Modelling Project 2

Quarkonia

Work to be done in weeks 5-11

Report due in 15/12/05

Presentation 16/12/05

Introduction And Aim

- Quarks are only seen in three quark (qqq) states (baryons) or quark-antiquark (qq) states (mesons) Confinement
- Can we model the meson states using non-relativistic quantum mechanics with a suitable potential?

Hydrogen- A Warm Up

- The hydrogen atom can be solved analytically
- You have been taught how to do this
- Only the potential differs from quarkonia
- An idea warm-up example

The Hydrogen Potential

- At all distances we expect a Coulomb like, 1/r, potential
- Take

$$V(r) = \frac{\alpha}{r}$$

ρ

Analytic Preamble H1

- Book Work for the H atom
  - Start with the Schrodinger equation
    - Remember to use the reduced mass
  - Pick a suitable coordinate system
    - Think about the symmetry of the problem
  - Solve the parts that the symmetry makes easy in terms of a set of basis functions
  - Obtain a differential equation for the remaining (radial) part

Analytic Preamble H2

- Understand the radial equation
  - Find the short distance behaviours
    - Typically try a power series solution
  - Find the long distance behaviours
    - Use substitutions like $\psi = r^p \phi$ to get rid of the first derivative term
    - Then look for the long distance solution
  - Look for a general solution
    - Power series solutions are the most likely
    - Try substitutions like $\psi = r^p e^{\alpha r} \phi$ to get a nice 2 term recurrence relation
    - Get the eigenvalues analytically

Pre-Numerical Analysis H

- Find the values of all the constants
  - The computer needs the numbers
  - Remember the c's and h's
- Pick suitable scales
  - Here, what is a sensible scale for r?
  - Set $r=\lambda\rho$ where λ is a length and ρ runs from 0 to 10 (say) across the region of interest
- Write the radial equation in terms of the scaled variables and corresponding constants
  - If you give the computer sensibly sized numbers, you'll have a better idea of the sort of numbers it should give you

Numerical Considerations H

- What type of problem do we have?
  - Ordinary differential equation
  - With two point boundary conditions
- Which approach should we use?
  - Shooting
    - \+ Intuitive
    - \+ Relatively easy to code
    - \- Best suited for single point boundary conditions
    - \+ Shoot and jiggle often runs reasonably fast
  - Relaxation
    - \+ best suited to two point boundary conditions
    - \+ can run very quickly
    - \- coding relatively involved

Shooting Methods

- Runge–Kutta probably simplest
  - What order?
    - Coding time vs. running time
      - 1^{st} order
        - » quick to code but slow to run
      - 4^{th} order
        - » Typical work horse
        - » Should be in everyone's library
- Accuracy Control
  - How will we estimate the accuracy of the solution?
  - When will we estimate the accuracy of the solution?
- Grid sizing
  - Fixed
    - Easy to code but not necessarily quick to run
  - Adaptive
    - Needs run-time accuracy estimation

Pick a Language

- For this project you will use a high level language
  - Visual Basic
    - You've used this before
    - Available on all lab PCs
    - Nice help pages on Chris Allton's home page
    - Only works under windows
  - Fortran
    - Prehistoric but still going
    - The scientific language of choice for many years
  - C
    - A bit more modern (but hardly cutting edge)
    - Universal
    - Available under Linux on PCs in 413

Intermediate Tasks/Check Points

- Check Point A: Analytic Preparation for H
  - Scaled radial equation with numerical coefficients
  - Clear statement of allowed values of all constants
- Check Point B: Numerical solution for H
  - A tested code implementing the method of your choice
  - A range of numerical eigenvalues and accuracy estimates

The Real Thing

- Quarkonia
- Quark/anti-quark bound state
  (rather than proton/electon bound state)
- Different potential
- Two particles may have same mass

The Potential

- At short distances we expect a Coulomb like, 1/r, potential
- At long distances a constant force gives confinement
  - This needs a linear potential
- Take

$$V(r) = \frac{\alpha}{r} + \beta r$$

Analytic Preamble Q1

- Book Work – just like the H atom
  - Start with the Schrodinger equation
    - Remember to use the reduced mass
  - Pick a suitable coordinate system
    - Think about the symmetry of the problem
  - Solve the parts that the symmetry makes easy in terms of a set of basis functions
  - Obtain a differential equation for the remaining (radial) part

Analytic Preamble Q2

- Understand the radial equation
  - Find the short distance behaviours
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    - Use substitutions like $\psi = r^p \phi$ to get rid of the first derivative term
    - Then look for the long distance solution
  - Look for a general solution
    - Power series solutions are the most likely
    - Try substitutions like $\psi = r^p e^{\alpha r} \phi$ to get a nice 2 term recurrence relation
    - If you're lucky get the eigenvalues analytically and forget the computer

Pre-Numerical Analysis Q

- Find the values of all the constants
  - The computer needs the numbers
  - Remember the c’s and h’s
- Pick suitable scales
  - Here, what is a sensible scale for r?
  - Set $r=\lambda\rho$ where λ is a length and ρ runs from 0 to 10 (say) across the region of interest
- Write the radial equation in terms of the scaled variables and corresponding constants
  - If you give the computer sensibly sized numbers, you’ll have a better idea of the sort of numbers it should give you

Numerical Considerations Q

Just as for hydrogen-

What method?

What language?

Intermediate Tasks/Check Points

- Check Point C: Eigenvalues for Quarkonia
  - Find the radial equation for quarkonia
  - Find the energy eigenvalues numerically for any given set of parameters in the potential
- Final Aim: Your Spectrum vs. The Data
  - Can you reproduce any part of the spectrum of meson masses?
  - What are the best parameters to use?
  - How good is the correspondence?

Assessment

- Report
  - Due 16/12/05
  - Counts for ½ of module mark
- Presentation
  - To be given on 15/12/05
  - Counts for 1/6 of the module mark
  - 20 minutes to the group
  - Powerpoint (or equivalent)

Shooting Methods

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Intermediate Tasks/Check Points

- Check Point A: Analytic Preparation
  - Scaled radial equation with numerical coefficients
  - Table of measured meson masses and quark content
- Check Point B: Utility Differential Eq'n Solver
  - A tested utility code implementing the method of your choice
  - This should be easily adaptable to solve a range of equations

(These strands may run concurrently)

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