Termodynamics and Transport in Improved Holographic QCD

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Work with E. Kiritsis, U. Gursoy, L. Mazzanti, G. Michalogiorgakis
0707.1324, 0707.1349 0804.0899, 0812.0792, 0903.2895, 0906.1890

Phenomenological Holography and QCD

Bottom-up Holography:

Construct a gravity background such that, using holography rules, we can model the properties of the desired gauge theory (e.g. AdS/QCD, Pomarol and Da Rold '05; Erlich *et al* '05).

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- 5 dimensions (4 space-time + radial (RG) direction)
- restrict to a small number of fields
- use hints from (what is known about) 5D non-critical string theory

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Aim: a realistic holographic model of the deconfined QGP (to compare with lattice and RHIC)

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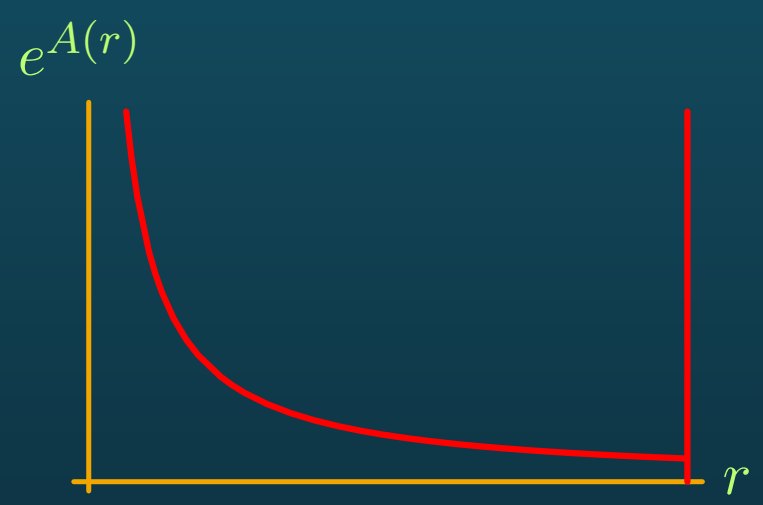
- Heavy quark diffusion

Precursors

AdS/QCD (Pomarol and Da Rold '05; Erlich *et al* '05)

- the metric is the only bulk field in the gauge sector
- The 5D spacetime is AdS_5 truncated by a IR cut-off with no explicit dynamics
  $\Rightarrow$ confinement is imposed via boundary conditions



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One step further: Soft-Wall AdS/QCD (Karch *et al* '06)

- hard cut-off replaced by a dilaton that grows in the UV
- the metric is still exactly AdS_5 (No mass gap; gluons are not confined)
- 2 bulk fields now, but still no dynamics (no action, no field equations): cannot compute free energies consistently.

Improved Holographic QCD

Gursoy, Kiritsis, FN 0707.1324, 0707.1349

- Make the soft-wall dynamical and interpret it as the holographic dual of the glue sector.
- Treat metric-dilaton as a dynamical system (including full backreaction).
- Breaking of conformal symmetry, mass gap, confinement, and all non-perturbative dynamics driven by the dilaton dynamics (aka the Yang-Mills coupling).

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Bulk Field Content $\Leftrightarrow$ Dimension 4 operators in Pure YM

4D Operator		Bulk field	Coupling
TrF^2	$\Leftrightarrow$	Φ	$\kappa N_c \int e^{-\Phi} TrF^2$
$T_{\mu\nu}$	$\Leftrightarrow$	$g_{\mu\nu}$	$\int g_{\mu\nu} T^{\mu\nu}$

$\lambda = \kappa N_c g_{YM}^2 = e^{\Phi}$ (finite in the large N limit).

The Setup

- only lowest dimension operators are considered
- action limited to 2-derivatives

$$S_E = -M_p^3 N_c^2 \left(\int d^5x \sqrt{-g} \left[R - \frac{4}{3}(\partial\Phi)^2 - V(\Phi) \right] \right)$$

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- $V(\Phi)$ fixed phenomenologically.
- N_c appears only as an overall factor.
  Effective Planck scale $\sim N_c^2$ is large.
- the parameter M_p can be fixed via thermodynamics in terms of $V(-\infty)$

Vacuum Solution

Non-constant $V(\Phi)$: solutions have running coupling.

$$ds^2 = e^{2A(r)}\left(dr^2 + \eta_{\mu\nu}dx^\mu dx^\nu\right), \quad \lambda(r) = e^{\Phi(r)}, \quad (E_{4D} \sim e^{A(r)})$$

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UV: $\quad e^A \to \infty,\ \lambda \to 0, \qquad V(\lambda) \sim \frac{12}{\ell^2}\left(1 + v_0\lambda + v_1\lambda^2 \ldots\right)$

Asymptotically AdS solution; $\ell^2_{ads} = 12/V(0)$ finite

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$$\beta(\lambda) \equiv \frac{d\lambda}{d\log E} = \frac{(d\lambda/dr)}{(dA/dr)}$$

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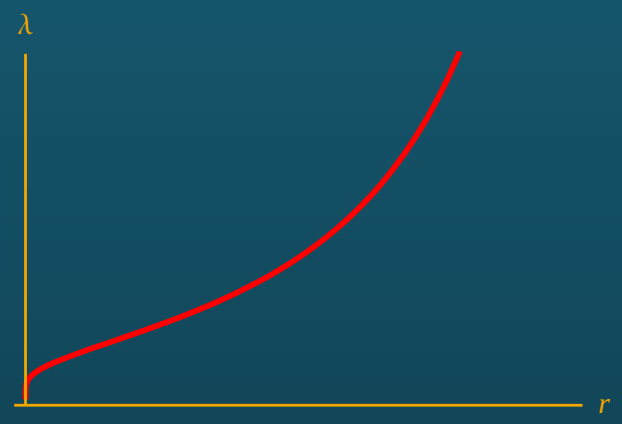

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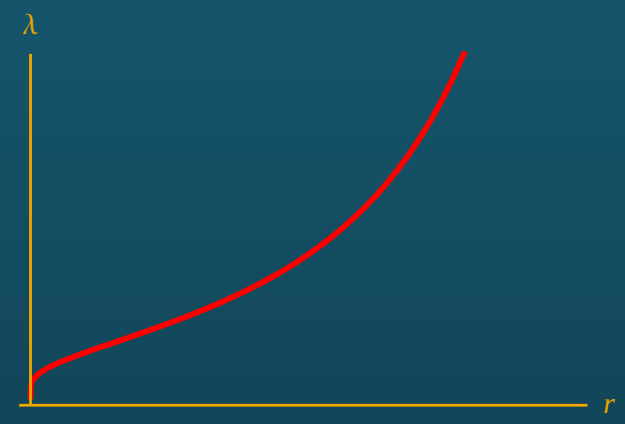

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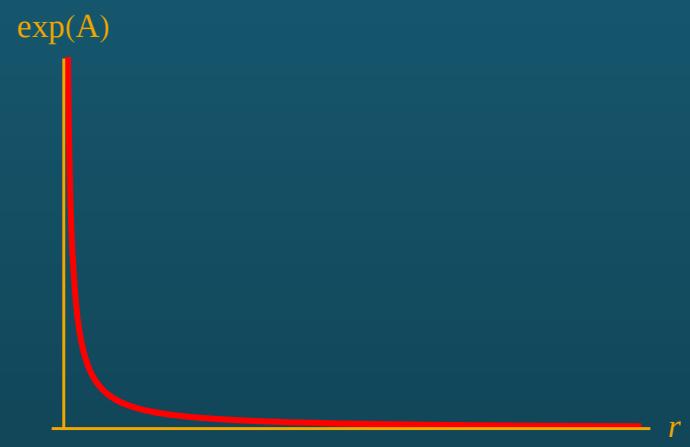

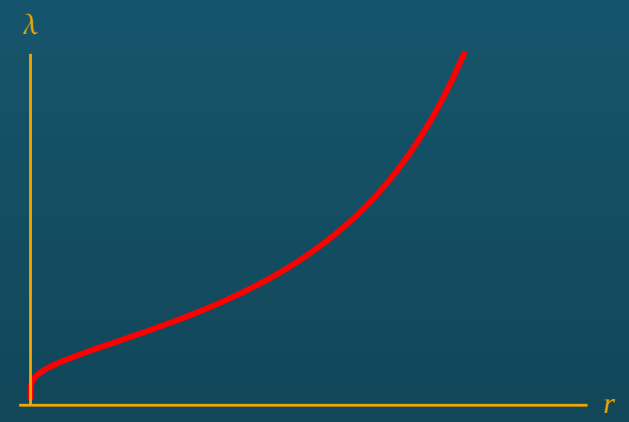

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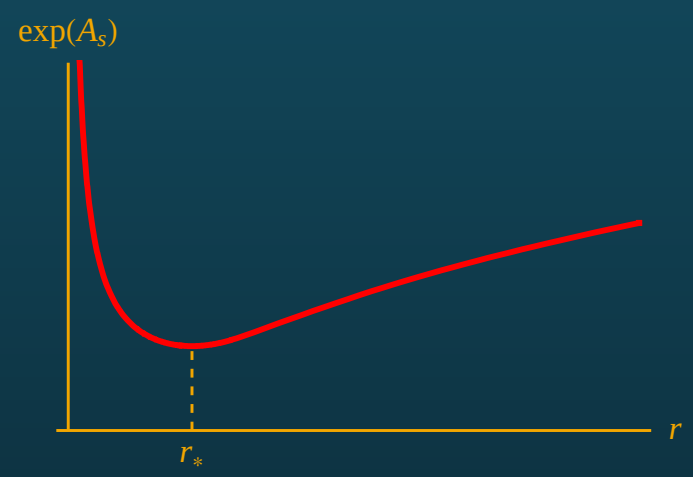

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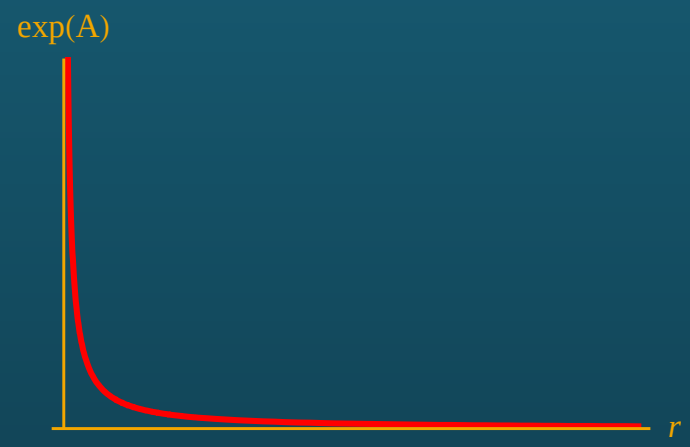

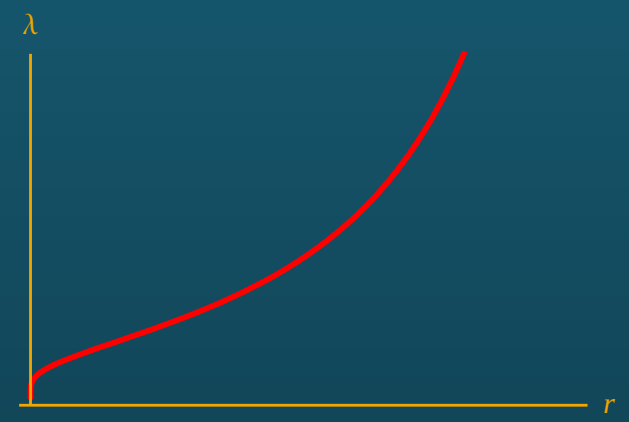

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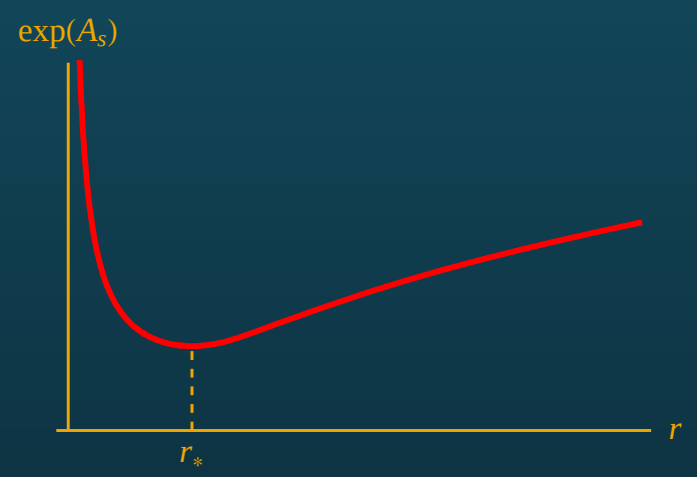

Wilson Loop area law with tension: $\sigma_c = \frac{1}{2\pi\ell_s^2} e^{2A(r_*)} \lambda^{4/3}(r_*)$

Finite Temperature

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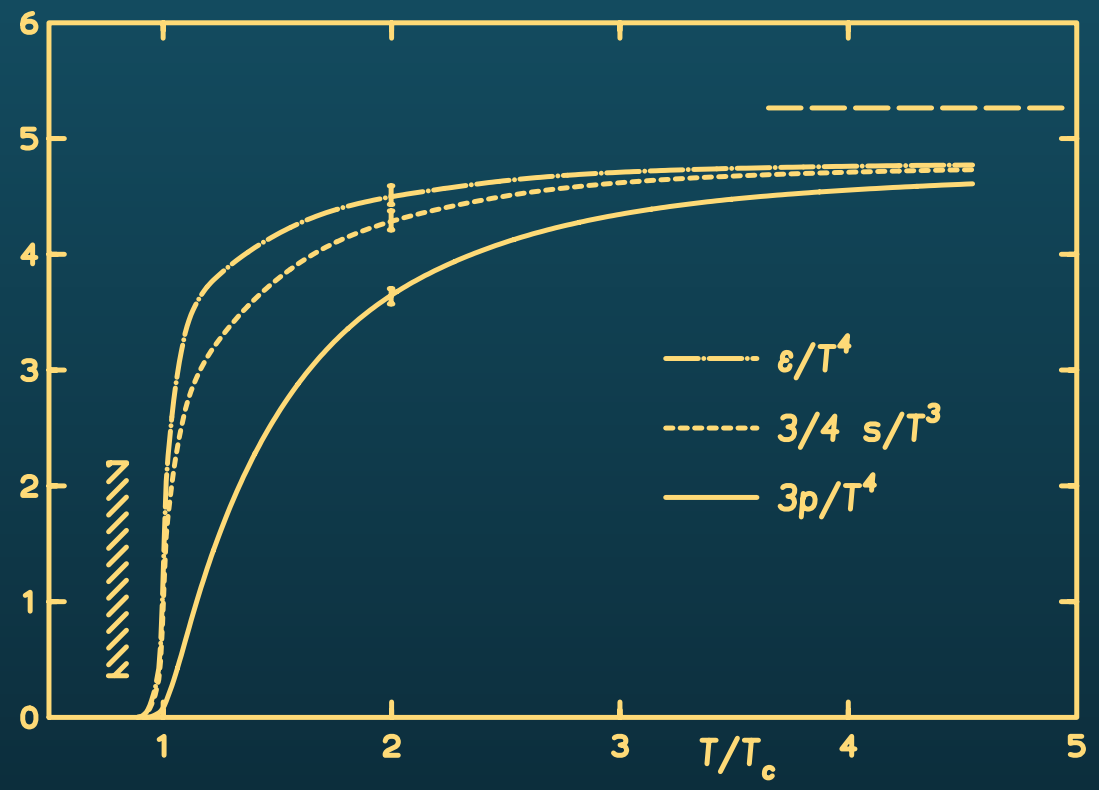
Can the holographic setup reproduce the phase transition and the YM equation of state found on the lattice?



Karsch, hep-lat/0106019

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- Thermal gas (Confined phase) :

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- Black hole (Deconfined phase - Gluon Plasma):

$$ds^2 = e^{2A(r)}\left(\frac{dr^2}{f(r)} + f(r)d\tau^2 + dx_i^2\right), \quad \Phi = \Phi(r), \quad 0 < r < r_h$$

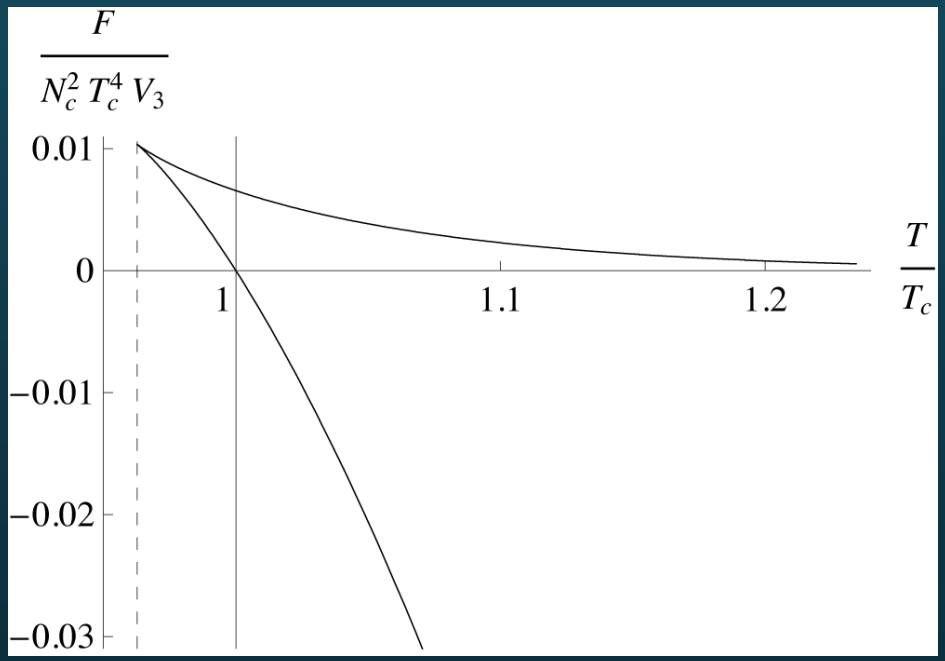
$$A(r) \underset{r\to 0}{\longrightarrow} A_0(r), \quad f(r) \underset{r\to 0}{\longrightarrow} 1, \quad f(r_h) = 0,$$

$$T = \frac{|\dot{f}(r_h)|}{4\pi}, \qquad S = (4\pi M_p^3 N_c^2 V_3)\ e^{3A(r_h)}$$

Deconfinement Transition

The gravity dual indeed displays a rich phase structure with a deconfininement phase transition:

- a minimum BH temperature T_{min};
- Two BH branches (big and small) for $T > T_{min}$;
- Big BHs dominates above the critical temperature $T_c > T_{min}$.



Thermodynamics of the deconfined phase

Thermodynamic observables are computed by geometric quantities:

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At high T we recover conformal behavior:

$$\frac{\epsilon}{T^4} \simeq \frac{3}{4}\frac{s}{T^3} \simeq 3\frac{p}{T^4} \rightarrow 3\pi^4 (M_p \ell)^3 N_c^2$$

Requiring a free gas with N_c^2 d.o.f. fixes M_p

$$(M_p \ell) = 1/(45\pi^2)^{1/3}$$

An Explicit Model

Gursoy, Kiritsis, Mazzanti, FN 0903.2859

Phenomenological potential, fixed small-λ and large-λ behavior.

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Other model parameters:

- M_p fixed by high-T regime
- ℓ_s/ℓ fixed at zero-T by matching $\sigma_c \sim (440\ MeV)^2$.

Matching Pure YM Thermodynamics

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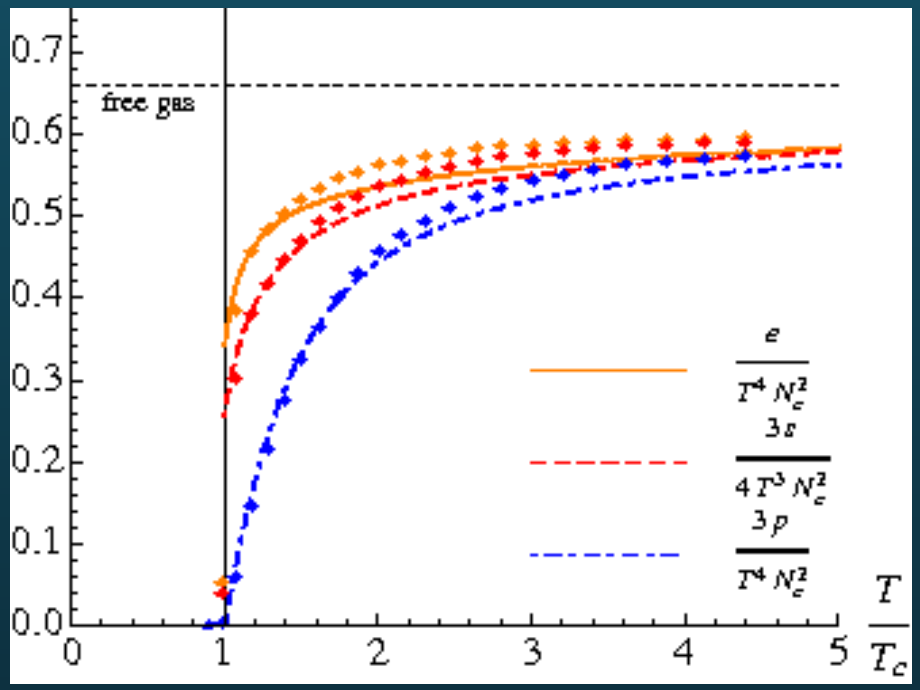
$$V_1 \simeq 14, \qquad V_3 \simeq 170$$



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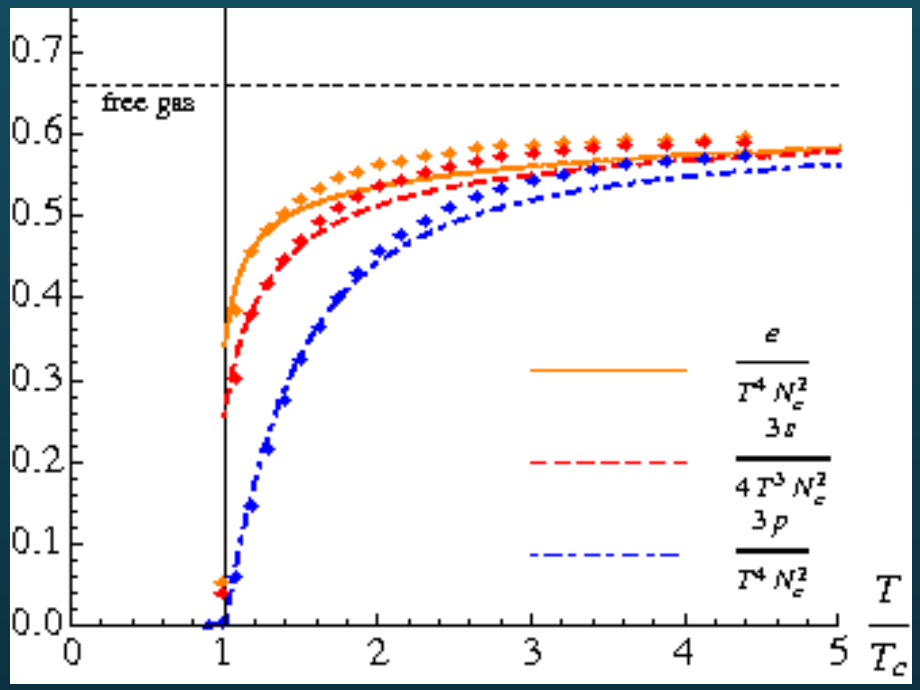

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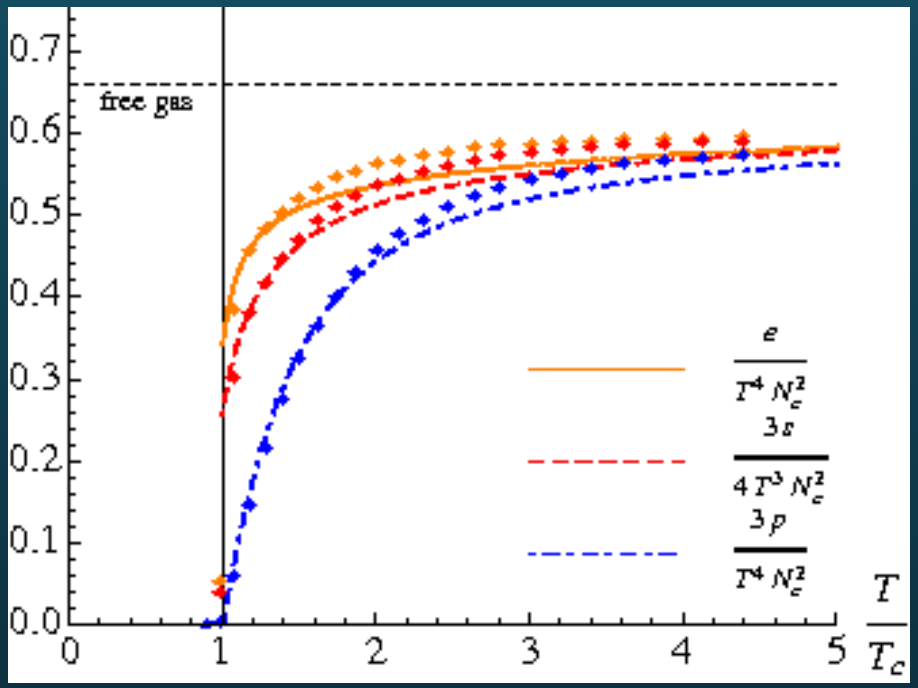

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Trace Anomaly and speed of sound

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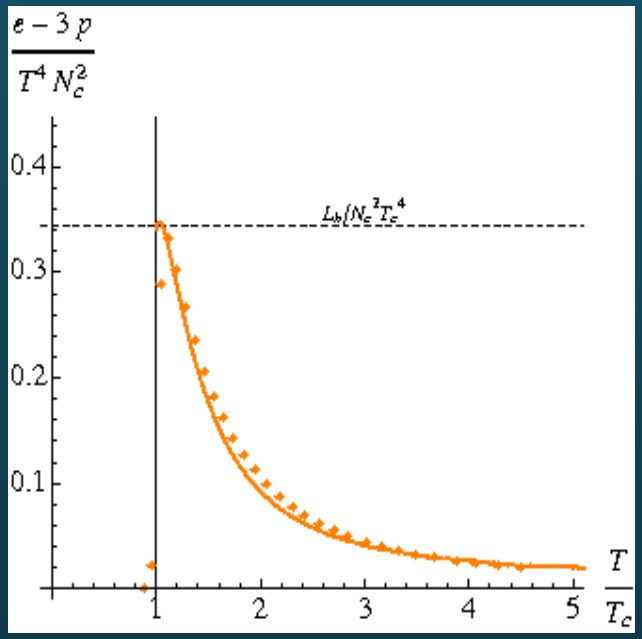

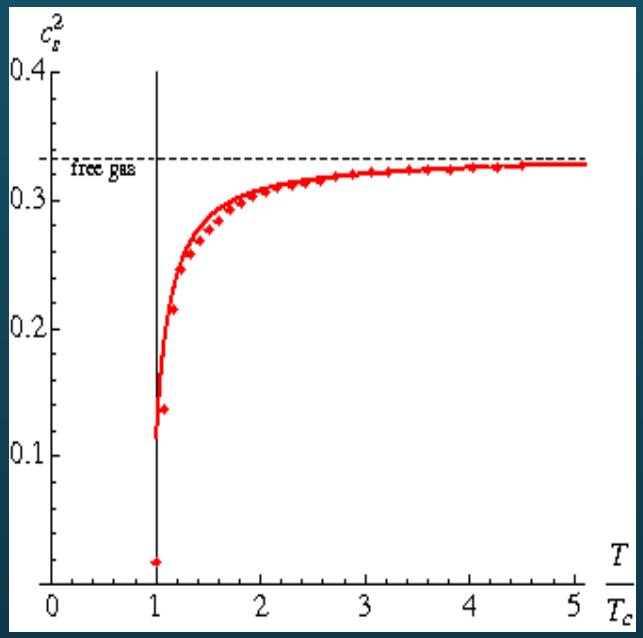

Hydrodynamic Transport

In the long-wavelength limit:

$$
\begin{aligned}
T^{\mu\nu} &= (\epsilon + p)\, u^\mu u^\nu + p\, g^{\mu\nu} \\
&\quad - P^{\mu i} P^{\nu j} \left[\eta \left(\partial_i u_j + \partial_j u_i - \frac{2}{3} g_{ij} \partial \cdot u \right) + \zeta\, g_{ij}\, \partial \cdot u \right]
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η : Shear viscosity.

- RHIC data consistent with very small η/s, $\sim 0.08 - 0.2$
- Closest match: Strong coupling holographic computation in $\mathcal{N} = 4$ SYM: $\eta/s = (4\pi)^{-1} \approx 0.08$
- IHQCD setup: same result as in $\mathcal{N} = 4$ SYM (universal in 2-derivative models)

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ζ : Bulk viscosity

- Vanishes in a conformal fluid (like $\mathcal{N} = 4$ plasma)
- How signigicant is ζ close to T_c?
- Computed holographically by the scalar fluctuation of the metric-dilaton system (Gubser *et al*, 08)

Bulk Viscosity

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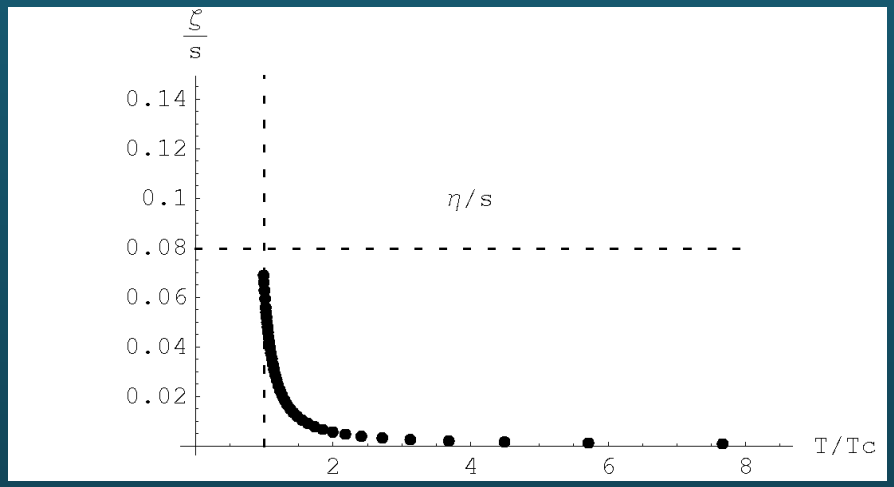

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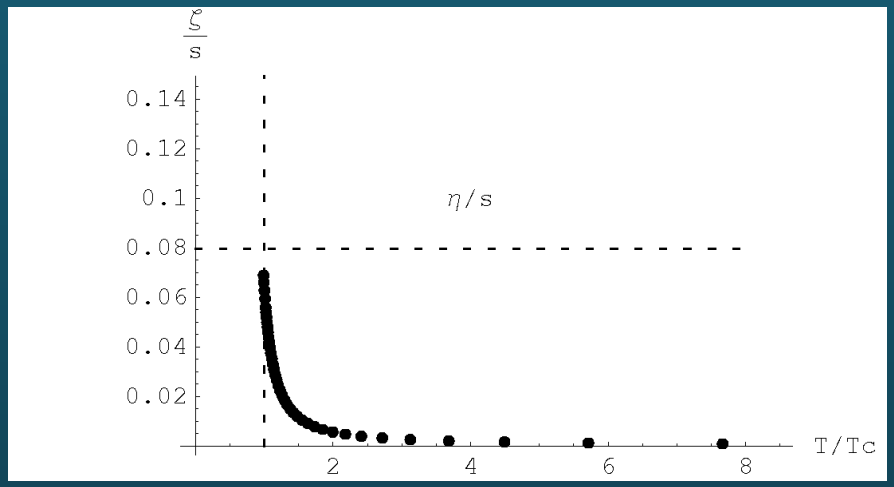
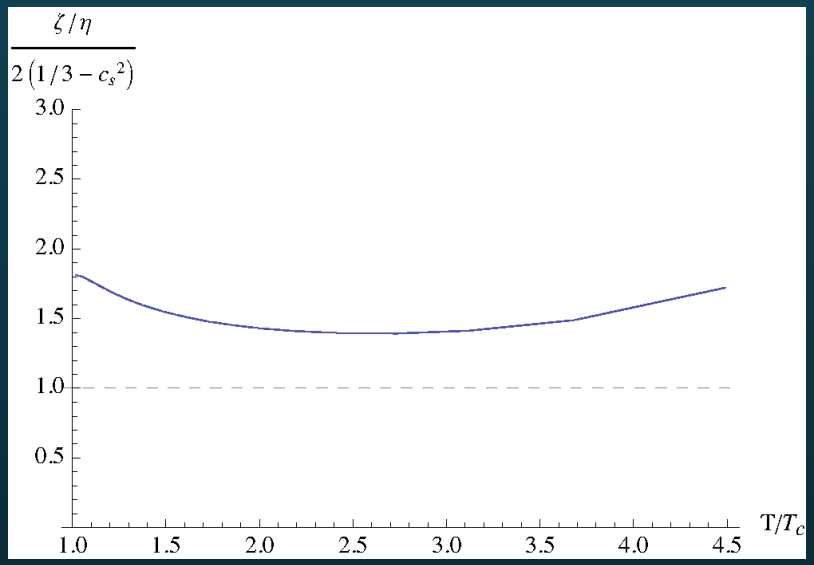
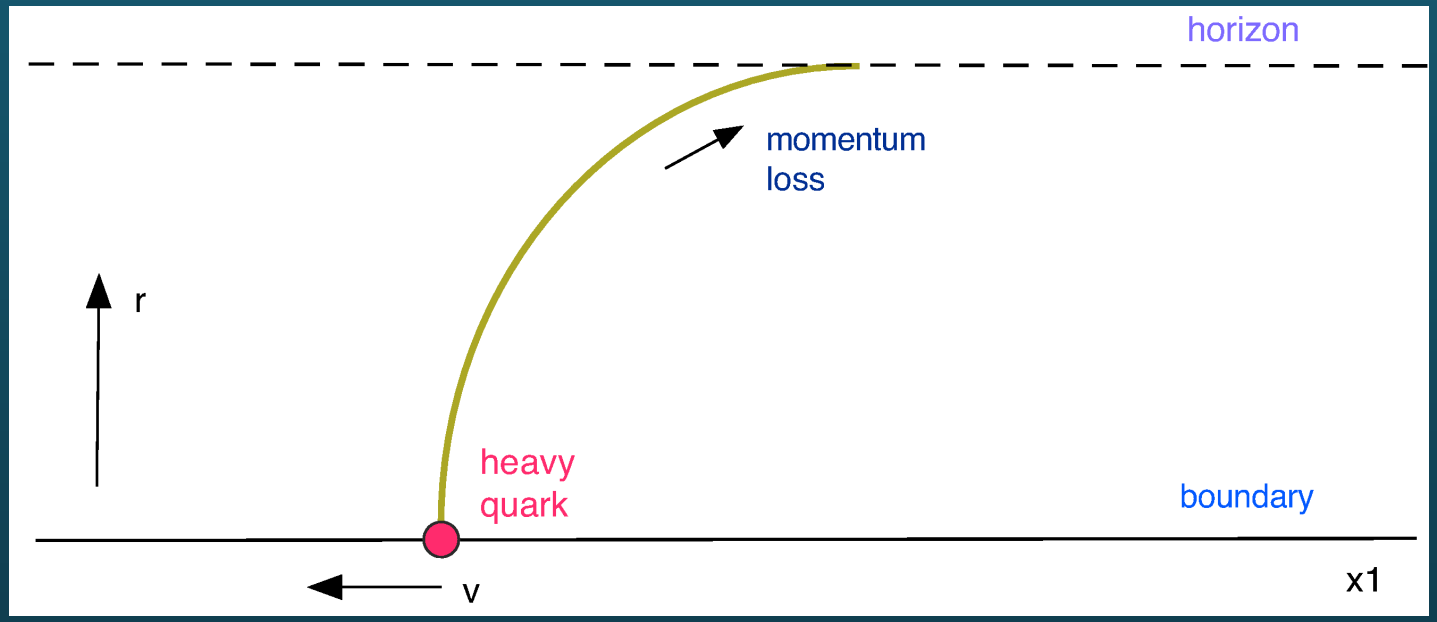


- Buchel's bound: $\zeta/\eta \geq 2(1/3 - c_s^2)$



Drag Force on a Heavy Quark

Trailing String picture:



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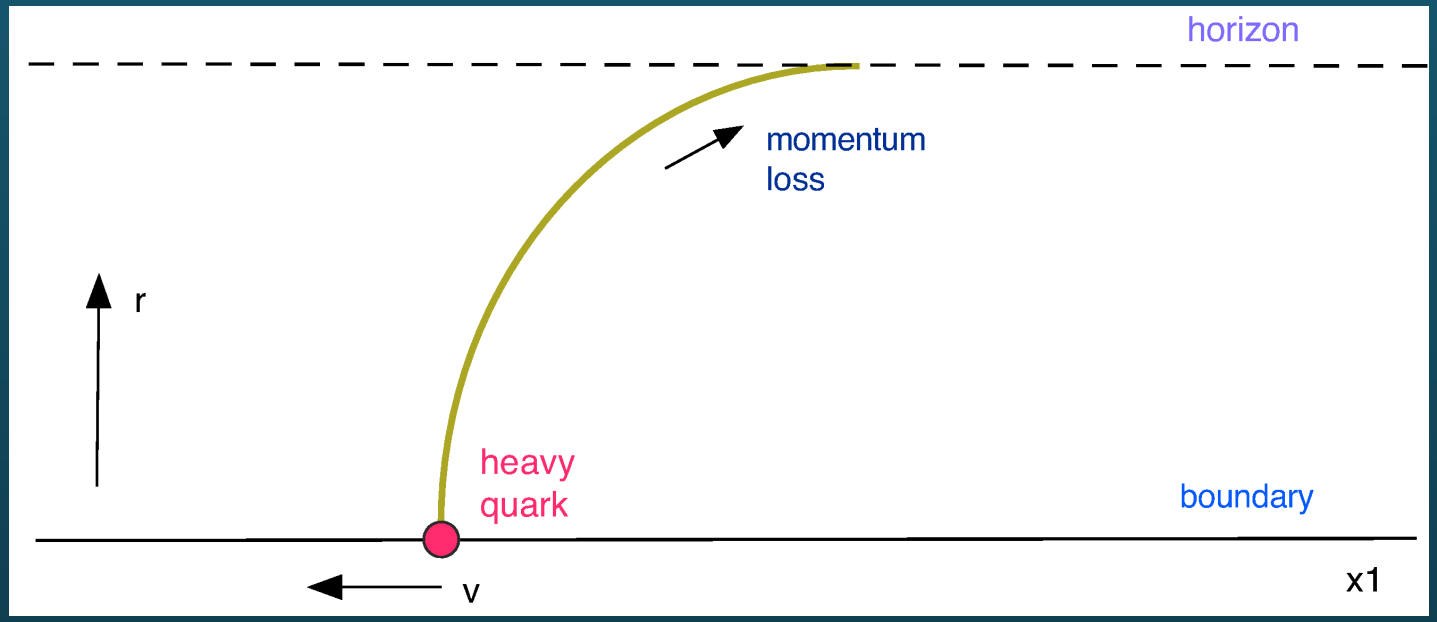

Drag force:

$$F = \frac{1}{v}\frac{dE}{dt} = -\frac{1}{2\pi\ell_s^2}\, v\, e^{2A(r_s)}\lambda(r_s)^{\frac{4}{3}}, \qquad f(r_s) = v^2$$

Drag Force on a Heavy Quark

Define diffusion time:

$$F = -p/\tau$$

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- $\tau(p) \approx const.$ for $p \gg m_q$.
- We obtain $\tau_{charm} \simeq 4.5 fm/c$ for $p \approx 10\, GeV$, consistent with RHIC analysis.

To Summarize...

Positive features:

- Strict relationships between confinement, mass gap and phase transitions.
- Smooth connection to the correct UV of the theory
- An explicit model that can reproduce at the quantitative level
  - glueball spectrum
  - thermodynamics
  - several transport properties

  of large-N pure Yang-Mills theory

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  of large-N pure Yang-Mills theory

Some drawbacks:

- No explicit construction of gauge theory as decoupling limit.
- No derivation of the gravity side from a fundamental theory
- The UV needs some rethinking (hints that the AdS asymptotics must be in the string frame)

Perspectives

Toward QCD

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.... stay tuned.

Spectrum at $T = 0$

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Numerical computation in the explicit HQCD model shows good agreement with lattice data

	HQCD	Lat. $N_c = 3$ (Chen *et al.*)	Lat. $N_c = \infty$ (Lucini, Teper)
$\frac{m_{0^{*++}}}{m_{0^{++}}}$	1.61	1.56(11)	1.90(17)
$\frac{m_{2^{++}}}{m_{0^{++}}}$	1.36	1.40(4)	1.46(11)
$\frac{T_c}{m_{0^{++}}}$	0.167	-	0.177

Vacuum Solution - UV

Non-constant $V(\Phi)$: solutions have running coupling.

$$ds^2 = e^{2A(r)}\left(dr^2 + \eta_{\mu\nu}dx^\mu dx^\nu\right), \quad \lambda(r) = e^{\Phi(r)}, \quad (E_{4D} \sim e^{A(r)})$$

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UV: $\quad e^A \to \infty,\ \lambda \to 0; \quad r \to 0$

$$V(\lambda) \sim \frac{12}{\ell^2}\left(1 + v_0\lambda + v_1\lambda^2 \ldots\right)$$

$$e^{A(r)} = \frac{\ell}{r}\Big(1 + \ldots\Big), \qquad \lambda \sim -\frac{1}{b_0 \log r\Lambda} + \ldots, \qquad b_0 = \frac{9}{8}v_0$$

$$\beta(\lambda) \equiv \frac{d\lambda}{d\log E} = -b_0\lambda^2 + \ldots$$

$\ell = \sqrt{V(0)/12}$ is the asymptotic AdS radius; Λ is an integration constant: the dynamically generated strong coupling scale.

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$$\Downarrow$$

- Confinement (Wilson Loop area law)
- Spectrum with mass gap and linear glueball trajectories $m_n^2 \sim n$