Flux tubes as strings

Mike Teper – Swansea '09

- some general remarks
- motivation : some theoretical and numerical results
- theory : Luscher-Weisz and Polchinski-Strominger
- lattice : spectrum of closed flux tubes in SU(N) gauge theories
  - fundamental flux in $D = 2 + 1$
  - k-strings in $D = 2 + 1$
  - fundamental flux in $D = 3 + 1$
- conclusions

Confining flux tubes in SU(N) gauge theories and their effective string theory description

– a long history –

- Veneziano amplitude
- 't Hooft large-N – genus diagram expansion
- Polyakov action
- Maldacena ... AdS/CFT/QCD ...

$\Longrightarrow$ at large N, flux tubes and perhaps the whole gauge theory can be described by a weakly-coupled string theory

But

string theories are theoretically problematic outside a critical dimension, e.g. $D = 26$ (bosonic) or $D = 10$ (SUSY)

Nonetheless

we *know* that effective string theories, for sufficiently long flux tubes must exist in lower dimensions,

e.g. for long Nielsen-Olesen flux tubes in a weakly-coupled Georgi-Glashow model

$\Longrightarrow$ effective bosonic string action for long flux tubes:

- M. Luscher, K. Symanzik, P. Weisz, Nucl. Phys. B173 (1980) 365; M. Luscher, Nucl. Phys. B180 (1981) 317.
- J. Polchinski, A. Strominger, Phys. Rev. Lett. 67 (1991) 1681.

$\Longrightarrow$ universal Luscher correction, e.g. potential between static sources

$$V(r) = \sigma r - \frac{\pi(D-2)}{24r} + O\left(\frac{1}{r^3}\right) \qquad r \gg \frac{1}{\sqrt{\sigma}}$$

and corresponding excitations.

analysis extended more recently to higher order terms :

- LSW next order : M. Luscher, P. Weisz, hep-th/0406205.
- PS next order : J. Drummond, hep-th/0411017; 0608109.

N. Hari Dass, P. Matlock, hep-th/0606265; 0608109; 0611215; arXiv:0709.1765

- LSW next next order : O. Aharony, E. Karzbrun, arXiv:0903.1927 ;

O. Aharony, Strings 09

$\Longrightarrow$ same as Nambu-Goto in flat space-time

Numerical calculations – an equally long history:

Euclidean $D = 3, 4$

- potential between static sources e.g. in $D = 3 + 1$

$$V(r) = -\frac{c_f \alpha_s(r)}{r} \qquad r \ll \frac{1}{\sqrt{\sigma}}$$

$$V(r) = \sigma r - \frac{\pi(D-2)}{24r} + O\left(\frac{1}{r^3}\right) \qquad r \gg \frac{1}{\sqrt{\sigma}}$$

and corresponding excitations.

- flux tubes wound around a spatial torus

$$E(l) = \sigma l - \frac{\pi(D-2)}{6l} + O\left(\frac{1}{l^3}\right) \qquad l \geq l_c = \frac{1}{T_c}$$

and corresponding excitations.

- ratios of Wilson loops vs Nambu-Goto

recent work :
e.g. Luscher, Weisz, Caselle, Gliozzi, Sommer, Necco, Kuti, Meyer, Bringoltz, Majumdar , Lucini, MT, Athenodorou ... and collaborators

older work – from early 80's :
e.g. Creutz, Copenhagen group, Michael, Schierholz, Bali, ... and collaborators

Here I will focus on the spectrum of flux tubes that are closed around a spatial torus of length l

— mainly D=2+1, but also D=3+1 —

- flux localised in 'tubes' $\forall l \geq l_c = 1/T_c$
- at $l = l_c$ there is a phase transition: first order for $N \geq 3$ in $D = 4$ and for $N \geq 4$ in $D = 3$
- is there a simple string description of the closed string spectrum for all possible lengths $l \geq l_c$?
- this is most plausible at $N \to \infty$ where complications such as mixing, e.g string $\to$ string + glueball, go away

Note: the static potential $V(r)$ describes the transition in r between UV (Coulomb potential) and IF (flux tubes) physics; potentially of great interest as $N \to \infty$.

The results shown here are from recent work in collaboration with Andreas Athenodorou and Barak Bringoltz :

◦ Spectrum of fundamental and $k > 1$ strings in $D = 2 + 1$ SU(N) gauge theories:
hep-th/0611286 ; arXiv:0802.1490 ; arXiv:0812.0334 ; arXiv:0709.0693 ; and in progress.

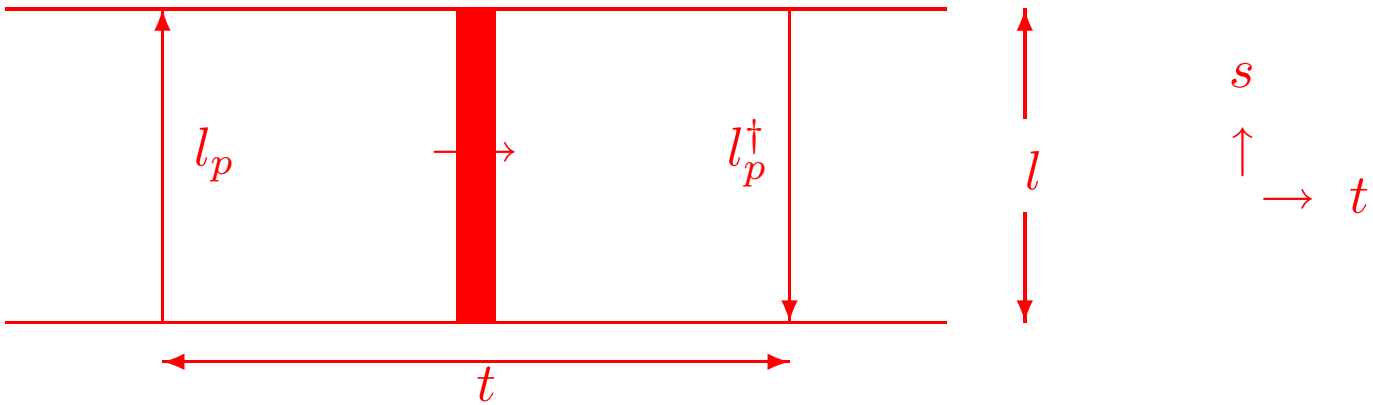
◦ Spectrum of fundamental and $k > 1$ strings in $D = 3 + 1$ SU(N) gauge theories:
in progress

[also see earlier work with Lucini, Meyer, Wenger]

So : calculate the mass of a confining flux tube winding around a spatial torus of length l, using correlators of Polyakov loops:

$$\langle l_p^\dagger(t) l_p(0)\rangle \overset{t\to\infty}{\propto} \exp\{-m_p(l)t\}$$

in pictures



where we expect,

$$m_p(l) \overset{l\to\infty}{=} \sigma l - c\frac{\pi(D-2)}{6l} + O\left(\frac{1}{l^3}\right)$$

with $c = 1$ if the only massless modes are from transverse translations

origin of the various terms:

○ linear confinement

$\Rightarrow \sigma l$

○ spontaneous breaking of (transverse) translation invariance

$\Rightarrow -\frac{\pi(D-2)}{6l}$

from the sum of zero-point energies of the massless (Goldstone) transverse oscillations – the Luscher correction

○ any other massless modes on the flux tube

$\Rightarrow$ further $O(1/l)$ contributions

$\Longrightarrow$

determine the coefficient of the $O(1/l)$ Luscher correction in order to determine whether a long flux tube is described by an "effective bosonic string theory"

simplest example of such a string theory is Nambu-Goto in flat space-time; this is a free string theory and it is 'sick' outside $D = 26$

but

the diseases are invisible if we focus on the sector of states built on a single long string

e.g. P. Olesen, PLB160 (1985) 144; J. Polchinski, A. Strominger, PRL67 (1991) 1681.

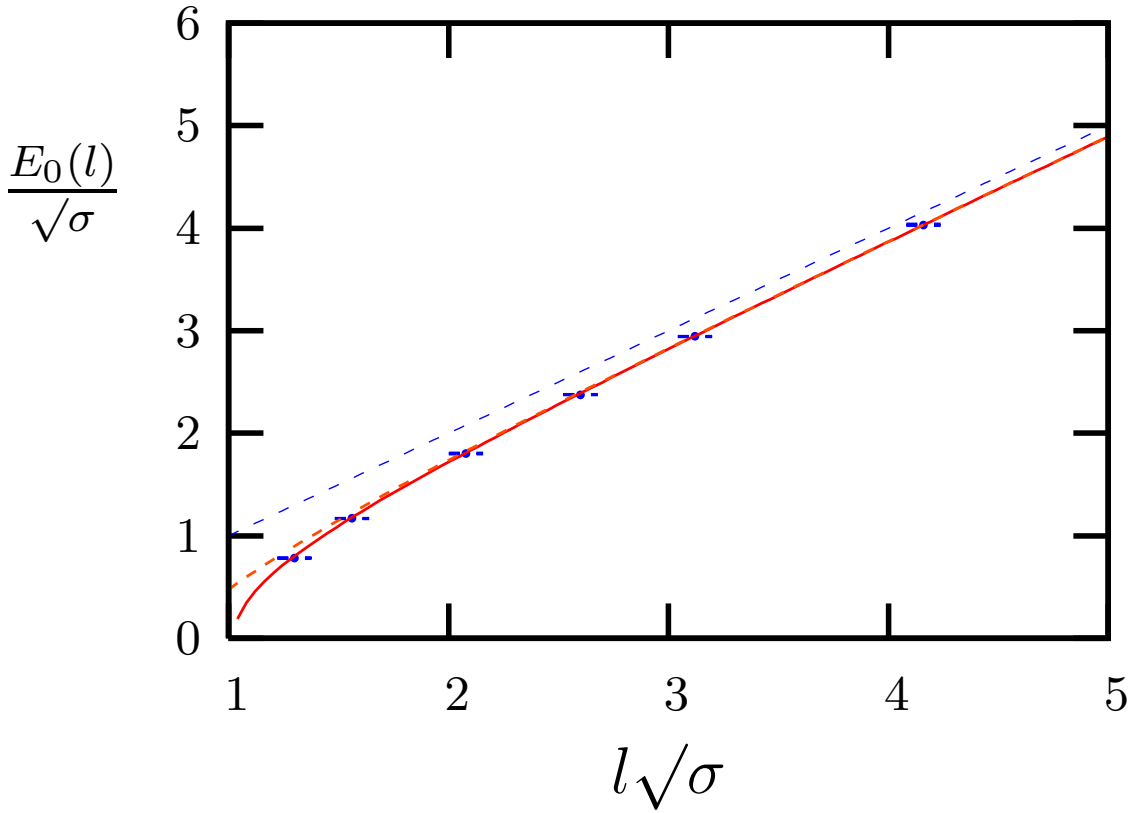
$\Longrightarrow$

the gound state energy: J. Arvis, PLB 127 (1983) 127

$$\begin{aligned} E_0(l) &= \sigma l \left(1 - \frac{\pi(D-2)}{3\sigma l^2}\right)^{\frac{1}{2}} \\ &= \sigma l - \frac{\pi(D-2)}{6l} + O\left(\frac{1}{l^3}\right) \quad : \quad l \geq \sqrt{\frac{\pi(D-2)}{3\sigma}} \end{aligned}$$

So : how well does all this describe the actual $E(l)$?

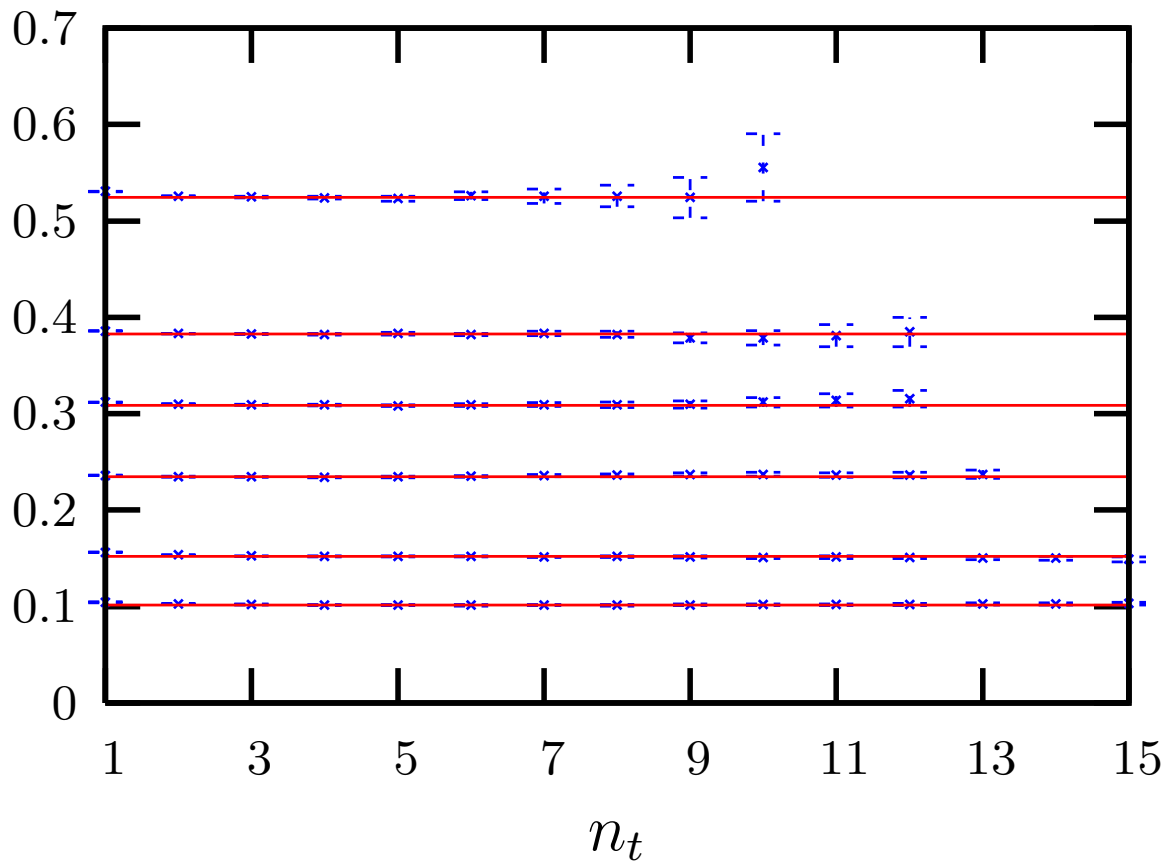
D=2+1 ; SU(5) $\quad a\sqrt{\sigma} \simeq 0.130 \quad ; \quad l_c\sqrt{\sigma} \simeq 1.07$



...Luscher: $E_0(l) = \sigma l - \frac{\pi}{6l}$; -Nambu-Goto:- $E_0(l) = \sigma l \left(1 - \frac{\pi}{3\sigma l^2}\right)^{\frac{1}{2}}$

how good are the energies?

$aE_{eff}(t) = -\ln C(t)/C(t-1)$



effective string theory – which universality class?

parameterise

$$E_0(l) = \sigma l - c_{eff}(l)\frac{\pi(D-2)}{6l}$$

and fit this to pairs of values of $E_0(l)$ for neighbouring values of l. Then if

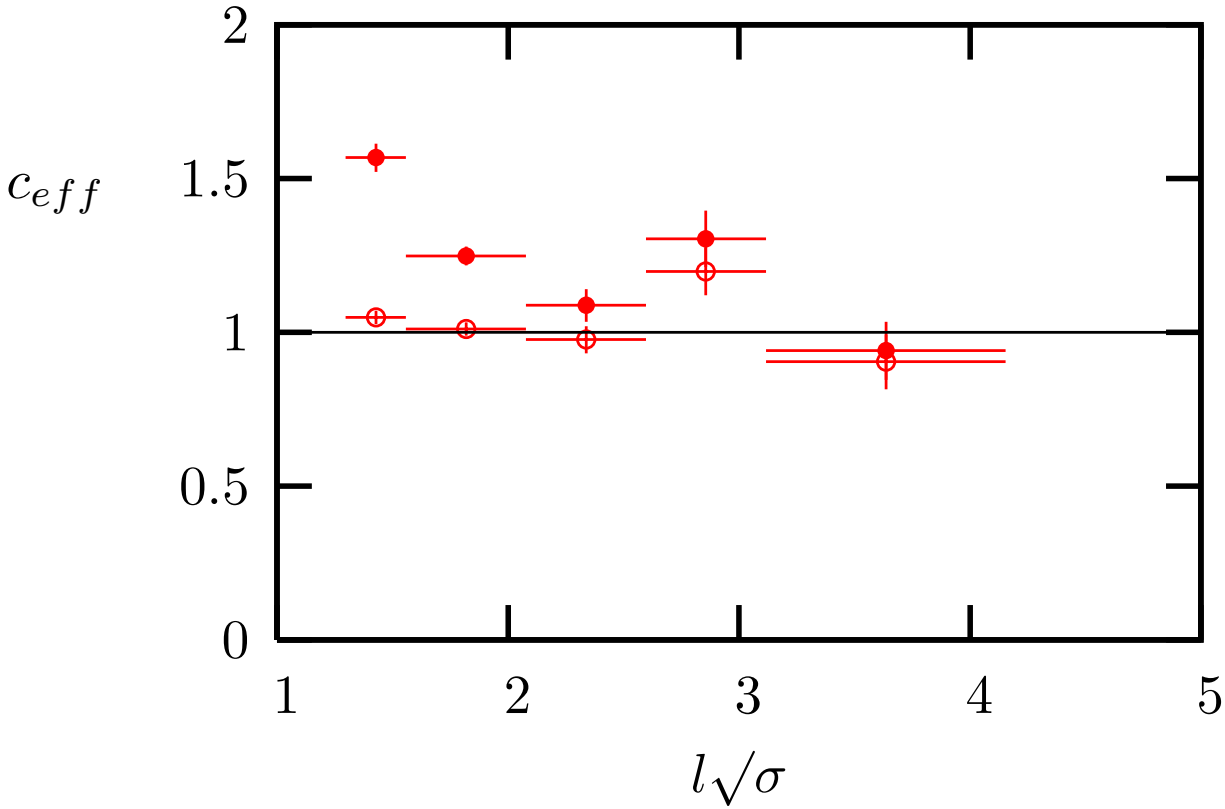
$$c_{eff}(l) \stackrel{l\to\infty}{\longrightarrow} c$$

the value of c corresponds to the central charge e.g. $c = 1$ for the bosonic string universality class.

and do the same for Nambu-Goto using:

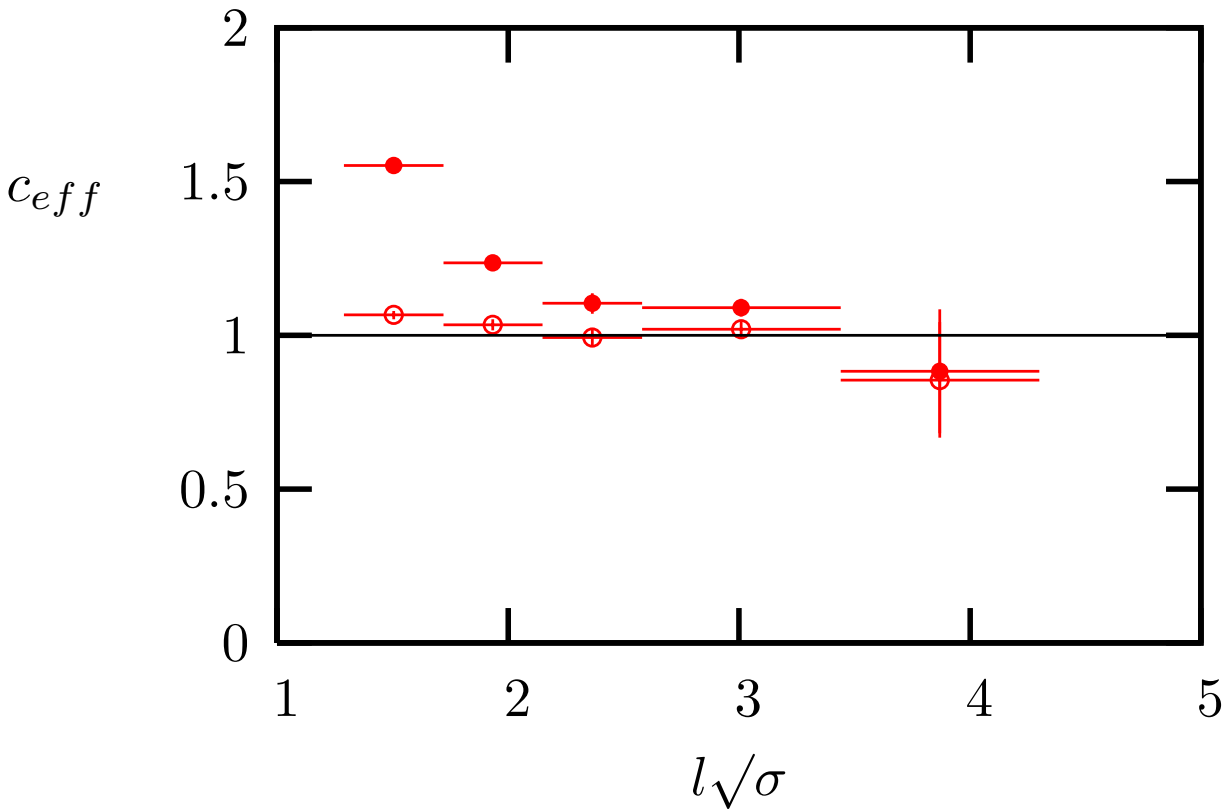
$$E_0(l) = \sigma l \left(1 - c_{eff}\frac{\pi}{3\sigma l^2}\right)^{\frac{1}{2}}$$

SU(5) : $l_c\sqrt{\sigma} \simeq 1.07$



c_{eff} : from Luscher ● , and from Nambu-Goto ○

similarly for SU(4) : $l_c\sqrt{\sigma} \simeq 1.08$



c_{eff} : from Luscher • , and from Nambu-Goto ○

$\Rightarrow$

• fitting the ground state energy of the flux tube with just the Luscher correction, we see good evidence that

$$c_{eff}(l) \stackrel{l\to\infty}{\to} 1$$

i.e. the only massless mode on the flux tube is the one associated with the spontaneous breaking of transverse translations

but

the significant deviation of $c_{eff}(l)$ from unity at smaller l, $\Rightarrow$ substantial higher order corrections to the Luscher term

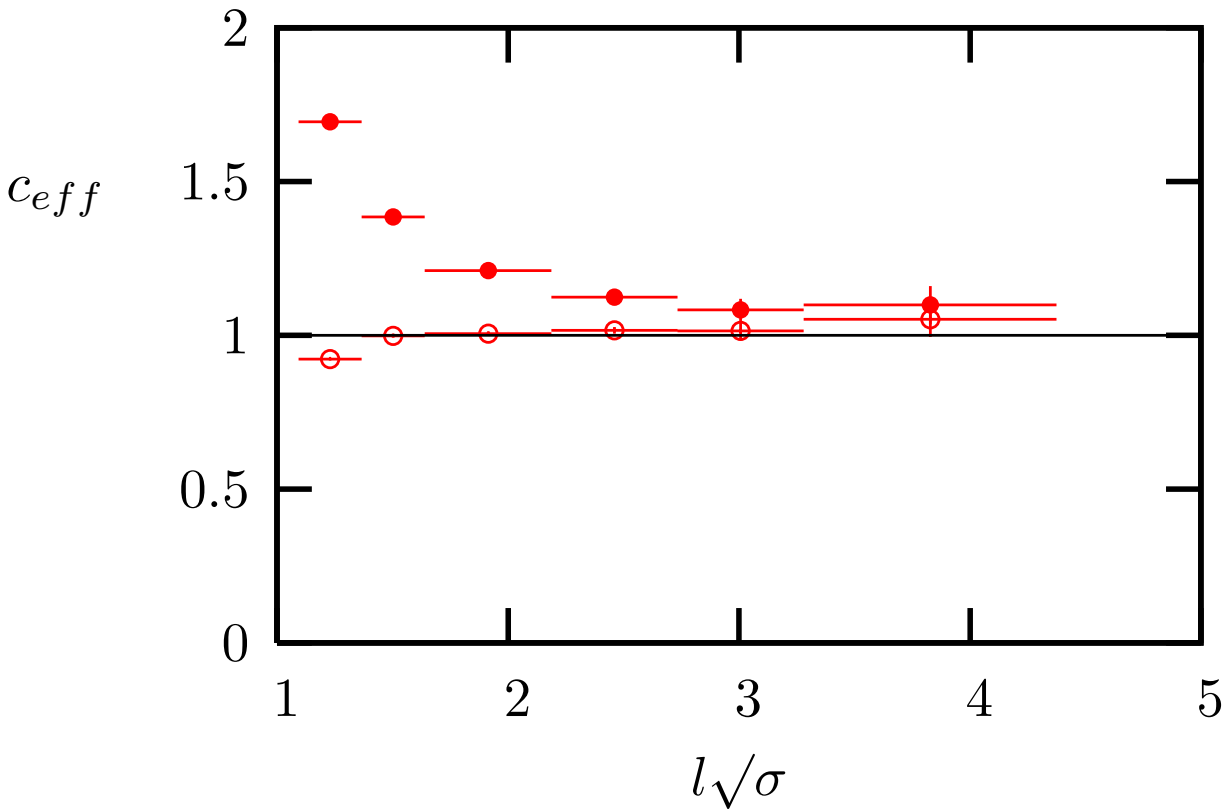
• by contrast we see that Nambu-Goto fits almost exactly all the way down to $l \sim l_c$, i.e.

$$c^{NG}_{eff}(l) \simeq 1 \quad \forall l$$

$\Rightarrow$ the confining flux tube behaves like a thin free string, even when it is a fat 'blob' that is hardly longer than it is wide

$\Rightarrow$ is this a large N effect?

small $N \longrightarrow$ SU(2) : $l_c\sqrt{\sigma} \simeq 0.95$



c_{eff} : from Luscher ● , and from Nambu-Goto ○

So it appears that the ground state flux tube energy, $E_0(l)$, is very close to that of a free bosonic string theory $\forall N$.

- Are there theoretical reasons to think that some of the corrections beyond the Luscher term are just like Nambu-Goto?

- What do the excited states look like?

(bosonic) strings in D=3,4?

e.g. J. Polchinski: hep-th/9210045

can long confining flux tubes in $D = 4$ SU(N) gauge theories be described by some string theory?

well

there are no consistent string theories in D=4 : we need D=26 for bosonic, and D=10 for SUSY strings

yes, but

while the properties of confining flux tubes are not that well known, because the physics is strongly coupled, there exist weakly coupled examples, such as Nielsen-Olesen vortices in the Abelian-Higgs model, that provide explicit examples of string like objects in D=4

so

effective string theories, for *long* strings, should certainly exist in $D = 4$

e.g.

a typical inconsistency in quantising a free bosonic string of length R in D=4 is a breakdown of Lorentz covariance: e.g. generators of rotations are anomalous

J. Arvis, Phys. Lett. 127B(1983)106

$$[L^i, L^j] = -L^{ij} + F(R)$$

but one sees that

$$F(R) \propto 1/R^2 \stackrel{R\to\infty}{\longrightarrow} 0$$

so that the inconsistencies disappear in any D for long enough strings

P. Olesen, Phys. Lett. 160B(1985)144

$\star$same for $D = 3$ $\star$

Nambu-Goto in flat space-time ; a free string theory

$$\int \mathcal{D}X e^{-\frac{i}{\sigma}\times \mathrm{Area}}$$

spectrum ($w = 1$):

$$E^2(l) = (\sigma\, l)^2 + 8\pi\sigma \left(\frac{N_L+N_R}{2} - \frac{D-2}{24}\right) + \left(\frac{2\pi q}{l}\right)^2 .$$

$2\pi q/l =$ total momentum along string;
$N_L, N_R =$ sum left and right oscillator ('phonon') energies;
state $= \prod_k a_k^{n_k} \prod_{k'} \tilde{a}_{'}^{n_{k'}}\, k|0\rangle$

$$N_L = \sum_{k>0} n_L(k)\, k, \quad N_R = \sum_{k'>0} n_R(k')\, k', \quad N_L - N_R = q$$

J. Arvis, Phys. Lett. 127B(1983)106

analysing effective string theories – a summary

• field theory approach (non-covariant 'gauge fixing' of the string theory) ; low-energy effective Lagrangian for the transverse displacement
M. Luscher, K. Symanzik, P. Weisz : Nucl. Phys. B173 (1980) 365; M. Luscher : Nucl. Phys. B180 (1981) 317

• covariant effective string approach; low-energy effective Lagrangian for a long string
J. Polchinski, A. Strominger : Phys. Rev. Lett. 67 (1991) 1681

In both approaches the starting point is to consider a long (open or closed) string of length l and to consider those corrections allowed by symmetry arguments (different in the two approaches) ordered in powers of $1/l$

lowest order ('Luscher correction') $\Rightarrow E_n = \sigma l + \frac{\pi}{l}\left(n - \frac{D-2}{6}\right)$
i.e. identical to Nambu-Goto to this order in $1/l$ for both $D = 2+1$ *and* $D = 3+1$

at the next order ...

• field theory approach

M. Luscher, P. Weisz : hep-th/0406205

• covariant effective string approach

J. Drummond : hep-th/0411017; N. Hari Dass, P. Matlock : hep-th/0612291

$\Rightarrow$

$$E_n = \sigma l + \frac{\pi}{l}\left(n - \frac{D-2}{6}\right) - \frac{\pi^2}{2\sigma l^3}\left(n - \frac{D-2}{6}\right)^2 + O(l^{-4})$$

i.e. still identical to Nambu-Goto to this order in $1/l$!

for $D = 2+1$ in both approaches *and* $D = 3+1$ in the Polchinski framework

and very recently ...

O. Aharony, E. Karzbrun : arXiv:0903.1927

- extends field theory approach to a further order in $1/\sigma l^2$

i.e. if we write

$$E_n(l) = \sigma l \left(1 + \frac{c_{1,n}}{\sigma l^2} + \frac{c_{2,n}}{(\sigma l^2)^2} + \frac{c_{3,n}}{(\sigma l^2)^3} + O\left(\frac{1}{l^8}\right)\right)$$

then in addition to older result that

- $c_{1,n} = \text{Nambu} - \text{Goto} \quad \forall D$
- $c_{2,n} = \text{Nambu} - \text{Goto} \quad D = 2 + 1$

they show that:

- $c_{2,n} = \text{Nambu} - \text{Goto} \quad \forall D$
- $c_{3,n} = \text{Nambu} - \text{Goto} \quad D = 2 + 1$
- $c_{3,0} = \text{Nambu} - \text{Goto} \quad \forall D$
- $\sum_n c_{3,n} = \text{Nambu} - \text{Goto} \quad \forall D$

and also something else that is very interesting ...

• calculate explicitly in a class of confining theories with a dual string theory description in a weakly curved background – *leading dependence* on curvature of background $\leftrightarrow$ to a certain order in the inverse 't Hooft coupling

$\Rightarrow$

○ all above contraints satisfied

and also

○ $c_{3,n} = \text{Nambu} - \text{Goto} \qquad \forall D$

$\Rightarrow$

there may indeed be extra constraints not captured in the effective field theory approach.

So Nambu-Goto interesting even if no-one expects it to be the whole story!

field-theoretic approach – a cartoon

• choosing a background confining string picks out a specific 'vacuum' that breaks spontaneously the translation symmetry transverse to the string

$\longrightarrow$

a Goldstone boson field h corresponding to transverse displacements

$\longrightarrow$

the effective action for h cannot depend on $\langle h \rangle$ (symmetry) and hence on h, so must depend only on $\partial_\alpha h$

$\longrightarrow$

the low energy physics will be given by an effective action that can be expanded inpowers of $\partial_\alpha h$ (just like a chiral Lagrangian)

$\longrightarrow$

the low-energy infrared physics should be described by this effective action.

• periodic string of length r with world sheet coords (static gauge)

$$z = (z_0, z_1), \qquad 0 \leq z_1 \leq r \; ; \; 0 \leq z_0 \leq T$$

and displacement vector $h(z)$ with effective action (D=2)

$$S = \sigma r T + S_0 + S_1 + \cdots , \qquad S_0 = \frac{1}{2} \int d^D z \, (\partial_a h \partial_a h)$$

one finds

$S_1 = \frac{1}{4} c_1 \int d^D z \, (\partial_a h \partial_a h) \, (\partial_b h \partial_b h) \, , \qquad c_1 = [l]^2$

$S_2 = \frac{1}{4} c_2 \int d^D z \, (\partial_a h \partial_b h) \, (\partial_a h \partial_b h) \, , \qquad c_2 = [l]^2$

since the couplings of S_i are increasing powers of length, they will contribute with increasing powers of $1/r$

- then obtain spectrum from:

$$Z = \int Dh e^{-S} = \sum_n e^{-E_n(r) T}$$

in powers of $1/r$

- to lowest order, i.e. Gaussian in ∂h, we have an exact expression

$$Z_0 = e^{-\sigma r T} \int Dh e^{-S_0} \sim e^{-\sigma r T} \eta(q)^{2-D}$$

where

$$\eta(q) \sim q^{\frac{1}{24}} \prod_{n=1}^{\infty} (1 - q^n) \quad ; \quad q = e^{-4\pi T/r}$$

is the Dedekind function

equating

$$Z_0 = \sum_n \omega_n e^{-E_n^0(r) T}$$

$\Rightarrow$

$$E_n^0(r) = \sigma r + \frac{4\pi}{r} \left\{ n - \frac{D-2}{24} \right\}$$

i.e. linear + Luscher correction with Nambu-Goto degeneracies

• at higher order, impose open-closed duality (Luscher), or closed-closed duality (Aharony), i.e. cylinder or torus world sheet, using

$$Z = \sum_n e^{-E_n(r)T} = \sum_{n'} e^{-E_{n'}(T)r},$$

and calculate Z in powers of S_i, and thus in powers of ∂h, and thus calculate $E_n(r)$ to a corresponding power of $1/r$, and $E_{n'}(T)$ to a corresponding power of $1/T$. The consistency of these results determines some of the unknown coefficients c_1, c_2, ...

Aside : accurate calculation of the energy spectrum as a function of l provides a *direct* way to determine the effective string action

Remarks:

○ the effective action arises from integrating all massive degrees of freedom; these will be characterised by a mass scale $M \sim O(\sqrt{\sigma})$ and so we only expect the effective action to be relevant for energies

$$\Delta E = E_n(r) - E_0(r) \ll \sqrt{\sigma}$$

and indeed only in this limit can we ignore the decay of excited states etc.

○ the massive modes should lead to extra states that do not look like phonon excitations of strings; crudely such a state might satisfy

$$\tilde{E}(r) - E_0(r) \simeq M \gg E_n(r) - E_0(r) \propto \frac{1}{r}$$

– at least or weak coupling : in strong coupling it is much less clear.

○ the formalism is easiest to justify when strings are weakly coupled – as at $N \to \infty$. It is only here – or in theories smoothly connected to such a theory – that one might hope that an effective string theory will completely describe a confining gauge theory or that a free string theory (NG in flat space-time) might have a role to play.

○ hence the stringy description of $k > 1$ flux tubes may be more complex than that of $k = 1$ fundamental flux tubes.

$\Longrightarrow$ What do we find? AA.BB,MT

- spectrum of closed loops of fundamental flux in $D = 2 + 1$ SU(N) gauge theories

- spectrum of closed loops of $k = 2$ flux in $D = 2 + 1$ SU(N) gauge theories

- spectrum of closed loops of fundamental flux in $D = 3 + 1$ SU(N) gauge theories

Strings in D=2+1 : quantum numbers

- length of string

- non-zero momentum $p = 2\pi q/l$ along string

$\longrightarrow$ requires a deformation along the string

$\longrightarrow$ so need non-trivial phonon excitation: $q = N_L - N_R$

- parity: $h(x) \to -h(x) \leftrightarrow a_k \to -a_k, \tilde{a}_k \to -\tilde{a}_k$

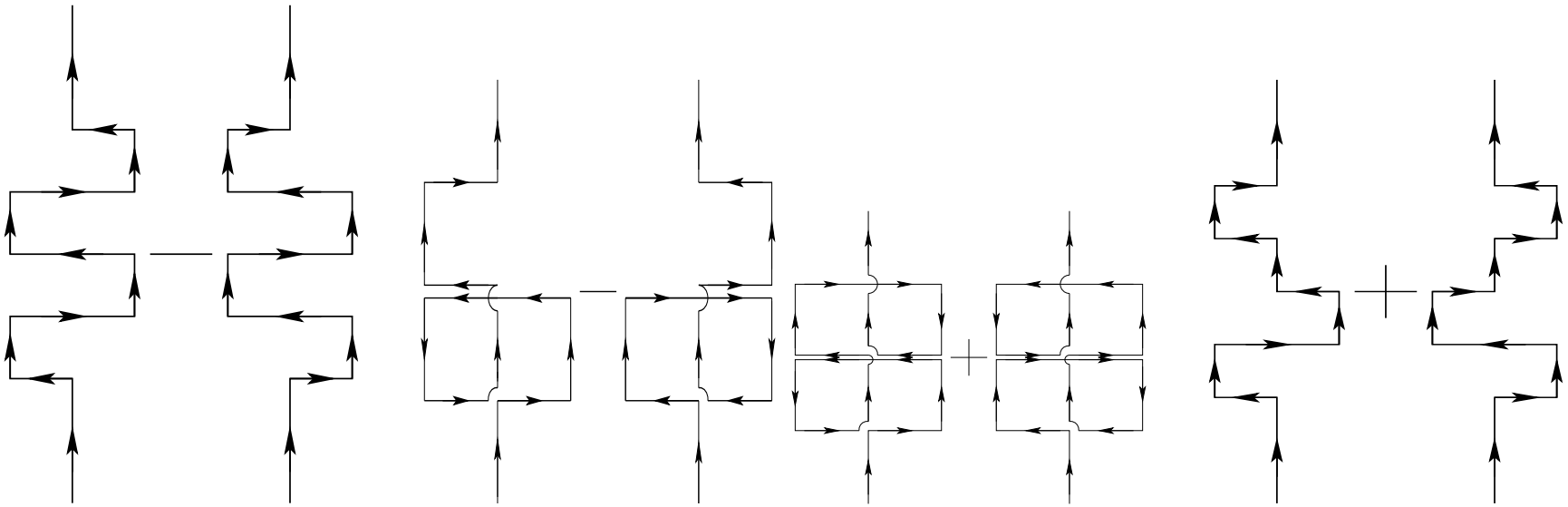
$\longrightarrow P = (-1)^{\sum_{k>0} n_L(k) + \sum_{k'>0} n_R(k')}$

- no rotations (2 space dimensions); transverse momentum uninteresting;

$C = \pm$ sectors degenerate, so charge conjugation uninteresting.

Excited States

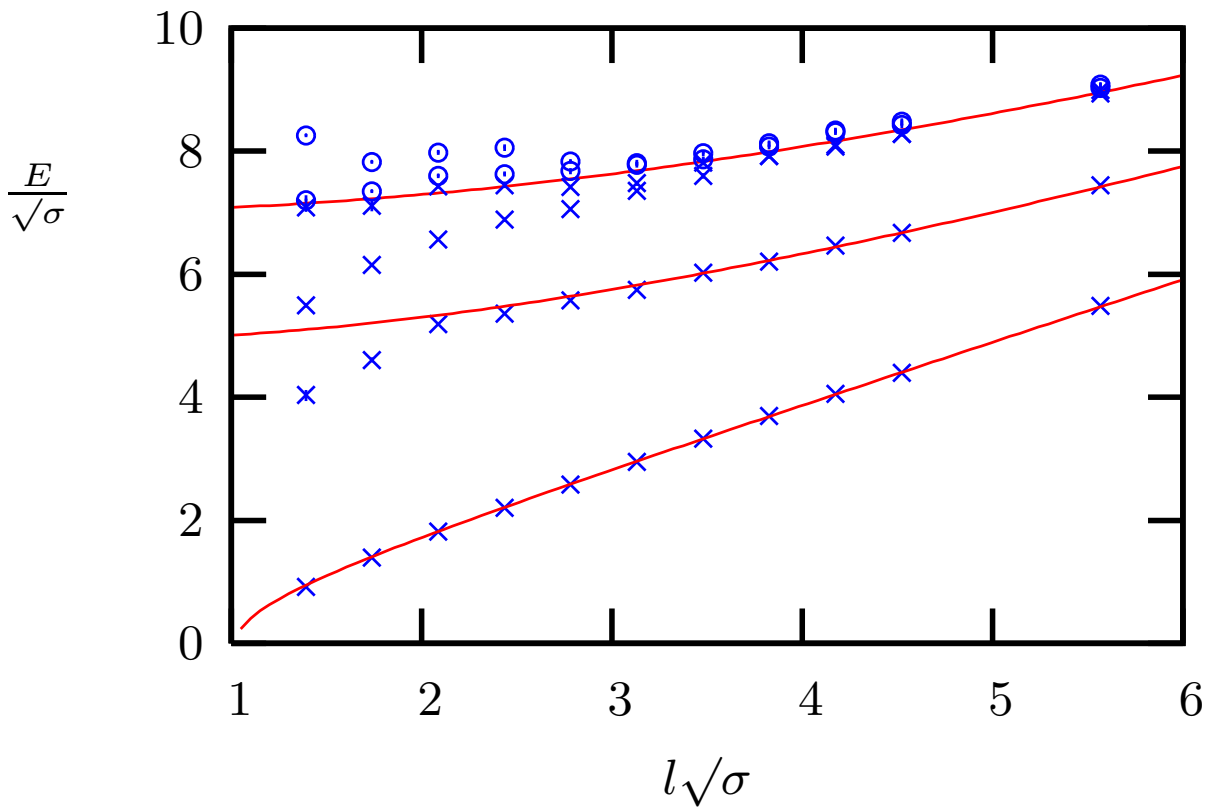
to have good overlaps onto excited string states, we need to include many more operators in our variational basis – in particular operators that 'look' excited and ones that have an intrinsic handedness so that we can construct $P = -$ as well as $P = +$, e.g.



typically we have 100-200 operators in our basis ...

SU(3) : $q = 0$ closed string spectrum

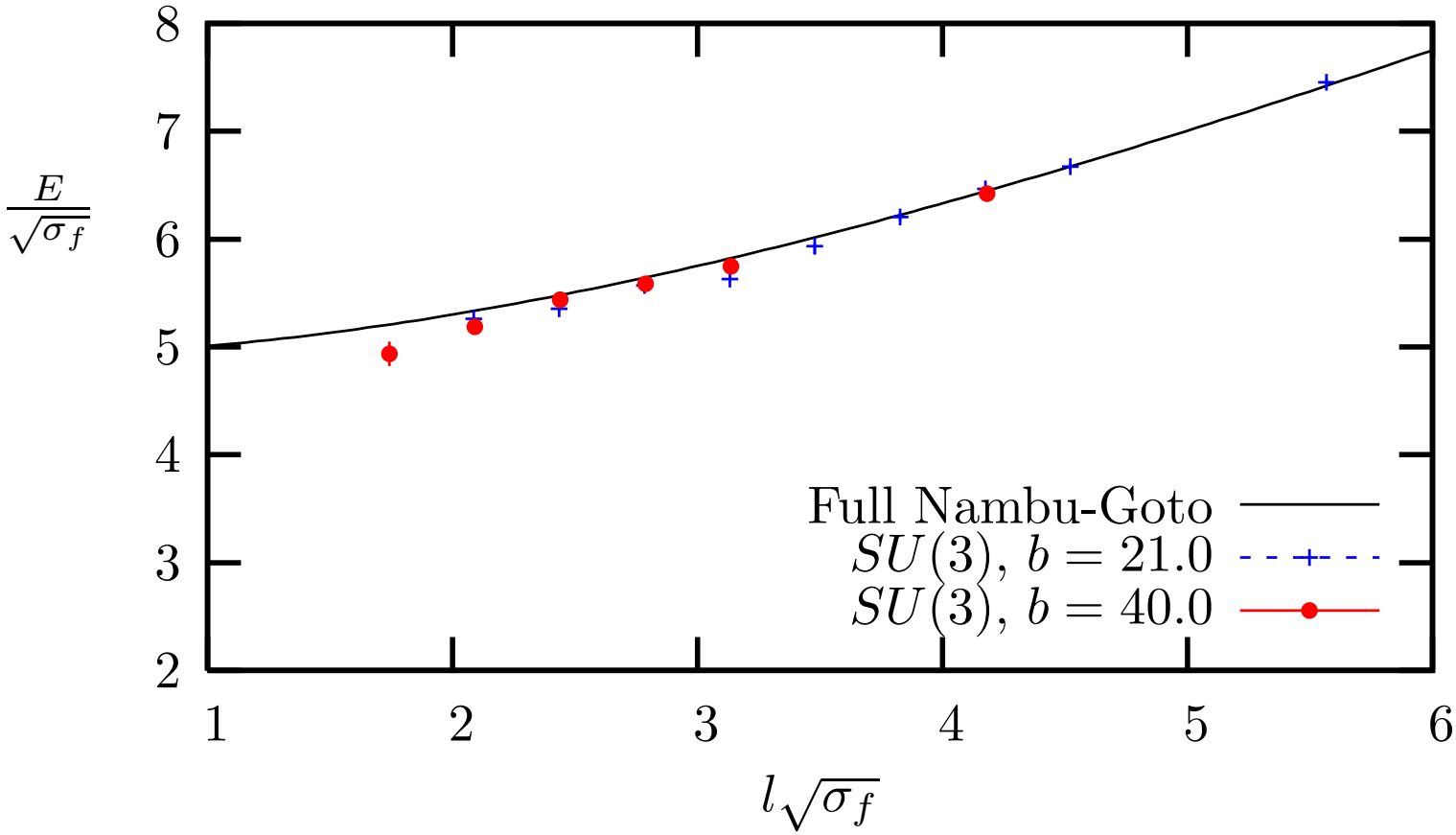
$a\sqrt{\sigma} \simeq 0.174 \quad ; \quad l_c\sqrt{\sigma} \simeq 1.0$



— : Nambu-Goto : σ from ground state

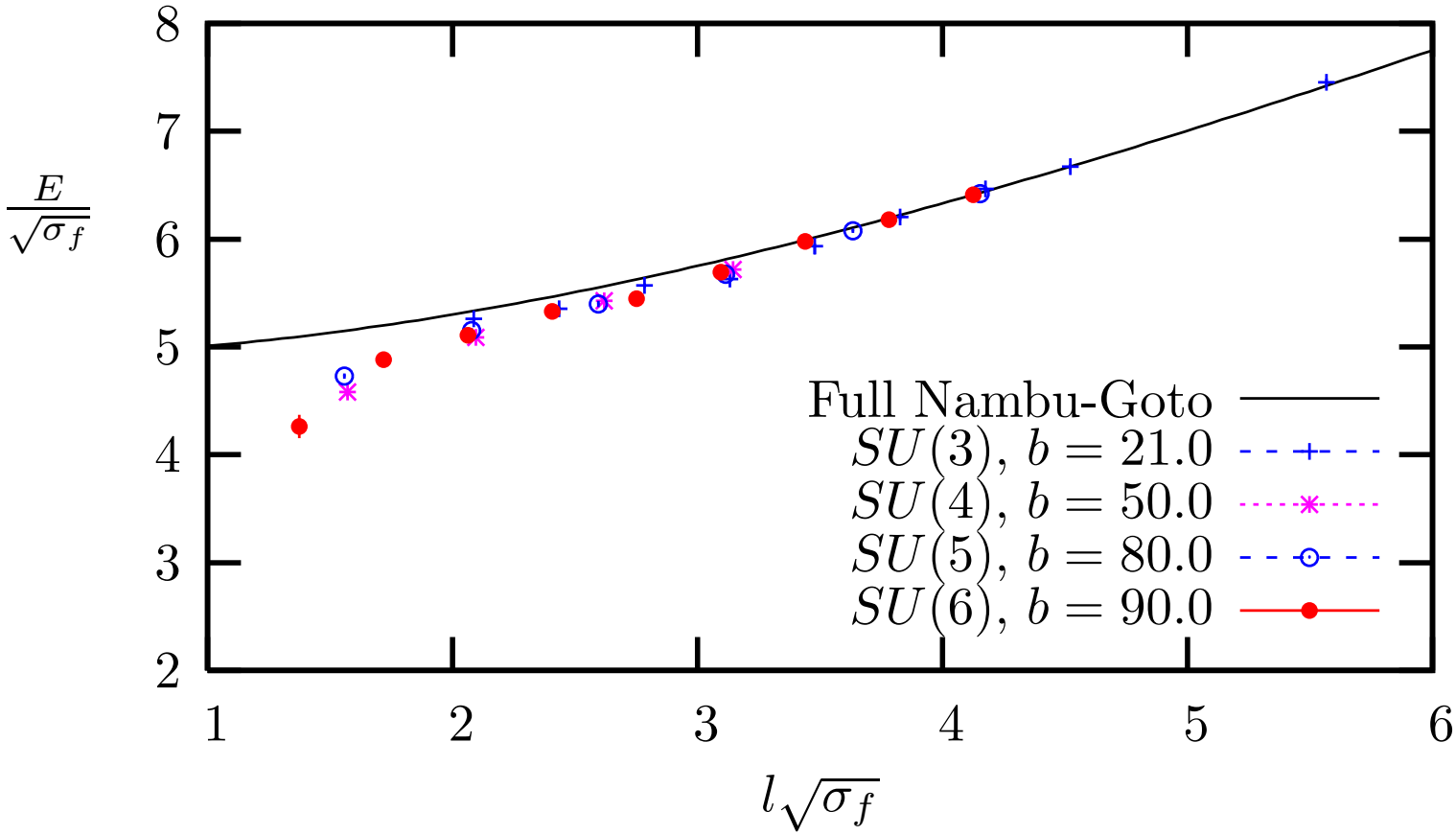
$\times$: +ve parity $\circ$: -ve parity

1st excited state : continuum physics?



no significant difference as $a \to a/2 \Rightarrow$ we have 'continuum' physics

1st excited state : large N physics ?



$\Rightarrow \quad SU(3) \simeq SU(\infty)$

content of lightest $q = 0$ NG states:

$\|0\rangle \quad \sim \quad$ background string	P=+, q=0
$a^R(k=1)a^L(k=1)\|0\rangle$	P=+, q=0
$a^R(k=2)a^L(k=2)\|0\rangle$	P=+, q=0
$a^R(k=1)a^R(k=1)a^L(k=1)a^L(k=1)\|0\rangle$	P=+, q=0
$a^R(k=2)a^L(k=1)a^L(k=1)\|0\rangle$	P=−, q=0
$a^R(k=1)a^R(k=1)a^L(k=2)\|0\rangle$	P=−, q=0

Since our lightest states have energies and degeneracies as in Nambu-Goto down to $l\sqrt{\sigma} \sim 2$, they are well-described by the above states.

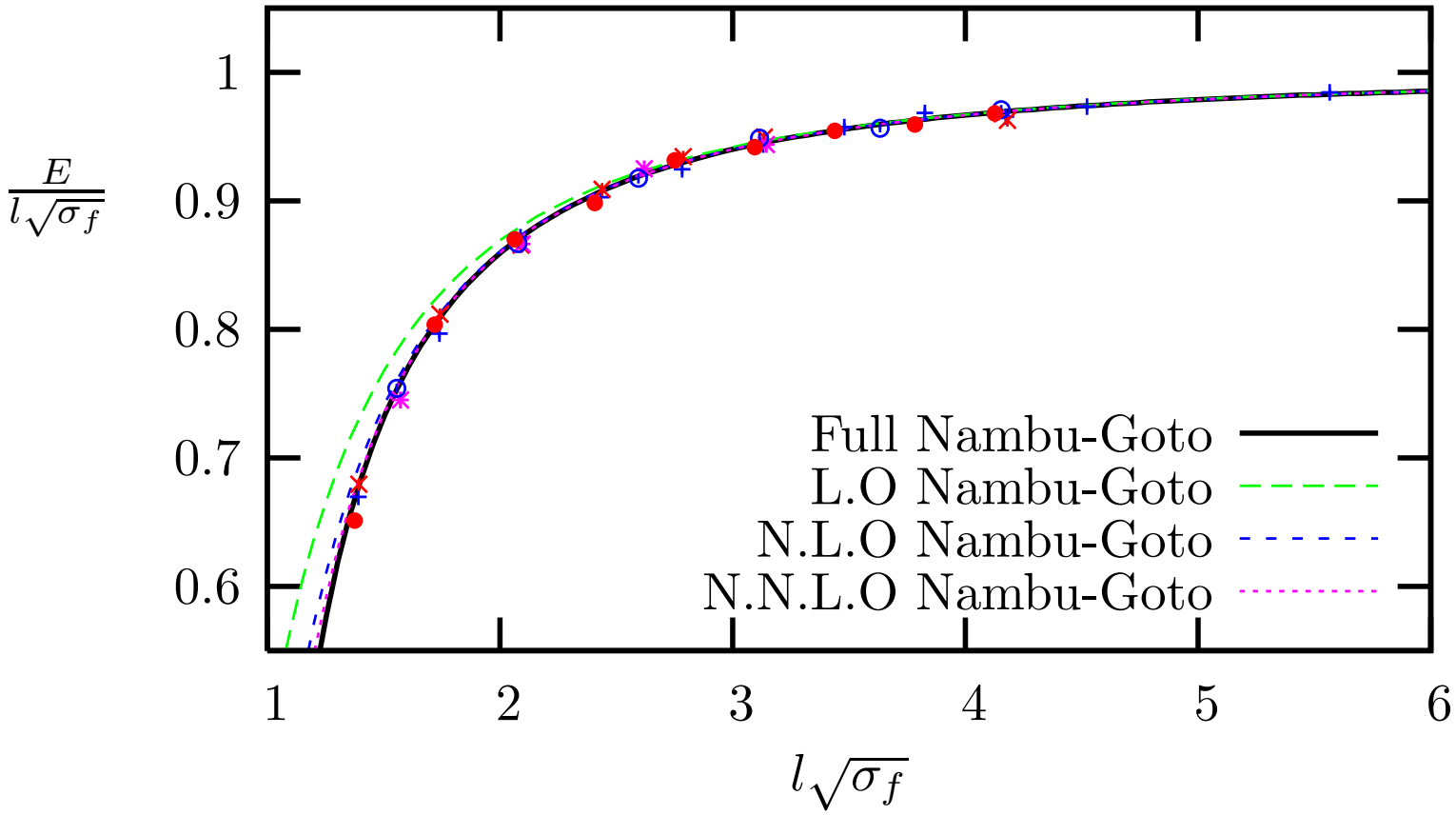
◇ Striking agreement with free string model, down to $l\sqrt{\sigma} \simeq 2$.

◇ Remarkable since $l\sqrt{\sigma} \simeq 2 \Rightarrow$ the flux tube is maybe only twice as long as it is wide – hardly an ideal 'string'.

◇ Is this just a manifestation of the fact that the first 3 or 4 terms in an expansion of $E_n(l)$ in powers of $1/\sigma l^2$ must be the same as Nambu-Goto?

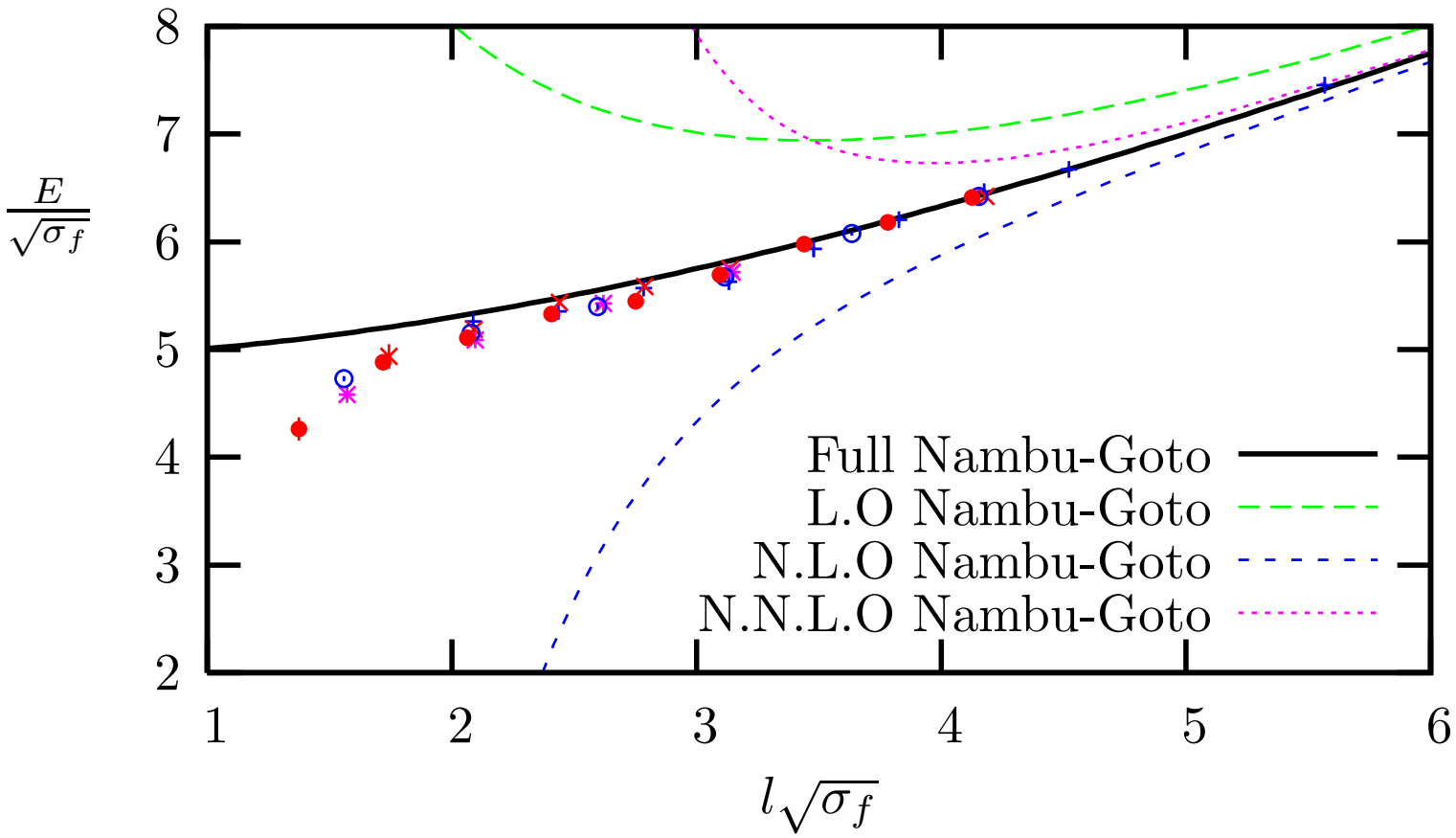
Nambu-Goto vs LSW, LW, D, AK ...

ground state



Nambu-Goto vs LSW, LW, D, AK ...

first excited state



Why ?

the covariant Nambu-Goto expression e.g. for $q = 0$,

$$E(l) = \sigma l \left(1 + \frac{8\pi}{\sigma l^2}\left(n - \frac{D-2}{24}\right)\right)^{\frac{1}{2}}$$

can only be expanded as a power series in $1/l\sqrt{\sigma}$ when

$$\frac{8\pi}{\sigma l^2}\left(n - \frac{1}{24}\right) \leq 1 \quad \leftrightarrow \quad l\sqrt{\sigma} \geq \sqrt{8\pi n} \sim 5\sqrt{n}$$

whereas in practice we have a very good fit by Nambu-Goto even down to

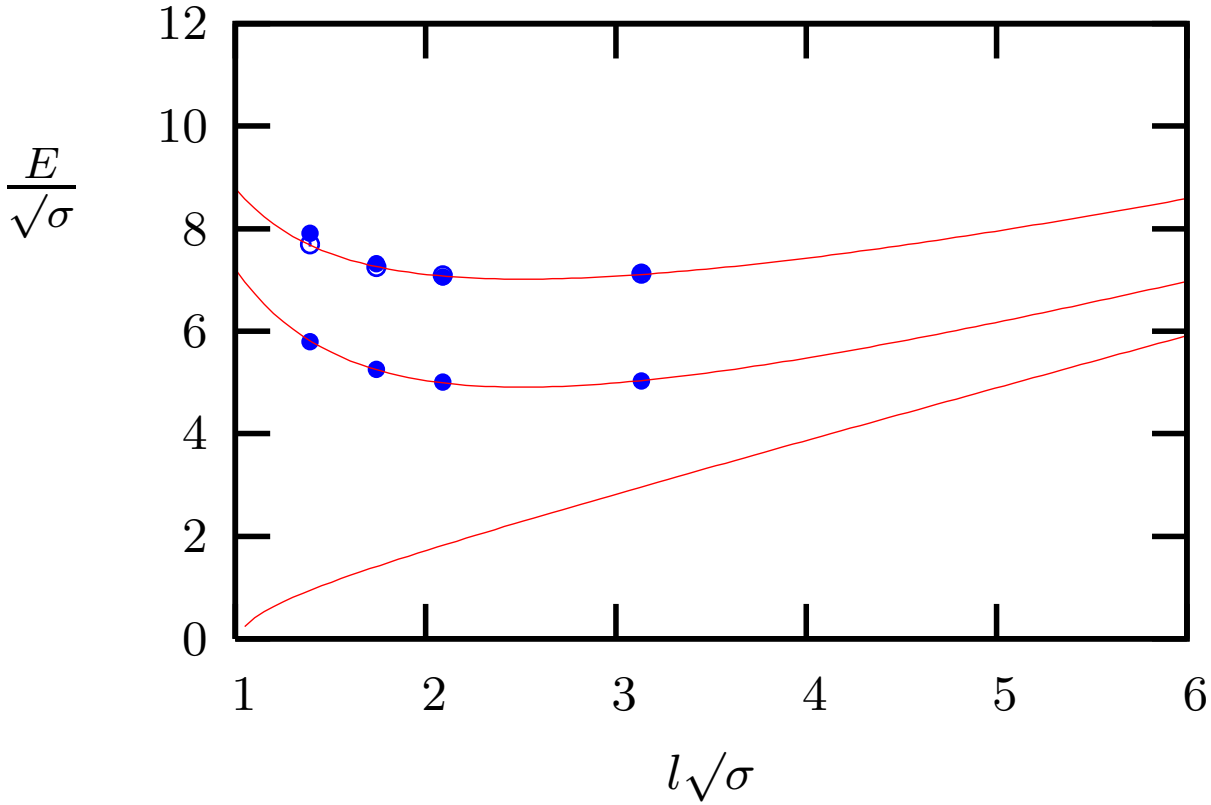
$$l\sqrt{\sigma} \sim 2 \ , \ n = 1, 2$$

which is well outside its radius of convergence

$\Rightarrow$

the agreement with NG that we see goes well beyond the range of validity of an expansion of $\mathcal{L}_{eff}$ in powers of derivatives: it makes a statement about $\mathcal{L}_{eff}$ to *all* orders in $1/\sigma l^2$

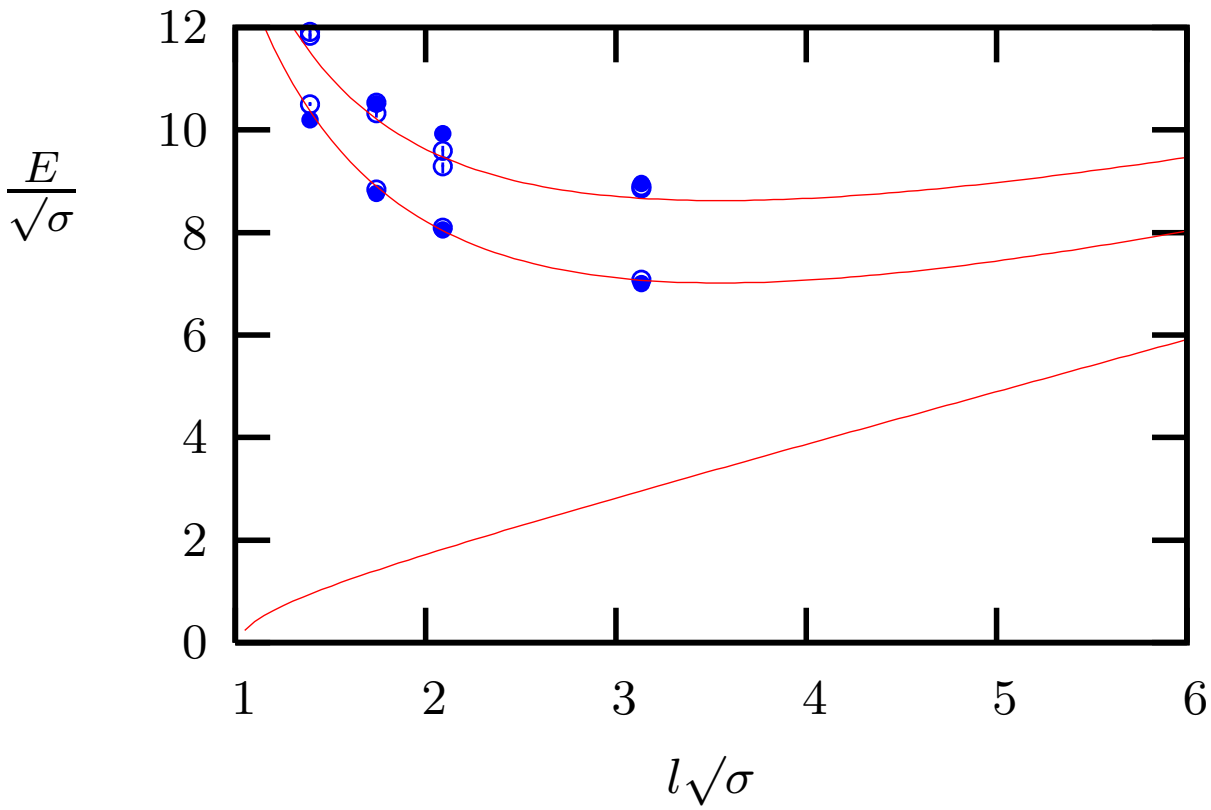
$q = 1$ spectrum : SU(3) at smaller a



• $P = -$; ∘ $P = +$

curves: NG predictions for $q = 1$ and also $q = 0$ ground state

$q = 2$ spectrum : SU(3) at smaller a



• $P = -$; ∘ $P = +$

curves: NG predictions for $q = 1$ and also $q = 0$ ground state

content of $q = 1, 2$ NG states:

$a^R(k=1)\|0\rangle$	P=$-$, q=1
$a^R(k=2)\|0\rangle$	P=$-$, q=2
$a^R(k=1)a^R(k=1)\|0\rangle$	P=$+$, q=2
$a^R(k=2)a^L(k=1)\|0\rangle$	P=$+$, q=1
$a^R(k=1)a^R(k=1)a^L(k=1)\|0\rangle$	P=$-$, q=1
$a^R(k=3)a^L(k=1)\|0\rangle$	P=$+$, q=2
$a^R(k=2)a^R(k=1)a^L(k=1)\|0\rangle$	P=$-$, q=2
$a^R(k=1)a^R(k=1)a^R(k=1)a^L(k=1)\|0\rangle$	P=$+$, q=2

$q \neq 0$ requires a deformation of the flux tube (otherwise it is translation invariant and hence $q = 0$) so in NG the ground state will have at least one phonon

• in D=2+1 SU(N) gauge theories, confining flux tubes belong to the universality class of a simple bosonic string theory

• more than that, the Nambu-Goto covariant free string spectrum
$E^2(l) = (\sigma\, l)^2 + 8\pi\sigma \left(\frac{N_L+N_R}{2} - \frac{D-2}{24}\right) + \left(\frac{2\pi q}{l}\right)^2 .$
accurately describes the spectrum down to values of $l\sqrt{\sigma}$ that are much below where an effective string theory expansion in $x = l\sqrt{\sigma}$, e.g.
$\frac{E_n}{\sqrt{\sigma}} = x\left(1 + \frac{c}{x^2}\right)^{\frac{1}{2}} = x + \frac{c}{2x} - \frac{c}{8x^3} + \cdots$
makes sense – i.e. far past its range of convergence.

• This suggests that we take the energy eigenstates to be those of the free NG string theory, as our starting point, and treat the deviations as a perturbation induced by some weak interactions e.g. between the phonons. There should be relations in the interaction energies within different NG eigenstates that depend on the nature of the interactions in this $D = 1 + 1$ phonon field theory.

• why is a simple bosonic string theory so good?

• usually we think of the flux tube as

either

some non-Abelian dual Nielsen-Olesen vortex, with a finite intrinsic width $\sim 1/\sqrt{\sigma}$;

and/or

a string in some '5D' gravity dual, dangling near some 'horizon' where the metric will have a highly non-trivial curvature, so that it projects to a flux tube of non-zero width on our '4D' boundary

• in either scenario, where are:

– the excited states due to excitations of the massive degrees of freedom generating the finite width?

– and the corrections to the stringy states from these massive degrees of freedom?

• very naively we might expect the mass scale of the lightest such extra states to be

$E(l) \sim E_0(l) + O(\sqrt{\sigma})$

and certainly no more than

$E(l) \sim E_0(l) + m_G \sim E_0(l) + 4\sqrt{\sigma}$

or maybe

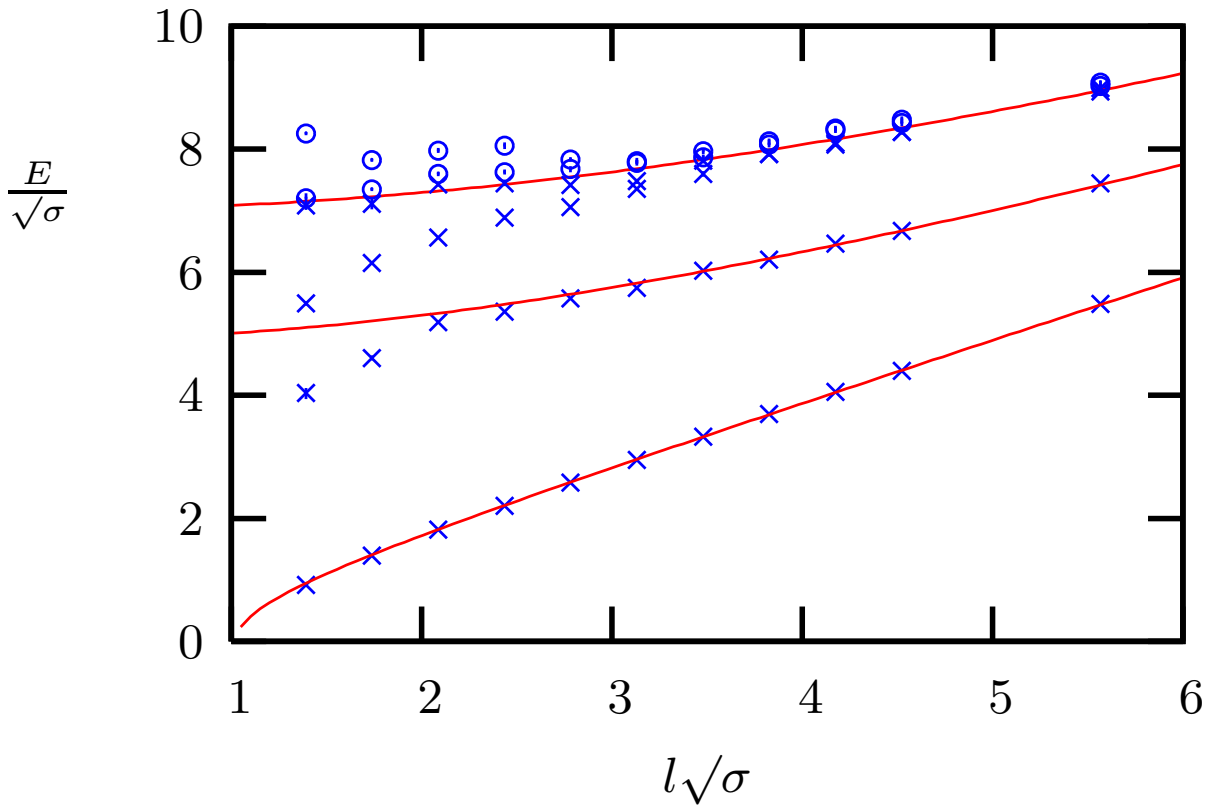
$E(l) \sim E_0(l) + \Delta m_G \sim E_0(l) + 2\sqrt{\sigma}$

at low $l\sqrt{\sigma}$ this should be one of the lightest excitations – but we do not see it in our spectra :

$\Rightarrow$ is this something special to flux tubes carrying fundamental flux ?

$\Rightarrow$ k-strings

recall : $q = 0$ closed string spectrum



nothing non-stringy in a range $E - E_{gs} \sim O(\sqrt{\sigma})$

k-strings

source = product of k fundamental sources

flux tube between such static sources = k-string

source may be screened by gluons from vacuum

gluons = adjoint so transform trivially under centre $\Rightarrow$ the screened source, always transforms under $z \in Z_N$ as
$\phi_k \to z^k \phi_k$

Thus k is a good quantum number.

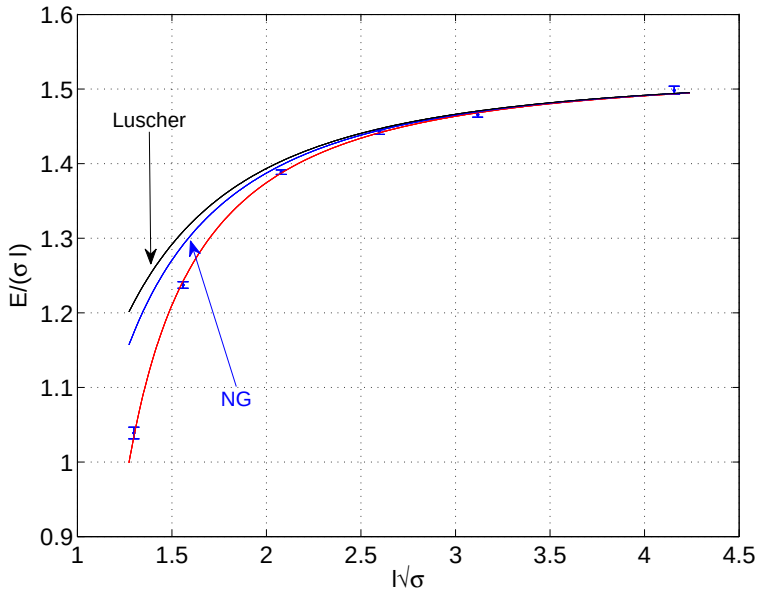
Typically a source will be screened to give the lightest string of given k.

One finds:

The lightest k-string is not composed of k separate fundamental strings, $\sigma_k = k\sigma_{k=1}$, but is a bound state with $\sigma_k < k\sigma_{k=1}$
e.g. for $k = 2$ in SU(4) one finds $\sigma_k \simeq 1.35\sigma_{k=1}$

$k = 2$ string corrections

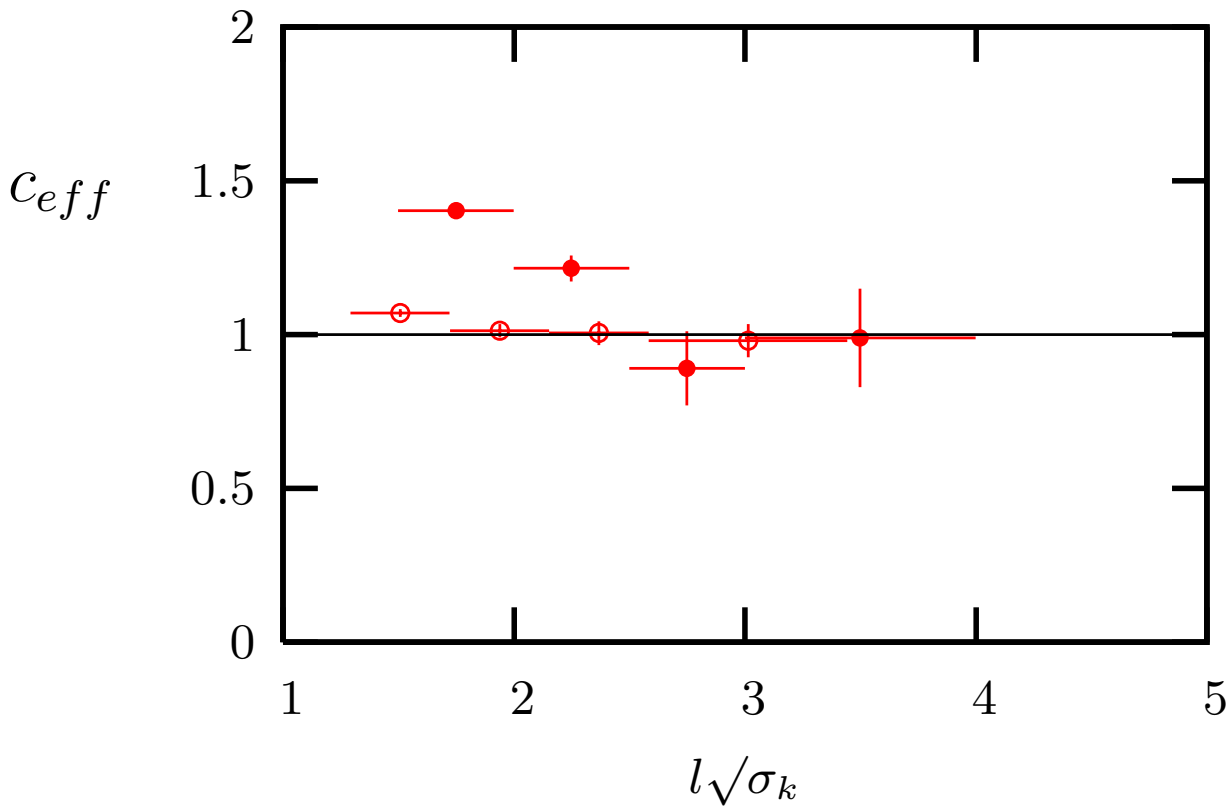
Nambu-Goto: $E_k(l) = \sigma_k l \left(1 - \frac{\pi(D-2)}{3\sigma_k l^2}\right)^{\frac{1}{2}}$ and Luscher $E_k(l) = \sigma_k l - \frac{\pi(D-2)}{6l}$



$\Rightarrow$

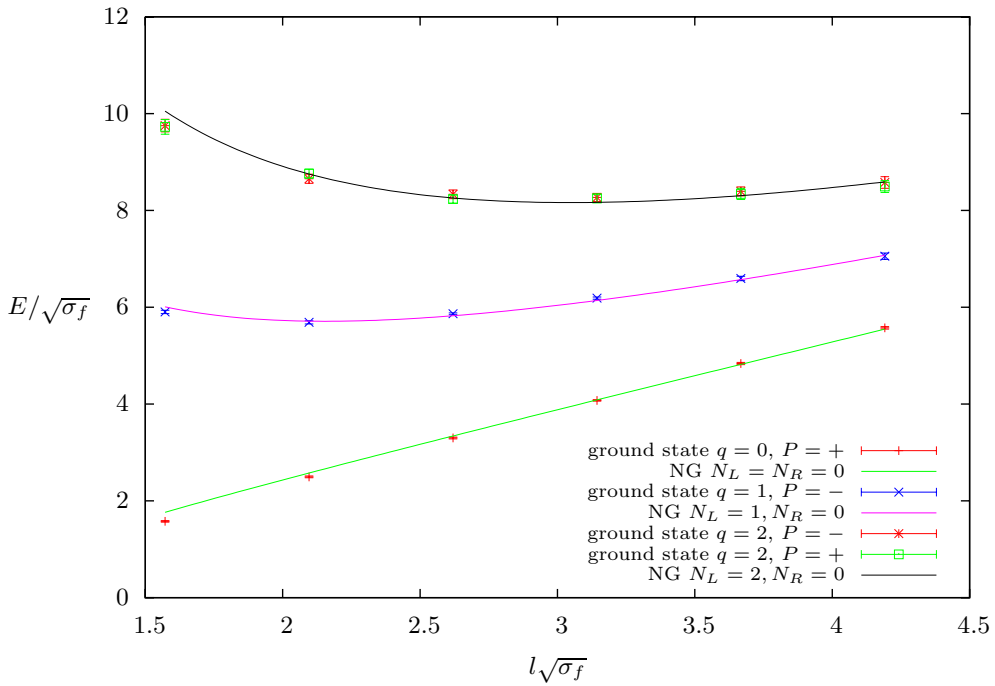
Much larger deviations at smaller l than for $k = 1$ flux tube : Nambu-Goto better, but not much better, than Luscher

SU(4) : Nambu-Goto effective charge, ○ $k = 1$ ● $k = 2$



⇒ $k = 2$ ground state flux tube is in the bosonic string universality class, but has larger corrections than the $k = 1$ flux tube for $l\sqrt{\sigma_k} \leq 2$

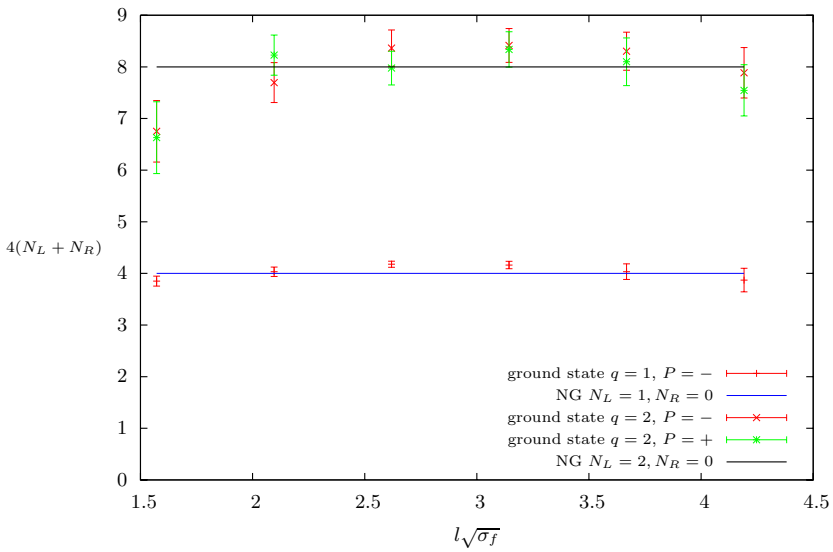
lightest $k = 2$ (antisymmetric) states with $q = 0, 1, 2$, for SU(4)



Lines are NG predictions with σ_k obtained by fitting the ground state.

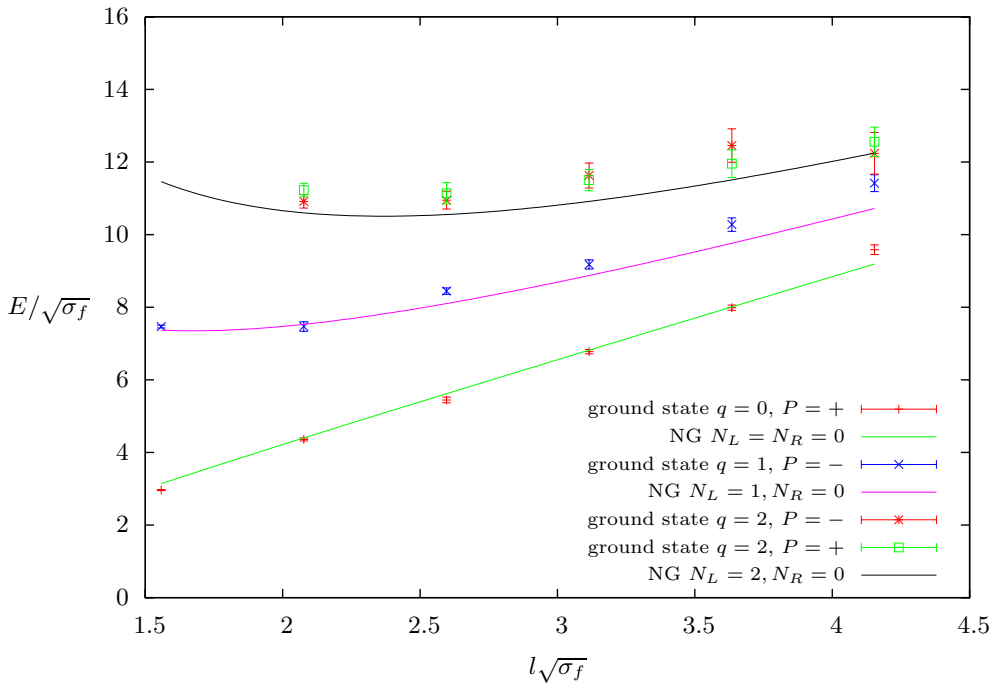
effective excitation numbers for $q = 1, 2$ using

$$\pi\sigma_k\{4(N_l + N_R)\}_{eff} = E_{gs}^2(q;l) - E_{gs}^2(0;l) - \left(\frac{2\pi q}{l}\right)^2$$



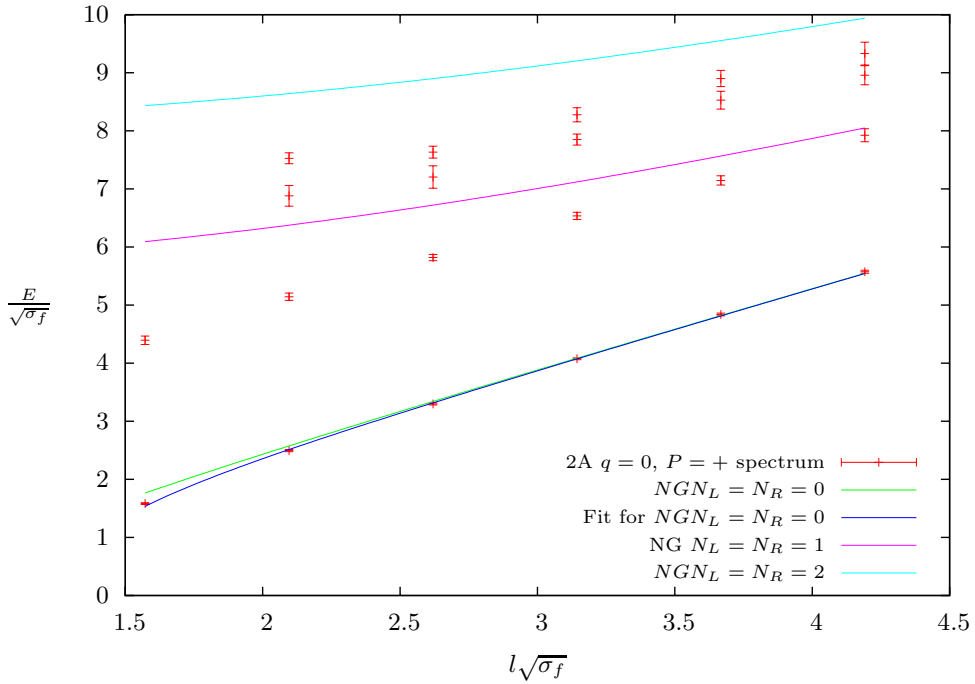
lines are NG

ASIDE: lightest $k = 2$ symmetric states with $q = 0, 1, 2$ for SU(4)



Lines are NG predictions with $\sigma_{k=2S}$ obtained by fitting the ground state.

spectrum light $k = 2$ (antisymmetric) states with $q = 0$ and $P = +$ for SU(4)



Lines are NG predictions with σ_k obtained by fitting the ground state.

• $q = 0$ excited states are far from NG – at best slowly converging to NG as $l \uparrow$.

• Is the first excited state perhaps not NG, but a 'breather' mode ? Comparing the $k = 1$ and $k = 2$ wave-functionals it in fact looks like it is a stringy NG mode

• So:
some states very close to NG (e.g. q=0,1,2 ground states) while other states (such as above) have large deviations
– this is ideal in a sense : it allows us to try and draw structural conclusions about the dynamics

e.g.

both the $q = 2$ $P = +$ ground state and the $q = 0$ first excited state have 2 lowest momentum phonons: the only difference is that for $q = 0$ they have opposite momenta while for $q = 2$ they have the same momentum
$\Rightarrow$ interactions between 2 phonons on a $k = 2$ string are small near threshold but large at 'high energies'.

So, in 2+1 dimensions :

◦ Confining flux tubes carrying fundamental flux wound around a spatial torus, are, to a very good first approximation, described by the free Nambu-Goto string theory, for all lengths for which there is confinement.

◦ When the flux is in a higher representation, e.g. $k = 2$, some states are still very Nambu-Goto-like, but others have large deviations.

The fact that the deviations are either large or small, enables us to infer something about the details of the string interactions.

D=3+1 ; 'preliminary'

Athenodorou,Bringoltz,Teper – in preparation

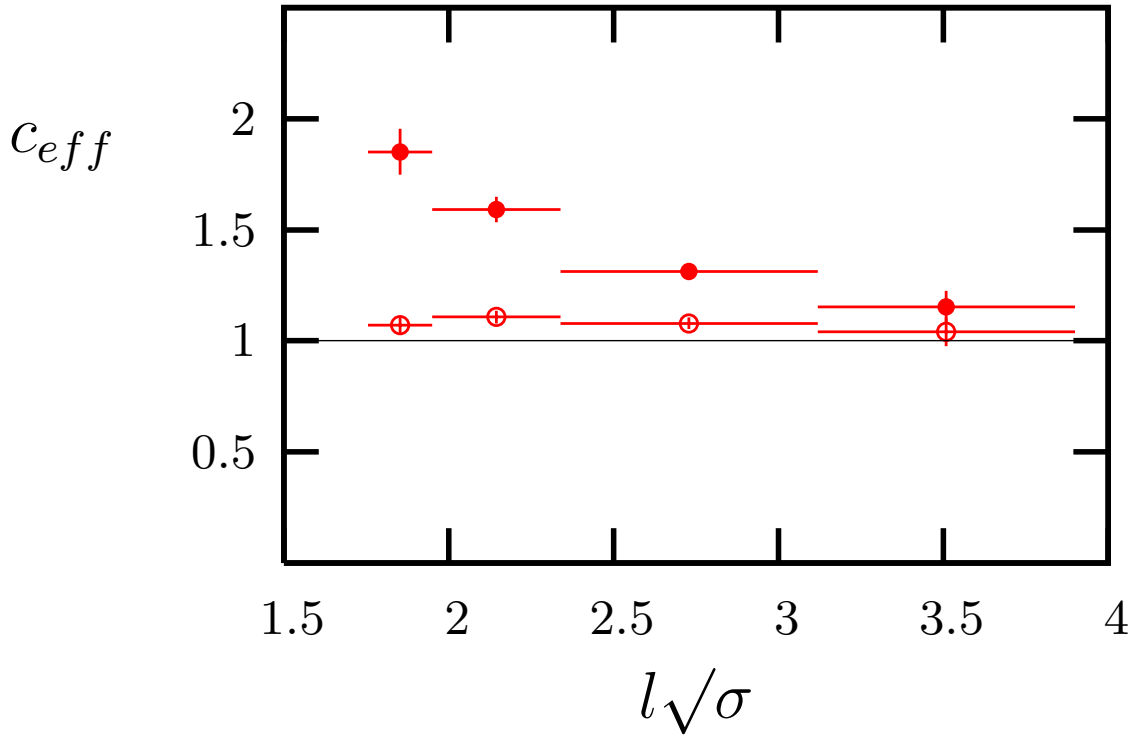
∘ SU(3) spectra at $a\sqrt{\sigma} \simeq 0.195$ and $a\sqrt{\sigma} \simeq 0.129$:
– continuum physics check

∘ SU(3) and SU(5) spectra at $a\sqrt{\sigma} \simeq 0.195$ and $a\sqrt{\sigma} \simeq 0.197$:
– large N check at fixed a

∘ SU(3) and SU(6) high statistics ground state calculations at $a\sqrt{\sigma} \simeq 0.195$ and $a\sqrt{\sigma} \simeq 0.202$:
– check power corrections at fixed a

Note: $l_c\sqrt{\sigma} \sim 1.1 \longrightarrow l_c\sqrt{\sigma} \sim 1.6$

string correction to ground state : SU(3) ; $a\sqrt{\sigma} \sim 0.2$



○ Nambu-Goto; ● Luscher

$\Longrightarrow$

bosonic string ; Nambu-Goto much better than Luscher but not as good as $D = 2 + 1$

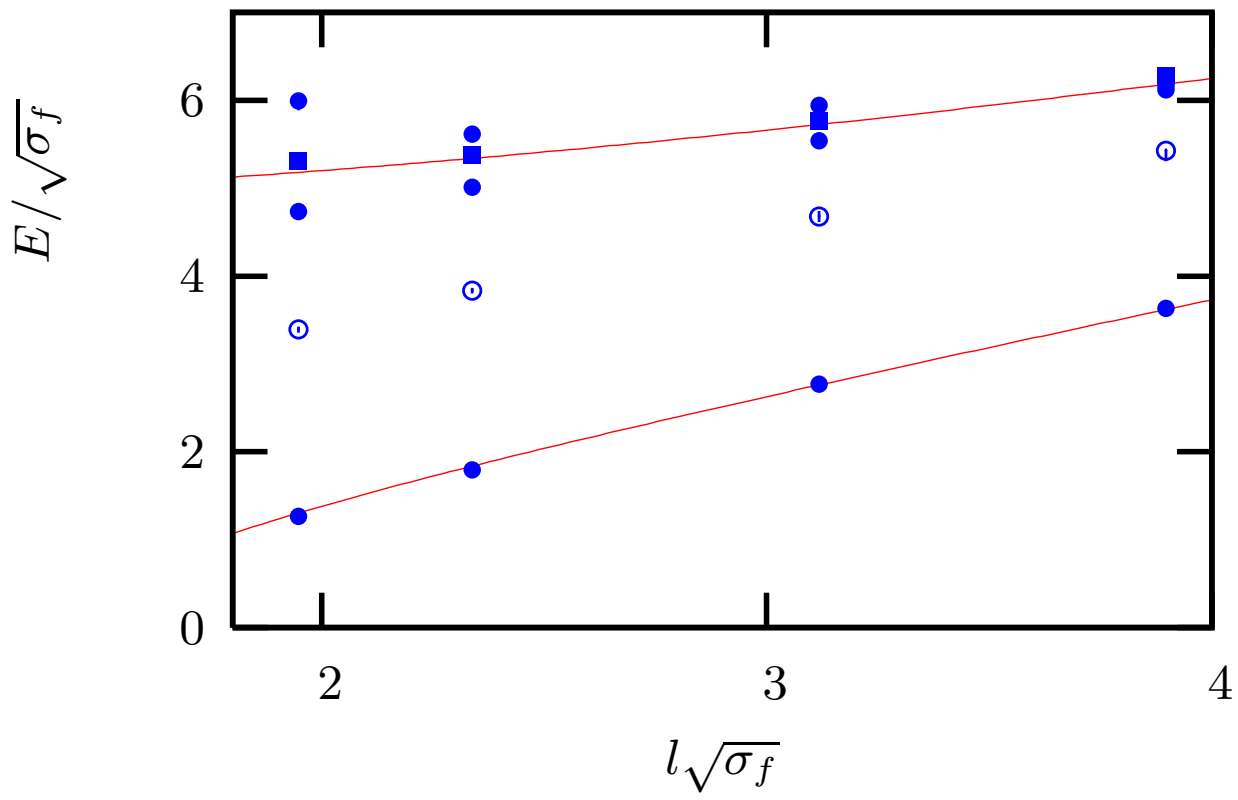
relevant string quantum numbers in 3+1 dimensions:

- length, l.
- momentum along string, $p = 2\pi q/l$.
- angular momentum around string axis, $J = 0, 1, 2...$
- $D = 2 + 1$ parity in plane orthogonal to string axis, P_ρ
- reflection 'parity' across this same plane, P_r

$\Rightarrow$

excitation spectrum (in progress)

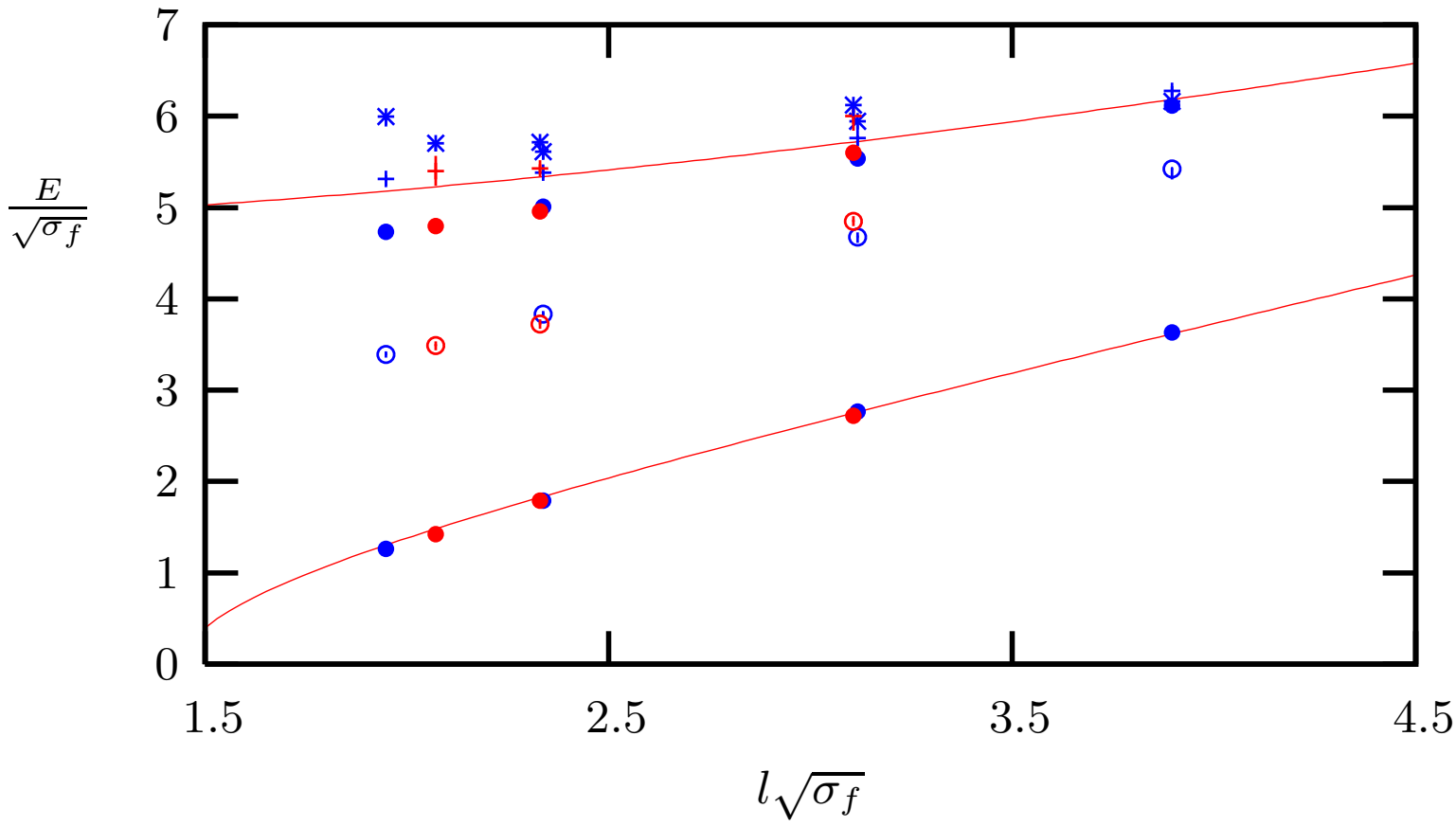
SU(3) at $a\sqrt{\sigma} \sim 0.2$; $q = 0$ spectrum



$\bullet$: $J = 0, P_\rho = P_r = +$; $\square$: $J = 2, P_\rho = P_r = +$; $\bullet$: $J = 0, P_\rho = -, P_r = +$; $\sim$ NG

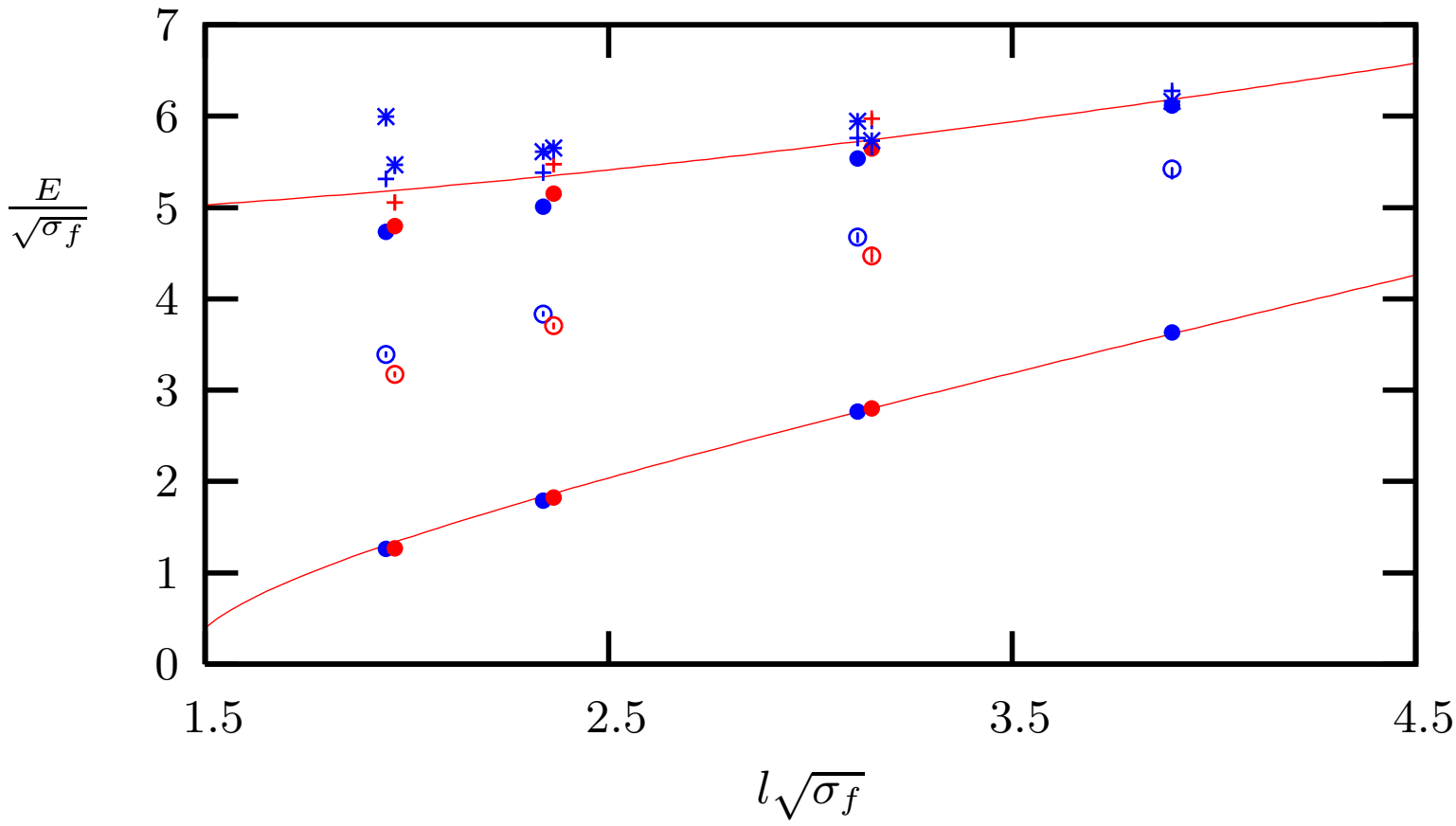
$\circ$: $J = 0, P_\rho = P_r = -$; $\leftarrow$ anomalous ?

SU(3) for $q = 0$: $a\sqrt{\sigma} \sim 0.2$ vs $a\sqrt{\sigma} \sim 0.13$



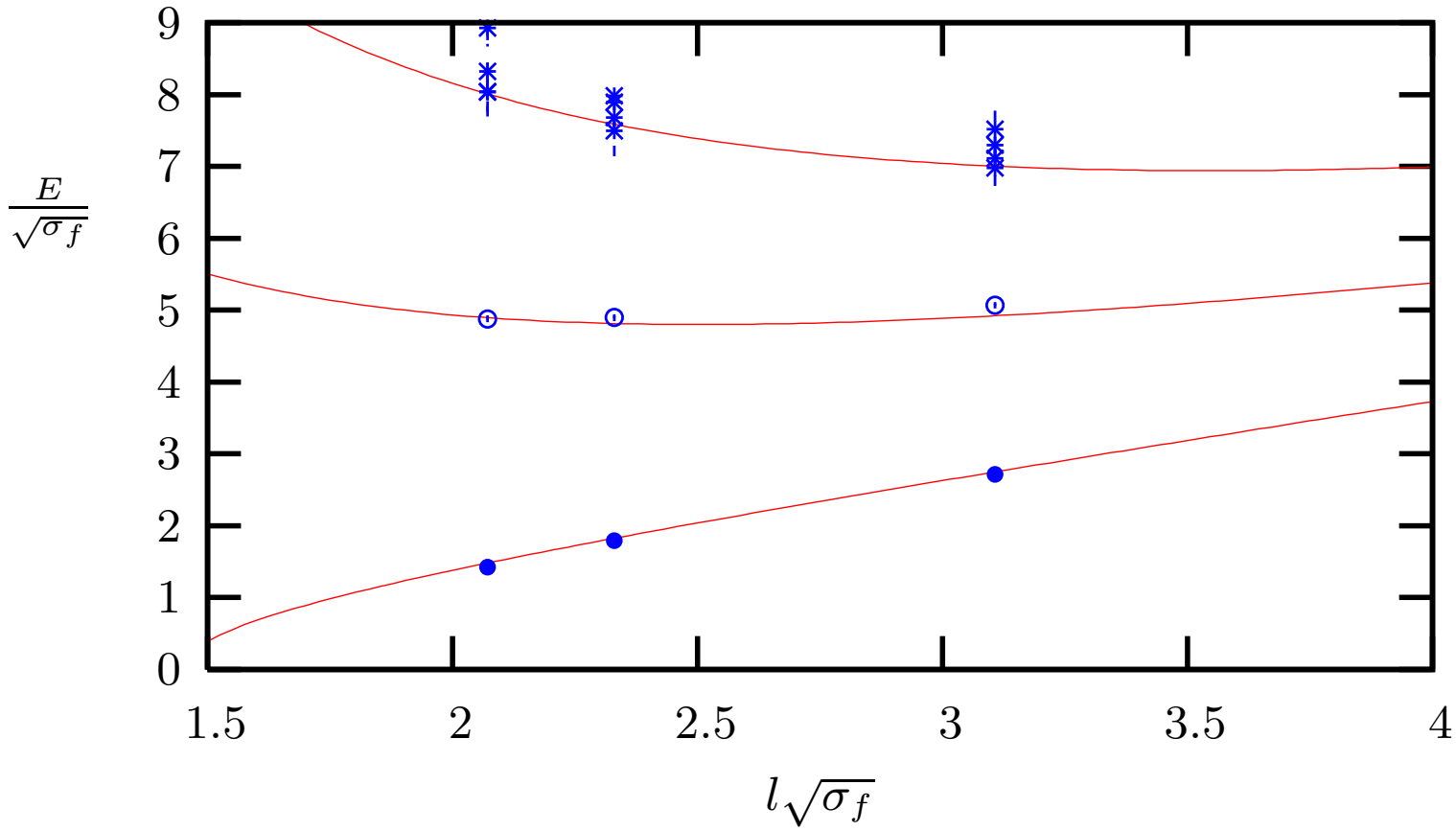
$\Longrightarrow$ lattice corrections negligible : 'anomaly' same

$q = 0$, $a\sqrt{\sigma} \sim 0.2$: SU(3) vs SU(5)



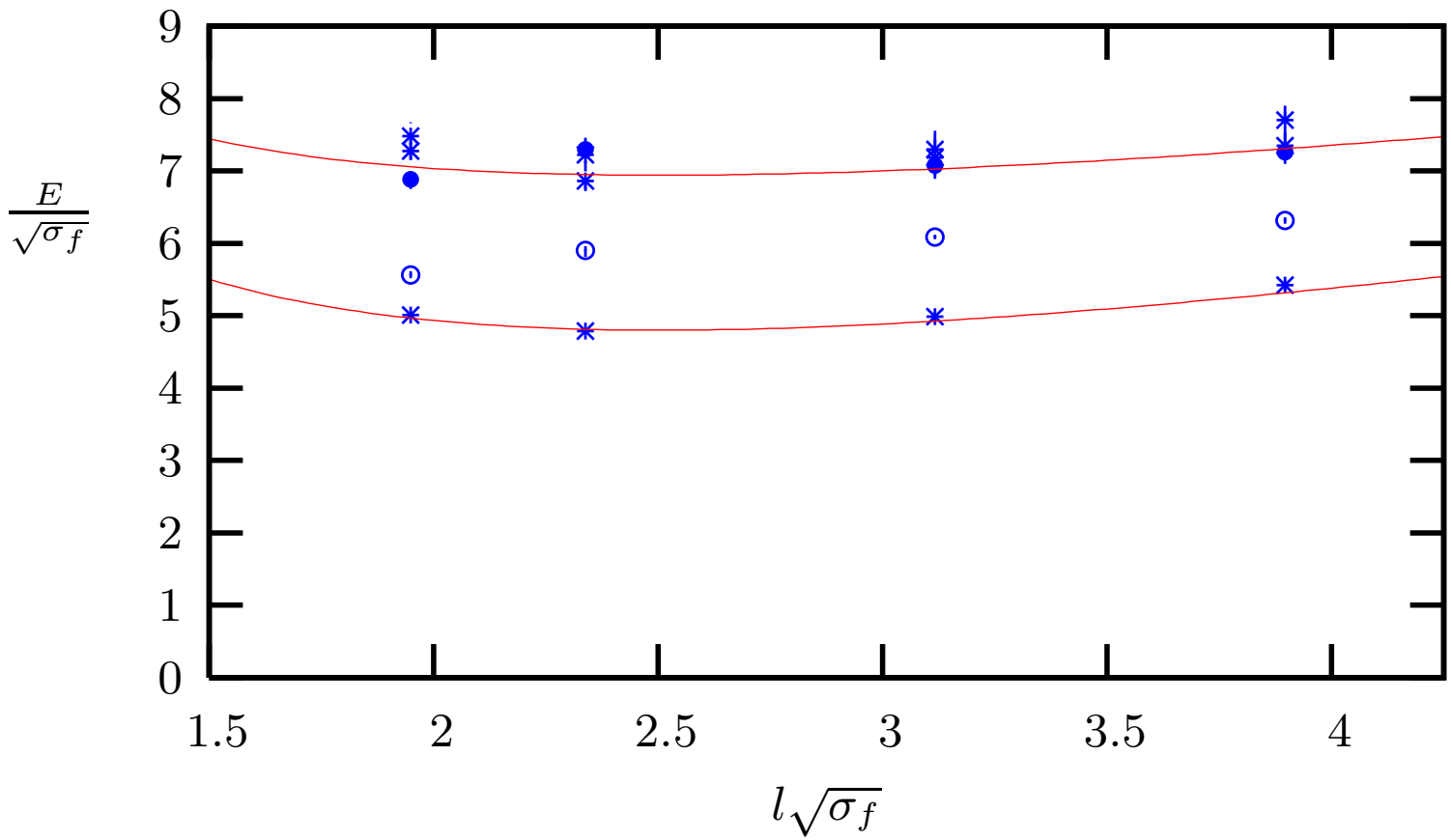
$\Longrightarrow$ finite N corrections small : 'anomaly' survives large-N

SU(3) at $a\sqrt{\sigma} \sim 0.13$: $q = 0, 1, 2$ ground states



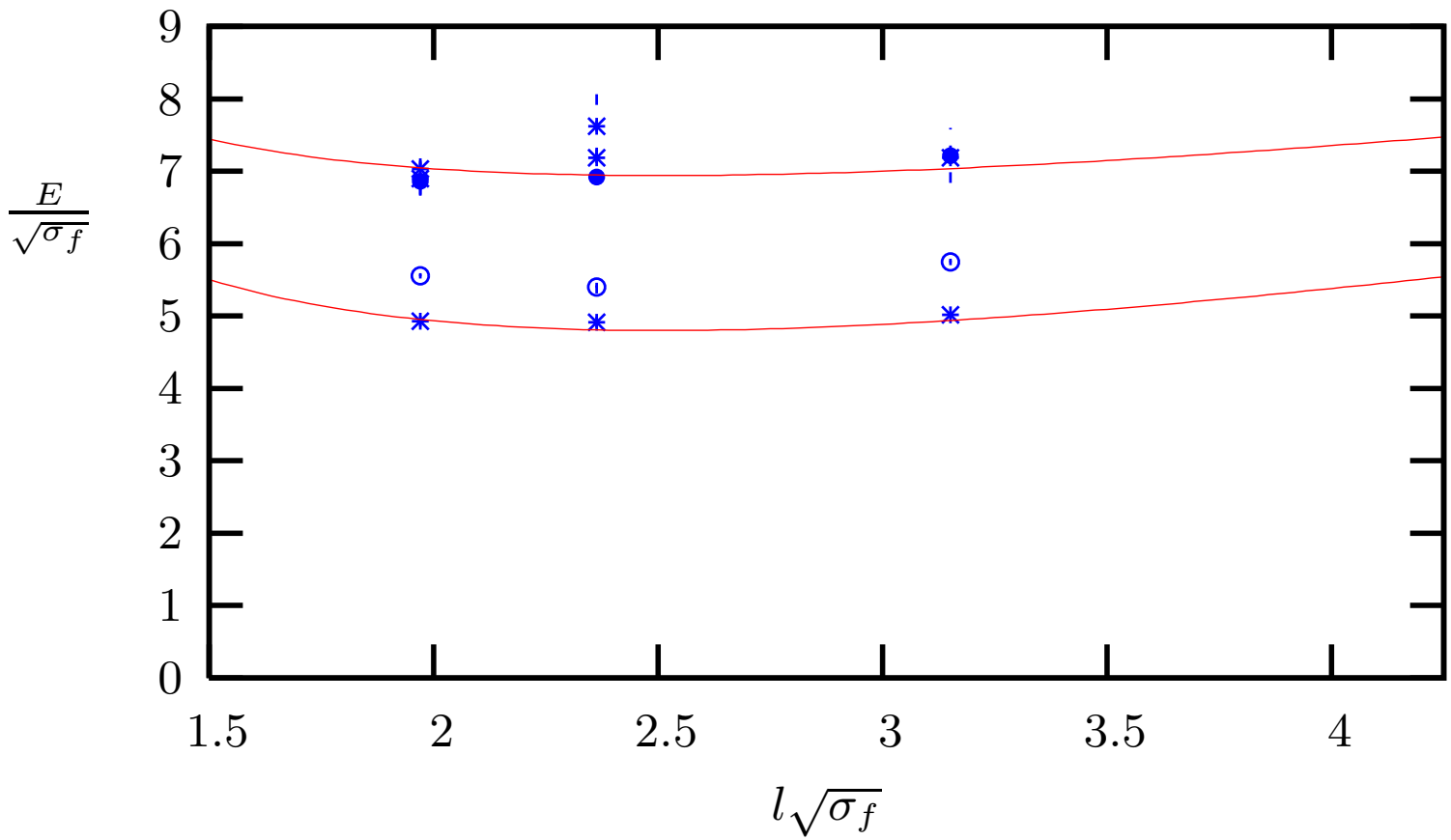
$\Longrightarrow$ remarkably like NG

SU(3) at $a\sqrt{\sigma} \sim 0.13$: $q = 1$ spectrum



$\Longrightarrow$ mostly $\sim$ NG, but $\circ$ $J = 0, P_p = -$ $\leftarrow$ anomalous (no cvgce ?!)

SU(5) $q = 1$ spectrum ?



$\Longrightarrow$ $\circ$ $J = 0, P_p = -$ state even more anomalous

So, in 3+1 dimensions :

∘ Most states in the spectrum of confining flux tubes carrying fundamental flux wound around a spatial torus are, to a very good first approximation, described by the free Nambu-Goto string theory, even for short lengths where the 'tube' is no more than a fat 'blob'.

∘ The low-l corrections to NG for these states are small, but more significant than in $D = 2 + 1$; although this may be in part reflect the fact that the critical deconfining length is larger in 3+1 than in 2+1 dimensions : $l_c/\sqrt{\sigma} \sim 1.1 \to 1.6$

∘ For certain quantum numbers, most visibly $J = 0$ $P_p(= P_r) = -$, the state is far from Nambu-Goto and approaches the latter for $l \uparrow$ only very slowly if at all – these 'anomalous' states clearly reflect non-stringy dynamics of some kind.

Conclusions

• In recent years there has been substantial progress in the description of confining flux tubes as strings – with substantial theoretical and numerical advances mutually reinforcing eahc other.

• Theoretically : we now know that in an effective string theory for flux tubes, if l is large enough that the gap $E_n(l) - E_0(l) \ll \sqrt{\sigma}$, then (+ some caveats) :

$$E_n(l) = \sigma l + \frac{c_1}{l} + \frac{c_2}{l^3} + \frac{c_3}{l^5} + O\left(\frac{1}{l^6}\right) \quad ; \quad c_{1,2,3} \equiv \mathrm{NG}$$

i.e. to this order, the 'low-energy' spectrum is identical with that of the free bosonic string theory.

• Numerically : we now know

∘ in $D = 2 + 1$ the spectrum of closed flux tubes that carry fundamental flux, is very close to Nambu-Goto $\forall N$ – and down to such small string

lengths, $l\sqrt{\sigma} \sim 2$, that an expansion of NG in $1/\sigma l^2$ no longer converges
$\rightarrow$ this striking feature cannot be easily addressed in the usual approach where one expands the low-energy effective string action, order-by-order in derivatives of transverse fluctuations, around a background straight string

◦ in the spectrum of $k = 2$ closed flux tubes most states are $\sim$ NG, but a few have large corrections, which enables us to conjecture something specific about the interactions: e.g. phonon interactions are small near threshold, but large at high cm energy.

◦ in $D = 3 + 1$ the spectrum of closed fundamental flux tubes is also mostly very close to NG, down to very small l, but there are a few states with particular quantum numbers that are far from NG and sometimes show little sign of converging to NG at large l;
$\rightarrow$ some mixture of stringy and massive mode excitations?

• It is also hard to see, within a traditional, bottom-up 'dual non-Abelian Nielsen-Olesen vortex' picture, why a short flux-tube that is just a fat blob should have a phonon-like spectrum of a non-interacting thin string;

$\longrightarrow$

even a short fat flux tube appears to know that it is really a string: evidence for a stringy dual?

– although this does not in itself resolve this puzzle, since non-trivial curvature should generate effects like those due to massive modes on the boundary.

• The fact that generically the corrections to the NG behaviour of individual states is either small or large, is in some sense ideal – far better than just having a variety of corrections all over the place.

It provides a potentially useful focus on the structure of the dynamics

• So where are the massive modes?