

# Alternative large $N_c$ baryons and holography

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July 7, 2009

# INTRODUCTION

# Classic large $N_c$ limits of QCD

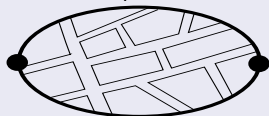
't Hooft; Veneziano '79

## How to take the limit

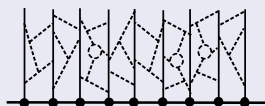
- $SU(3) \rightarrow SU(N_c), N_c \rightarrow \infty$
- Fundamental Flavors  $q^i \rightarrow q^i$

## Hadrons

Mesons  $\bar{q}_i q_i$ : string picture



Baryons  $\epsilon_{i_1 \dots i_{N_c}} q^{i_1} \dots q^{i_{N_c}}$ : solitons



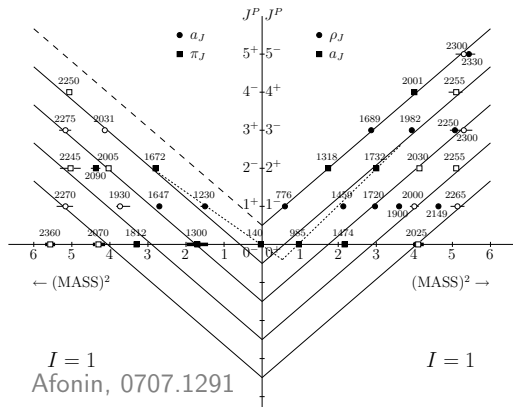
Pomarol

# Mesons and Regge trajectories

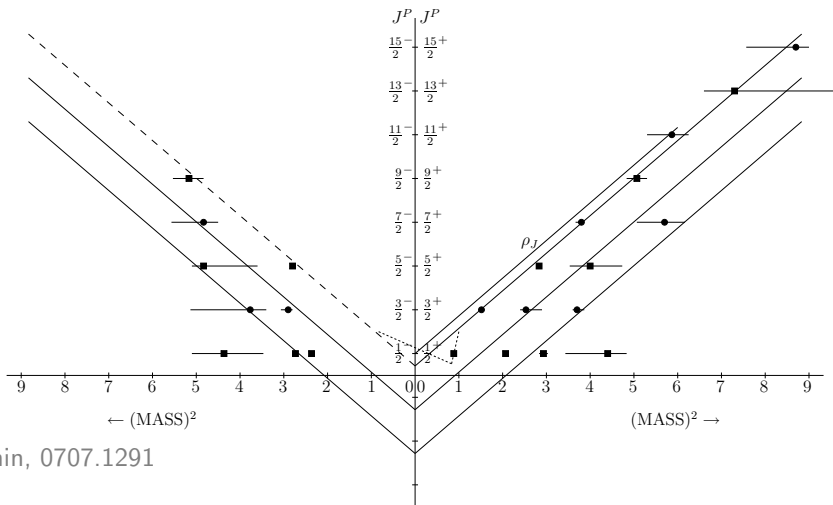
$$\alpha' \simeq 1 \text{ GeV}^{-2}$$

$$J = \alpha_0 + \alpha' M^2$$

$$M = \bar{q}_i q^i$$

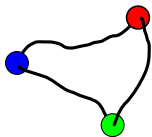


# Baryons and Regge trajectories

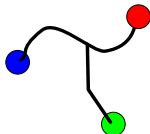


Afonin, 0707.1291

# What is a baryon in QCD?



Two-quark interactions  $\mathcal{B} = \epsilon_{ijk} q^i q^j q^k$



Baryon vertex  $\mathcal{B} = \epsilon_{ijk} q^i q^j q^k$



Baryon formed by a quark-diquark state  
 $\mathcal{B} = \epsilon_{ijk} q^i q^j Q^k$

# Other large $N_c$ limits of QCD

Corrigan and Ramond '79; Armoni, Shifman and Veneziano '03

## How to take the limit

- $SU(3) \rightarrow SU(N_c), N_c \rightarrow \infty$
- Fundamental Flavors  $\Psi_{[ij]} = \frac{1}{2}\epsilon_{ijk}q^k \rightarrow \Psi_{[ij]}; N_{AS} \leq 5$
- More flavors:  $q^i$
- Chiral version Rytov, Sannino '05

## Hadrons

- 'Light' mesons:  $\bar{q}_i q^i, \bar{\Psi}^{[ij]} \Psi_{[ij]}$
- 'Heavy baryons':  $\epsilon_{i_1 \dots i_{N_c}} q^{i_1} \dots q^{i_{N_c}}$
- 'Heavy' mesons:  $\epsilon_{i_1 i_2 \dots i_{N_c-2} i_{N_c-1} i_{N_c}} \bar{\Psi}^{[i_1 i_2]} \dots \bar{\Psi}^{[i_{N_c-2} i_{N_c-1}]} q^{i_{N_c}}$
- 'Light' baryons:  $\Psi_{[ij]} q^i q^j$

# Planar equivalence and Regge trajectories

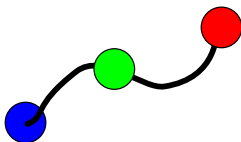
Armoni and Patella, 0901.4508

- Supersymmetric theory:

$$\bar{q}_i e^{i \int A} q^j \rightarrow \bar{q}_i \int dx e^{i \int^x A} \lambda_j^i(x) e^{i \int_x A} q^j + \mathcal{O}(1/R)$$

- Equivalence:  $\lambda_j^i \rightarrow \Psi_{[ij]}$
- Quarks joined by fermionic/unoriented string

$$\bar{q}_i e^{i \int A} q^j \rightarrow q_i \int dx e^{i \int^x A} \Psi_{[ij]}(x) e^{i \int_x A} q^j + \mathcal{O}(1/R)$$





- Natural framework for large  $N_c$  gauge theories at strong coupling
- Mesons appear as fluctuations of probe branes in the geometry
- Usual baryons appear as heavy objects: D-branes

How are light baryons realized?

# HOLOGRAPHIC MODEL

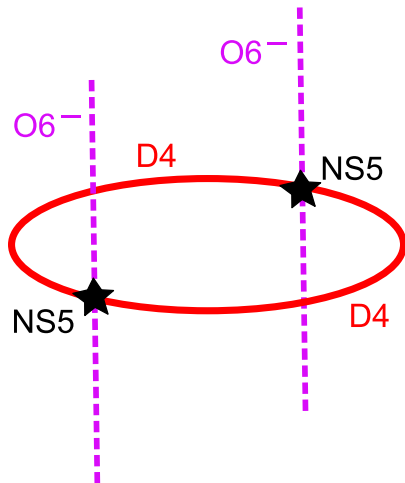
- $\mathcal{N} = 4$   $SU(N_c)$  SYM +  $\mathcal{N} = 2$  hypermultiplets  $Q, \tilde{Q}$
- Global symmetries:  $SU(2)_L \times SU(2)_R \times U(1)_R \times U(N_f)$

fields	$SU(N_c) \times U(N_f)$	$(j_R, j_L)_R$
$Q$	$(\mathbf{1}, \mathbf{1})$	$(1/2, 0)_1$
$\Phi_1, \Phi_2$	$(\mathbf{N}_c^2 - \mathbf{1}, \mathbf{1})$	$(1/2, 1/2)_0$
$\Phi_3$	$(\mathbf{N}_c^2 - \mathbf{1}, \mathbf{1})$	$(0, 0)_{\pm 2}$
$Q$	$(\overline{\mathbf{N}}_c, \mathbf{N}_f)$	$(1/2, 0)_0$
$\tilde{Q}$	$(\mathbf{N}_c, \overline{\mathbf{N}}_f)$	$(1/2, 0)_0$

- Supersymmetric projection  $\mathbf{Z}_2 \subset SU(2)_L$
- $\Phi_1, \Phi_2 \rightarrow$  antisymmetric

# D-brane setup

Park and Uranga, hep-th/9808161



- $2N_c$  D4 branes
- $N_f$  D6s on top of  $O6^-$
- $N_f \ll N_c$  (quenched)
- Projection made in T-dual ( $D3+D7/O7+\mathbf{Z}_2$ )

# Field theory from D-branes

	0	1	2	3	4	5	6	7	8	9
$D3$	X	X	X	X	.	.	.	.	.	.
$O7/D7$	X	X	X	X	.	.	X	X	X	X
$Z_2$	X	X	X	X	X	X	.	.	.	.

## Chan-Paton factors

- Orbifold projection  $X \rightarrow \pm \gamma_p X \gamma_p^{-1}$

$$\gamma_3 = \begin{pmatrix} iI_{N_c} & \\ & -iI_{N_c} \end{pmatrix}, \quad \gamma_7 = \begin{pmatrix} iI_{N_f} & \\ & -iI_{N_f} \end{pmatrix},$$

- Orientifold projection  $X \rightarrow \pm \omega_p X^T \omega_p^{-1}$

$$\omega_3 = \begin{pmatrix} & I_{N_c} \\ -I_{N_c} & \end{pmatrix}, \quad \omega_7 = \begin{pmatrix} & I_{N_f} \\ I_{N_f} & \end{pmatrix},$$

- D3 brane fields

$$\begin{array}{ll} A_{0123} & \text{Adjoint } SU(N_c) \\ X_{45} & \text{Adjoint } SU(N_c) \\ X_{67}, X_{89} & \text{Antisymmetric } SU(N_c) \end{array}$$

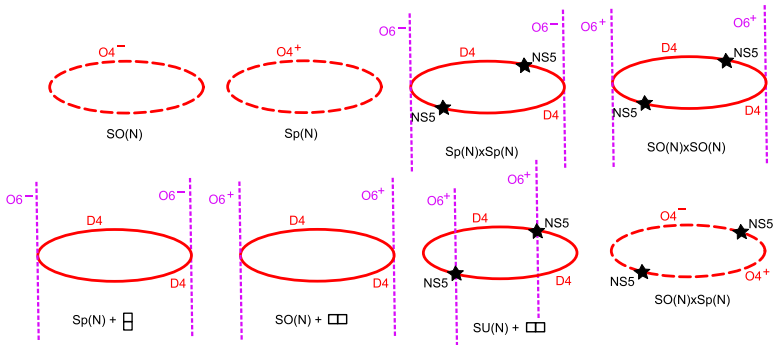
- D3/D7 spectrum

$$H^A \quad (\mathbf{N}_c, \bar{\mathbf{N}}_f)$$

- D7 brane fields ( $A_{0123}, X_{45} \leftrightarrow A_{6789}$  parity odd)

$$\begin{array}{ll} A_{0123} & \text{Adjoint } SU(N_f) \\ X_{45} & \text{Adjoint } SU(N_f) \\ A_{6789} & \text{Antisymmetric } SU(N_f) \end{array}$$

# Other examples



## BPS hypermultiplet $j_L = \ell/2, n$

- Lowest component:  $\Delta = 2j_R + R/2$
- Bosonic degrees of freedom  $8(\ell + 1)$

- Bosons:

$\Delta$	$(j_R, j_L)_R$	#
$\ell + 2$	$(\frac{\ell}{2} + 1, \frac{\ell}{2})_0$	2
$\ell + 3$	$(\frac{\ell}{2}, \frac{\ell}{2})_{\pm 2} \oplus (\frac{\ell}{2}, \frac{\ell}{2})_0$	$1 \oplus 2$
$\ell + 4$	$(\frac{\ell}{2} - 1, \frac{\ell}{2})_0$	2

- Fermions:

$\Delta$	$(j_R, j_L)_R$	#
$\ell + 5/2$	$(\frac{\ell+1}{2}, \frac{\ell}{2})_{\pm 1}$	2
$\ell + 7/2$	$(\frac{\ell-1}{2}, \frac{\ell}{2})_{\pm 1}$	2



# BPS Spectrum: gravity dual

$$AdS_5 \times \mathbf{RP}^5 + \text{O7/D7 on } \mathbf{RP}^3 \subset \mathbf{RP}^5$$

## D7 brane fluctuations

- Isometry  $\mathbf{RP}^3$ :  $SO(4)/\mathbf{Z}_2 \simeq SU(2)_L/\mathbf{Z}_2 \times SU(2)_R$
- Rotation on plane transverse to D7:  $U(1)_R$
- Supersymmetries:  $(j_R, j_L)_R = (1/2, 0)_1$

D7 KK modes on  $\mathbf{S}^3 \subset \mathbf{S}^5$ :  $A, \Phi, \Psi$  enter in hypermultiplets with  $j_L = \ell/2$

$\Delta$	mode
$\ell + 2$	$A_-^{\ell+1}$
$\ell + 5/2$	$\Psi_-^\ell$
$\ell + 3$	$\Phi^\ell, A^\ell$
$\ell + 7/2$	$\Psi_+^{\ell-1}$
$\ell + 4$	$A_+^{\ell-1}$

Kruczenski, Mateos, Myers and Winters, hep-th/0304032; Kirsch hep-th/0607205

# Projection of the bosonic BPS spectrum

Parity odd modes  $\ell = 2k + 1$  ( $j_L = k + 1/2$ ) are projected to the antisymmetric rep. of  $SU(N_f)$

$$\Phi^{2k+1} \rightarrow -(\Phi^{2k+1})^T, \quad A^{2k+1} \rightarrow -(A^{2k+1})^T, \quad A_{\pm}^{2k+2} \rightarrow -(A_{\pm}^{2k+2})^T.$$

## Example

$$\mathcal{B}^{ab} = Q_i^a \Phi_1^{[ij]} Q_j^b$$

- $\mathcal{B}$  ( $\Delta = 3, R = 0$ ) is the lowest component of a  $j_L = 1/2$  multiplet
- Symmetric in flavor  $\Rightarrow$  antisymmetric in  $SU(2)_R \Rightarrow j_R = 1/2$
- Antisymmetric in flavor  $\Rightarrow$  symmetric in  $SU(2)_R \Rightarrow j_R = 3/2$
- BPS condition  $\Delta = 2j_R + R/2 \Rightarrow$  antisymmetric in flavor

- Massless string excitations - BPS operators:  $\sim m_q/\sqrt{\lambda}$  (e.g.  $\mathcal{B}^{[ab]}$ )
- Small string excitations - non-BPS operators:  $\sim m_q/\lambda^{1/4}$  (e.g.  $\mathcal{B}^{(ab)}$ )
- Large classical strings - semiclassical regime:  $\sim m_q$
- D3 on  $\mathbf{RP}^3$  - Pfaffian operators/heavy mesons:  $\sim N_c m_q/\sqrt{\lambda}$
- D5 on  $\mathbf{RP}^5$  - Baryon operator:  $\sim N_c m_q$

## Meson-baryon degeneracy

- Mesons and baryons lie on parallel 'Regge trajectories' (defined by  $\ell$ )
- Degenerate in the semiclassical limit  $\sim m_q/\sqrt{\lambda}$  or  $\sim m_q/\lambda^{1/4} \ll m_q$ .
- $SO(5)$  symmetry: mass  $\sim n + \ell \Rightarrow$  accidental degeneracy

- Alternative baryons and mesons have a common description in bulk
- In the semiclassical limit mesons and baryons are degenerate
- The results do not depend on the  $AdS_5$  part of the geometry
- In a confining theory, a similar construction will lead to equal Regge slopes