CONFORMALITY OR CONFINEMENT? (IR)RELEVANCE OF TOPOLOGICAL EXCITATIONS

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Large N at Swansea

based on arXiv:0709.3269 [hep-th] (Magnetic bions) and
joint work with E. Poppitz: Conformality or confinement?

Related ideas from earlier collaborations:
M. Shifman, L. G. Yaffe, E. Poppitz

GOAL

- My goal since 2007 has been to develop a new tool-box and set of methods to study 4d asymptotically free non-abelian gauge theories.

- I believe that with the right techniques, there should not be dramatic (difficulty) differences in addressing the dynamics of YM, QCD-like, chiral or supersymmetric gauge theories....

- Common lore: SUSY: Very friendly, beautiful. YM: Formal. QCD-like: Hard, leave it to lattice folks! (chiral limit still hard!) Non-susy chiral gauge theories: Even lattice does not work in practice. (See no evil,....)

Motivations

- Dynamics of YM, QCD-like and chiral theories
- Problem of electro-weak symmetry breaking (EWSB), scenarios in TeV scale physics. (where Higgs is a composite) to be tested at LHC.
- technicolor: Scaled-up QCD fails with EW precision data, fails to produce acceptable spectrum. (walking, conformal,....)
- Theories with interesting long distance behavior, scale invariance.

Outline

- Progress in understanding of confinement mechanism(s) in vector-like and chiral theories

- Generalized QCD (different physical IR behavior, scale invariance)

- Conformality or confinement? (and what determines it?)

- Chiral gauge theories (No time unfortunately.)

Necessity of new methods.

Undoubtedly, we need new techniques and perspectives about non-perturbative gauge theory, chiral or vector-like.

Progress in SUSY QCD is well-known. However, the dynamics of theories without elementary scalars is vastly different (and more interesting) from the SUSY examples.

They deserve study of their own.

A broad-brush overview of some very recent progress.

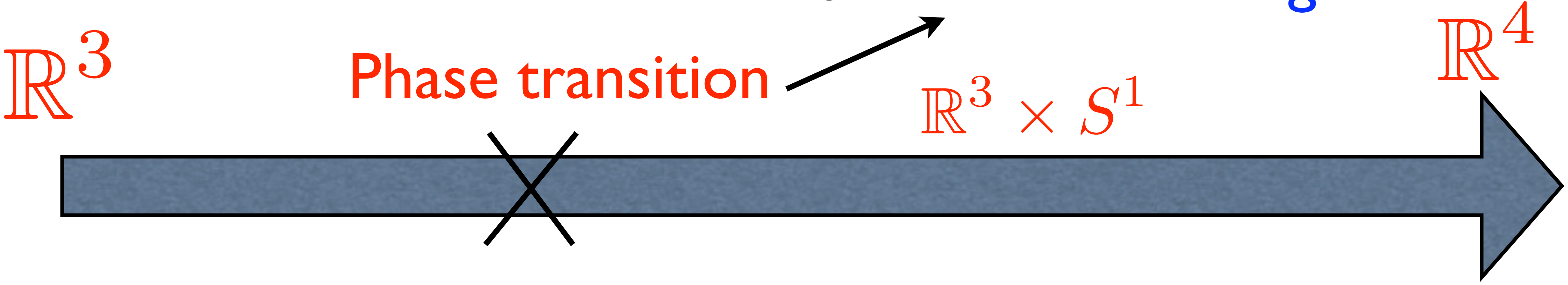
Theories on $\mathbb{R}^4$ (way to hard)

Keep locally d=4 such as $\mathbb{R}^3 \times S^1$

Take advantage of circle (as control parameter), i.e, AF and weak coupling.

Traditional: thermal setting.

Bad for our goal.



Use circle compactifications with periodic b.c. for fermions or center-stabilizing deformations.

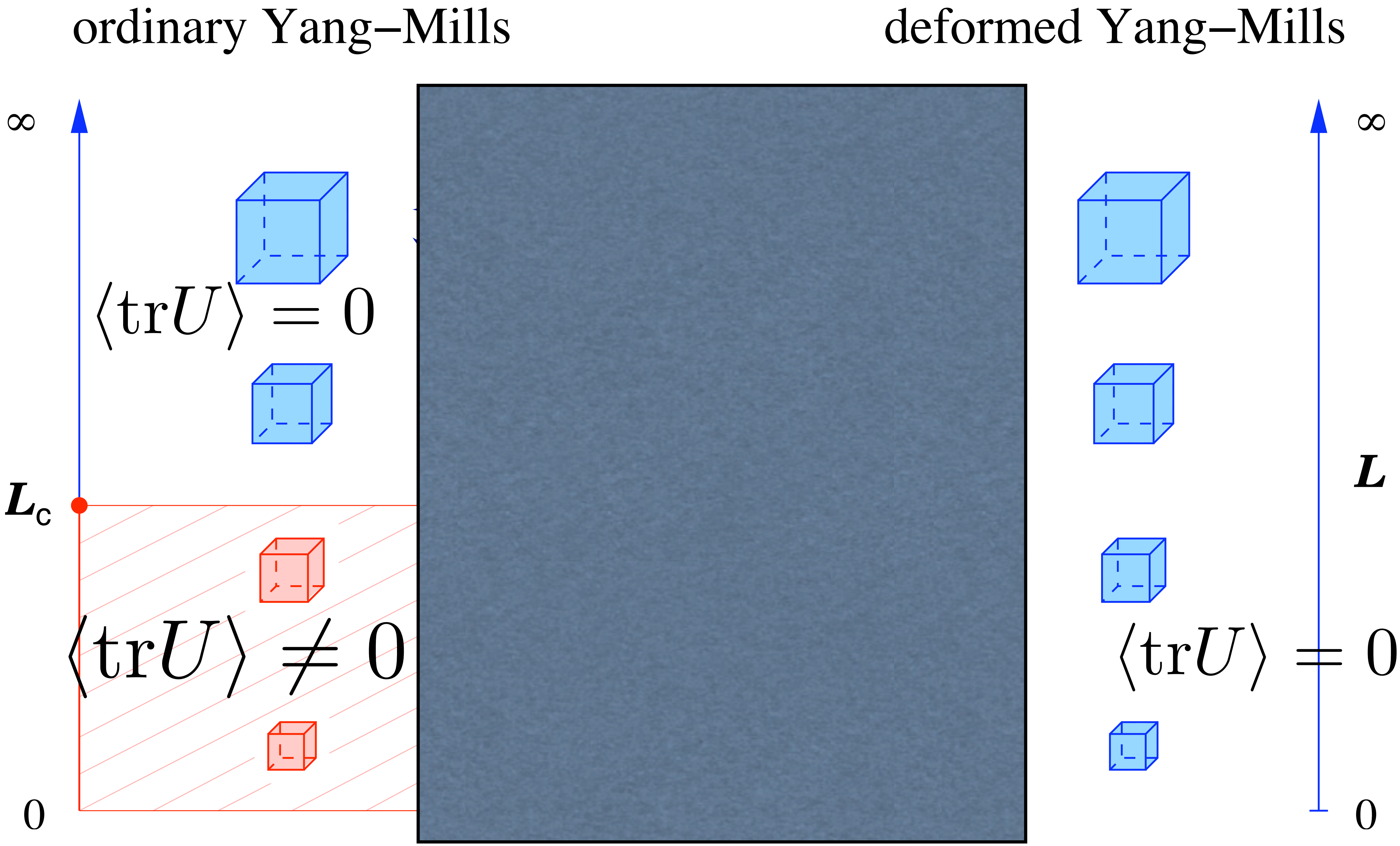
NEW METHODS

1) Twisted Partition Function

2) Deformation theory

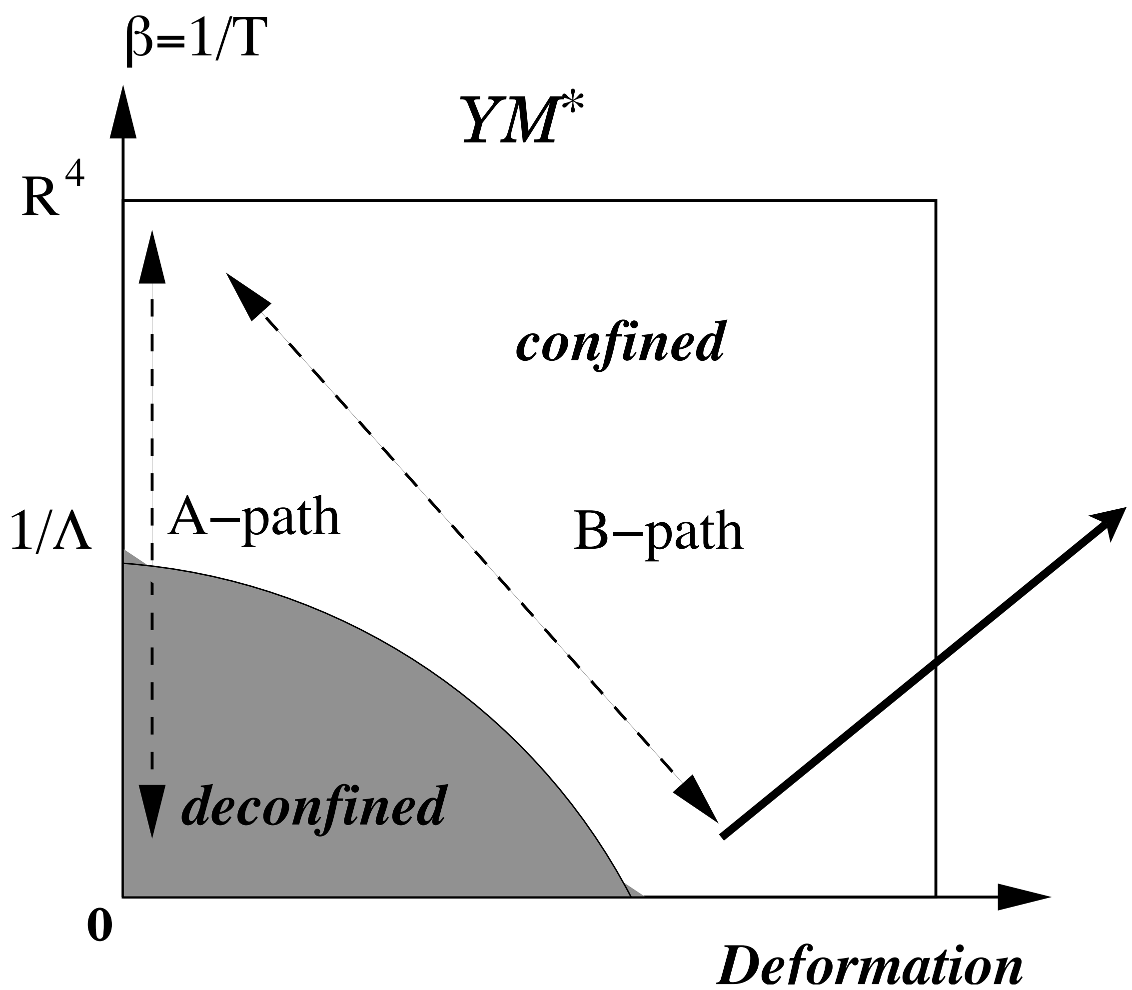
a small step in the desired direction: One of the two always guarantees that small and large circle physics are connected in the sense of center symmetry and confinement.

Order Parameter/center symmetry



What is the relation between ordinary and deformed theory?

Raison d'être of deformation theory at finite N

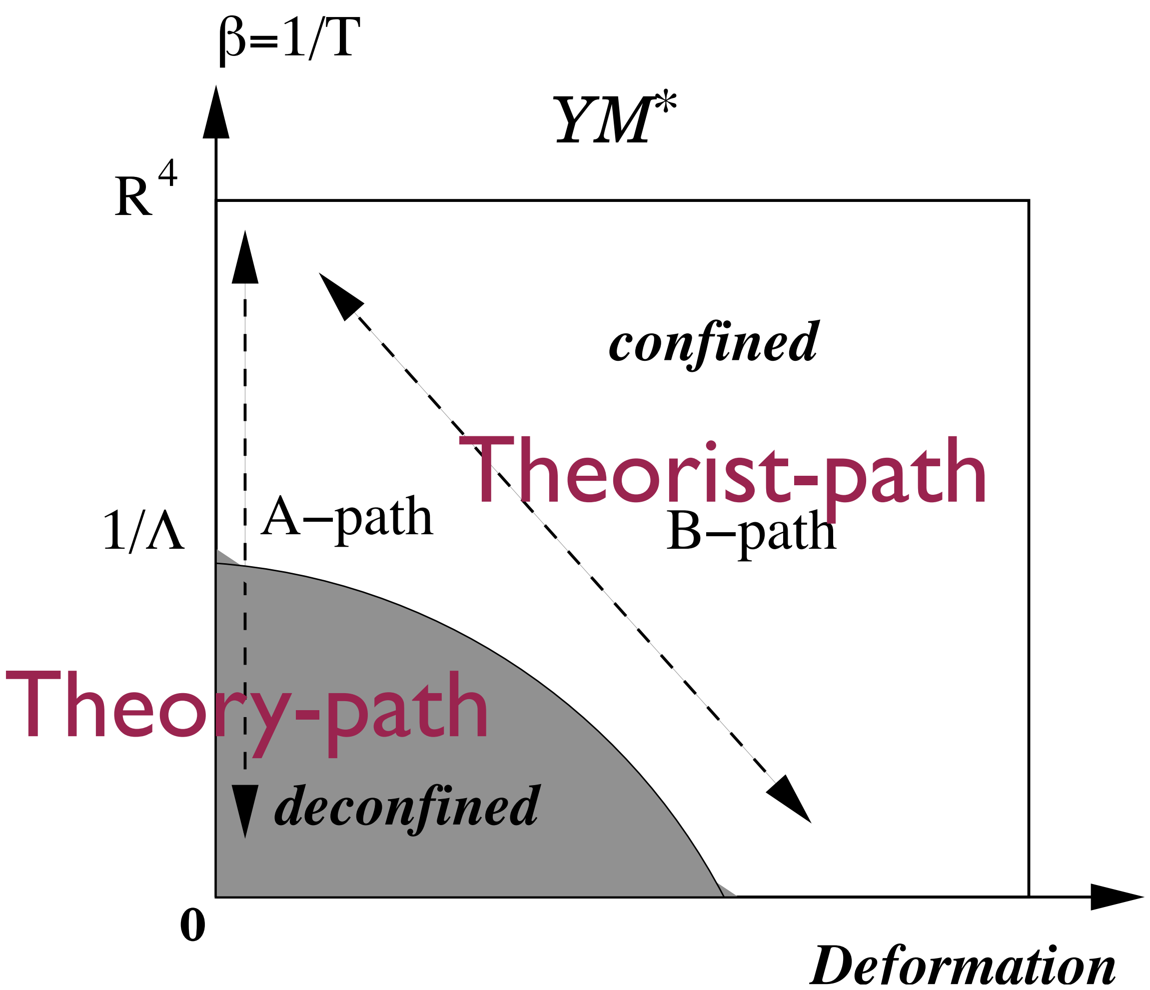


Small volume theory becomes solvable in the same sense as Polyakov model or Seiberg-Witten theory by abelian duality.

One can show the mass gap and linear confinement. Although the region of validity does not extend to large circle, it is continuously connected to it with no gauge invariant order parameter distinguishing the two regimes.

Deformed YM theory at finite N

Shifman-MU, Yaffe-MU



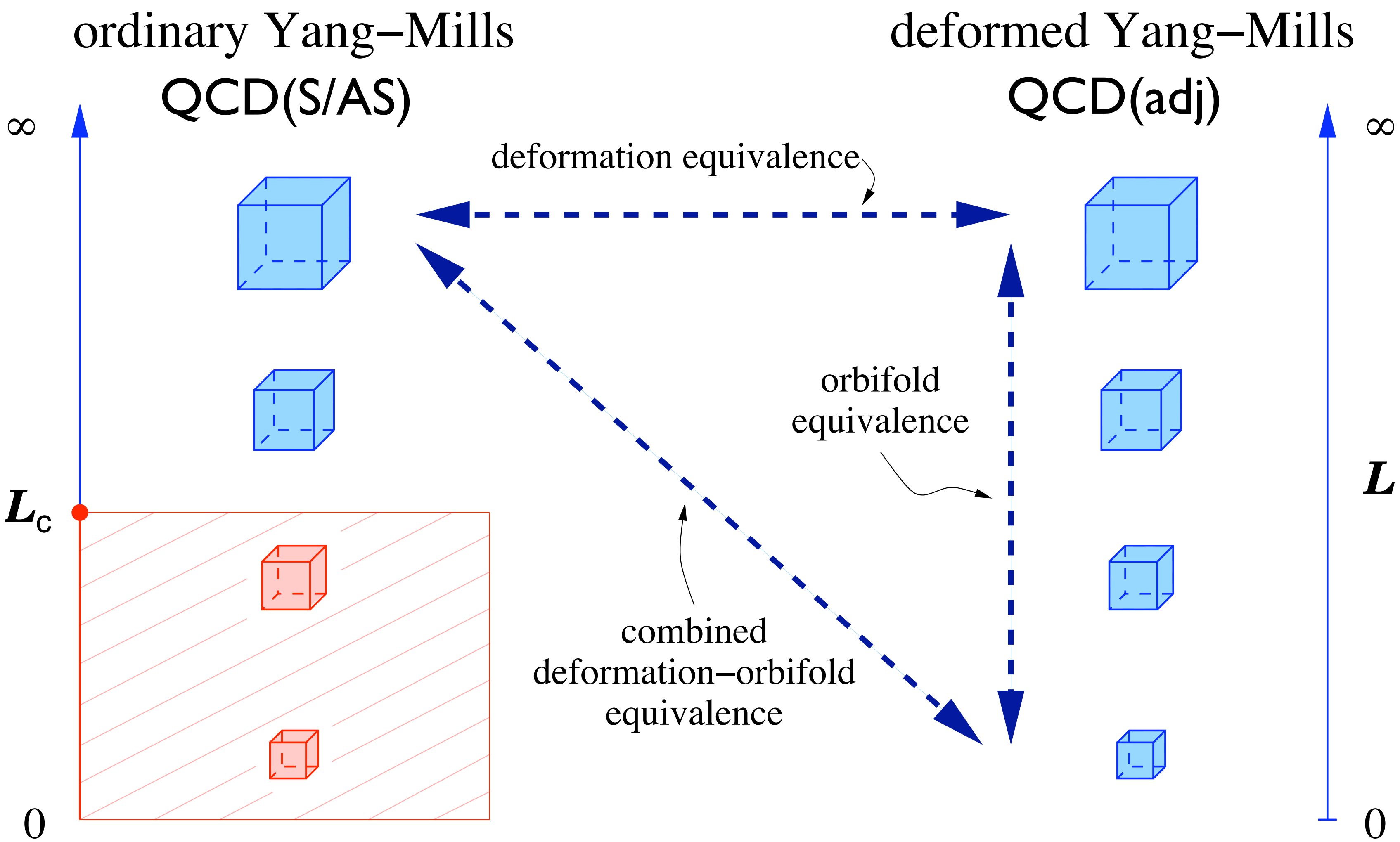
$$S^{\mathrm{YM}^*} = S^{\mathrm{YM}} + \int_{R^3 \times S^1} P[U(\mathbf{x})]$$

$$P[U] = A \frac{2}{\pi^2 \beta^4} \sum_{n=1}^{\lfloor N/2 \rfloor} \frac{1}{n^4} \left| \mathrm{tr}\,(U^n) \right|^2$$

Lattice studies by Ogilvie, Myers, Meisinger backs-up the smoothness conjecture.

Ogilvie, Myers also independently proposed the above deformations to study phases of partially broken center.

Raison d'être of deformation theory at infinite N



80's: EK, QEK, TEK...
Eguchi-Kawai, EK,
Gonzalez-Arroyo, Okawa, TEK,
Bhanot, Heller, Neuberger, QEK,
Tan et.al.

Last few years:
QCD(adj), deformations:

Pavel Kovtun, MU, Yaffe,
Shifman, MU.
Barak Bringholtz, Sharpe,
D'elia et.al,
Bedaque et.al.

At large N, volume independence is an exact property, a theorem. Although I will not use/discuss it in this talk, I will mention it as display of homage to my source of inspiration. [Solution of small volume theory implies the solution of the theory on R4.] (My) Failure to make progress on reduced large N theory was eventually responsible for the things that I will tell you from now on.

SU(N) QCD(adj)

$$S = \int_{R^3 \times S^1} \frac{1}{g^2} \mathrm{tr}\left[\frac{1}{4} F_{MN}^2 + i\bar{\lambda}^I \bar{\sigma}^M D_M \lambda_I\right]$$ short distance

Center Z_{N_c}

Chiral $(SU(n_f) \times Z_{2N_c n_f})/Z_{n_f}$

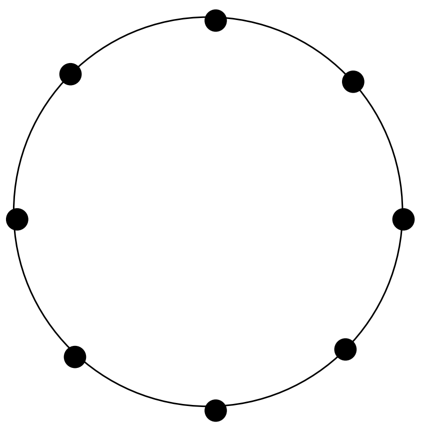
Let us solve it by using twisted partition function.

BSM application: minimal walking

$n_f < 5.5$ AF-boundary

Spatial Wilson line/non-thermal Polyakov loop

With deformation or pbc for adjoint fermions, eigenvalues repel. Minimum at



$$U = \mathrm{Diag}(1, e^{i2\pi/N}, \ldots, e^{i2\pi(N-1)/N})$$

$$\langle \mathrm{tr} U \rangle = 0$$

At weak coupling, the fluctuations are small, a "Higgs regime"

$$SU(N) \to [U(1)]^{N-1}$$

Georgi-Glashow model with compact adjoint Higgs field.

Compactness implies N types of monopoles, rather than N-1.

Reminder: Abelian duality and Polyakov model

Free Maxwell theory is dual to the free scalar theory.

$$F = *d\sigma$$

The masslessness of the dual scalar is protected by a **continuous shift symmetry**

$$U(1)_{\text{flux}} : \ \sigma \to \sigma - \beta$$

Topological current vanishes by Bianchi identity.

Noether current of dual theory:

$$\mathcal{J}_\mu = \partial_\mu \sigma = \tfrac{1}{2}\epsilon_{\mu\nu\rho}F_{\nu\rho} = F_\mu$$

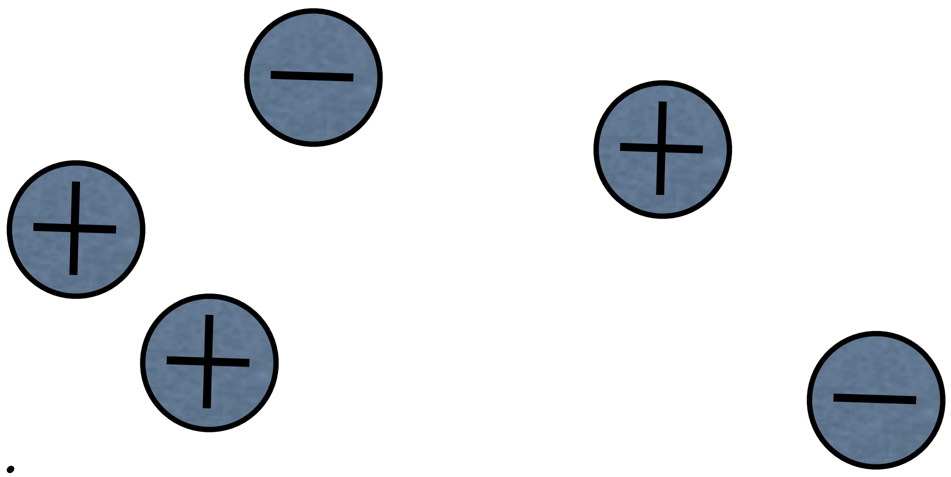
Its conservation implies the absence of magnetic monopoles in original theory

$$\partial_\mu \mathcal{J}_\mu = \partial_\mu F_\mu = 0$$

Proliferation of monopoles

The presence of the monopoles in the original theory implies reduction of the continuous shift symmetry into a discrete one. Polyakov mechanism.

$$\partial_\mu \mathcal{J}_\mu = \partial_\mu F_\mu = \rho_m(x)$$



The dual theory

$$L = \frac{1}{2}(\partial\sigma)^2 - e^{-S_0}(e^{i\sigma} + e^{-i\sigma})$$

Physics of Debye mechanism

Discrete shift symmetry: $\sigma \to \sigma + 2\pi$

$U(1)_{\rm flux}$ if present, forbids (magnetic) flux carrying operators.

Topological excitations in QCD(adj)

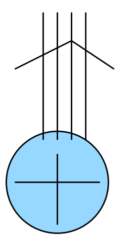
$$\left(\int_{S^2} F, \ \int_{R^3 \times S^1} F\tilde{F} \right)$$

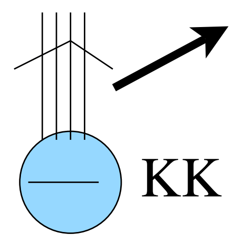
Magnetic Monopoles

Magnetic Bions

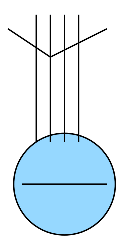
relevant index theorems
Callias 78
Nye-A.M.Singer, 00
E. Weinberg 80
Poppitz, MU 08
Atiyah-M.I.Singer 75

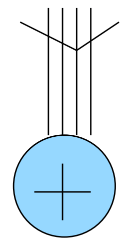
fermionic zero modes


BPS
(1, 1/2)


KK
(–1, 1/2)

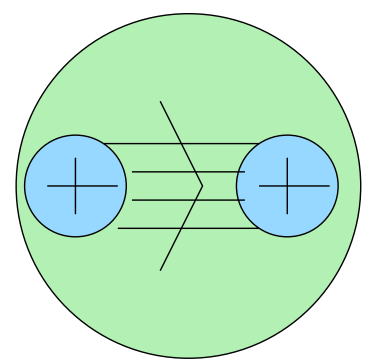
$e^{-S_0} e^{i\sigma} \det_{I,J} \psi^I \psi^J,$

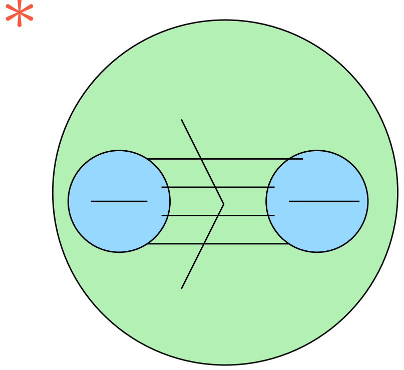

$\overline{\text{BPS}}$
(–1, –1/2)


$\overline{\text{KK}}$
(1, –1/2)

$e^{-S_0} e^{i\sigma} \det_{I,J} \bar{\psi}^I \bar{\psi}^J$

$(\mathbb{Z}_2)_*$


(2,0)


(–2, 0)

$$e^{-2S_0} (e^{2i\sigma} + e^{-2i\sigma})$$

Discrete shift symmetry : $\sigma \longrightarrow \sigma + \pi \qquad \psi^I \longrightarrow e^{i\frac{2\pi}{8}} \psi^I$

Dual Formulation of QCD(adj)

$$L^{\mathrm{dQCD}} = \frac{1}{2}(\partial\sigma)^2 - b\, e^{-2S_0}\cos 2\sigma + i\bar{\psi}^I\gamma_\mu\partial_\mu\psi_I + c\, e^{-S_0}\cos\sigma(\det_{I,J}\psi^I\psi^J + \mathrm{c.c.})$$

magnetic bions

Non-selfdual

magnetic monopoles

Self-dual

Same mechanism in N=1 SYM.

Also see Hollowood, Khoze, ... 99

Earlier in N=1 SYM, the bosonic potential was derived using supersymmetry and SW-curves, F, M theories, field theory methods. However, the physical origin of it remained elusive till this work.

$$m_\sigma \sim \frac{1}{L}e^{-S_0(L)} = \frac{1}{L}e^{-\frac{8\pi^2}{g^2(L)N}} = \Lambda(\Lambda L)^{(8-2N_f^W)/3} \;,$$

Proliferation of magnetic bions

Crucial earlier work: van Baal et.al. and Lu, Yi, 97

Conformality or confinement:

Conceptually, two problem of out-standing importance in gauge theories:

Mechanisms of confinement and conformality:
What distinguishes two theories, one just below the conformal boundary and confines, and the other slightly above the conformal window boundary? In other words, why does a confining gauge theory confine and why does an IR-CFT, with an almost identical microscopic matter content, flows t a CFT?

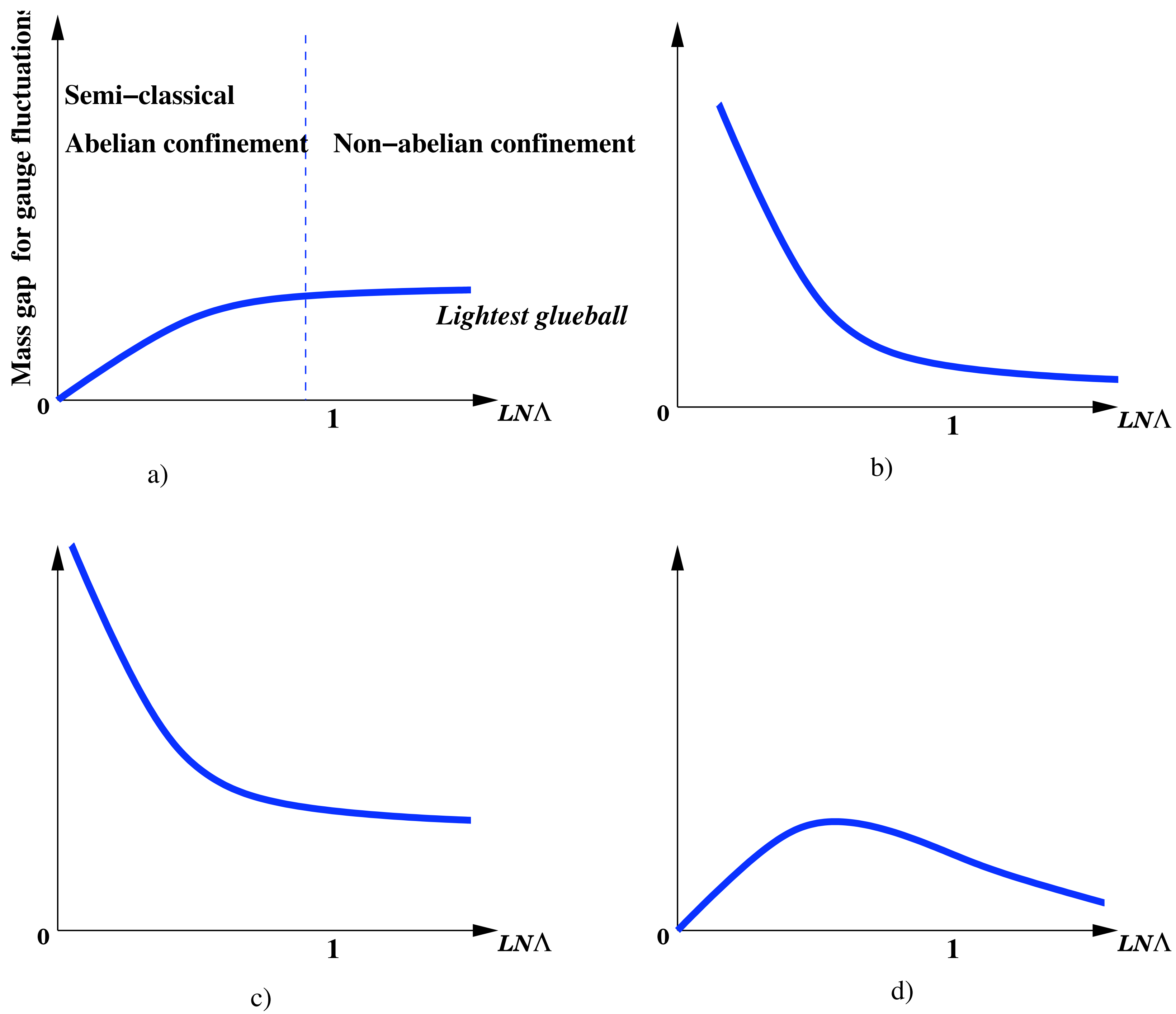
Lower boundary of conformal window: What is the physics determining the boundary of conformal window?

Map the problem to the mass gap for gauge fluctuations:

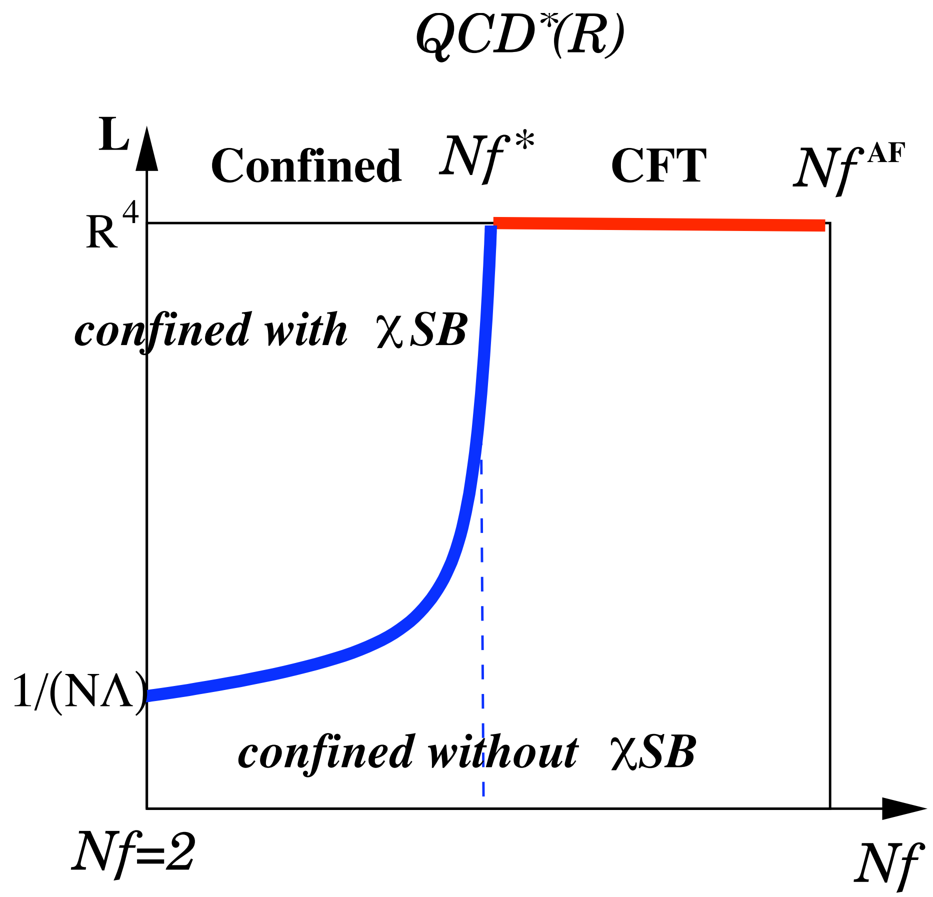
$$m^{-1}_{\text{gauge fluc.}}(\mathbf{R}^4) = \begin{cases} \text{finite} & N_f < N_f^* \qquad\qquad \text{confined} \\ \infty & N_f^* < N_f < N_f^{AF} \qquad \text{IR}-\text{CFT} \end{cases}$$

A priori, not a smart strategy.

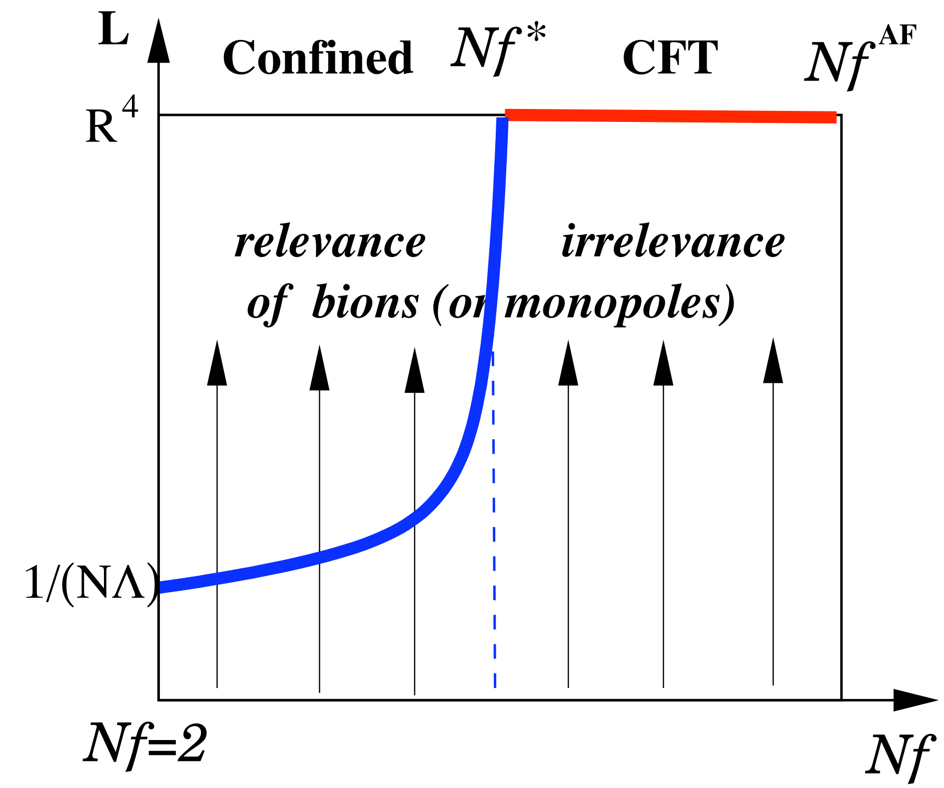
Mass gap for gauge fluctuations



Main Idea of our proposal



a)



b)

(IR)relevance of topological excitations

Conjecture: (E. Poppitz and M.Ü)

In a given center symmetric gauge theory on $\mathbf{R}^3 \times \mathbf{S}^1$, if the topological excitations become non-dilute and the mass gap in the semiclassical window increases with increasing L, this theory eventually flows to a confining gauge theory on $\mathbf{R}^4$ with a finite mass gap. If the mass gap for gauge fluctuations decreases with increasing L, diluting the topological excitations in the semiclassical regime, such theories will (most likely) flow to IR-CFT on $\mathbf{R}^4$, with a vanishing mass gap; if, on the other hand, χSB is triggerred outside of the semiclassical regime, when $L\Lambda \sim 1$, they will flow to confining theories on $\mathbf{R}^4$.

Crucial data: Index theorem on R3*S1, the knowledge of mechanism of confinement, and one-loop beta fun.

QCD(F/S/AS/Adj):Estimates and comparisons

Below, I will present the estimates based on this idea and compare it various other approaches. In particular:

1) Truncated SD (ladder, rainbow) approximation.

QCD(F): Appelquist, Lane, Mahanta, and Miransky: Usually, believed to be an overestimation. Two-index cases: Sannino, Dietrich.

2) NSVZ-inspired conjecture: Sannino, Ryttov.

Crucial data for 1) and 2): Two-loop (or conjectured all orders) beta function, anomalous dimension of fermion bilinear.

Does not usefully applies to chiral gauge theories.

QCD(S/AS/Adj):Estimates and comparisons

N	Deformation theory (bions)	Ladder (SD)-approx.	NSVZ-inspired: $\gamma = 2/\gamma = 1$	N_f^{AF}
3	2.40	2.50	1.65/2.2	3.30
4	2.66	2.78	1.83/2.44	3.66
5	2.85	2.97	1.96/2.62	3.92
10	3.33	3.47	2.29/3.05	4.58
∞	4	4.15	2.75/3.66	5.5

Table 1: Estimates for lower boundary of conformal window in QCD(S), $N_f^* < N_f^D < 5.5\left(1 - \frac{2}{N+2}\right)$.

N	Deformation theory (bions)	Ladder (SD)-approx.	NSVZ-inspired: $\gamma = 2/\gamma = 1$	N_f^{AF}
4	8	8.10	5.50/7.33	11
5	6.66	6.80	4.58/6.00	9.16
6	6	6.15	4.12/5.5	8.25
10	5	5.15	3.43/4.58	6.87
∞	4	4.15	2.75/3.66	5.50

Table 2: Estimates for lower boundary of conformal window in QCD(AS), $N_f^* < N_f^D < 5.5\left(1 + \frac{2}{N-2}\right)$.

N	Deformation theory (bions)	Ladder (SD)-approx.	NSVZ-inspired: $\gamma = 2/\ \gamma = 1$	N_f^{AF}
any N	4	4.15	2.75/3.66	5.5

Table 3: Estimates for lower boundary of conformal window in QCD(adj), $N_f^* < N_f^W < 5.5$. In QCD(adj), we count the number of Weyl fermions as opposed to Dirac, since the adjoint representation is real.

“(Minimal) Walking technicolor”

QCD(F)

N	Deformation theory (monopoles)	Ladder (SD)-approx.	Functional RG	NSVZ-inspired: $\gamma = 2/\gamma = 1$	N_f^{AF}
2	5	7.85	8.25	5.5/7.33	11
3	7.5	11.91	10	8.25/11	16.5
4	10	15.93	13.5	11/14.66	22
5	12.5	19.95	16.25	13.75/18.33	27.5
10	25	39.97	n/a	27.5/36.66	55
∞	$2.5N$	$4N$	$\sim (2.75 - 3.25)N$	$2.75N/3.66N$	$5.5N$

Table 1: Estimates for lower boundary of conformal window for QCD(F), $N_f^* < N_f^D < 5.5N$

Most-likely, our approach is a lower bound on the lower boundary in this case.

Kinematics, dynamics, and universality

$$\begin{aligned}\text{adjoint Weyl} &= \text{AS Dirac} = \text{S Dirac} = N \times (\text{F Dirac}) \\ &= [\text{AS Weyl}, N \times (\overline{\text{F}}\ \text{Weyl})] = [\text{S Weyl}, N \times (\overline{\text{F}}\ \text{Weyl})]\ , \end{aligned} \tag{1}$$

$$\xi^{AF}(\mathcal{R}) = \left\{ \frac{N_f^{AF,D}(\text{F})}{N},\ N_f^{AF,D}(\text{AS/S/BF}),\ N_f^{AF,\text{W}}(\text{Adj}) \right\}\ , \tag{2}$$

Kinematic

$$\lim_{N\to\infty} \xi^{AF}(\mathcal{R}) = 5.5 \tag{3}$$

$$\xi^{*}(\mathcal{R}) = \left\{ \frac{N_f^{*,D}(\text{F})}{N},\ N_f^{*,D}(\text{AS/S/BF}),\ N_f^{*,\text{W}}(\text{Adj}) \right\}\ . \tag{4}$$

Dynamics

$$\begin{aligned}
&\text{Ladder (or functional RG)}: && \lim_{N\to\infty} \xi^*(\text{F}) = 4\ (\text{or} \sim 3), \qquad \lim_{N\to\infty} \xi^*(\text{S/AS/BF/Adj}) = 4.15\ (\text{n/a}), \\
&\text{NSVZ} - \text{inspired}, \gamma = 2: && \lim_{N\to\infty} \xi^*(\text{F}) = \lim_{N\to\infty} \xi^*(\text{S/AS/BF/Adj}) = 2.75\ , \\
&\text{NSVZ} - \text{inspired}, \gamma = 1: && \lim_{N\to\infty} \xi^*(\text{F}) = \lim_{N\to\infty} \xi^*(\text{S/AS/BF/Adj}) = 3.66\ , \\
&\text{Deformation theory}: && \lim_{N\to\infty} \xi^*(\text{F}) = 2.5, \qquad \lim_{N\to\infty} \xi^*(\text{S/AS/BF/Adj/chiral}) = 4\ .
\end{aligned} \tag{5}$$

Conclusions

- The dynamics of QCD(adj) with circle compactification (pbc) is a true courtesy of the theory. We should take advantage of it, especially on lattice. This may, however, require us facing the chiral limit more directly.

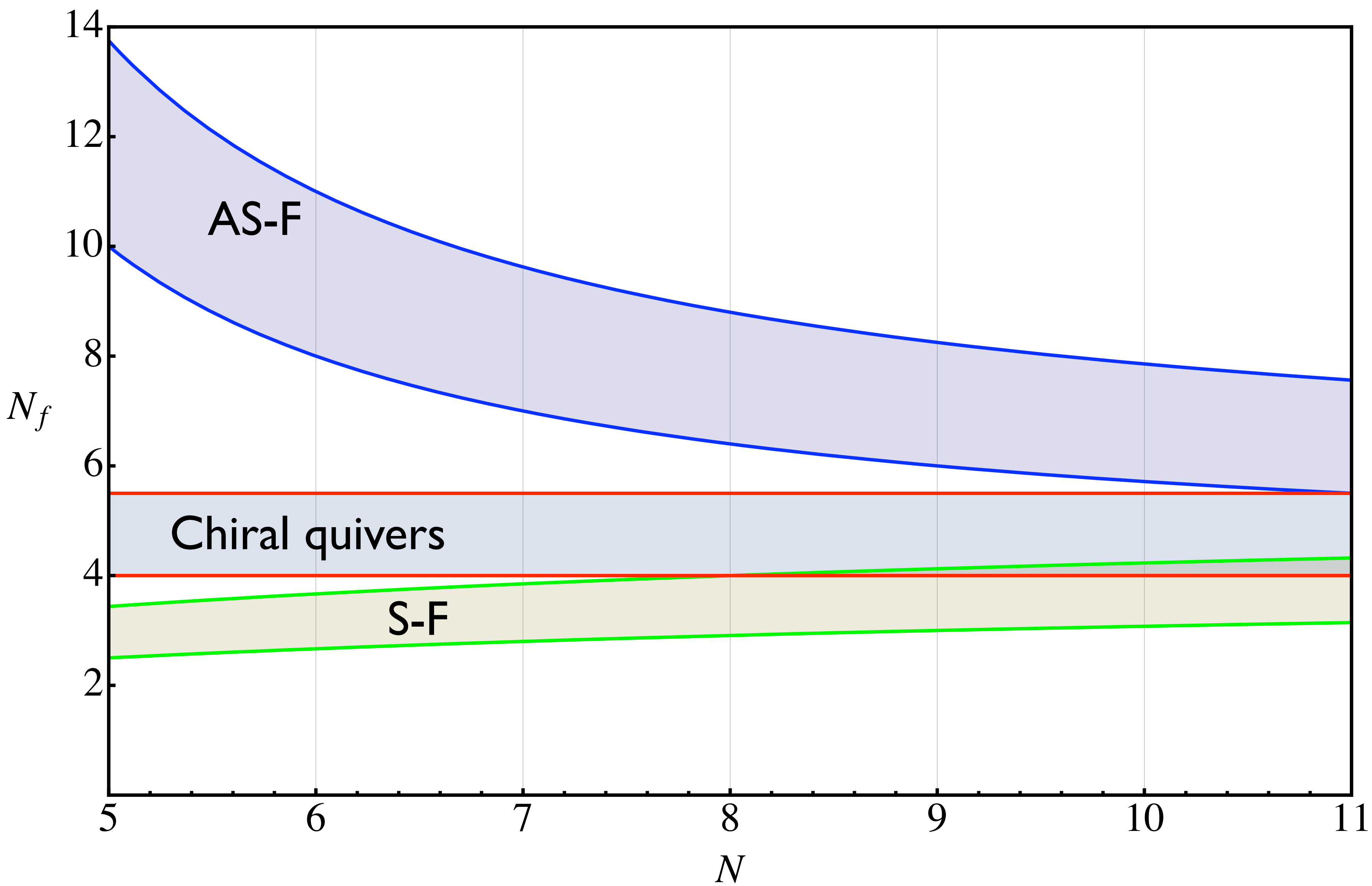
- There is now a window through which we can look into non-abelian gauge theories and understand their internal goings-on. Whether the theory is chiral, pure glue, or supersymmetric is immaterial. We always gain a semi-classical window (in some theories smoothly connected to R4 physics.)

- Deformation theory is complementary to lattice gauge theory. Sometimes lattice is more powerful, and sometime otherwise. Currently, DT is the only dynamical framework for chiral gauge theories. It may also be more useful in the strict chiral limit of vector-like theories.

- We learned the existence of a large class of new topological excitation through this program in the last two years.

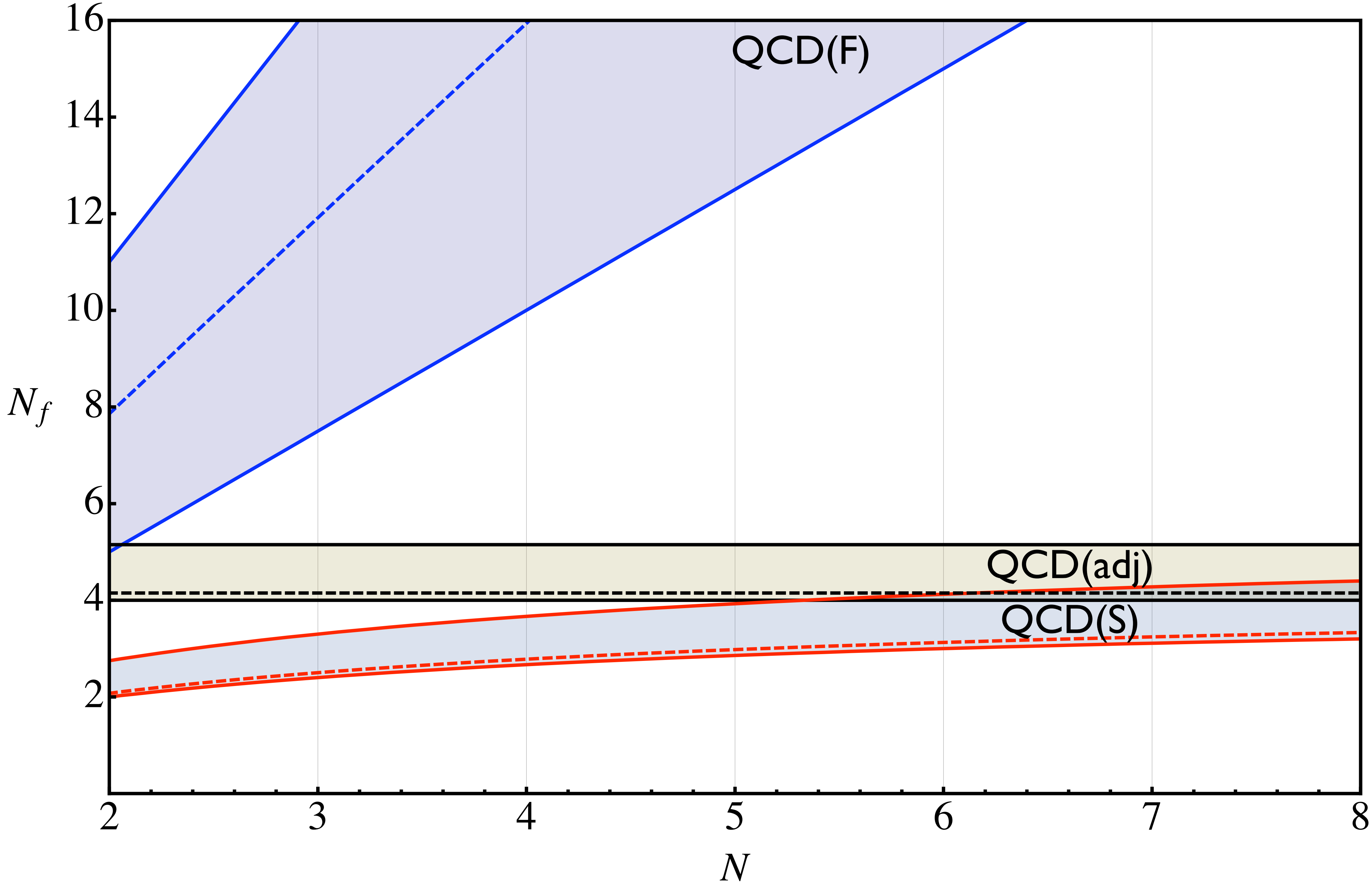
Supporting material

A lower bound on the lower boundary Chiral theories



No known earlier estimates. Could some of these theories be useful in addressing EWSB, and satisfy EW-precision?

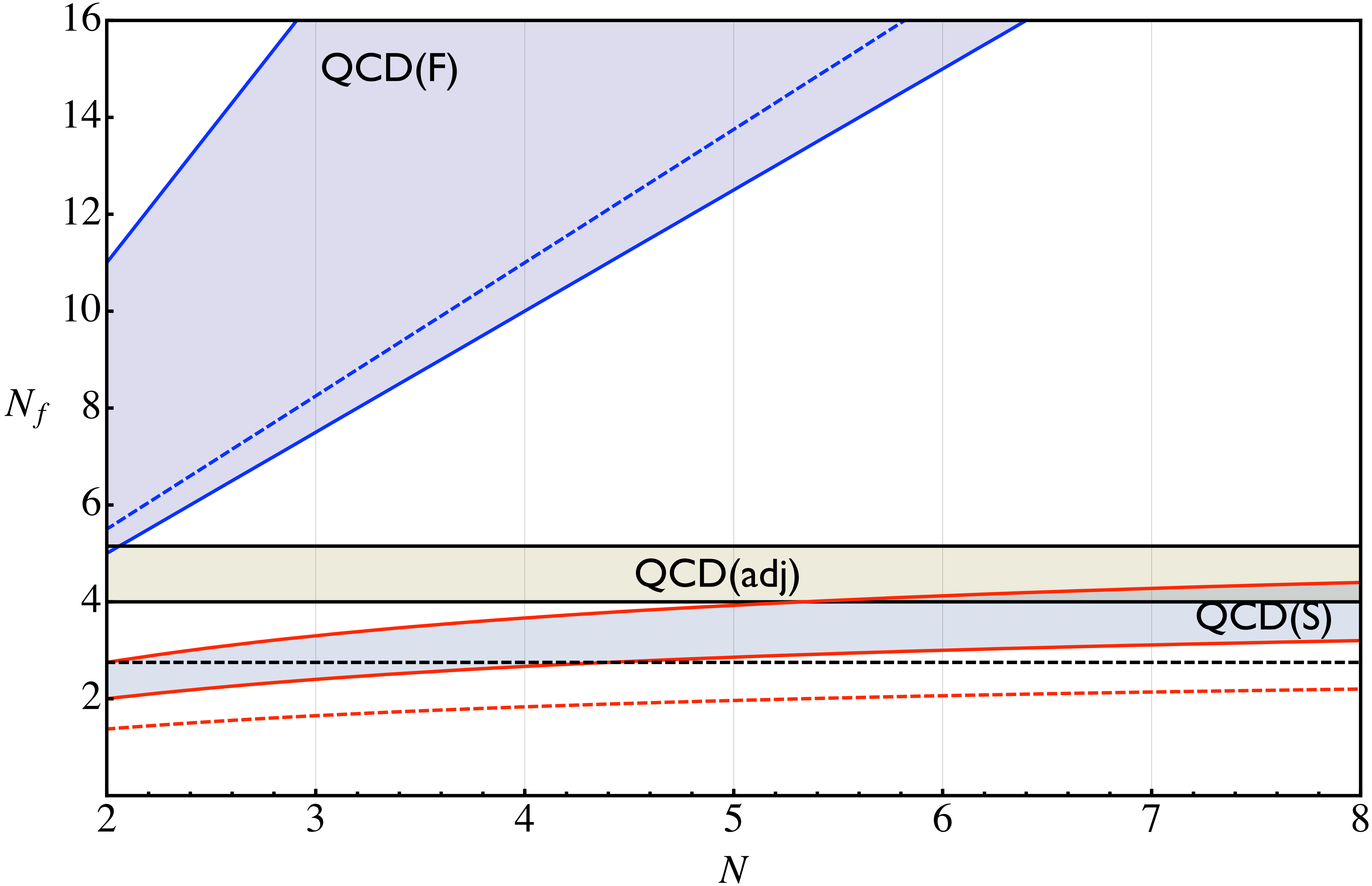
A lower bound on the lower boundary from deformation theory



Dashed line: Truncated SD (ladder, rainbow) approximation.

QCD(F):Miransky, Appelquist, Lane, Mahanta,... believed to be an overestimation.
Two-index: Sannino et.al.

A lower bound on the lower boundary from Deformation th.



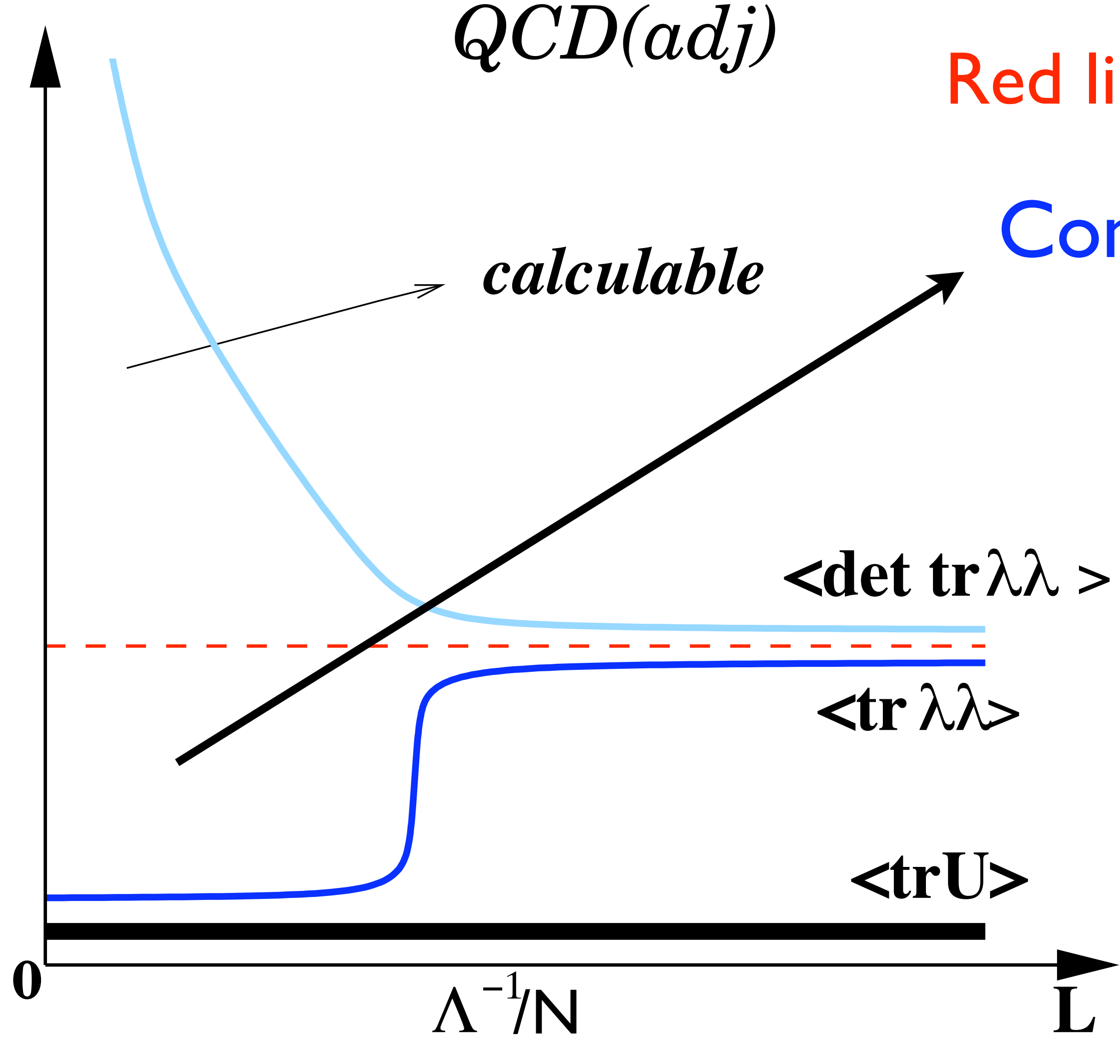
Dashed lines: SUSY-inspired approach of Ryttov and Sannino.

Large mismatch with 2-index estimates

A) Mass gap in gauge sector due to magnetic bion mechanism, so is linear confinement, and stable flux tubes.

B) Discrete chiral symmetry is always broken.

C) Continuous chiral symmetry is unbroken at small radius, hence massless fermions in the spectrum.



Red line N=1SYM.

Confinement without chiral symmetry breaking

Region of validity of dual formulation

- $LN_c\Lambda \ll 1$ why not $L\Lambda \ll 1$? (Surprising..)
- Separation of scales between W-bosons and dual photons.
- Deeper reason: Large N volume independence EK reduction
- The large N volume independence holds provided unbroken center symmetry. Thus, the chiral transition scale must move to arbitrarily small radius at large N.
- New dynamical scale in QCD: Λ^{-1}/N
- Invisible in gravity approximation to gauge/string duality or holographic QCD (or whatever) as well as perturbation theory,