

Is Large N_c QCD a string theory?

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Schrödinger's cat?

Is Large N_c QCD ~~a~~ an approximate string theory for sufficiently excited states?

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Schrödinger's cat?

QCD & the Hagedorn Spectrum

The Hagedorn Spectrum

- The Hagedorn spectrum refers to a hadronic spectrum in which the number of hadrons with mass less then m grows exponentially with m: $N(m) \sim m^{-2b} \exp(m/T_H)$, where T_H, the Hagedorn temperature is a mass parameter.

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- It has been argued that hadrons have a Hagedorn spectrum.

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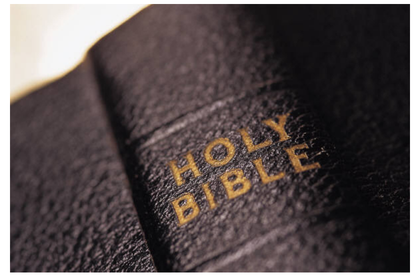
  - Thermodynamically T_H corresponds to an upper bound on the temperature of hadronic matter.

There is *some* phenomenological support for both of these

Get data from the **PARTICLE DATA BOOK**



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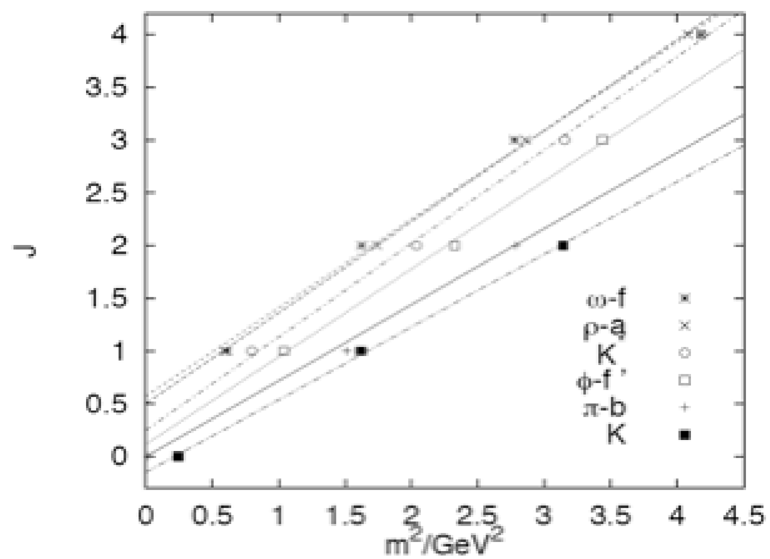

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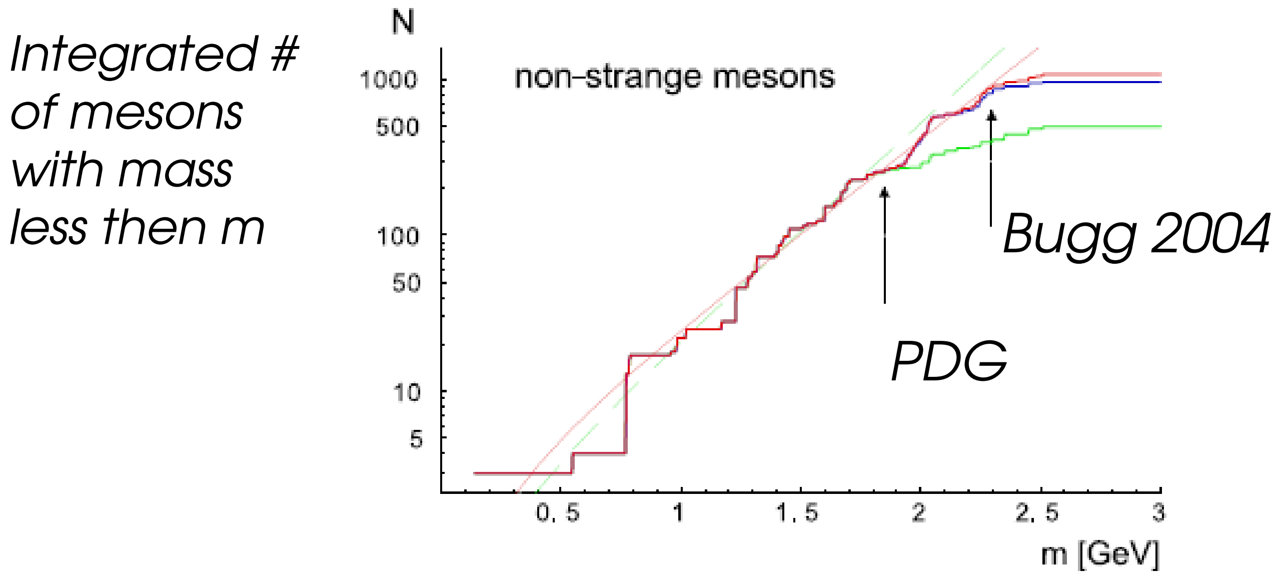
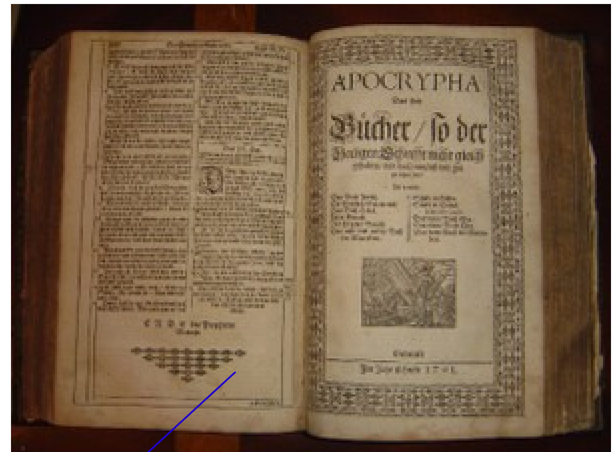
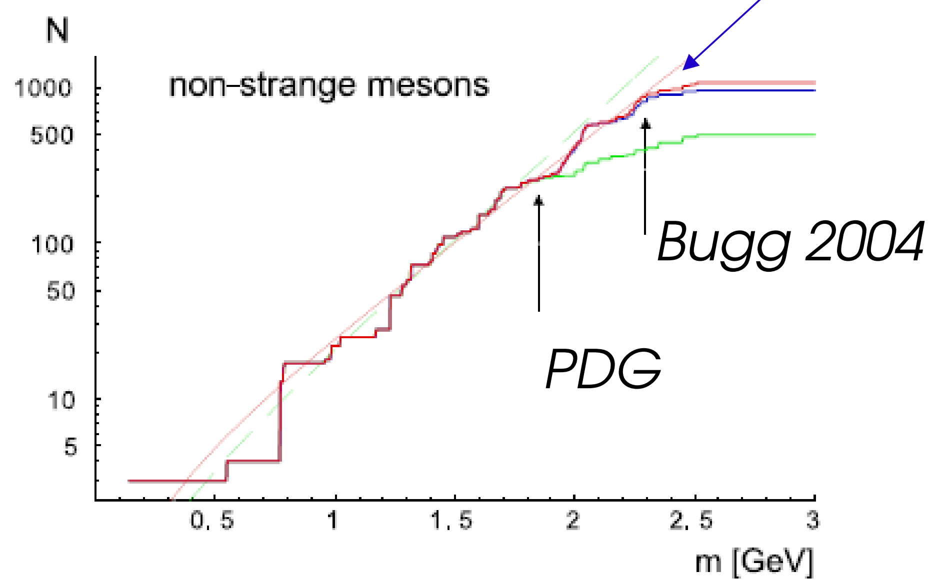


From Broniowski, Florkowski and Glozman 2004

Some data should be viewed as being from the apocrypha



Integrated # of mesons with mass less then m



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Some data should be viewed as being from the apocrypha

This apparent exponential growth in the empirical hadron spectrum may be misleading for reasons which may become clear later in the talk

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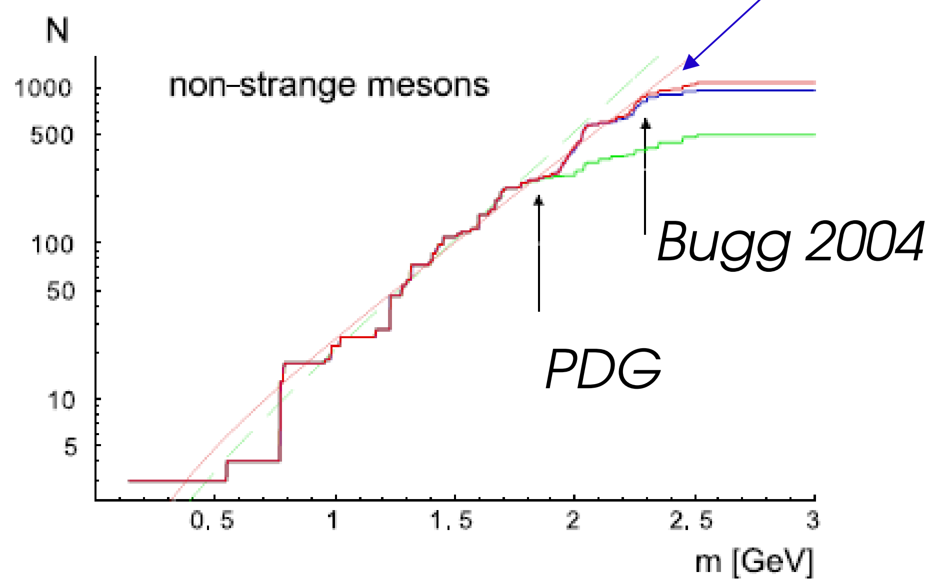

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- At $T > T_H$ the energy density diverges implying the temperature cannot be achieved
  - The QCD phase transition thus is thought to occur for temperatures at or below T_H .

QCD Strings

- Theoretical side:
  - Strings are thought to emerge in QCD as flux tubes.
    - Some theory reasons why an area law for Wilson loops may be expected (eg. strong coupling limit, but is it relevant?)
    - Lattice evidence in quenched QCD for *static flux tubes*

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    - Lattice evidence in quenched QCD for *static flux tubes*
  - This evidence is essentially static
    - In a real sense this is not a real probe of the hadronic string which is essentially dynamic.
    - Key issue is to find ways to probe the dynamics of a stringy picture of QCD. Hadron spectroscopy does this.

Phenomenology and QCD Strings

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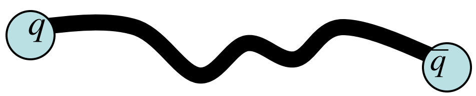
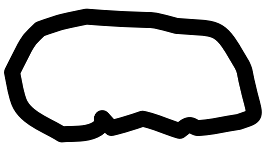
  - Picture becomes valid when excitations are high enough so that the length of the flux tube is much bigger than its width and string motion dominates
  - Dynamics of flux tubes give excitation spectrum of hadrons.
    - Mesons are open strings.



    - Glueballs are closed strings.



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  - However these diseased states are low-lying
  - One might hope that a theory which approaches a string theory in four dimensions might work for highly excited states

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  - Spectral results for hadrons (Regge, Hagedorn) depend on hadron masses which are well defined if decays are suppressed.
    - Meson & glueball widths are suppressed at large as N_c^{-1} and N_c^{-2} respectively.
  - Flux tubes break due to the creation of quark-antiquark pairs. This destroys a simple string picture.
    - This is suppressed by a factor of $1/N_c$.

Rate of string breaking

$$\Gamma_{2-body}(m) \sim \frac{\Lambda \sigma m}{N_c}$$

(Casher, Neuberger, Nussinov 1979)

Proportional to mass and inversely with *Nc*

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  - Assume here that $N_c \to \infty$ limit taken first so stringy description survives for high lying states.

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A similar question was asked by the current speaker when he heard that a different former Republican president had Alzheimer's

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- The folklore is that QCD (and all confining theories) will have a Hagedorn spectrum with an exponentially growing number of hadrons
- So far as I know this folklore is based on the notion that QCD *does* become an effective string theory.
  - I know of no previous demonstration from within QCD that is in fact true.
  - Thus a key issue is how can you tell if large Nc QCD really does behave this way.

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    - Within these assumptions it is possible to demonstrate that the *N(m)* grows exponentially.
  - Thermodynamics of QCD's metastable phase (which exists at large Nc) which are testable on the lattice.
    TDC Phys. Lett B 636 (2006) 81.

Theoretical Argument

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  - Confinement (in the sense of only color neutral states in the physical space).
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- Motivated via Witten's 1979 argument for showing that at large *Nc* there are an infinite number of mesons/glueballs with any quantum number

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Consider a zero momentum correlation function of a local color singlet operator J of dimension D and which at large N_c is " color indivisible" in having a single color trace. For simplicity consider a scalar.

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$$\Pi(Q^2) \equiv i \int d^4x \, e^{iq\cdot x} \langle T[J(x)J(0)] \rangle \quad Q^2 = -q^2$$

$$\Pi(Q^2) = \sum_n \frac{\left| \langle n | J | vac \rangle \right|^2}{Q^2 + m_n^2}$$

Large N_c

hadrons

$$\rightarrow const \; Q^{2(D-2)} Log(Q^2)$$

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Large Q^2 (Asymptotic Freedom)

Two forms are incompatible if the sum over hadrons is finite: *the large Nc result with any finite number of poles goes to Q^{-2} at large Q.* **Ergo there must be an ∞ number of mesons with this quantum #.**

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- Note that this argument shows that important qualitative features of the spectrum can be deduced from correlation functions in the asymptotically free (perturbative regime)

Sufficient conditions for a Hagedorn spectrum

TDC arXiv:0901.0494

- Goal is to show that a Hagedorn spectrum really exists in large Nc QCD and its cousins.
  - A rigorous mathematical theorem is NOT derived
  - However it can be shown that a Hagedorn spectrum must arise if certain assumptions we commonly make about correlation functions are correct. These involve the applicability of perturbation theory to short distance correlators.

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  - Exploit in any way the notion of confinement as an unbroken center symmetry
    - It does exploit the notion of confinement as the requirement that all physical states be color singlet.


WARNING
ACHTUNG
警告
Crocodiles inhabit this area – attacks may cause injury or death
• Keep away from the water's edge and do not enter the water.
• Take extreme care when launching and retrieving boats.
• Do not clean fish or leave fish waste near the water's edge.
• Camp well away from the water.
Queensland Government

In what follows ordering of limits plays a crucial, if implicit role. The large *Nc* limit will always be taken prior to any other.

Witten's argument for an infinite number of mesons required the large Nc spectral form at the outset

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  - Assumes a bound on the correction to the leading the perturbative expression for the short-distance behavior of a class of correlation functions.
  - Uses the exponential growth in the number of currents (of a certain type) with mass dimension

- The key ingredient: a matrix of correlators *in (Euclidean) configuration space* for a set of single-color-trace currents which (for simplicity) will be taken to scalars & pseudoscalars

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- There is such a matrix associated with any set of currents: $S = \{ J_1, J_2, J_3, J_4, \ldots, J_n \}$

$$\overset{\leftrightarrow}{\Pi}_{ab}(r) = \int ds \, \rho_{ab}(s) \, \Delta(r, \sqrt{s})$$

$$= \sum_n c^*_{a,n} \, c_{b,n} \Delta(r, m_n)$$

$$\rho_{ab}(s) = \sum_n c^*_{a,n} \, c_{b,n} \delta(s - m_n^2)$$

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Fact that currents only make single meson states follows from large Nc planarity + confinement (in the sense of only singlet states) for single color trace operators

A useful lemma

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Follows from the preceding form + basic properties of the scalar propagator, Δ, and the Log structure.

Form should be familiar to anybody trying to extract excited hadron masses from the lattice.

Implication of the lemma

$$-\frac{d\,\mathrm{Tr}\left(\log\left(\overleftrightarrow{\Pi}(r)\right)\right)}{d\,r} \geq \sum_{j}^{\dim\left(\overleftrightarrow{\Pi}\right)} m_j$$

If one has an infinite sequence of sets of operators $S_1, S_2, S_3, \ldots S_n \ldots$ where

1. The size of the set (i.e. $\dim(\Pi_n)$) grows exponentially in n: $\dim(\Pi_n) \geq A^n$)

2. There exists constants p, n_0 r_0 such that

$$-\frac{1}{\dim\left(\left(\overleftrightarrow{\Pi}_n(r)\right)\right)} \left.\frac{d\,\mathrm{Tr}\left(\log\left(\overleftrightarrow{\Pi}_n(r)\right)\right)}{d\,r}\right|_{r=r_0} \leq n\frac{p}{r}$$

for all $r \leq r_0$ and $n \geq n_0$ then a Hagedorn spectrum must exist

Why is this so?

$$n\frac{p}{r} \geq -\frac{1}{\dim(\overset{\leftrightarrow}{\Pi}_n(r))}\left.\frac{d\,\mathrm{Tr}\left(\log\left(\overset{\leftrightarrow}{\Pi}_n(r)\right)\right)}{d\,r}\right|_{r=r_0} \geq \frac{1}{\dim(\overset{\leftrightarrow}{\Pi}_n(r))}\sum_{k}^{\dim(\Pi_n)} m_k = \langle m \rangle$$

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Recall *N(m)* is the number of mesons of mass less then *m* . Assume that N(m) is at least linear in *m*. Then $\langle m\rangle > m_0/2$ where $N(m_0)=dim(\Pi)$

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$$n\frac{p}{r_0} \geq n\frac{p}{r} \geq \frac{m_0}{2} \quad \text{or} \quad n\frac{2p}{r_0} \geq m_0$$

$$N\left(n\frac{2p}{r_0}\right) \geq N\left(m_0\right) \geq A^n$$

***N* is monotonic**

$$N\left(n\frac{2p}{r_0}\right) \geq A^n$$

thus

$$N(m) \geq e^{m/T_H}$$ with

$$T_H \leq \frac{2p}{r_0 \log(A)}$$

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uses the two conditions stated above +explicit assumption that *N* is at least linear. This latter assumption is clearly true self-consistently

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We need to find sets of operators satisfying these two conditions

A useful set of currents for QCD with fundamental quarks:

$$S_n = \{J_1, J_2, J_3, \ldots, J_{2^n}\} \qquad \overbrace{J_a = \bar{q}\,\mathrm{O}_1\mathrm{O}_2\cdots\mathrm{O}_n q}^{\text{Single color trace}}$$

$$\text{where} \quad \mathrm{O}_p = \mathrm{O}_+, \mathrm{O}_- \qquad \underbrace{\mathrm{O}_+ = \underbrace{\tfrac{1}{N_c^2} F_{\mu\nu} F^{\mu\nu}}_{\substack{\text{summed over}\\ \text{Lorentz indices}\\ \text{unsummed over}\\ \text{color}}} \qquad \mathrm{O}_- = \underbrace{\tfrac{1}{N_c^2} F_{\mu\nu} \tilde{F}^{\mu\nu}}_{\substack{\text{summed over}\\ \text{Lorentz indices}\\ \text{unsummed over}\\ \text{color}}}}_{\text{Color octet at large Nc}}$$

Thus

$$S_1 = \{\bar{q}O_+q, \bar{q}O_-q\}$$

$$S_2 = \{\bar{q}O_+O_+q, \bar{q}O_+O_-q, \bar{q}O_-O_+q, \bar{q}O_-O_-q\}$$

$$\begin{aligned} S_3 = \{&\bar{q}O_+O_+O_+q, \bar{q}O_+O_+O_-q, \bar{q}O_+O_-O_+q, \bar{q}O_+O_-O_-q, \\ &\bar{q}O_-O_+O_+q, \bar{q}O_-O_+O_-q, \bar{q}O_-O_-O_+q, \bar{q}O_-O_-O_-q\} \end{aligned}$$

$$\cdots$$

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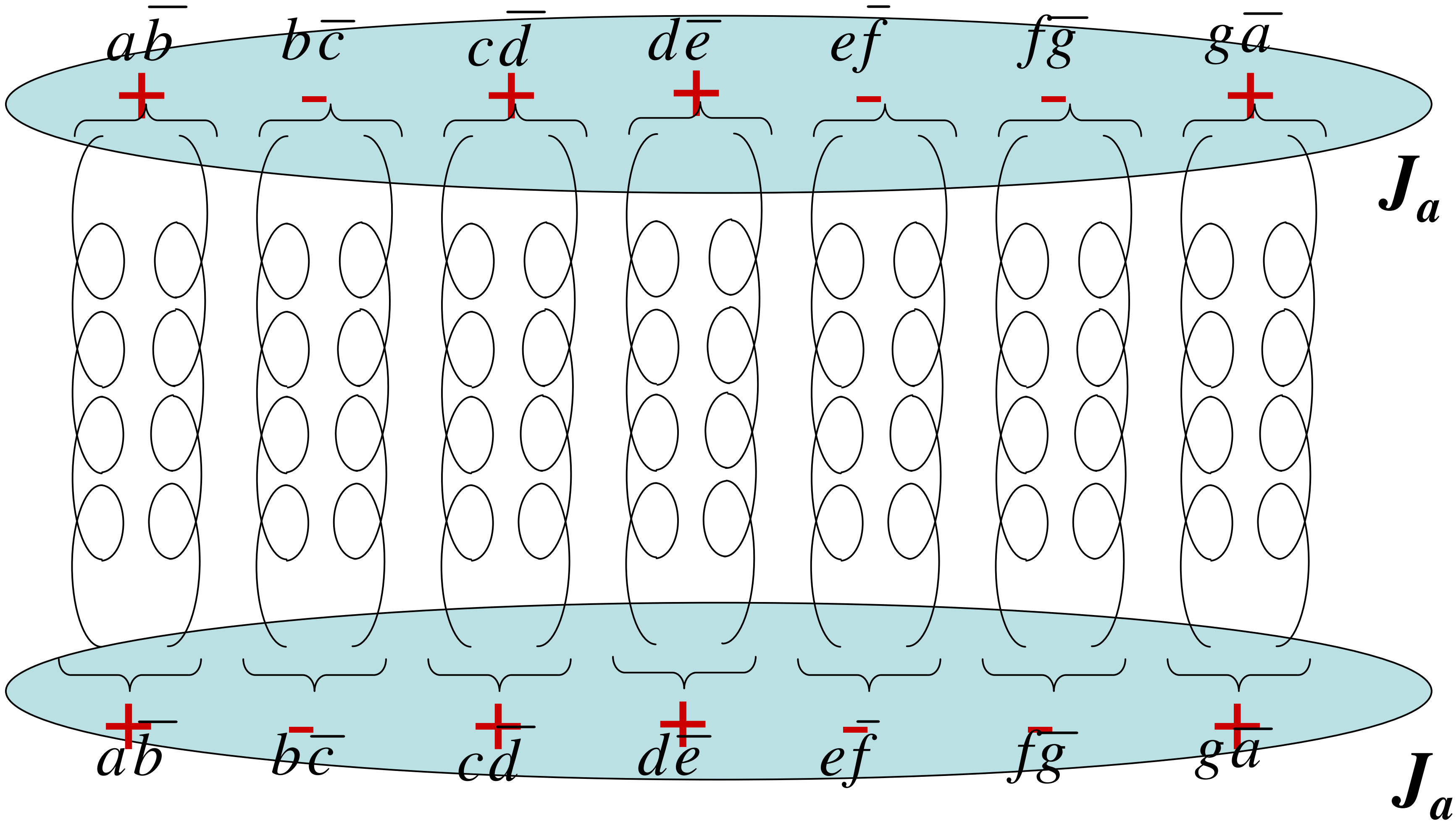
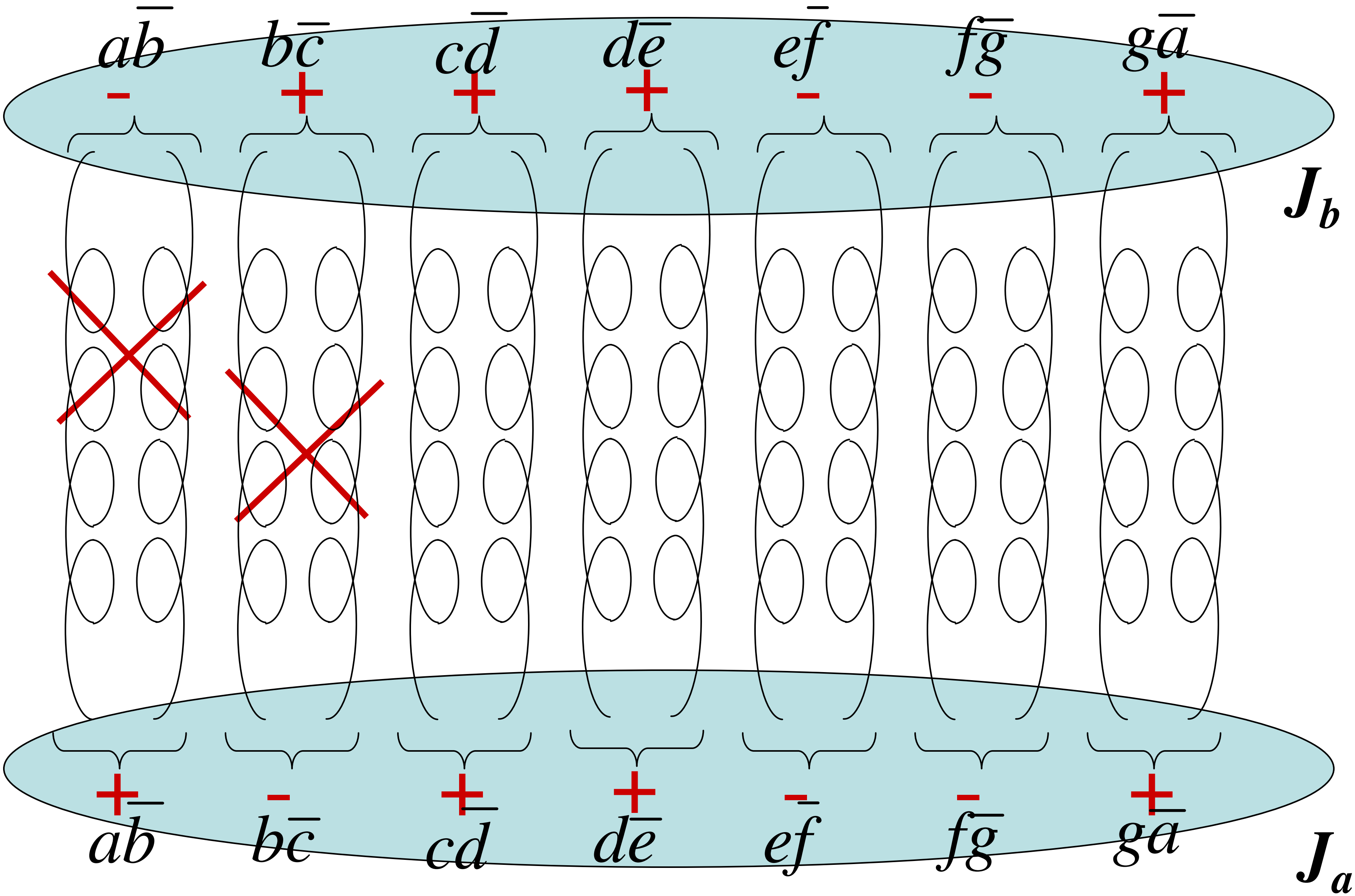
$$\cdots$$

Note that the number of states grows like 2^n---for obvious reasons; this satisfies one of the 2 conditions needed for a Hagedorn spectrum

For sufficiently small r theory acts asymptotically free and correlation functions are given by their free field values;



A contribution to a typical non-zero (diagonal) correlator asymptotically



An off-diagonal correlator vanishes asymptotically

Thus Π is diagonal and easy to compute---indeed each diagonal ME can be determined by dimensional analysis.

$$-\frac{1}{\dim\left(\overleftrightarrow{\Pi}_n\right)}\frac{d\,\mathrm{Tr}\left(\log\left(\overleftrightarrow{\Pi}_n(r)\right)\right)}{d\,r} = n\,\frac{8+\frac{6}{n}}{r}$$

For asymptotically small r

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A useful parameterization

Due to interactions

$$-\frac{1}{\dim\left(\overleftrightarrow{\Pi}_n\right)}\frac{d\,\mathrm{Tr}\left(\log\left(\overleftrightarrow{\Pi}_n(r)\right)\right)}{d\,r}=n\frac{8+\frac{6}{n}}{r}+R(r,n)$$

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Generically you might expect this not to be true due to combinatoric factors which grow with n.

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Fortunately, the planarity of the large *Nc* limit (which is taken first!) greatly restricts the combinatorics.

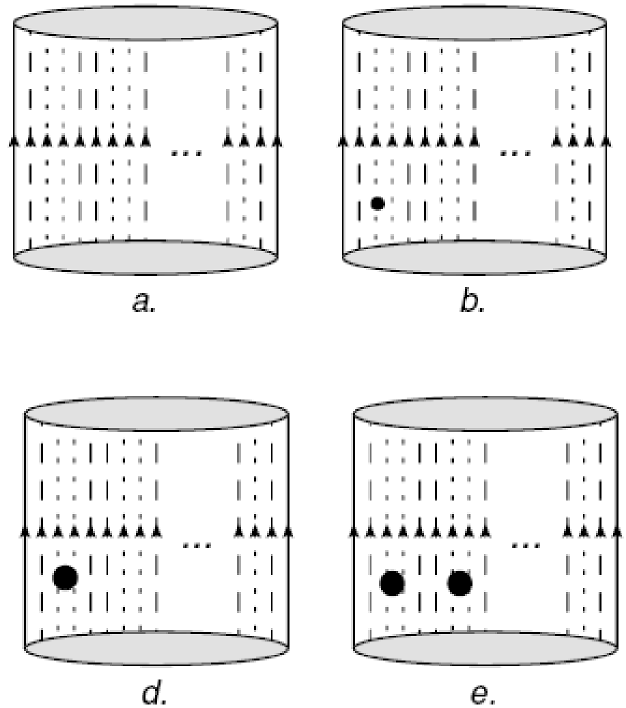
This plus the point-to-point nature of the correlator--- which implies that the contributions of interactions factorize and the logarithmic nature of the relevant quanity ensures that working up to any fixed order in RG improved perturbation theory all perturbative contributions to $R(r,n)$ are linear in n (or slower).

Why is this?

$$\overleftrightarrow{\Pi}_n(r)=\overleftrightarrow{\Pi}_n^{free}(r)\left(1+\overleftrightarrow{C}(r)\right)$$

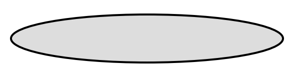
$$R(r,n)=\partial_r\frac{\sum_{j\geq 1}\frac{(-1)^j}{j}\,\mathrm{tr}\left(\overleftrightarrow{C}^j(r)\right)}{\dim\left(\overleftrightarrow{\Pi}_n\right)}$$

$$=\partial_r\frac{\mathrm{tr}\left(\overleftrightarrow{C}(r)\right)-\frac{1}{2}\mathrm{tr}\left(\overleftrightarrow{C}^2(r)\right)+\frac{1}{3}\mathrm{tr}\left(\overleftrightarrow{C}^3(r)\right)}{\dim\left(\overleftrightarrow{\Pi}_n\right)}$$



One or more gluon exchanges between distinct pairs

One or more gluon exchanges within a single pair



a current in the set S_n

+ parity pair of gluons

- parity pair of gluons

Why is this?

$$\vec{\Pi}_n(r) = \vec{\Pi}_n^{free}(r)\left(1+\vec{C}(r)\right)$$

$$R(r,n) = \partial_r\left(\sum_{j\geq 1}\frac{(-1)^j}{j}\,\mathrm{tr}\left(\vec{C}^j(r)\right)\right)\left(\frac{1}{\dim\left(\vec{\Pi}_n\right)}\right)$$

$$= \partial_r\left(\mathrm{tr}\left(\vec{C}(r)\right) - \tfrac{1}{2}\mathrm{tr}\left(\vec{C}^2(r)\right) + \tfrac{1}{3}\mathrm{tr}\left(\vec{C}^3(r)\right) + \ldots\right)\left(\frac{1}{\dim\left(\vec{\Pi}_n\right)}\right)$$

Contribution of type *b.* is ~*n* since there are *n* lines

Contribution of type *b.* is ~n^2 since there are n^2 pairs of lines

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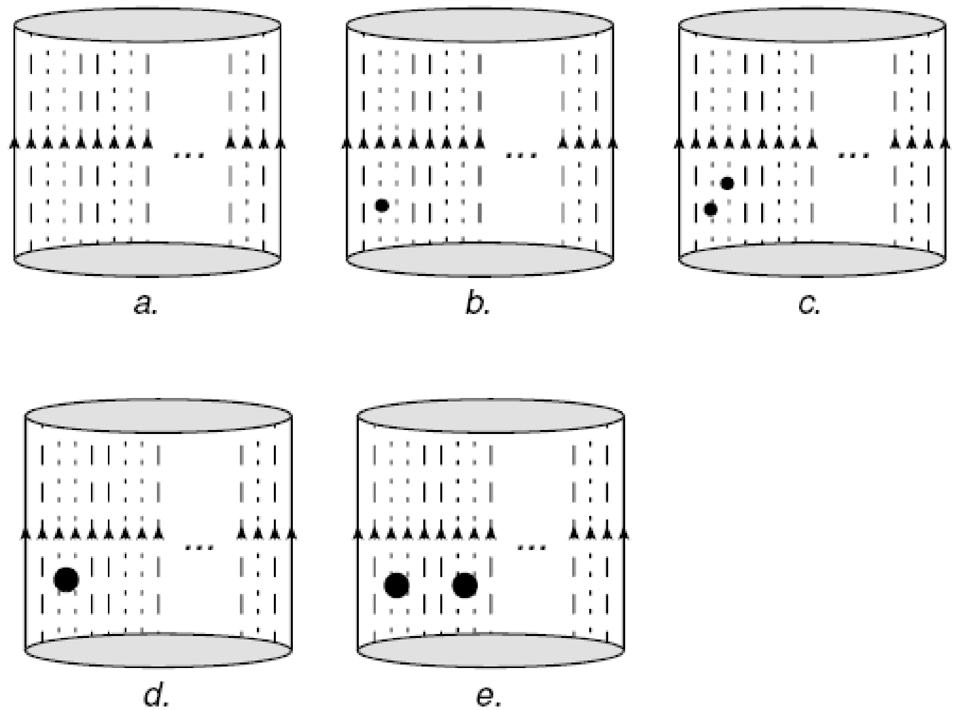

These cancel yielding a contribution ~*n*

This type of cancelation is generic always yielding ~*n*

The upshot: ***up to any order***, l ,in perturbation theory for all $n>l$

Universal constants independent of n and r.

$$R(r,n)=\frac{\sum_{i=2}^{l}\left(c_i+\frac{d_i}{n}\right)\Big(\overbrace{N_c\alpha_s(\mu^2)}^{\text{The ‘t Hooft coupling}}\Big)^i}{r}\quad,\quad \mu^2=r^{-2}$$

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The 't Hooft coupling

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This is the condition for a Hagedorn spectrum!!!

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- Demonstration depends, however, on the applicability of pertrubation theory for correlation functions at short distance.
- This assumption is completely standard---however it is not mathematically rigorous.
  - The asymptotic nature of the perturbative expansion makes it very difficult to strengthen this argument into a rigorous theorem
  - This is hardly surprising: truly rigorous results in QCD are very rare. Asymptotic freedom has never been proved rigorously

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- This assumption is not invented for the purpose of showing a Hagedorn spectrum.
  - No explicit assumption about stringy dynamics is made.
  - Confinement is only assumed in the sense that all physical states are color singlets. **No explict assumptions made about area law of Wilson loops or unbroken center symmetry.**

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  - For mesons with non-scalar quantum numbers
    - Modify the dispersion relation; still build currents with insertions of scalar and pseudscalar pairs
  - For glueballs
    - Operators are traces of products $O_{\pm}$ operators. Since the trace is cyclic the number of operators does not grow as 2^n. It does grow exponentially however. The number of operators is $(2^n + 2n - 2)/n$ for n prime; and larger otherwise*. This is sufficient.

*I thank Michael Cohen for pointing this out

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  - Theories in other dimensions greater then 1+1(there are some subtleties here)
  - QCD with adjoint fermions in 1+1 (using fermion bilinears as $O\pm$ —these still involve a single color trace)

What does this teach us?

- Helps confirm what we already believed---that highly excited states in large Nc QCD look stringy
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  - The evidence for this only involve aspects of QCD which are superficially quite removed from the issue of stringy dynamics
- May give some insight into the nature of confinement.

What is confinement?

There are two distinct notions of confinement

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A cartoon

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What is confinement?

- There are two distinct notions of confinement
  - The idea that all physically isolated states are color singlets
  - Unbroken center (Zn) symmetry; area law for Wilson loop
- The first notion is what we mean when we say that QCD is confining
- The second applies to a cartoon world
  - Doesn't apply to real QCD with 3 colors and 2½ light flavors
  - Where it does apply, it allows the use of the Polyakov line as an order parameter for confinement/ deconfinement transition.

The argument given here for a Hagedorn spectrum does require confinement in the sense of no colored physical states it does NOT use center symmetry in any explicit way

Real QCD does not have a center symmetry, due to the influence of quarks. However, as it happens quark effects are suppressed at large *Nc*. At large *Nc*, there is an *emergent* center symmetry.

Does a Hagedorn spectrum *require* confinement in the sense of an unbroken center symmetry---albeit an emergent symmetry at large *Nc*?

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  - QCD with quarks in the fundamental representation
    - This does NOT have a manifest center symmetry.
    - But there is an obvious emergent center symmetry since at large Nc quarks don't matter

- The argument also gives a Hagedorn spectrum in:
  - QCD with quarks in the two-index antisymmetric represenation (orientifold large Nc limit)
    - This does NOT have a manifest center symmetry.
    - Moreover quarks DO matter at large Nc
    - However, this has a more subtle emergent center symmetry at large Nc (due to the orientifold equivalence with adjoint case A Armoni, M. Shifman & M. Unsal Phys. Rev. D77, 045012 (2008)).

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  - Theories with other gauge groups (eg. orthogonal groups)
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  - However
    - Mesons are not narrow in this limit

      $$\Gamma \sim N_f/N_c \sim O(1)$$

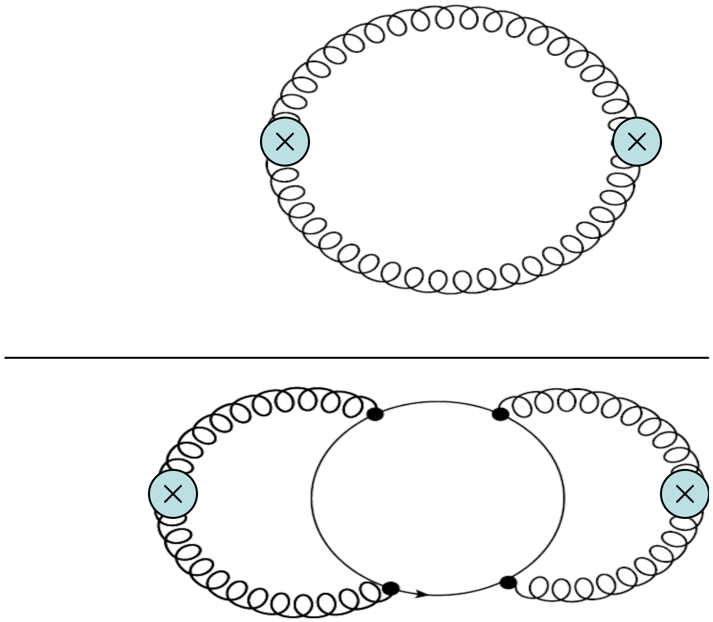
    - Glueballs are not narrow in this limit

      $$\Gamma \sim N_f^2 / N_c^2 \sim O(1)$$

Why are gluon widths ~ $O(1)$?

Gluon-meson mixing amplitude is $(Nf\,/Nc)^{1/2} \sim O(1)$

Skeleton Feynman diagram



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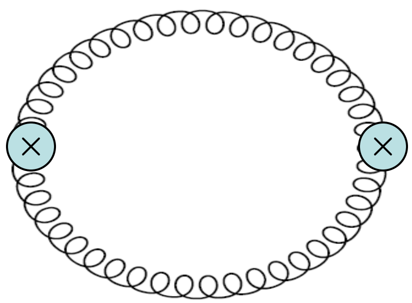
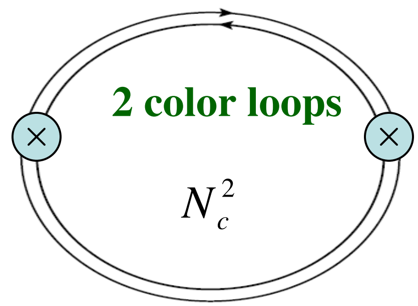
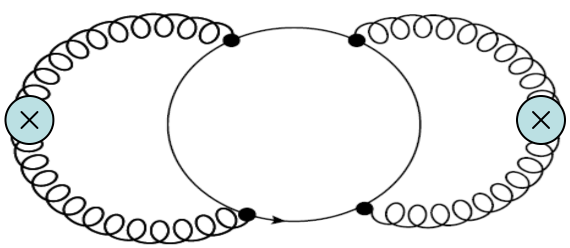
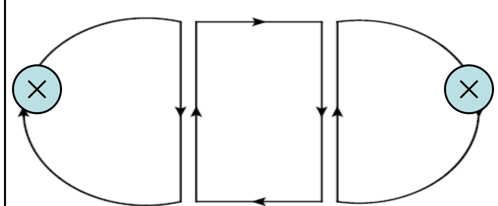


`t Hooft color flow diagram







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1 quark loop N_f

4 coupling constants N_c^{-2}

Total $N_c N_f$

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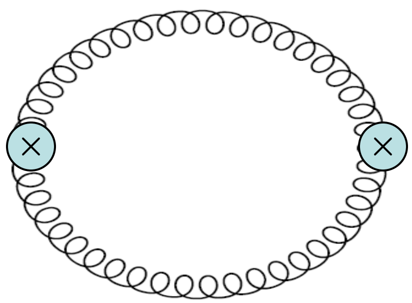

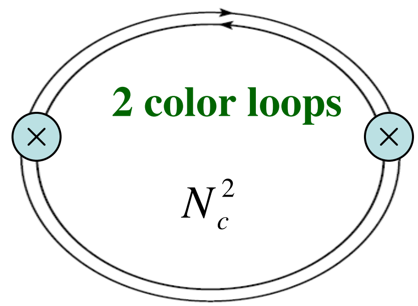

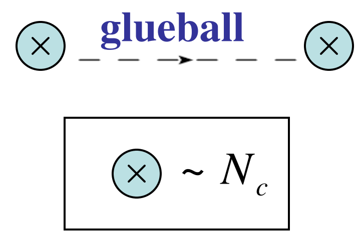

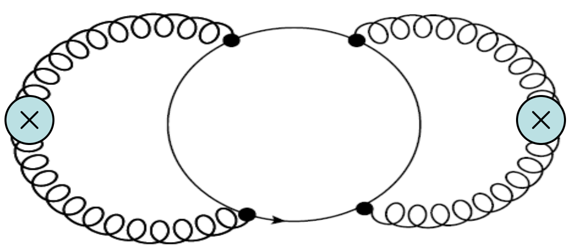

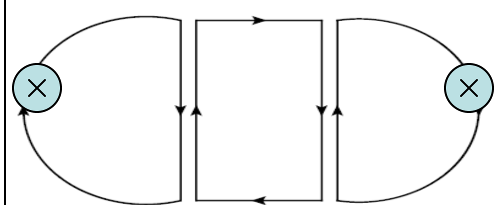

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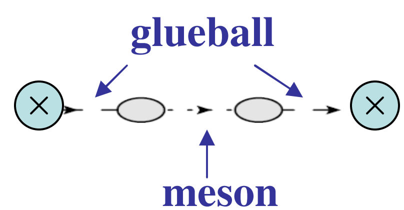
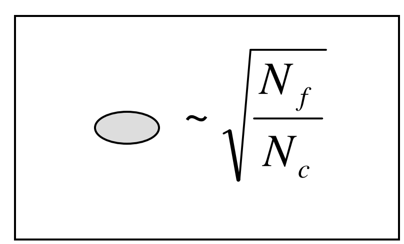
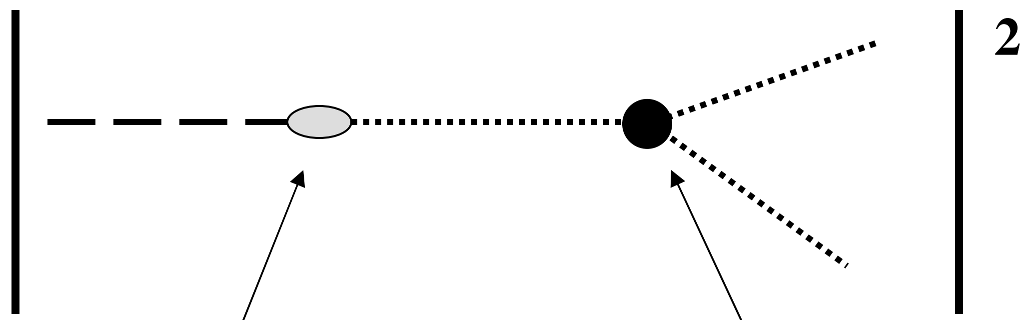




Contribution to gluon widths due to mixing into mesons



$$\Big|\ \cdots\ \Big|^{2} \sim \frac{N_f^2}{N_c^2} \sim O(1)$$

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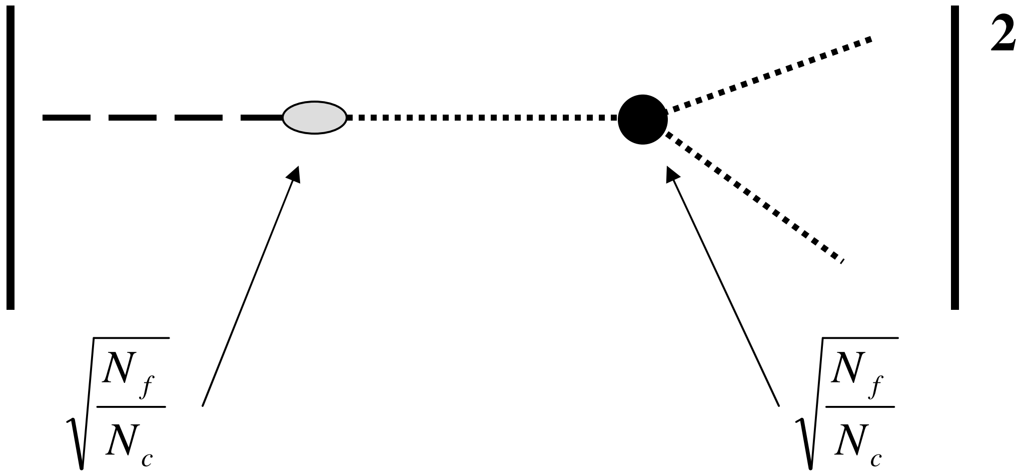

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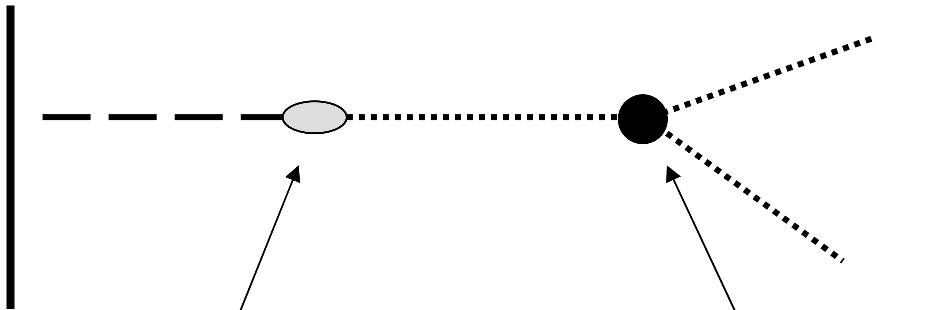

$$\left| \quad \cdots \quad \right|^2 \sim \frac{N_f^2}{N_c^2} \sim O(1)$$

$$\sqrt{\frac{N_f}{N_c}} \qquad \sqrt{\frac{N_f}{N_c}}$$

- In Veneziano limit neither mesons or glueballs are narrow.
- Masses are not well-defined at large Nc and thus Hagedorn spectrum does not exist.

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- Is there a theorem lurking somewhere that this must be true?
- Does this sat something deep about confinement---at least of the cartoon variety?