PH207 - Solid State Physics

Problem set N. 5

Handed out 24/02 – Due in 2/05

1. For the phonon dispersion curve of a linear diatomic lattice the maximum frequency of the acoustic branch is 10^{12} Hz and the forbidden gap is 3 meV wide. Calculate the lowest allowed frequency of the optical branch ($e = 1.6 \times 10^{-19} C$; $h = 6.63 \times 10^{-34} Js$).
[*5 marks*]

2. The dispersion relation between the angular frequency ω and the wave number k of a wave transmitted through a linear monoatomic lattice can be written in the form

$$\omega = \omega_m \sin(ka/2)$$

What is the physical significance of ω_m? Calculate the phonon group velocity and discuss its behaviour at long wavelengths and at boundary of the first Brillouin zone.
[*5 marks*]

3. When considering a linear diatomic lattice the amplitudes C and D of vibration of the atoms with masses m and M ($m < M$) can be written in terms of the angular frequency ω and wave number k as

$$\begin{aligned} C\left[2\mu - m\omega^2\right] &= 2\mu D \cos(ka/2) \\ D\left[2\mu - M\omega^2\right] &= 2\mu C \cos(ka/2) \end{aligned}$$

where μ is the force constant. Given that the angular frequencies in the acoustic and optic branches are $(2\mu/M)^{1/2}$ and $(2\mu/m)^{1/2}$ at the Brillouin zone boundary, show that the light atoms are stationary in the acoustic branch and the heavier atoms are stationary in the optical branch. Furthermore, show that as $k \to 0$ in the acoustic branch the light and the heavy atoms move with the same amplitude.
[*10 marks*]