PH207 - Solid State Physics

Problem sheet N. 1

Handed out 09/02 – Due in 16/02

1. Consider a square lattice with "atoms" at the corners and at the centre (this is the 2-dimensional equivalent of the body centred lattice). In the approximation that "atoms" are hard circles, compute the packing fraction of this lattice.
   [*2 marks*]

2. Same as question 1, for an hexagonal lattice ("atoms" at the corners and at the centre of an hexagon).
   [*3 marks*]

3. The primitive lattice translation vectors of a BCC lattice are

   $$\vec{a_1} = a(-\hat{x} + \hat{y} + \hat{z})/2 \qquad \vec{a_2} = a(\hat{x} - \hat{y} + \hat{z})/2 \qquad \vec{a_3} = a(\hat{x} + \hat{y} - \hat{z})/2$$

   Show that the reciprocal lattice is of the FCC type, with spacing $2\pi/a$.
   [*5 marks*]

4. Show that the reciprocal of the reciprocal lattice is the original lattice. You might find useful the following vector identities:

   $$(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = (\vec{a} \cdot (\vec{b} \times \vec{d}))\vec{c} - (\vec{a} \cdot (\vec{b} \times \vec{c}))\vec{d}$$
   $$\vec{a} \cdot \vec{b} \times \vec{c} = \vec{b} \cdot \vec{c} \times \vec{a} = \vec{c} \cdot \vec{a} \times \vec{b}$$

   [*5 marks*]

5. The primitive translation vectors of an hexagonal closed packed lattice are given by

   $$\vec{a}_1 = a\hat{x} \qquad \vec{a}_2 = a(\hat{x} + \sqrt{3}\hat{y})/2 \qquad \vec{a}_3 = c\hat{z}$$

   Compute the volume of the first Brillouin zone. (hint: how does the volume of a cell depends on the choice of the primitive translation vectors?)
   [*5 marks*]