TWO-LEVEL LASERS

A Numerical Study

INTRODUCTION

- Many physical laws are characterised by an exponential decay of an observable (radiative decay, c absorption of gases by the human body...)
- It is important to be able to model those phenomena
- Lasers are a non-so trivial example of exponential laws
- Details on laser physics are the subject of specialistic courses, here we have more modest goals

AIM

- Solving a system of differential equation and comparing with analytical solution
- You will have access to the material of PH-206 (you will need the Range-Gutta solver)
- The material provided is in VisualBasic, you are free to use another programming language, but there is little/no code support for that

LASERS

- Are cheap technology, we can find them in CD readers, bar code readers, pointers etc
- Still an interesting physical system that combines non-trivial optical, quantum-mechanical and statistical physics phenomena
- The main property is the emission of a COHERENT and NARROW beam of light

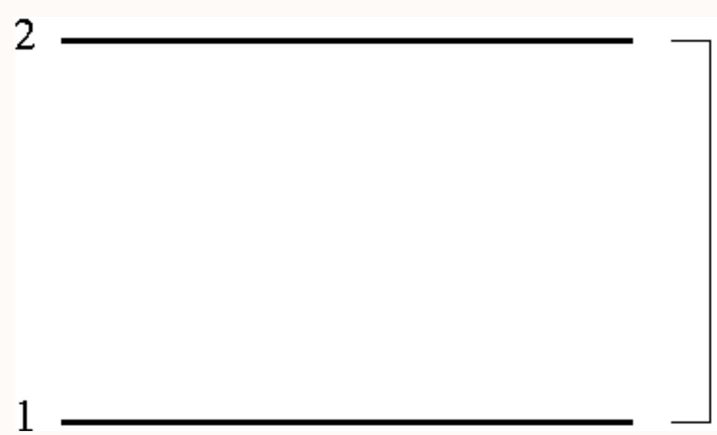
PROPERTIES OF A LASER

A simple laser system is made of

1. An amplifying medium, which produces the beam via stimulated emission
2. A pump, which puts the atoms in the excited level
3. A resonator, which sustain the production of the beam

A MODEL FOR A TWO-LEVEL SYSTEM

- At equilibrium most of the atoms are in level 1
- Transition from 1 to 2 by absorbing a photon
- Transition from 2 to 1 by emitting a photon



$$h\nu = h(\nu_2 - \nu_1)$$

EMISSION OF PHOTONS

- Spontaneous emission: an atom excited into level 2 emits a photon of energy $h\nu = h(\nu_2 - \nu_1)$ and decays into level 1
- Stimulated emission: an atom in level 2 absorb a photon of energy $h\nu$ and emit two identical photons

  To produce a coherent beam, we need stimulated emission

PUMPING

- At a given temperature, in thermodynamical equilibrium the fraction of the atoms in the upper level is determined by the Boltzmann distribution
- To put atoms in level 2, we must create a non-equilibrium state
- Even if we start from a system with all atoms in level 2, stimulated and spontaneous emission will soon
- Stationary non-equilibrium state by providing energy

LIGHT FROM THE LASER

- After pumping, most of the atoms are in level 2 (population inversion)
- At some point, some atoms must decay via spontaneous emission
- Spontaneous emission triggers stimulated emission, generating a cascade of identical photons
- Photons accumulated in the medium, but some of them are lost through the laser cavity → beam!

MILESTONE 1: DECAY

Consider the differential equation

$$\frac{d}{dt}A(t) = -\gamma A(t), \qquad A(0) = A_0$$

1. Solve it analytically
2. Solve it numerically Range-Gutta
3. Analyse the error as a function of the step size of the algorithm

MILESTONE 2: PUMPING

Consider now the differential equation

$$\frac{d}{dt}A(t) = -\gamma A(t) + \gamma A_0 \ , \qquad A(0) = 0$$

1. Solve it with the initial condition given
2. What is the difference with the previous case
3. Why this case describe pumping?
4. How do you model a pump?

MILESTONE 3: COUPLING

Write down the equation for a two-level system in which atoms decay from level 2 to 1 and atoms in 1 are pumped into 2 at a rate proportional to their numbers (the two rates are different).

1. For a system with number of atom N, what's the constraint between N1 (number of atoms in 1) and N2 (number of atoms in 2)?

2. What's the stationary state?

3. How the stationary population is affected by the ratio of the rates?

4. Solve numerically the equations for a) initial condition in which all the atoms are in 1; b) initial condition in which all the atoms are in 2.

LASER EQUATIONS

A laser can be described by the following equations:

$$\frac{d}{dt}n_p(t) = Bn_p(t)n_a(t) - \gamma_c n_p(t)$$
$$\frac{d}{dt}n_a(t) = -Bn_p(t)n_a(t) - \gamma_a n_a(t) + R$$

where n_p is the number of phonons, n_a is the number of atoms, γ_c is the cavity loss rate, γ_a is the population decay rate, R is the pumping rate into the higher level and B is a constant describing stimulated emission (Einstein coefficient)

MODELLING THE LASER

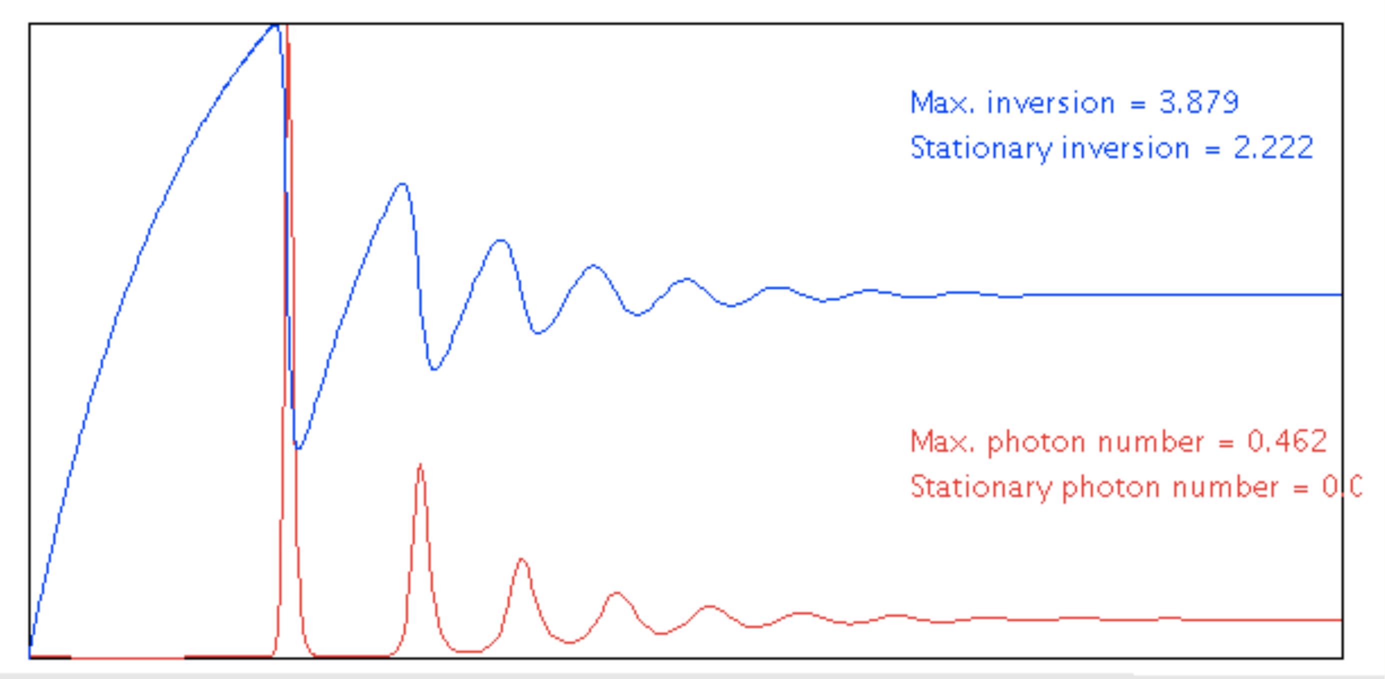
Write a VB program that solves the equations of the laser system

Make it flexible, in such a way that it is possible to explore different physical situations (e.g. low/high cavity loss rate, low/high R etc.)

TESTING

Use the following parameters:

$$\gamma_c = 200\text{MHz} \ , \gamma_a = 2MHz \ , B = 90MHz \ , R = 10MHz$$



You should reproduce this image. Why do you expect those oscillations?

FINAL STAGE

Study the lasers in several regimes (small/big cavity loss, small/big R... in various combinations)

Which range of parameters would you choose for building an efficient laser? Justify your answer with the computer simulations