Two topics in large N: walls and strings

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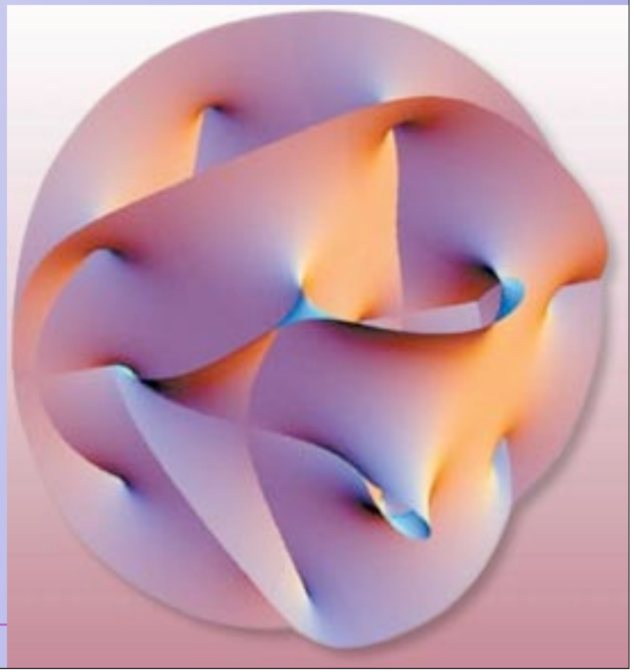
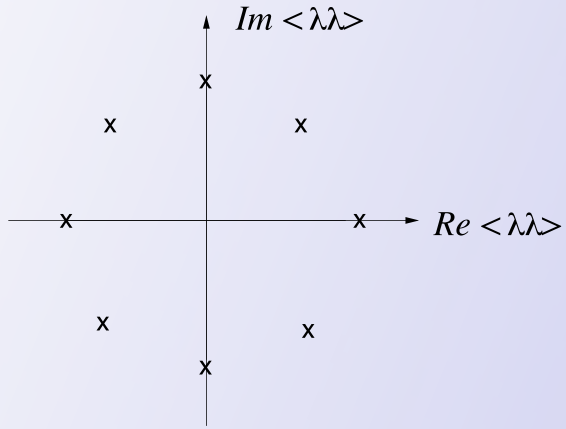


Quantum fusion of well-separfated domain walls in super-Yang-Mills

$$\mathcal{L} = -\frac{1}{4g^2} G^a_{\mu\nu} G^{\mu\nu\, a} + \frac{i}{g^2} \bar{\lambda}^a_{\dot\alpha} D^{\dot\alpha\beta} \lambda^a_\beta$$



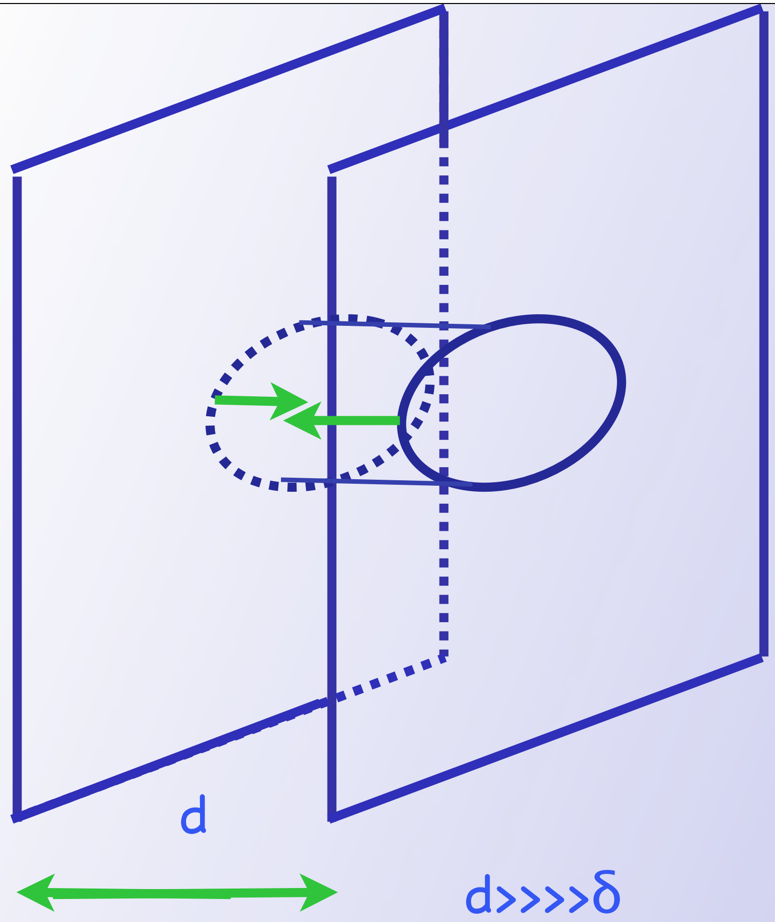
N vacua for SU(N)

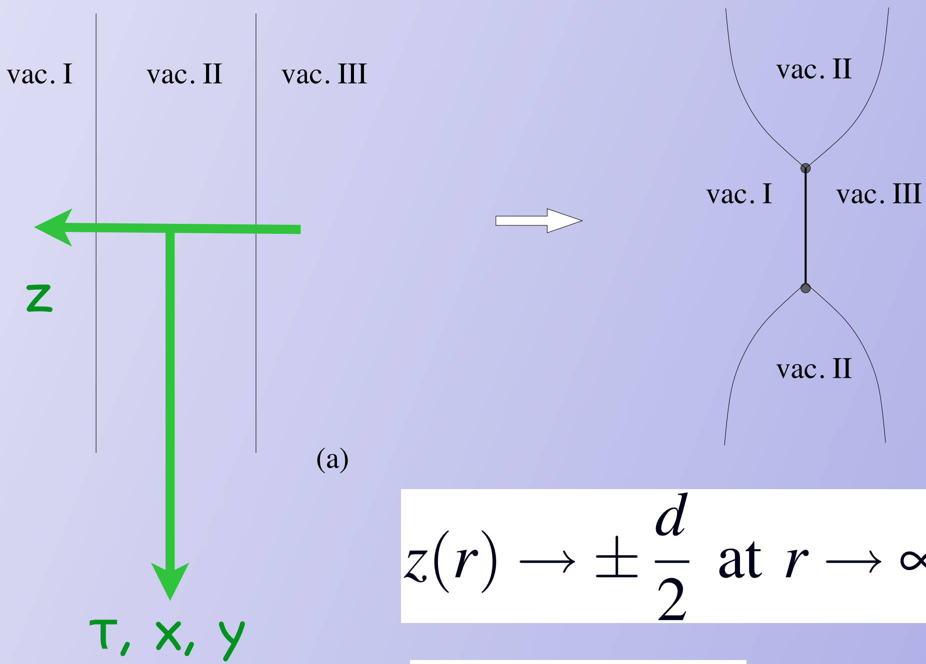
✷ N vacua labeled by $\langle\lambda\lambda\rangle = -6N\Lambda^3 \exp(2\pi i k/N)$

$$T = \frac{N}{8\pi^2} \left| \langle Tr\lambda^2 \rangle_{vac\, f} - \langle Tr\lambda^2 \rangle_{vac\, i} \right|$$

$$T_k = N^2 \Lambda^3 \sin\left(\frac{\pi k}{N}\right)$$

$$\boxed{2T_1 - T_2 = \frac{\pi^3}{N}\Lambda^3, \qquad T_1 = \pi N \Lambda^3}$$



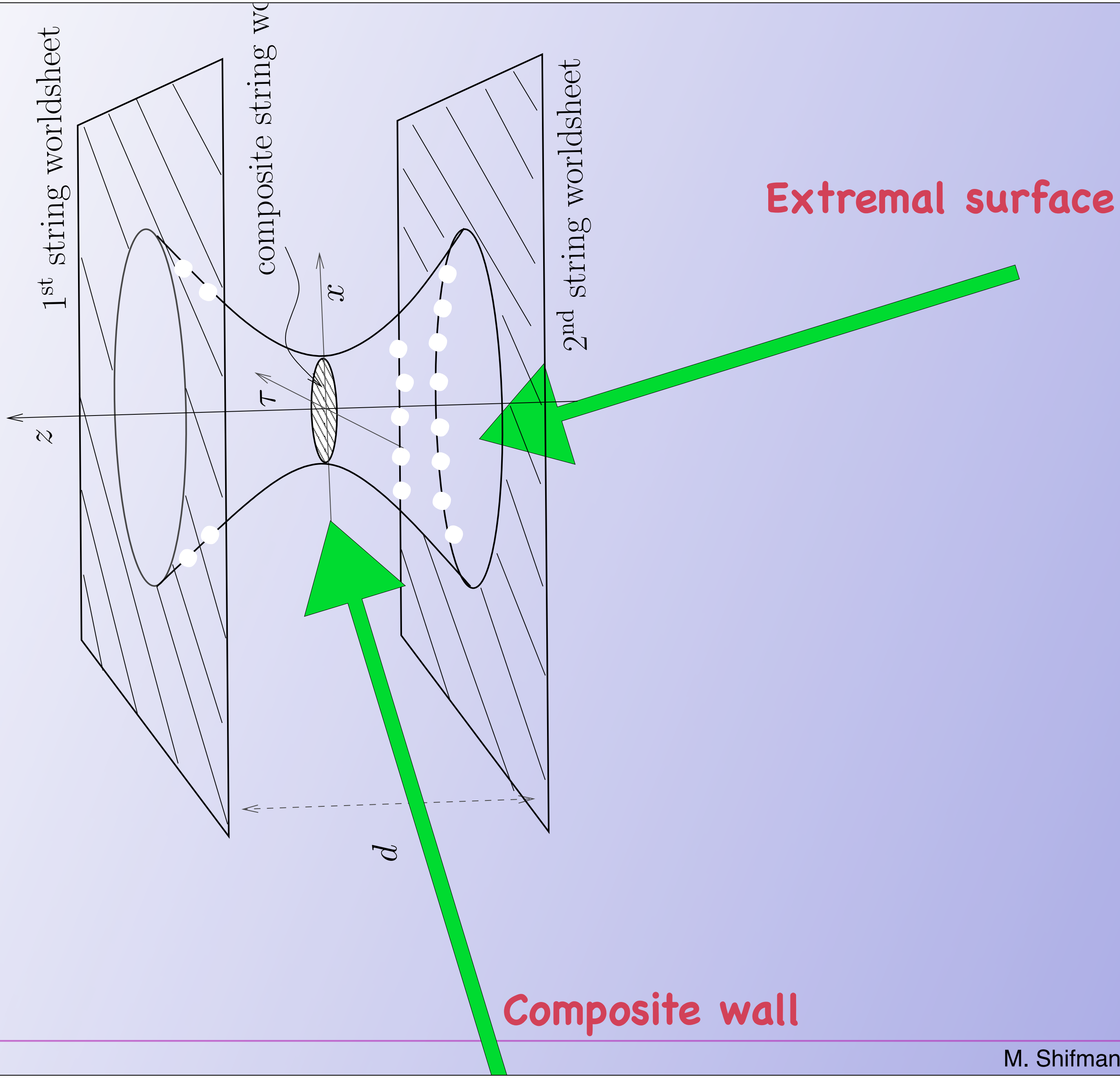


$$z(r) \longrightarrow \pm \frac{d}{2} \text{ at } r \longrightarrow \infty$$

$$\Delta z = 0$$

$$z = \frac{d}{2}\left(1 - \frac{r_*}{r}\right), \qquad r > r_*$$

$z=0, \quad r<r_*$


1st string worldsheet
composite string wo
2nd string worldsheet
x
z
τ
d
Extremal surface
Composite wall

$$S=(T_2-2T_1)\,\frac{4\pi}{3}\,r_*^3+2T_1\int_{r_*}^{\infty}4\pi r^2 dr\,\frac{z'^2}{2}$$

$$=\quad -(2T_1-T_2)\,\frac{4\pi}{3}\,r_*^3+T_1\,\pi\,r_*\,d^2$$

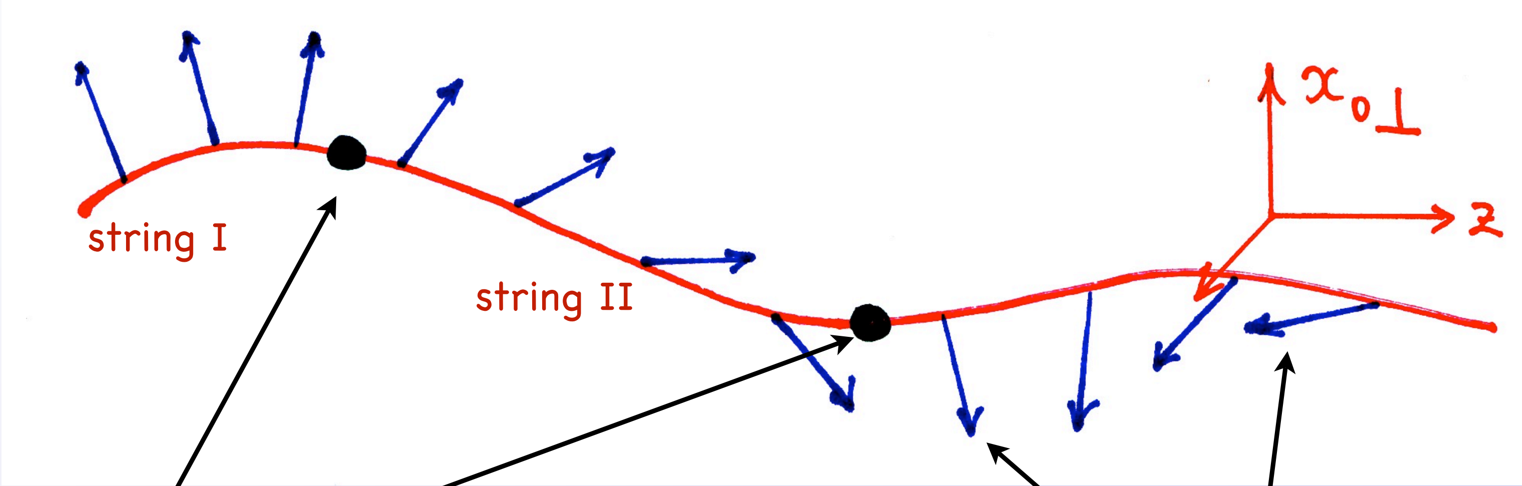
$$\boxed{r_*=\frac{d}{2}\sqrt{\frac{T_1}{2T_1-T_2}}}\quad\Longrightarrow\quad\boxed{S_B=\frac{\pi}{3}\,T_1 d^3\sqrt{\frac{T_1}{2T_1-T_2}}}$$

$$\Gamma=C\,\exp(-S_B)$$

What we find for super-Yang-Mills?

$$\Gamma \sim \exp\left[-\frac{\pi}{3}N^2(\Lambda d)^3\right].$$

- In many theories one can switch on magnetic fluxes inside the brane (electric fluxes on the brane)
- If the flux is larger than critical, elementary branes are stabilized (work in progress)



kinks (confined monopoles)

orientation of chromomagnetic flux
in group space fluctuates and
averages to zero!

$\boxed{\text{Basic theory:}}$

✽ N=2 U(N) gauge group, N flavors

gluons + 2 gluinos + adjoint scalars

+ U(1) Fayet-Iliopoulos term ξ

$$S = \int d^4x \left\{ \frac{1}{4g^2} F^2_{\mu\nu} + \frac{1}{g^2} |\partial_\mu a|^2 + \bar{\nabla}_\mu \bar{q}_A \nabla_\mu q^A + \bar{\nabla}_\mu \tilde{q}_A \nabla_\mu \bar{\tilde{q}}^A \right. \quad + \ \ldots$$

$$\left. + \frac{g^2}{8} \left(|q^A|^2 - |\tilde{q}_A|^2 - \xi \right)^2 + \frac{g^2}{2} \left| \tilde{q}_A q^A \right|^2 + \frac{1}{2} \left(|q^A|^2 + |\tilde{q}^A|^2 \right) \left| a + \sqrt{2} m_A \right|^2 \right\},$$

$|\bar{q} T^a q|^2$

FI stabilization

- $\tilde{q}=0 \Rightarrow$ BPS
- Bulk theory is fully Higgsed
- Color+Flavor locking
- Common mass term eliminated by shifting a

If $\xi \neq 0$

$$\langle q^{kA} \rangle = \sqrt{\xi} \begin{pmatrix} 1 & \ldots & 0 \\ \ldots & \ldots & \ldots \\ 0 & \ldots & 1 \end{pmatrix}$$

$T = 2\pi\xi$

$$\langle \bar{\tilde{q}}^{kA} \rangle = 0$$

$$k = 1, ..., N, \qquad A = 1, ..., N_f,$$

$$\sqrt{\xi} \begin{pmatrix} e^{i\alpha} & 0 \\ 0 & 1 \end{pmatrix}$$

Non-Abelian string, $SU(N)/SU(N-1)\times U(1)=CP(N-1)$

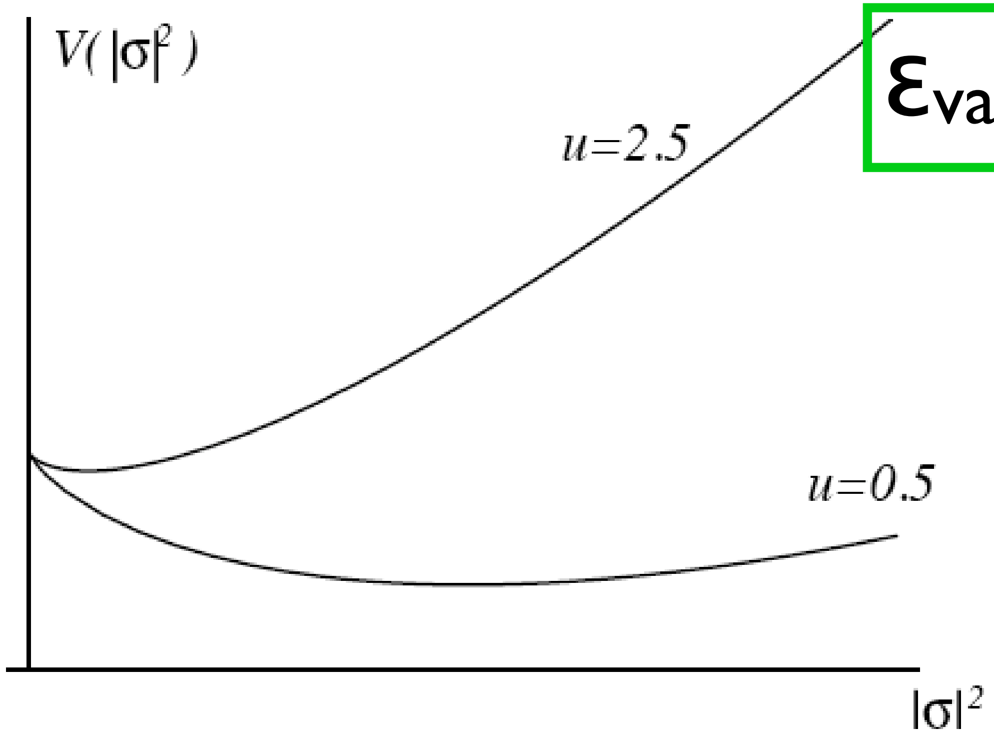
Now, add $W=\mu a^2$, breaking N=2 down to N=1

$$
\begin{aligned}
L_{\text{heterotic}} \quad &= \zeta_R^{\dagger}\, i\partial_L\, \zeta_R + \left[\gamma\, g_0^2\, \zeta_R\, G_{i\bar{j}}\, (i\, \partial_L \phi^{\dagger\, \bar{j}})\psi_R^i + \text{H.c.}\right] \\
&- g_0^4\, |\gamma|^2\, \left(\zeta_R^{\dagger}\, \zeta_R\right) \left(G_{i\bar{j}}\, \psi_L^{\dagger\, \bar{j}}\, \psi_L^i\right) \\
&+ G_{i\bar{j}}[\partial_\mu \phi^{\dagger\, \bar{j}}\, \partial_\mu \phi^i + i\bar{\psi}^{\bar{j}} \gamma^\mu D_\mu \psi^i] \\
&- \frac{g_0^2}{2} \left(G_{i\bar{j}} \psi_R^{\dagger\, \bar{j}}\, \psi_R^i\right) \left(G_{k\bar{m}} \psi_L^{\dagger\, \bar{m}}\, \psi_L^k\right) \\
&+ \frac{g_0^2}{2} \left(1 - 2 g_0^2 |\gamma|^2\right) \left(G_{i\bar{j}} \psi_R^{\dagger\, \bar{j}}\, \psi_L^i\right) \left(G_{k\bar{m}} \psi_L^{\dagger\, \bar{m}}\, \psi_R^k\right)
\end{aligned}
$$

$$\omega = \begin{cases} \text{const}\,\sqrt{r_0}\;\dfrac{g_2^2\mu}{m_W}\,, & \text{small}\;\;\mu \\[2ex] \text{const}\,\sqrt{r_0\,\ln\dfrac{g_2^2\mu}{m_W}}\,, & \text{large}\;\;\mu\,. \end{cases}$$

$$\boxed{u = \frac{8\pi}{N}\,|\omega|^2}$$

$$\boxed{2|\sigma|^2 = \Lambda^2\,e^{-u}\,, \qquad \sigma = \frac{1}{\sqrt{2}}\,\Lambda\,\exp\left(-\frac{u}{2}+\frac{2\pi\,i\,k}{N}\right), \qquad k = 0, ..., N-1}$$



$$\varepsilon_{\rm vac} = \frac{N}{4\pi}\Lambda^2\left(1 - e^{-u}\right)$$

u =∞ finish: N vacua fuse.

$\sigma \to 0$. Conformal limit ???

Conclusions:

Part 1.

- Two distant elementary wall can fuse through quantum tunneling. $\Gamma \sim e^{-N^2}$. Fluxes tend to stabilize;

Part 2.

- Heterotic N=(0,2) CP(N-1) model solved in 1/N
- SUSY broken , but 2D confinement persists